UNIVERSITÄT BERN

α_s FROM EVENT-SHAPES

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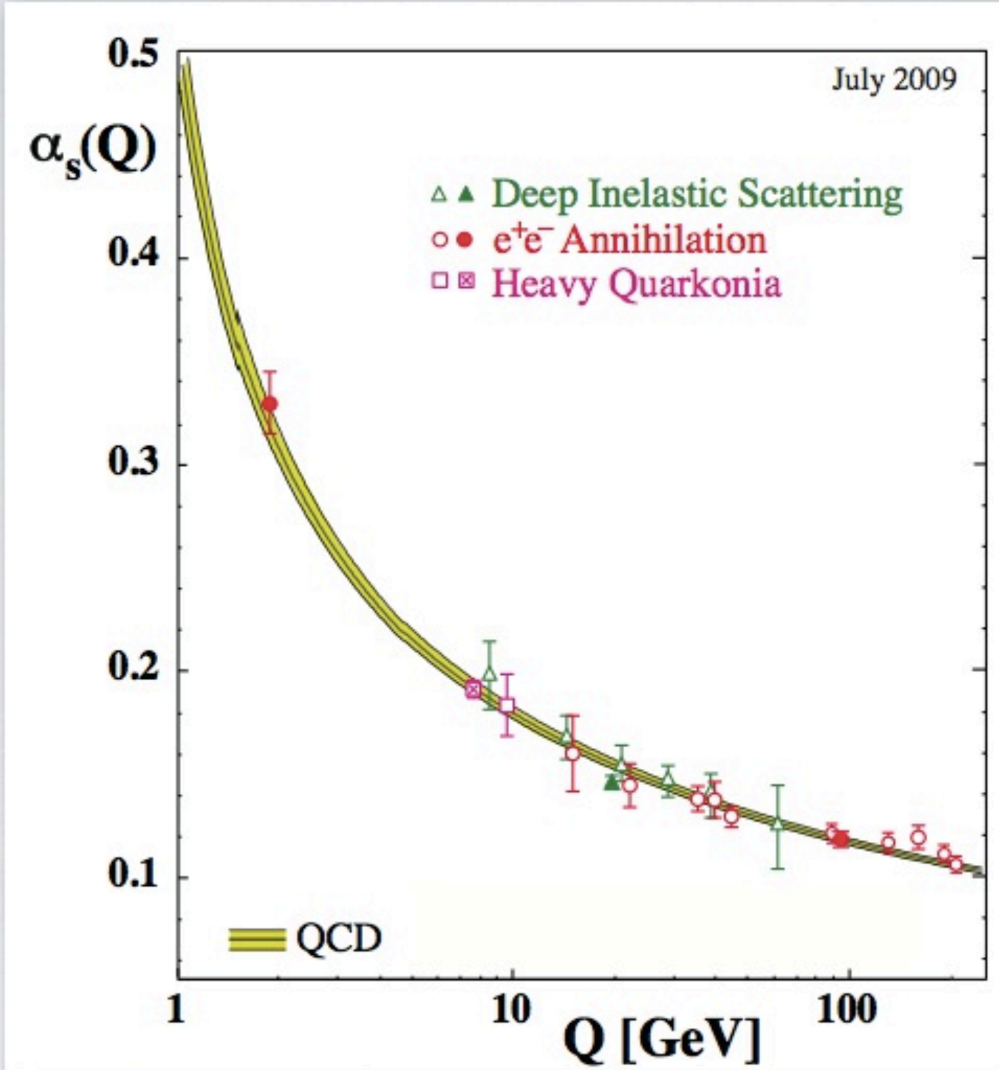


S. Bethke

Determinations of α_s at various energies beautifully confirm the scale-dependence predicted by QCD and provide evidence for asymptotic freedom.

A lot of progress in the α_s determination of over the last years, in particular for event shapes.

- NNLO fixed order, resummations up to N^{3}LL, detailed studies of hadronisation, ...

However, tensions among the most precise values of $\alpha_s(M_Z)$, e.g.

$0.1135(10)$ (thrust) Abbate et al. '10

$0.1142(23)$ (DIS, F_2) Blümlein et al. '06

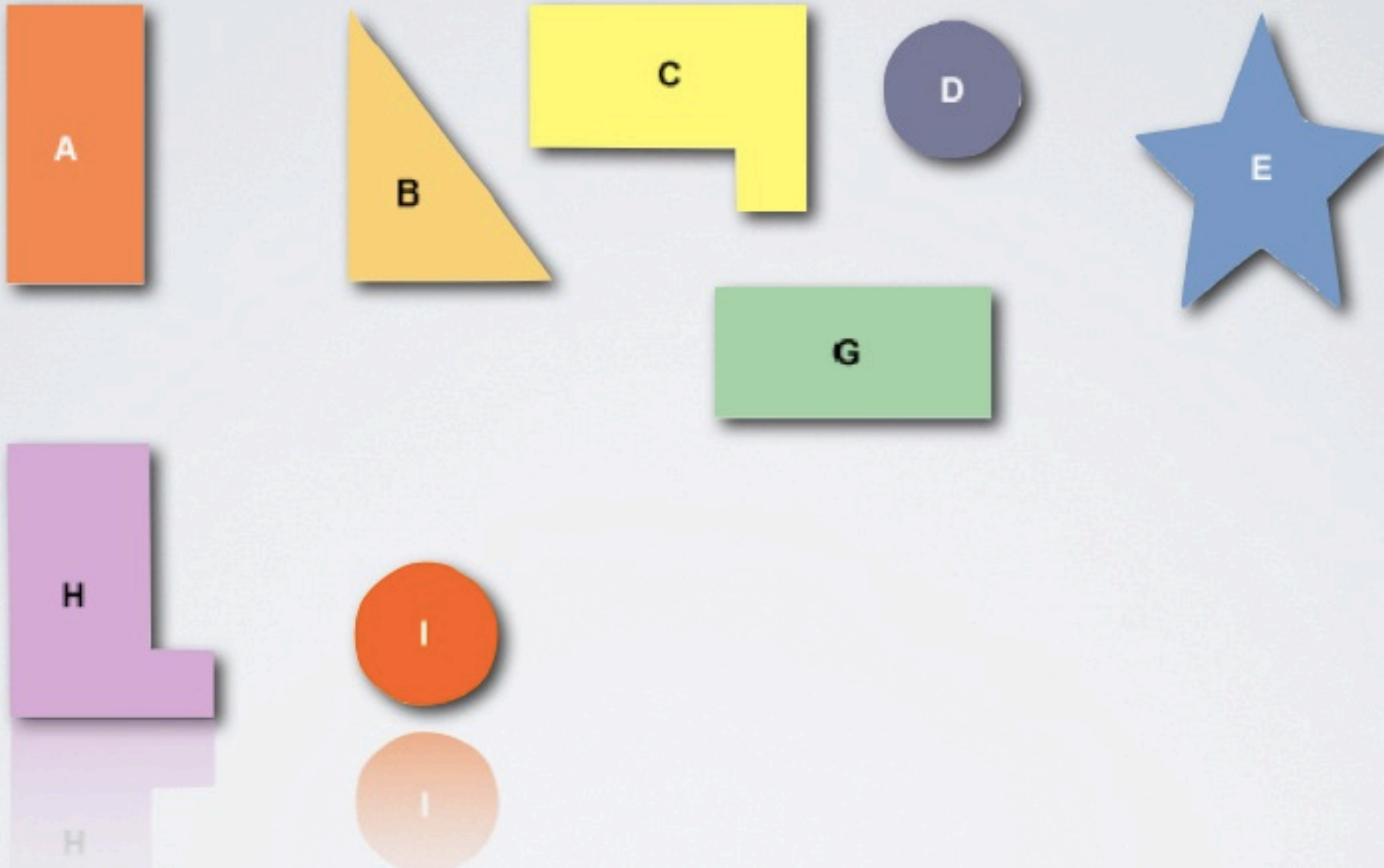
$0.1175(25)$ (3-jet rate) Dissertori et al. '09

$0.1183(8)$ (lattice) HPQCD '08

$0.1212(14)$ (τ-decays) Pich '10

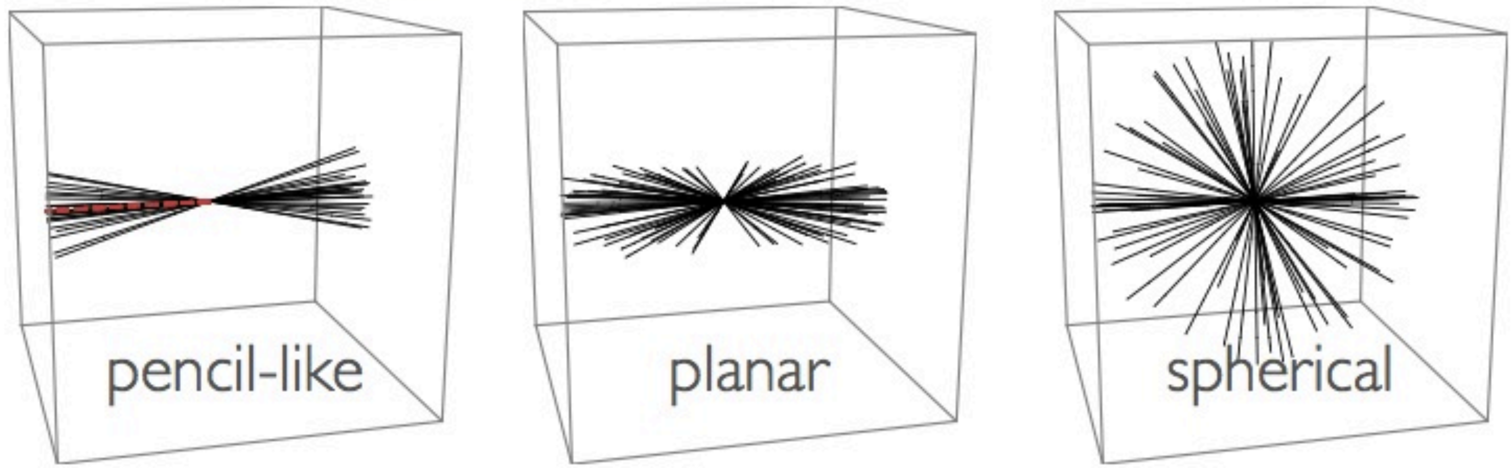
In the following, I will discuss the extraction from event shapes. Some remarks about other $\alpha_s(M_Z)$ determinations

- τ-decays: large difference between fixed-order and contour-improved perturbation theory. Beneke and Jamin '08 argue that one should use FOPT, and obtain 0.1180(8)
- lattice: there are now determinations with different actions and different methods:
  - JLQCD '10: 0.1181(3)(+14/–12) (overlap fermions)
  - PACS-CS '09: 0.1205(8)(5)(+0/–17) (Wilson fermions)
- NNLO PDF fits find both low 0.1135(14) (ABKM09) and high values ~0.117 (MSTW08, NNPDF21).



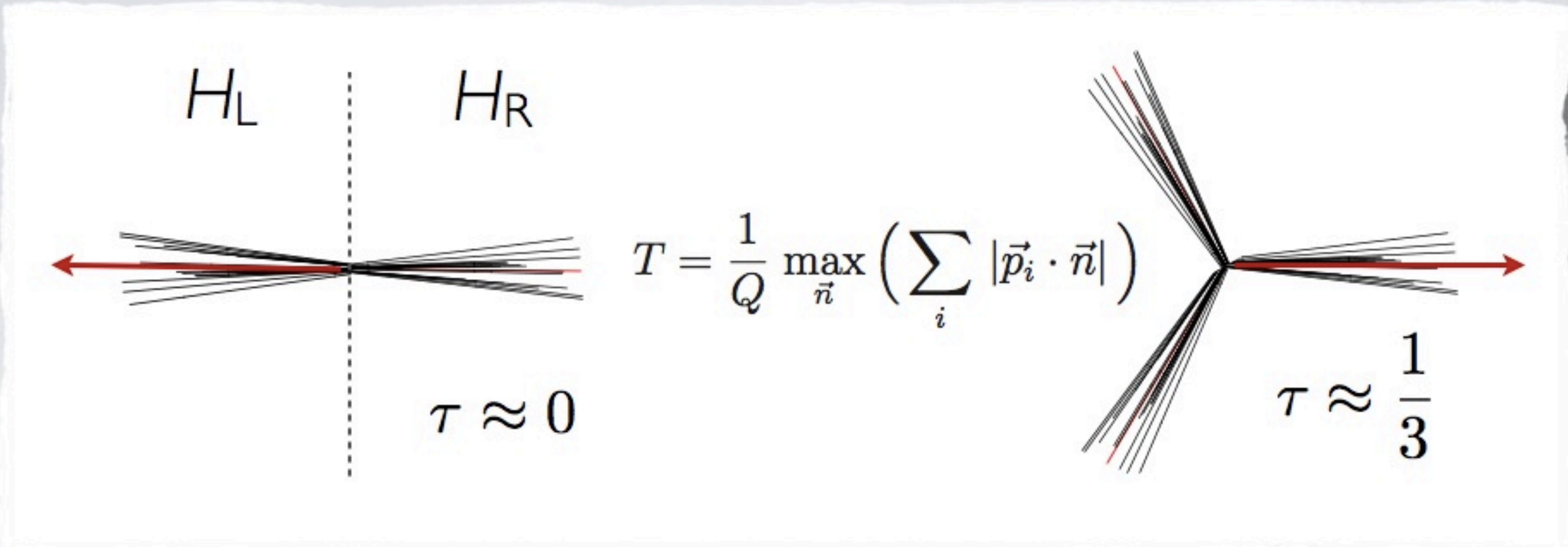
EVENT SHAPES

EVENT-SHAPE VARIABLES



- Parameterize geometric properties of energy and momentum flow in high energy collisions.
  - Inclusive observables: can be calculated in perturbation theory, hadronisation effects are suppressed at high energy.
- Canonical event shape is thrust *T*

THRUST $T = 1 - \tau$



The fraction of three-jet events is proportional to α_s.

OTHER CLASSIC EVENT SHAPES

Heavy "jet" mass:

$$M_H = \frac{1}{Q^2} \max\left(M_L^2, M_R^2\right)$$

$$M_{L/R}^2 = \Big(\sum_{i\in L/R} p_i\Big)^2$$

invariant mass of particles in hemishpere

Broadenings:

- total $\quad b_T = b_L + b_R$
- wide $\quad b_W = \max\left(b_L, b_R\right)$

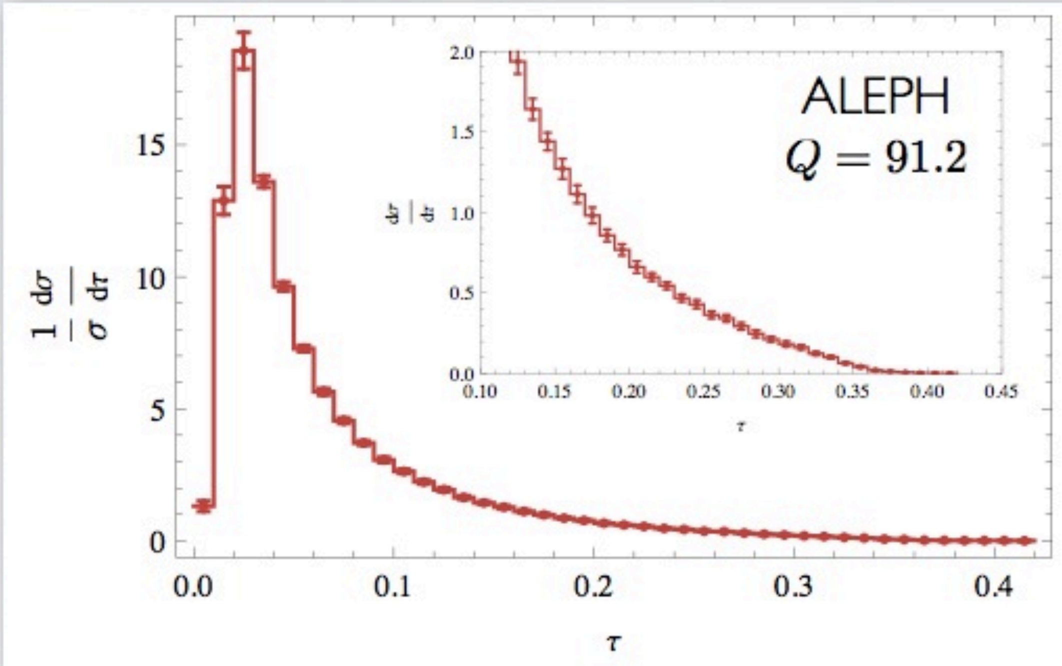
$$b_{L/R} = \frac{1}{2} \sum_{i\in L/R} |\vec{p}_i \times \vec{n}_T|$$

sum of transverse momenta in hemishpere

C-parameter

Jet rate y_{23} (or y_{34}, y_{45}) for resolution parameter y of given jet algorithm

HIGH-QUALITY DATA!

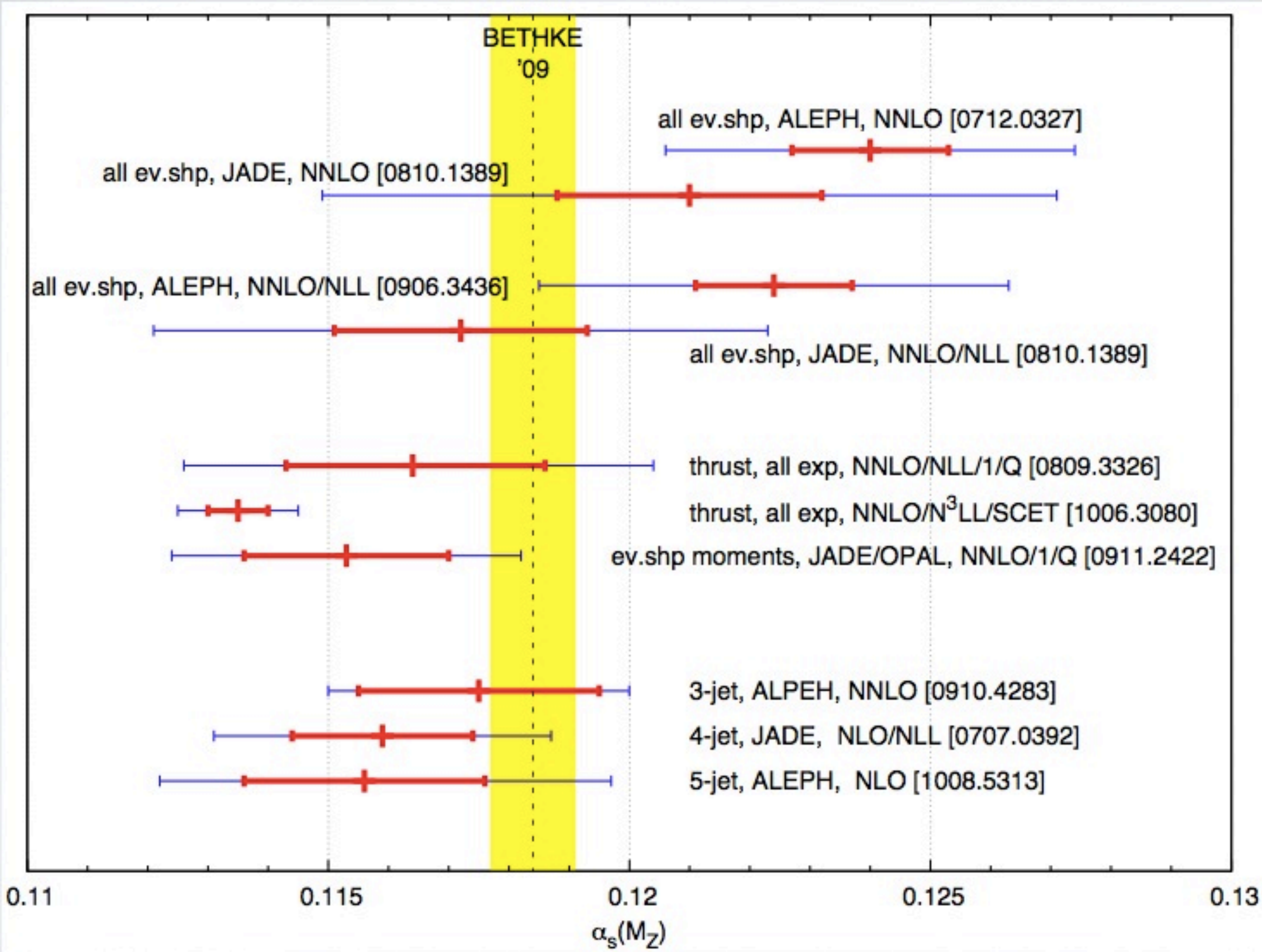


- Similar precision by the other LEP experiments. Same level of precision for other event-shapes.
- Lower energy (14-55GeV) data with good statistics is available as well JADE (recently reanalyzed), TASSO, AMY

THEORY DEVELOPMENTS

- Fixed order results to NNLO Gehrmann-De Ridder, Gehrmann, Glover, Heinrich '07; Weinzierl '08
- N^3LL resummations for two event shapes
  - thrust TB, Schwartz '08, heavy jet mass Chien, Schwartz '10
  - 2-loop soft function Kelley et al. '11; Hornig et al. '11; Monni et al. '11
- All-order factorization theorem for broadening Chiu, Jain, Neill, Rothstein 11; TB, Bell, Neubert '11
  - NNLL, once necessary perturbative computations are performed.
- Computation of the 5-jet rate to NLO Frederix et al. '10

Many new α_s extractions based these results!



from G. Salam at α_s workshop 2011.

ANALYSIS INPUT

EXP. DATA

one or several shapes
one or several exp's

Fixed order

NNLO for 3-jet region
NLO for 4 and 5 jets

Resummation

LL for jet rates
NLL for T, B, C, y_{23}
N^3LL for T and M_H

Hadronisation effects

estimate using MC generator
or from data at different Q

ANALYSIS TYPES

- **LEP style**
  - combined analysis of thrust T, broadenings B_{T} and B_{W}, heavy jet mass M_{H}, C-parameter and y_{23}.
  - NNLO or NNLO + NLL, MC hadronisation
- **JET rates**
  - 3 jets at NNLO, 4 or 5 jets at NLO, no resummation
- **Using SCET**
  - NNLO + N^3LL, but only T and M_{H}, hadronisation from data (or no hadronisation)

LEP STYLE ANALYSES

- NNLO + MC hadronisation

$$\alpha_s(M_Z) = 0.1240 \pm 0.0008\ (\mathrm{stat}) \pm 0.0010\ (\mathrm{exp}) \pm 0.0011\ (\mathrm{had}) \pm 0.0029\ (\mathrm{theo})$$

Dissertori et al. , ALEPH data, '07

$$\alpha_s(M_Z) = 0.1210 \pm 0.0007\ (\mathrm{stat}) \pm 0.0021\ (\mathrm{expt}) \pm 0.0044\ (\mathrm{had}) \pm 0.0036\ (\mathrm{theo})$$

Bethke et al. , JADE, '08

$$\alpha_s(M_Z) = 0.1201 \pm 0.0008\ (\mathrm{stat}) \pm 0.0013\ (\mathrm{exp}) \pm 0.0010\ (\mathrm{had}) \pm 0.0024\ (\mathrm{theo})$$

OPAL '11

- NNLO + NLL + MC hadronisation

- $\alpha_s(M_Z) = 0.1224 \pm 0.0009\ (\mathrm{stat}) \pm 0.0009\ (\mathrm{exp}) \pm 0.0012\ (\mathrm{had}) \pm 0.0035\ (\mathrm{theo})$

Dissertori et al. , ALEPH data, '09

$$\alpha_s(M_Z) = 0.1172 \pm 0.0006\ (\mathrm{stat}) \pm 0.0020\ (\mathrm{exp}) \pm 0.0035\ (\mathrm{had}) \pm 0.0030\ (\mathrm{theo})$$

Bethke et al. , JADE, '08

$$\alpha_s(M_Z) = 0.1189 \pm 0.0008\ (\mathrm{stat}) \pm 0.0016\ (\mathrm{exp}) \pm 0.0010\ (\mathrm{had}) \pm 0.0036\ (\mathrm{theo})$$

OPAL '11

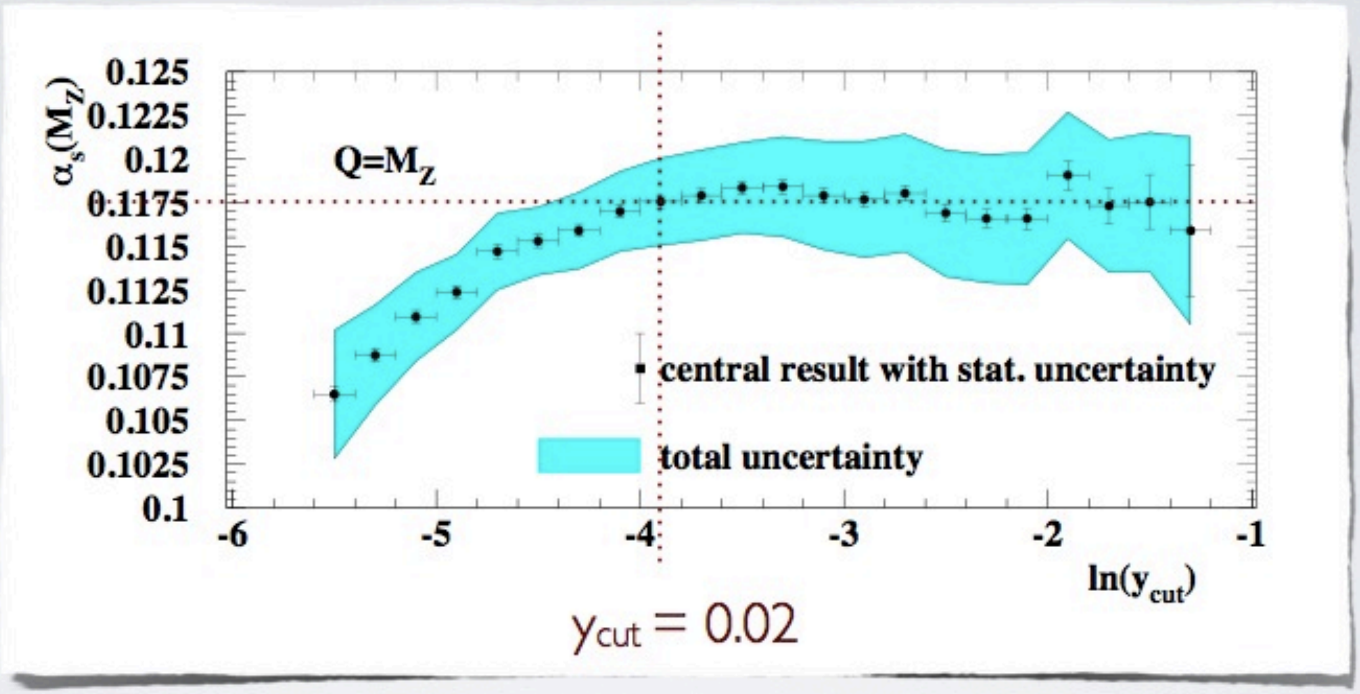
NNLO + NLL

- Values compatible, but not competitive with world average for α_s
- Even at NNLO perturbative uncertainties dominate!
- NLL resummation not enough: perturbative uncertainty increases after resummation!
- Hadronisation estimated by running different shower MCs and comparing parton and hadron level.
  - MC hadronisation might not be relevant for correcting perturbative computations, since shower works with hard cut-off
  - shower was tuned to event shapes: bias on α_s
  - for thrust, hadronisation effects extracted from data come out *much* larger than typical MC hadronisation effects!

NEW EXTRACTIONS FROM JET-RATES

FROM 3-JET RATE AT NNLO

Dissertori et al., arXiv:0910.4283, ALEPH data



For $y_{\text{cut}} = 0.02$:

$$\alpha_s(M_Z) = 0.1175 \pm 0.0004\ (\text{stat}) \pm 0.0019\ (\text{exp}) \pm 0.0006\ (\text{had}) \pm 0.0014\ (\text{theo})$$

$$= 0.1175 \pm 0.0020\ (\text{exp}) \pm 0.0015\ (\text{theo})$$

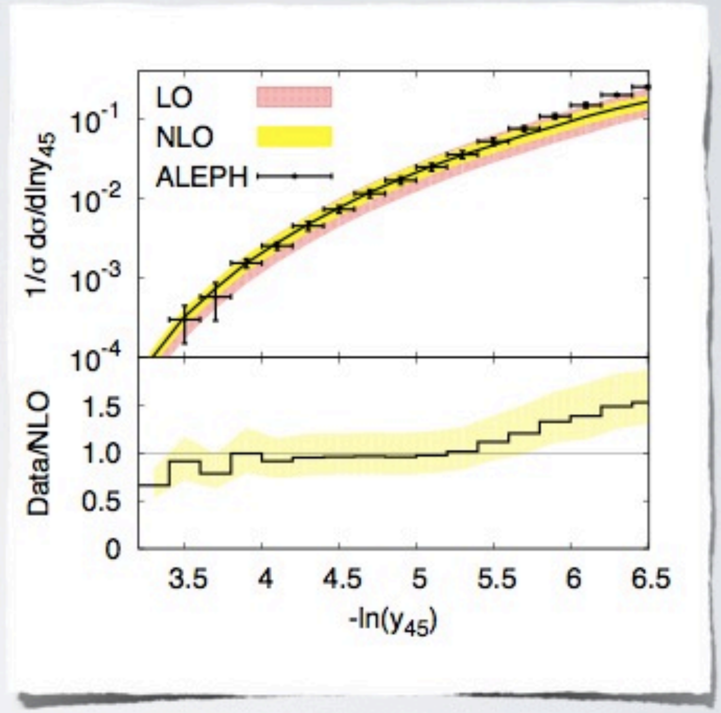
Desirable to perform resummation to stabilize results down to lower y_{cut}

FROM 5-JET RATE AT NLO

Ferderix et al., arXiv:1008.5313, ALEPH data

- 5-jet rate $\propto \alpha_s^3$ quite sensitive to the value of α_s
- Hadronisation corrections on α_s small, use SHERPA to estimate (shower matching crucial)
- Result from combining LEP I & II

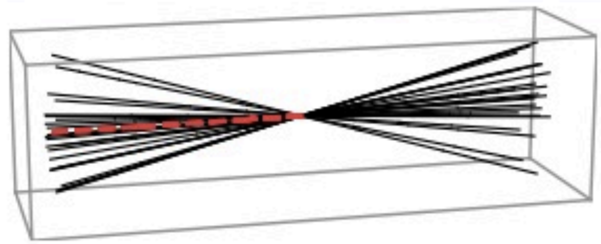
$$\alpha_s(M_Z) = 0.1156^{+0.0041}_{-0.0034}$$



N^3LL+NNLO USING SCET

RESUMMATION FOR THRUST

$$T = \max_{\mathbf{n}} \frac{\sum_i |\mathbf{p}_i \cdot \mathbf{n}|}{\sum_i |\mathbf{p}_i|}$$



$$1 - T \approx \frac{M_1^2 + M_2^2}{Q^2}$$

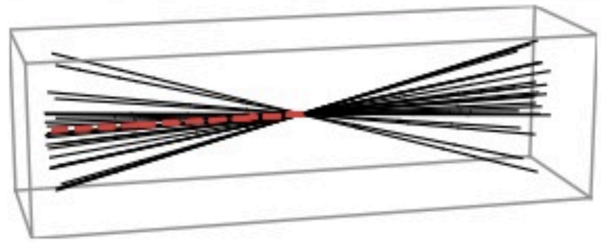
- The perturbative result for the thrust distribution contains logarithms $\alpha_s^n \ln^{2n} \tau$, where $\tau = 1\text{-}T$.
  - Near the end-point $\tau \to 0$ the logarithmic terms dominate.
- Using SCET one can derive a factorization theorem

$$\frac{1}{\sigma_0}\frac{d\sigma}{d\tau} = H(Q^2,\mu) \int dM_1^2 \int dM_2^2\, J(M_1^2,\mu)\, J(M_2^2,\mu)\, S_T(\tau Q - \frac{M_1^2 + M_2^2}{Q}, \mu)$$

Q^2	$\gg$	$M_1^2 \sim M_2^2 \sim \tau Q^2$	$\gg$	$\tau^2 Q^2$
hard		collinear		soft

RESUMMATION FOR THRUST

$$T = \max_{\mathbf{n}} \frac{\sum_i |\mathbf{p}_i \cdot \mathbf{n}|}{\sum_i |\mathbf{p}_i|}$$



$$1 - T \approx \frac{M_1^2 + M_2^2}{Q^2}$$

- Obtained NNNLL resummed distribution matched to NNLO TB and Schwartz '08. Fit to LEP data gives

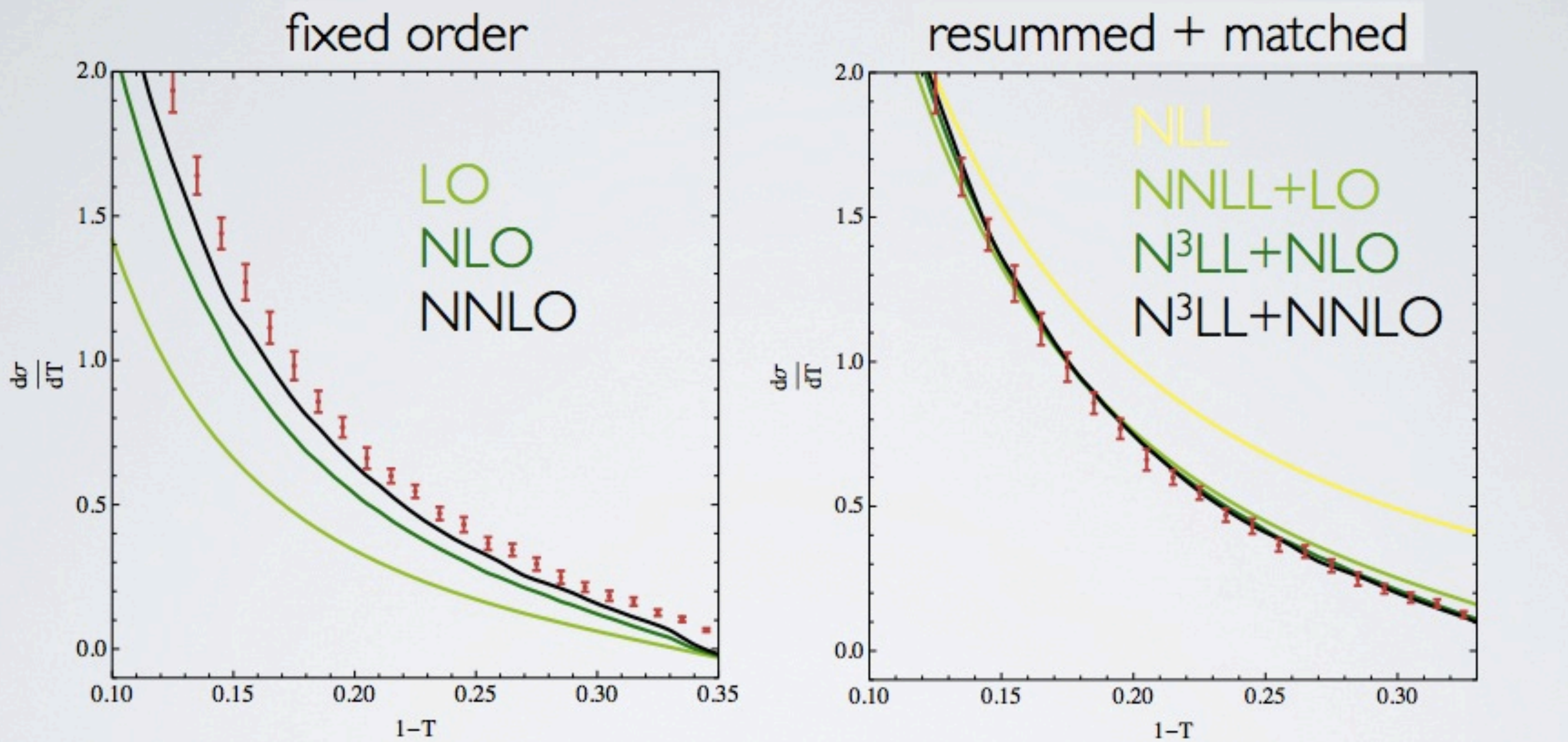
$$\begin{aligned} \alpha_s(m_Z) &= 0.1172 \pm 0.0010(\mathrm{stat}) \pm 0.0008(\mathrm{sys}) \pm 0.0012(\mathrm{had}) \pm 0.0012(\mathrm{pert}) \\ &= 0.1172 \pm 0.0022\,. \end{aligned}$$

estimated with Pythia & Ariadne, but hadronisation effect not included

- Similar factorization theorem and same accuracy also for heavy-jet mass. Chien and Schwartz '10

$$\begin{aligned} \alpha_s(m_Z) &= 0.1220 \pm 0.0014\ (\mathrm{stat}) \pm 0.0013\ (\mathrm{syst}) \pm 0.0022\ (\mathrm{had}) \pm 0.0009\ (\mathrm{pert}) \\ &= 0.1220 \pm 0.0031 \end{aligned}$$

RESUMMED VS. FIXED ORDER



- For PDG '05 value $\alpha_s(M_Z)=0.1176$
- This is the region relevant for the α_s determination

Abbate et al. have performed a global fit to all available thrust data. They fit simultaneously for hadronisation effects which result in a shift of the thrust distribution.

- Find much larger hadronisation effects than estimated by PYTHIA

$$\alpha_s(m_Z) = 0.1135 \pm (0.0002)_{\mathrm{expt}} \pm \underbrace{(0.0005)_{\Omega_1}}_{\text{(hadronisation)}} \pm (0.0009)_{\mathrm{pert}}$$

Abbate, Fickinger, Hoang, Mateu and Stewart 1004.4894

- 3.6σ lower than world average 0.1184(7) Bethke '09
- Important to validate this with other event shapes!
  - ongoing work on a similar analysis for M_H
- Moment fit gives $\alpha_s = 0.1153 \underbrace{\pm 0.0017}_{\text{(expt)}} \underbrace{\pm 0.0023}_{\text{(th)}}$ Gehrmann, Jacques, Luisoni '09.

TREATMENT OF HADRONISATION

Following Korchemsky '98, Abbate et al. use shape-function to parameterize non-perturbative corrections

$$S_\tau(k,\mu) = \int \mathrm{d}k' \; \underbrace{S_\tau^{\mathrm{part}}}_{\text{perturbative}}(k-k',\mu) \; \underbrace{S_\tau^{\mathrm{mod}}}_{\text{non-perturbative}}(k')\,,$$

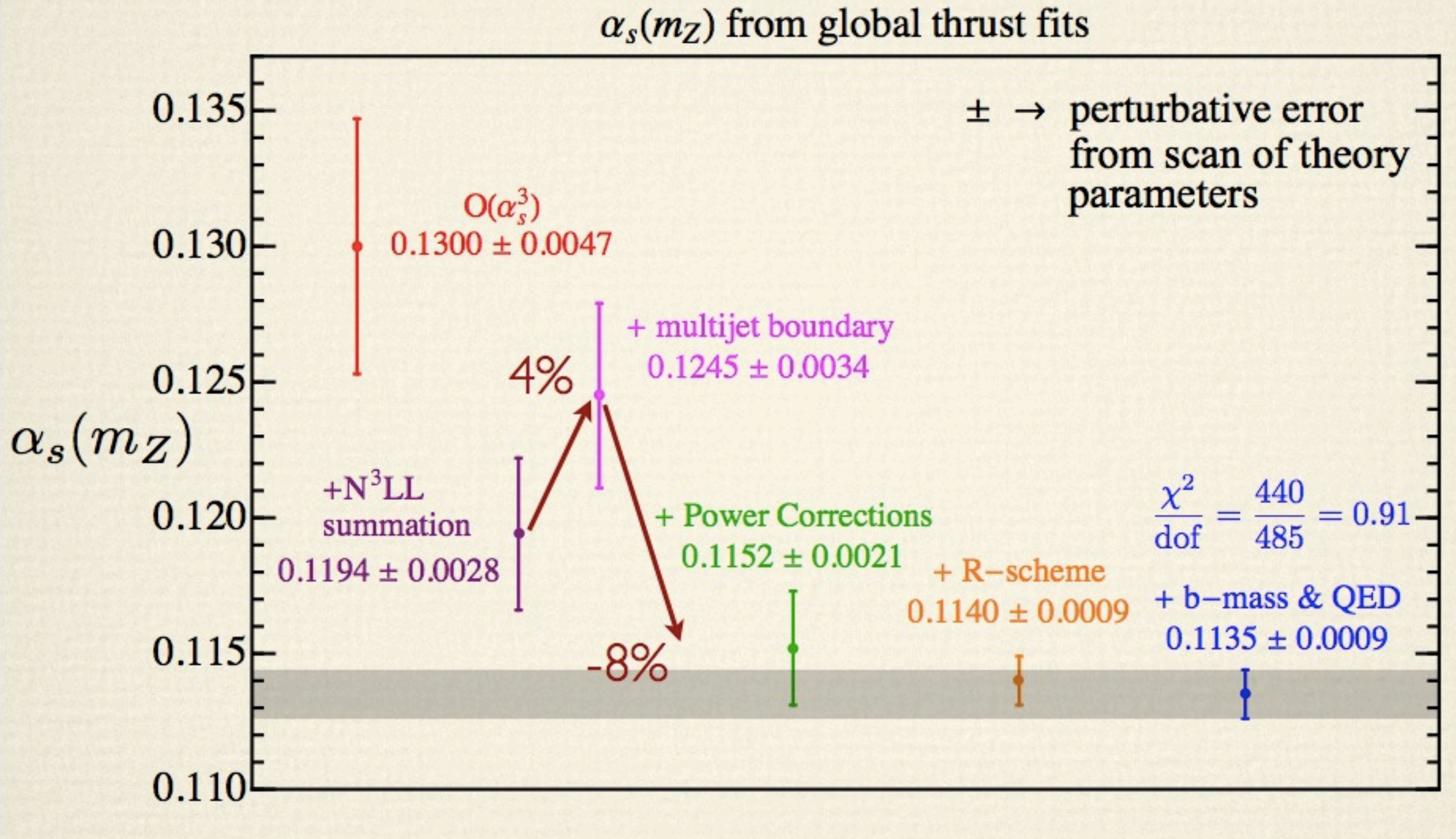
In the fit region $k \sim Q\,\tau \gg \Lambda_{\mathrm{QCD}}$, main effect comes from first moment

$$2\bar{\Omega}_1 = \int \mathrm{d}k'\, k'\, S_\tau^{\mathrm{mod}}(k')$$

which leads to a shift of the distribution. Not a model, can check for the effects of higher moments. [Also analytic coupling model by Dokshitzer and Webber '95 predicts shift.]

SIZE OF THE CORRECTIONS

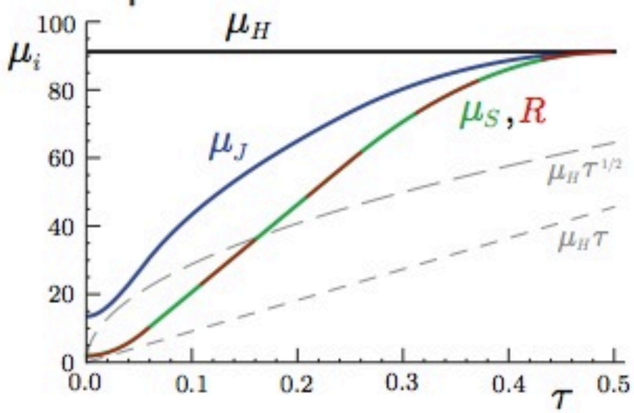
plot from R. Abbate's talk at loopfest '11



SCALE SETTING ISSUES (I)

At $\tau = 1 - T = 1/2$ the thrust distribution goes to zero. Abbate et *al*. enforce this by choosing the scales in the logarithms such that resummation switches itself off at the end-point.

"profile functions"



Experiment	Energy	BS results [20]	our BS profile	default profile
ALEPH	91.2 GeV	0.1168(1)	0.1170	0.1223
ALEPH	133 GeV	0.1183(37)	0.1187	0.1235
ALEPH	161 GeV	0.1263(70)	0.1270	0.1328

Quite a big difference on α_s, around 4%, despite the fact that the α_s fit only goes up to $\tau = 1/3$. Uncertainty associated with choice of profile function?

SCALE SETTING ISSUES (II)

Can either resum spectrum or cumulant $\boldsymbol{\Sigma}$

$$\Sigma(\tau) = \int_0^\tau \mathrm{d}\tau' \, \frac{1}{\sigma} \frac{\mathrm{d}\sigma}{\mathrm{d}\tau}(\tau') \,.$$

and get result for event fraction in bin [τ_1 , τ_2] as difference

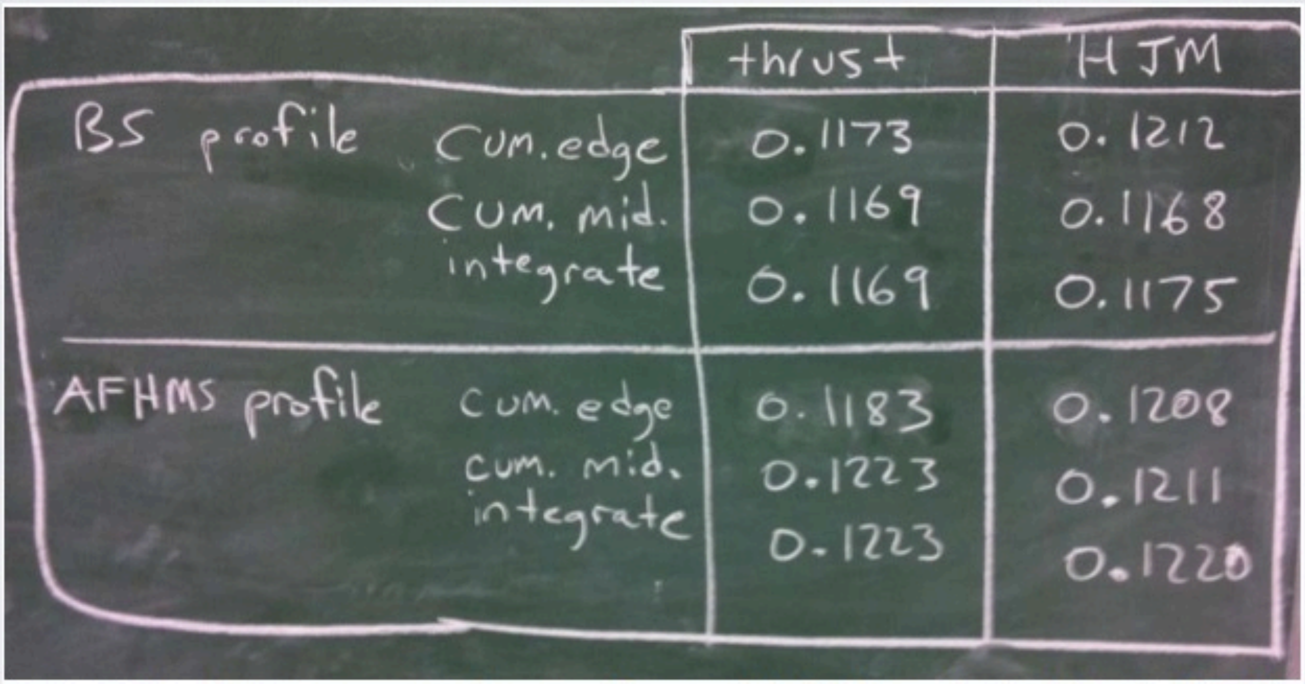
$$\Sigma(\tau_2, \mu_i(\tilde{\tau}_2)) - \Sigma(\tau_1, \mu_i(\tilde{\tau}_1))$$

Alternatively, one can just integrate spectrum over bin

$$\int_{\tau_1}^{\tau_2} d\tau \frac{1}{\sigma} \frac{d\sigma}{d\tau}(\tau, \mu_i(\tau))$$

SCALE SETTING ISSUES (II)

To the order of the computation, the different prescriptions are equivalent, but lead to ~3% differences in the extracted α_s

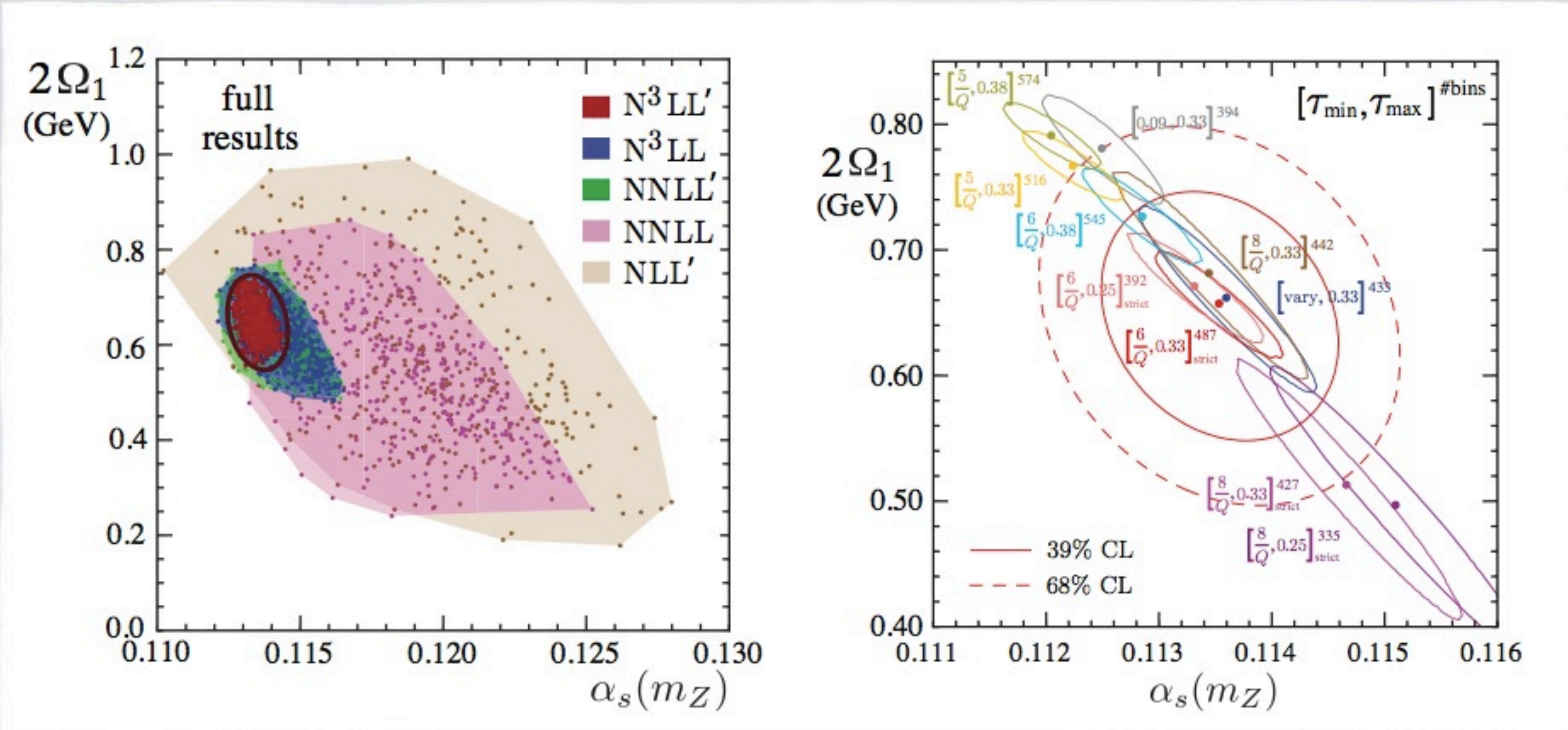


		thrust	HJM
BS profile	cum. edge	0.1173	0.1212
	cum. mid.	0.1169	0.1168
	integrate	0.1169	0.1175
AFHMS profile	cum. edge	0.1183	0.1208
	cum. mid.	0.1223	0.1211
	integrate	0.1223	0.1220

preliminary numbers, from V. Mateu's talk at SCET workshop

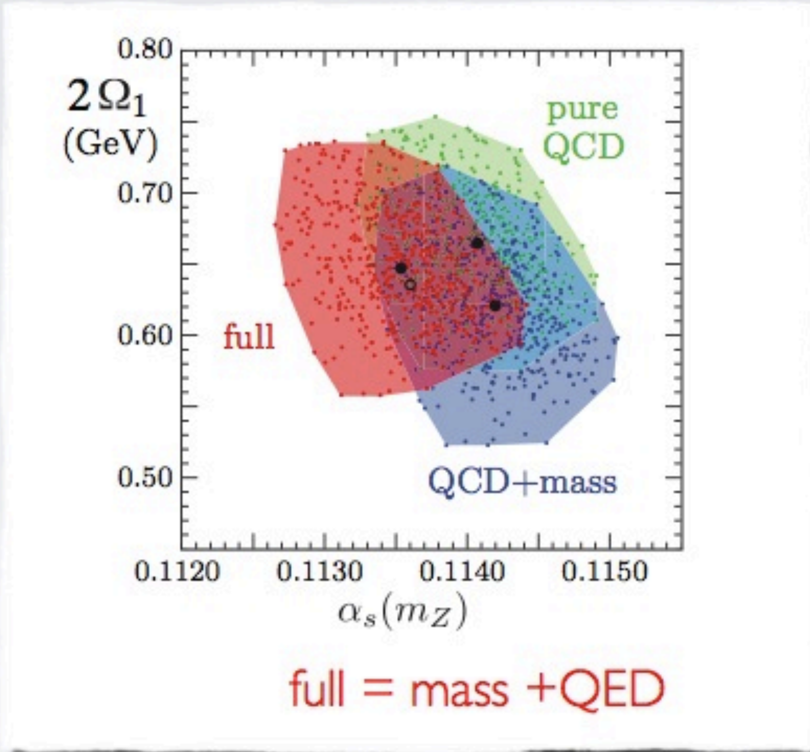
Abbate et al. argue that use of cumulant (with scales set at the edges) leads to spurious contribution of peak region in the tail.

FIT AND FITRANGE



Correlation between extracted value of α_s and lower edge of fit range.

QUARK MASS EFFECTS

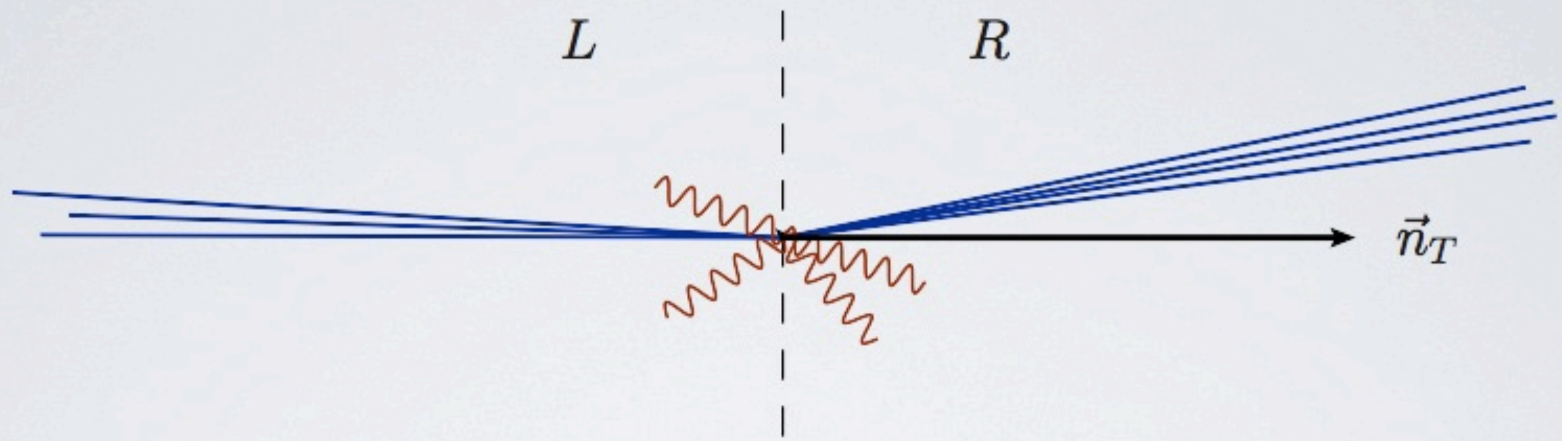


- Quark mass effects at LO + NLL. Negligibly small effect on α_s.
- ALEPH analysis includes heavy quarks at NLO, finds +1.0% mass effect at LEPI.

RESUMMATION FOR JET BROADENING

TB, Bell, Neubert, arXiv:1104.4108

JET BROADENING IN e^+e^-



- Broadening measures momentum relative to the thrust axis

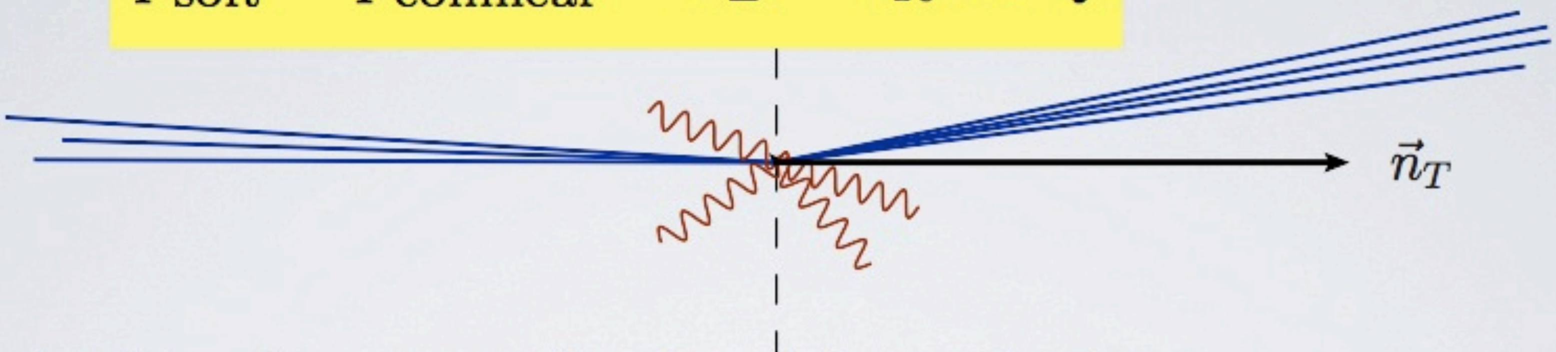
$$b_L = \frac{1}{2}\sum_i |\vec{p}_i^{\perp}| = \frac{1}{2}\sum_i |\vec{p}_i \times \vec{n}_T|$$

- Measured are the total and wide broadening

$$b_T = b_L + b_R; \qquad\qquad b_W = \max(b_L, b_R)$$

FACTORIZATION

$$p_{\text{soft}}^{\perp} \sim p_{\text{collinear}}^{\perp} \sim b_L \sim b_R \ll Q$$



- Factorization theorem for small broadening

$$\frac{1}{\sigma_0}\frac{d^2\sigma}{db_L db_R} = H(Q^2,\mu)\int db_L^s \int db_R^s \int d^{d-2}p_L^{\perp} \int d^{d-2}p_R^{\perp}$$
$$\boxed{\mathcal{J}_L(b_L - b_L^s, p_L^{\perp}, \mu)\, \mathcal{J}_R(b_R - b_R^s, p_R^{\perp}, \mu)}\boxed{\mathcal{S}(b_L^s, b_R^s, -p_L^{\perp}, -p_R^{\perp}, \mu)\,.}$$

Chiu, Jain, Neill and Rothstein '11, Bell, TB, Neubert '11

- Jet recoils against soft radiation!
- *J* and *S* suffer from coll. anomaly, analytic regulator

LAPLACE AND FOURIER SPACE

- Have derived all-order form of anomalous Q-dependence

$$\frac{1}{\sigma_0}\frac{d^2\sigma}{d\tau_L\,d\tau_R} = H(Q^2,\mu)\int_0^\infty dz_L\int_0^\infty dz_R\,\left(Q^2\bar{\tau}_L^2\right)^{-F_B(\tau_L,z_L,\mu)}\left(Q^2\bar{\tau}_R^2\right)^{-F_B(\tau_R,z_R,\mu)}$$
$$\times\, W(\tau_L,\tau_R,z_L,z_R,\mu)\,.$$

TB, Bell, Neubert, arXiv:1104.4108

- One-loop anomaly coefficient is

$$F_B(\tau,\mu) = \frac{C_F\alpha_s}{\pi}\left(\ln\mu\bar{\tau}_L + \ln\frac{\sqrt{1+z^2}+1}{4}\right)$$

- To NLL tree-level jet and soft functions are sufficient

$$W(\tau_L,\tau_R,z_L,z_R,\mu) = \frac{z_L}{\left(1+z_L^2\right)^{3/2}}\,\frac{z_R}{\left(1+z_R^2\right)^{3/2}}$$

NLL RESULT

- Because of the simple τ dependence the Mellin inversion can be done analytically. Result for total broadening:

$$\frac{1}{\sigma_0}\frac{d^2\sigma}{db_T} = H(Q^2,\mu)\frac{e^{-2\gamma_E\eta}}{\Gamma(2\eta)}\frac{1}{b_T}\left(\frac{b_T^2}{\mu^2}\right)^{2\eta} I^2(\eta)$$

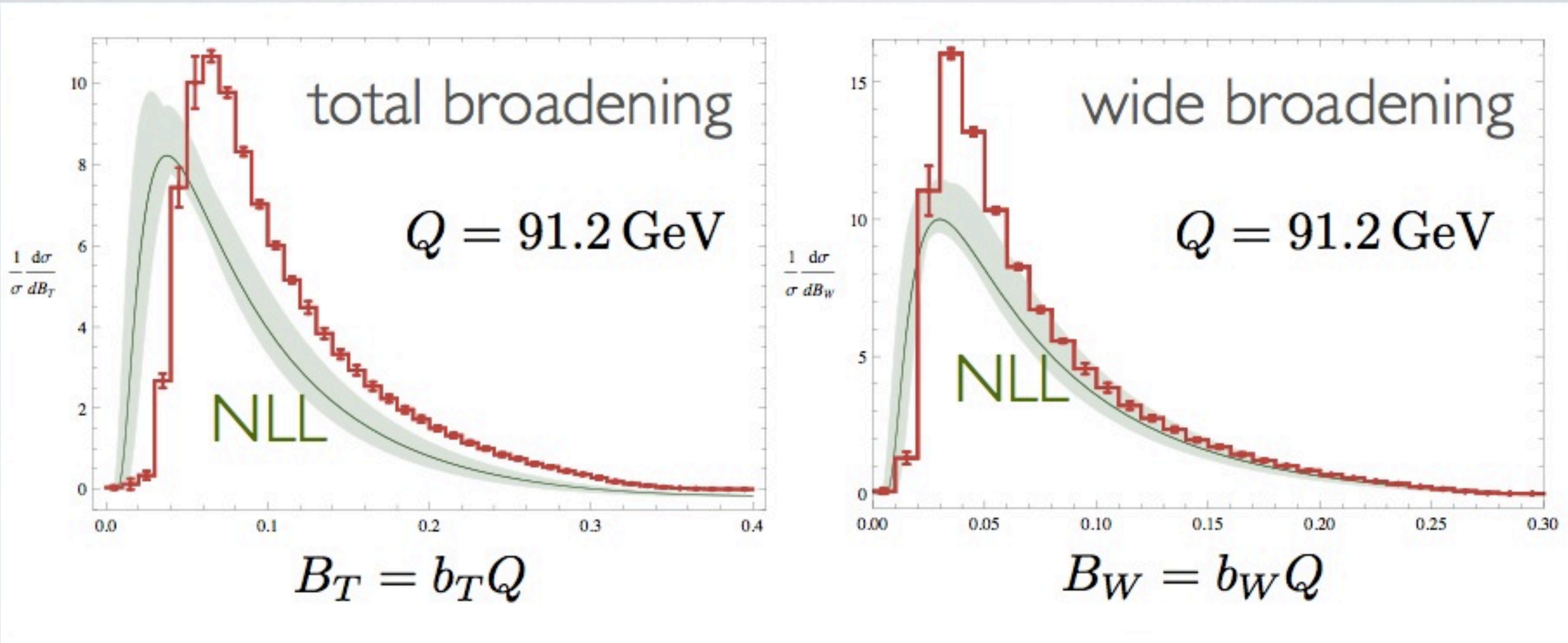
- with

$$I(\eta) = \int_0^\infty dz\,\frac{z}{(1+z^2)^{3/2}}\left(\frac{\sqrt{1+z^2}+1}{4}\right)^{-\eta} \quad \text{and} \quad \eta = \frac{C_F\alpha_s(\mu)}{\pi}\ln\frac{Q^2}{\mu^2}$$

- Equivalent to the result of Dokshitzer, Lucenti, Marchesini and Salam '98.

- Factor $I^2(\eta)$ missing in Catani, Turnock and Webber '92 and also in Chiu, Jain, Neill and Rothstein 1104.0881.

COMPARISON TO ALEPH DATA



- NLL is *not* a very good description of data.
- To combine with NNLO fixed order, and to extract α_s from a fit to data want at least NNLL.
    - Scale unc. of NNLO+NLL larger than NNLO alone. Dissertori et al. '09

NNLL?

- Have operator definitions for the jet and soft functions, e.g.

$$\frac{\pi}{2}(\not{n})_{\alpha\beta}\,\mathcal{J}_L(b,p^\perp) = \sum\!\!\!\!\!\!\int_X (2\pi)^d\,\delta(\bar{n}\cdot p_X - Q)\,\delta^{d-2}(p_X^\perp - p^\perp)$$
$$\delta\Big(b - \frac{1}{2}\sum_i |p_i^\perp|\Big)\,\langle 0|\chi_\alpha(0)|X\rangle\langle X|\bar{\chi}_\beta(0)|0\rangle$$

- For NNLL we need
  - one loop jet and soft functions and
  - two-loop anomaly function *F*, obtained e.g. from 2-loop divergence of the soft function.

CONCLUSIONS

- Hadronisation effects on event shapes are significant. Cannot rely on MC hadronisation model for precise α_s determination.
  - jet rates are less sensitive to hadronisation
- Important to validate result of N^3LL global analysis by Abbate et al. with other event shapes.
  - N^3LL global heavy-jet mass analysis is under way. Abbate et al. + Schwartz, in progress
  - All-order factorization theorem for broadening is available, N^2LL after necessary perturbative computations