PDF in Parton Shower

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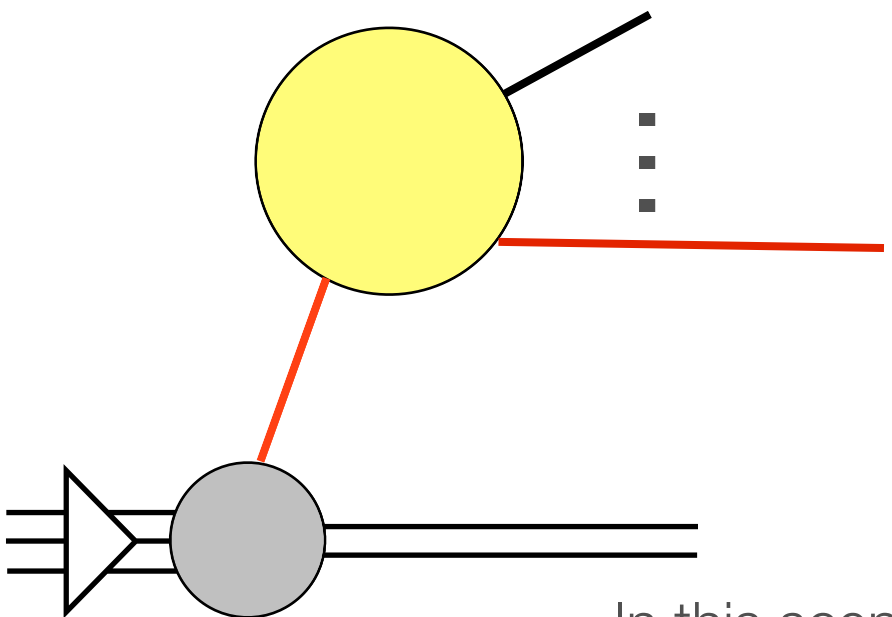
Heavy Quark in Parton Shower



In the standard parton showers (Herwig and Pythia) the mass of the heavy quarks are explicitly considered. The phase space and the splitting kernels depend on the mass parameters.

In this scenario the heavy quarks (charm, bottom) comes from the proton and makes some hard interaction and then propagates to the final state.

One can try to use massless quark in the initial state but massive in the final state. But this violates gauge invariance and it makes hard (or impossible) to define hard matrix elements.

We have to take care about the heavy flavor mass in the initial state if we consider masses in the final state.

Heavy Flavor in PDF

- The parton distribution function (PDF) depends on the heavy flavor masses in a very simple way. The PDF of the charm and bottom quarks disappear under the scale $\mu_F = m_{c,b}$.

$$f_{c/A}(\eta, \mu_F^2 < m_c^2) = f_{b/A}(\eta, \mu_F^2 < m_b^2) = 0$$

- There is no intrinsic charm and bottom, they are generated via evolution from gluon splittings.

- The threshold depends only on the mass but it might depend on other parameters like the momentum fraction.

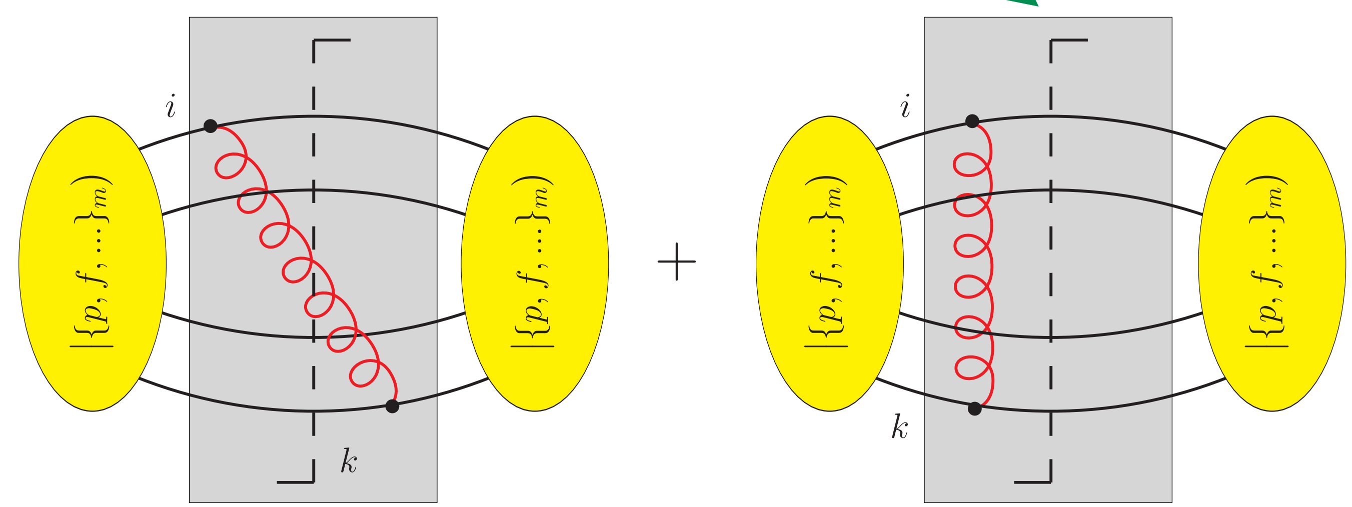
- The matching conditions are derived from fully inclusive cross sections (DIS) and it is a questions if they work for exclusive quantities or in parton showers. At LHC we use the PDFs mostly in exclusive processes.

Shower Evolution

Using the factorization properties of the QCD the approximated order by order calculation can be organized according to

$$\mathcal{U}(t,t') = 1 + \int_{t'}^{t} d\tau\, \mathcal{U}(t,\tau)\big[\mathcal{H}_I(\tau) - \mathcal{V}(\tau)\big]$$

$$\frac{d}{dt}\mathcal{U}(t,t') = \left[\mathcal{H}_I(t) - \mathcal{V}_I(t)\right]\mathcal{U}(t,t')$$



resolved radiations *unresolved radiation*

From the unitary condition:

$$\big(1\big|\mathcal{V}(t) = \big(1\big|\mathcal{H}_{\mathrm{I}}(t)$$

The shower form of the solution is

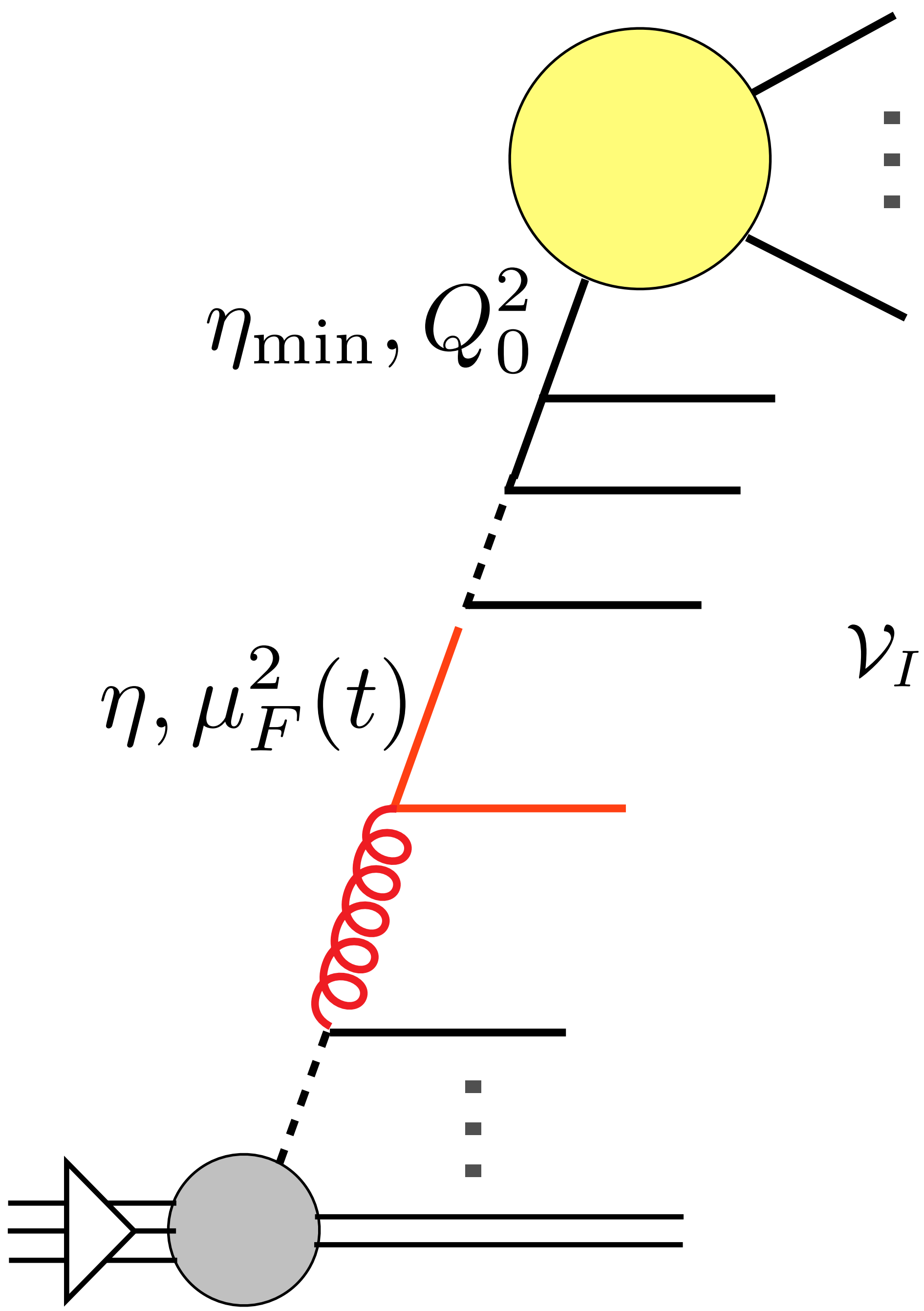
$$\mathcal{U}(t,t') = \mathcal{N}(t,t') + \int_{t'}^{t} d\tau\, \mathcal{U}(t,\tau)\, \mathcal{H}_I(\tau)\, \mathcal{N}(\tau,t')$$

and the Sudakov operator is

$$\mathcal{N}(t,t') = \mathbb{T}\exp\left(-\int_{t'}^{t} d\tau\, \mathcal{V}(\tau)\right)$$

Heavy Quark: Massless Treatment

The heavy quark mass is considered only in the PDF functions. In the shower kinematics they are considered as massless partons.



Every heavy quark turns to gluon in the shower evolution. This is ensured by the splitting kernel. The splitting kernel goes to infinity as the scale approaches to the threshold scale.

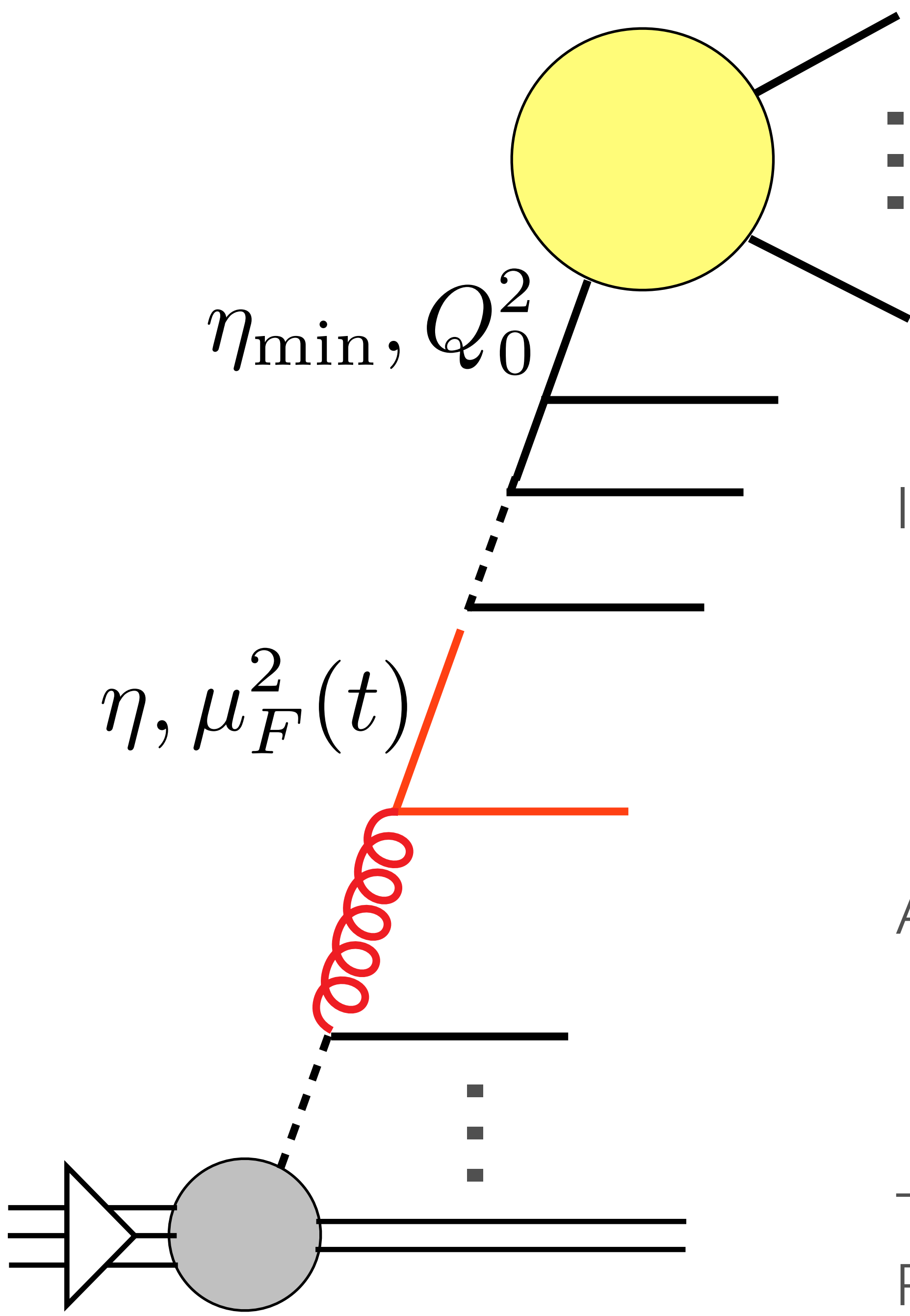
$$\mathcal{V}_I(t) \sim \int_\eta^1 \frac{dz}{z} \frac{f_{g/A}\left(\frac{\eta}{z}, \mu_F^2\right)}{f_{b/A}(\eta, \mu_F^2)} S_{qg}(z, y) \xrightarrow{\mu_F^2 \to m_b^2} \infty$$

$$f_{b/A}(\eta, \mu_F^2) \xrightarrow{\mu_F^2 \to m_b^2} 0$$

The MSbar PDF and DGLAP looks consistent with the shower evolution.

Heavy Quark: Massive Treatment

Since the shower tries to generate fully exclusive multi-parton final states we should consider the heavy flavor masses explicitly in the splitting kernels and in the phase space.



$$\mathcal{V}_I(t) \sim \int_\eta^{z_+} \frac{dz}{z} \frac{f_{g/A}\left(\frac{\eta}{z}, \mu_F^2\right)}{f_{b/A}(\eta, \mu_F^2)} S_{qg}(z,y)$$

$$\xrightarrow{\mu_F^2 \to \mu_{F,\min}^2} \text{finite}$$

In this case the allowed phase space region is

$$\eta < z < z_+$$

$$\mu_F^2 > \mu_{F,\min}^2 \sim y = \frac{1}{2\eta}\left[1 - \eta - \sqrt{(1-\eta)^2 - 4\eta\mu_b}\right]$$

At small scales

$$z_+ \xrightarrow{\mu_F^2 \to \mu_{F,\min}^2} \eta \qquad \mu_{F,\min}^2 > m_b^2$$

The threshold is at higher scale in the phase space then in the PDF and it depends on the momentum fraction.

MSbar PDF is not consistent with the shower evolution.

Sudakov Exponent

The operator of the virtual and unresolved radiations can be obtained from the unitary condition and that leads to the following:

$$\left(1\middle|\mathcal{V}(t)\middle|\{p,f,c\}_m\right) = \frac{y\,2n_{\mathrm{a}}\cdot n_{\mathrm{b}}}{4(2\pi)^2}\sum_{f'}\int dz\;\frac{1-y^2\nu_{\mathrm{a}}\nu_{\mathrm{b}}}{1-z^2y^2\hat{\nu}_{\mathrm{a}}\,\nu_{\mathrm{b}}}\;\frac{n_{\mathrm{c}}(a)}{n_{\mathrm{c}}(\hat{a})}\;\frac{f_{\hat{a}/A}(\eta_{\mathrm{a}}/z,\mu_F^2)}{f_{a/A}(\eta_{\mathrm{a}},\mu_F^2)}$$

Pure collinear

$$\times\Big\{C(\hat{f}_{\mathrm{a}},f')\,[\overline{w}_{\mathrm{aa}}(\{\hat{f},\hat{p}\}_{m+1})-\overline{w}^{\mathrm{eikonal}}_{\mathrm{aa}}(\{\hat{f},\hat{p}\}_{m+1})]$$

$$+\,\theta(f'=\mathrm{g})\sum_{k\neq\mathrm{a}}C_{\mathrm{a},k}(\{c\}_m)\int\frac{d\phi}{2\pi}\,\overline{w}^{\mathrm{soft}}_{\mathrm{a}k}(\{\hat{f},\hat{p}\}_{m+1})\Big\}$$

$$+\,(\text{final state splittings})$$

Soft and soft-collinear

This looks like the real part in the DGLAP equation....

DGLAP Equation with Masses

The parton distribution function should obey

$$\mu_F^2 \frac{d\, f_{a/A}(\eta_{\rm a}, \mu_F^2)}{d\mu_F^2} \approx \sum_{\hat a} \int \frac{dz}{z}\; f_{\hat{a}/A}(\eta_{\rm a}/z, \mu_F^2)\, \frac{y\, z\, 2n_{\rm a}\cdot n_{\rm b}}{4(2\pi)^2}\, \frac{1 - y^2 \nu_{\rm a}\nu_{\rm b}}{1 - z^2 y^2 \hat{\nu}_{\rm a}\, \nu_{\rm b}}\, \frac{n_{\rm c}(a)}{n_{\rm c}(\hat a)}$$
$$\times \Big\{ C(\hat{f}_{\rm a}, f')\, \overline{w}_{\rm aa}(\{\hat f, \hat p\}_{m+1}) + \text{virtual terms} \Big\}$$

This is complicated and involves many non-leading contributions. We want to apply some approximations to get as simple equation as the standard DGLAP.

First we have to decide what the μ_F is. It is the shower evolution scale. From another analysis we have found that the best choice for shower evolution variable is the *virtuality of the splitting divided by the mother parton energy*. That is

$$\mu_F^2 = y\frac{\eta_{\rm min}}{\eta} 2n_a \cdot n_b$$

y : normalized virtuality

η_{min} : momentum fraction at the hard interaction

η : momentum fraction of the mother parton

DGLAP Equation with Masses

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$$\times \left\{ C(\hat{f}_\mathrm{a}, f')\, \overline{w}_\mathrm{aa}(\{\hat{f}, \hat{p}\}_{m+1}) + \text{virtual terms} \right\}$$

This is complicated and involves many non-leading contributions. We want to apply some approximations to get as simple equation as the standard DGLAP.

Approximations: We assumes that the scale at hard interaction is large. Then

$2n_a \cdot n_b$ is large and $y \ll 1$

We neglect the quark masses compared to 1, but we keep every terms that is proportional to

$$\frac{m^2}{\mu_F^2}$$

DGLAP Equation with Masses

We try to write the evolution equation as

$$\mu_{\mathrm{F}}^2 \frac{d}{d\mu_{\mathrm{F}}^2} f_{a/A}(\eta_{\mathrm{a}}, \mu_{\mathrm{F}}^2) = \sum_{\hat{a}} \int \frac{dz}{z} \, \frac{\alpha_{\mathrm{s}}(\mu_{\mathrm{F}}^2)}{2\pi} P_{a\hat{a}}(z, \eta_{\mathrm{a}}/z, \mu_{\mathrm{F}}^2) f_{\hat{a}/A}(\eta_{\mathrm{a}}/z, \mu_{\mathrm{F}}^2)$$

The splitting kernel depend on the momentum fraction of the mother parton and on the evolution scale.

The real part of the splitting kernels can be obtained from shower splitting kernels but the virtual contributions are fixed by plus prescription and the momentum and flavor sum rules.

The kernel also depends on the momentum fraction of the incoming parton of the hard interaction (at the beginning of the shower)

$$\xi = \frac{m^2}{\mu_{\mathrm{F}}^2} \eta_{\mathrm{min}}$$

Modified Splitting Kernels

After some algebra can get the following functions:

$$P_{\rm qq}(z,\eta/z) = C_{\rm F}\left[\frac{2}{(1-z)_+} - (1+z) - 2z\,\frac{\xi}{\eta}\right]\theta\left(z > 1 - \frac{\eta}{\xi}\right) + (\gamma_q + \Delta\gamma_q)\,\delta(1-z)$$

$$P_{\rm gg}(z,\eta/z) = 2C_{\rm A}\left[\frac{1}{(1-z)_+} - 1 + \frac{1-z}{z} + z(1-z)\right] + (\gamma_{\rm g} + \Delta\,\gamma_{\rm g})\,\delta(1-z)$$

$$P_{\rm qg}(z,\eta/z) = T_{\rm R}\left[1 - 2\,z\,\left(1 - z - \frac{\xi}{\eta}\right)\right]\theta\left(z < 1 - \frac{\xi}{\eta}\right)$$

$$P_{\rm gq}(z,\eta/z) = C_{\rm F}\left[\frac{1+(1-z)^2}{z} - 2z\frac{\xi}{\eta}\right]\theta\left(\frac{1-z}{z^2} < \frac{\xi}{\eta}\right)$$

The virtual contributions are fixed by the sum rules. Actually it not that trivial because we have three sum rules and only two parameter to fix.

Sum Rules

Momentum sum rules:

$$\sum_a \int_0^1 dz\, z P_{a,b}(z, \hat{\eta}) = 0$$

This can fix both virtual contribution:

$$\gamma_{\mathrm{g}} + \Delta\gamma_{\mathrm{g}} = \frac{11}{6} C_A - \frac{2T_R}{3} \sum_q \theta\left(\frac{\xi_q}{\eta} < 1/4\right) \sqrt{1 - 4\frac{\xi_q}{\eta}} \left(1 + 2\frac{\xi_q}{\eta}\right)$$

$$\gamma_{\mathrm{q}} + \Delta\gamma_{\mathrm{q}} = \frac{3}{2} C_{\mathrm{F}} + 2C_F \log\left(1 + \frac{\xi_q}{\eta}\right) + C_F \frac{\frac{\xi_q}{\eta}\left(2 + \frac{\xi_q}{\eta}\right)}{2\left(1 + \frac{\xi_q}{\eta}\right)^2}$$

Flavor sum rule:

$$\int_0^1 dz\, P_{q,q}(z, \hat{\eta}) = 0$$

This fixes the $\gamma_q + \Delta\gamma_q$ constant and agrees with the results above. To get every sum rules right, it was important to choose the evolution variable carefully.

Modified DGLAP

- From $\gamma_g + \Delta\gamma_g$ one can see that the heavy flavor thresholds depend on the momentum fraction and they are at

$$\mu_F^2 > 4m_q^2 \frac{\eta_{\min}}{\eta}$$

- In the standard PDF the thresholds are at

$$\mu_F^2 > m_q^2$$

- In the massless limit we have the standard DGALP equations.

- The modified pdf is a function of three variable

$$f_{a/A}(\eta_{\mathrm{a}}, \mu_F^2, \eta_{\min})$$

What PDF to use in shower? How to fit PDF for parton shower?

LO* and LO**

- LO* and LO** are leading order fits with relaxing the momentum sum rules.
- The idea was to match some shower cross section to the corresponding NLO cross sections.
- Relaxing the sum rules could be dangerous because that implicitly changes the Sudakov exponent and the shower sums up the NLL collinear logs wrongly.
- It would be more useful to fit PDFs to shower cross sections, but the PDF dependence in the shower evolution is too complicated.

Perturbative Shower

The shower evolution equation is

$$\mathcal{U}(t,t') = \mathcal{N}(t,t') + \int_{t'}^{t} d\tau\, \mathcal{U}(t,\tau)\, \mathcal{H}_I(\tau)\, \mathcal{N}(\tau,t')$$

Exponentiates the ratio of PDFs

Depends on the PDF ratio

Actually the shower evolution doesn't depend strongly on the PDF fit, only the running is the important. One can show that

$$\mathcal{U}(t,t') \approx \mathcal{F}(t)\mathcal{U}_{\text{pert}}(t,t')$$

PDF factor

Perturbative shower

where

$$\mathcal{U}_{\text{pert}}(t,t') = \mathcal{N}_{\text{pert}}(t,t') + \int_{t'}^{t} d\tau\, \mathcal{U}_{\text{pert}}(t,\tau)\, \mathcal{H}_{\text{pert}}(\tau)\, \mathcal{N}_{\text{pert}}(\tau,t')$$

Here everything is independent of the PDF functions. Thus, the perturbative shower generates the partonic cross sections and the physical cross section is convolution of the PDFs and the partonic cross section.

Fitting Shower PDF

1. Let us pick a LO PDF with proper evolution, it can be completely ad-hoc PDF, it is not necessary to be an actual fit to data. This choice defines a shower scheme (SS). With this PDF one can do the shower evolution,

$$\big|\rho_{\rm SS}(t_2)\big) = \mathcal{U}_{\rm SS}(t_2,t_1)\big|\rho_{\rm SS}(t_1)\big)$$

2. But this is not the physical state what we want. We physical states with another PDF functions.

$$\big|\rho(t_2)\big) = \underset{\text{Fitting function}}{\mathcal{F}(t_2)}\, \underbrace{\mathcal{F}_{\rm SS}^{-1}(t_2)\mathcal{U}_{\rm SS}(t_2,t_1)\big|\rho_{\rm SS}(t_1)\big)}_{\approx\, |\rho_{\rm pert}(t_1))}$$

One can fit this to the data obtaining shower PDF.

3. Now set

$$\mathcal{F}_{\rm SS}(t) = \mathcal{F}(t)$$

4. Goto step 1 and repeat the procedure until the PDF is stable.

Conclusion

- We derived a modified DGLAP evolution equation for the heavy flavors from shower equation.
- The evolution equations includes the thresholds effects and they are consistent with the phase space constraint.
- Momentum and flavor sum rules are simultaneously satisfied.
- It works only with a special shower evolution variable, but we obtained the same variable form another analysis. It is the virtuality normalized by the mother parton energy.
- It would be interesting to derive and understand the result in field theory...
- ... but first we need precise field theory definition of the parton shower.