PDF Correlations-benchmarking for the PDF4LHC working group

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PDF correlations

- Consider a cross section X(a), a function of the Hessian eigenvectors
- i^{th} component of gradient of X is

$$\frac{\partial X}{\partial a_i} \equiv \partial_i X = \frac{1}{2}(X_i^{(+)} - X_i^{(-)})$$

- Now take 2 cross sections X and Y
  - or one or both can be pdf's
- Consider the projection of gradients of X and Y onto a circle of radius 1 in the plane of the gradients in the parton parameter space
- The circle maps onto an ellipse in the XY plane
- The angle ϕ between the gradients of X and Y is given by

$$\cos\varphi = \frac{\vec{\nabla} X \cdot \vec{\nabla} Y}{\Delta X \Delta Y} = \frac{1}{4 \Delta X \, \Delta Y} \sum_{i=1}^{N} \left(X_i^{(+)} - X_i^{(-)}\right)\left(Y_i^{(+)} - Y_i^{(-)}\right)$$

- The ellipse itself is given by

$$\left(\frac{\delta X}{\Delta X}\right)^2 + \left(\frac{\delta Y}{\Delta Y}\right)^2 - 2\left(\frac{\delta X}{\Delta X}\right)\left(\frac{\delta Y}{\Delta Y}\right)\cos\varphi = \sin^2\varphi$$

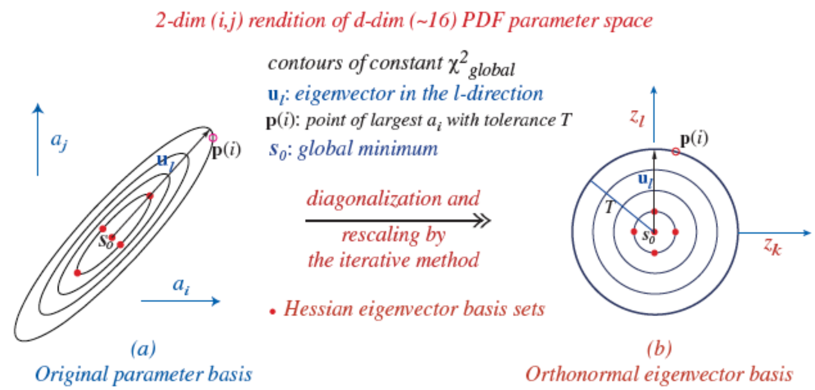

Figure 28. A schematic representation of the transformation from the pdf parameter basis to the orthonormal eigenvector basis.

•If two cross sections are very correlated, then cosϕ~1
•…uncorrelated, then cosϕ~0
•…anti-correlated, then cosϕ~-1

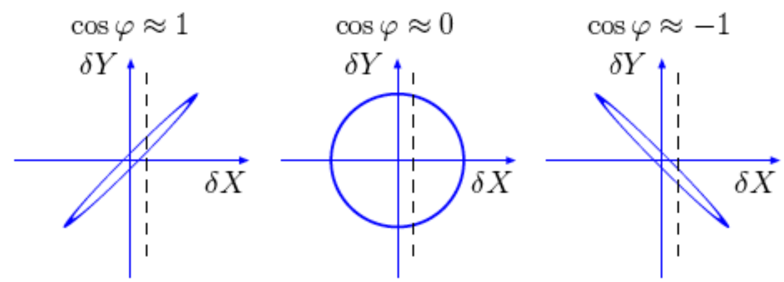

Figure 1: Dependence on the correlation ellipse formed in the $\Delta X - \Delta Y$ plane on the value of the correlation cosine $\cos\varphi$.

…from PDF4LHC report (CTEQ6.6)

Process	σ	PDF (asym)	PDF (sym)	$\alpha_s(m_Z)$ error	combined	orrelation
$\sigma_{W^+} * BR(W^+ \to l^+\nu)[nb]$	6.057	+0.123/-0.119	0.116	0.045	0.132	0.87
$\sigma_{W^-} * BR(W^- \to l^-\nu)[nb]$	4.106	+0.088/-0.091	0.088	0.029	0.092	0.92
$\sigma_{Z^o} * BR(Z^o \to l^+l^-)[nb]$	0.9469	+0.018/-0.018	0.018	0.006	0.0187	1.00
$\sigma_{t\bar{t}}[pb]$	156.2	+7.0/-6.7	6.63	4.59	8.06	-0.74
$\sigma_{gg\to Higgs}(120\ GeV)[pb]$	11.59	+0.19/-0.23	0.21	0.20	0.29	0.01
$\sigma_{gg\to Higgs}(180\ GeV)[pb]$	4.840	+0.077/-0.091	0.084	0.091	0.124	-0.47
$\sigma_{gg\to Higgs}(240\ GeV)[pb]$	2.610	+0.054/-0.058	0.056	0.055	0.078	-0.73

Table 5: Benchmark cross section predictions and uncertainties for CTEQ6.6 for $W^\pm, Z, t\bar{t}$ and Higgs production (120, 180, 240 GeV) at 7 TeV. The central prediction is given in column 2. Errors are quoted at the 68% c.l.. Both the symmetric and asymmetric forms for the PDF errors are given. In the next-to-last column, the (symmetric) form of the PDF and $\alpha_s(m_Z)$ errors are added in quadrature. In the last column, the correlation cosine with respect to Z production is given.

The values of ΔX, ΔY, and $\cos\varphi$ are also sufficient to estimate the PDF uncertainty of any function $f(X,Y)$ of X and Y by relating the gradient of $f(X,Y)$ to $\partial_X f \equiv \partial f/\partial X$ and $\partial_Y f \equiv \partial f/\partial Y$ via the chain rule:

$$\Delta f = \left|\vec{\nabla} f\right| = \sqrt{(\Delta X\ \partial_X f\ )^2 + 2\Delta X\ \Delta Y\ \cos\varphi\ \partial_X f\ \partial_Y f + (\Delta Y\ \partial_Y f)^2}. \qquad (9)$$

Used for LHC Higgs searches

Procedure for the LHC Higgs boson search combination in summer 2011

(LHC Higgs Combination Group Report)

A. Armbruster[1], K. A. Assamagan[2], S. Banerjee[3], G. Gomez-Ceballos[4], M. Chen[5], G. Cowan[6], K. Cranmer[7], S. Ferrag[8], E. Gross[9], J. Huston[10], A. Korytov[5], C. Mariotti[11], L. Moneta[12], W. J. Murray[13], G. Petrucciani[14], J. Qian[2], G. Schott[15], V. Sharma[14], G. E. Steele[8], R. Tanaka[16], F. Tarrade[17], O. Vitells[9], and H. Wang[4]

[1] *University of Michigan, Ann Arbor, MI, USA*

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Correlations for Higgs Working Group

Backgrounds

	Z	W	ZZ	WW	WZ	Wγ	WQQ	ZQQ	ggWW	ggZZ	ttbar	tW	tb	tbq
Z	1	0.95	0.67	0.70	0.95	0.9	0.43/0.53	0.08	-0.67	-0.75	-0.74	-0.81	0.59	-0.29
W	0.95	1	0.52/0.69	0.60/0.71	0.88/1.0	0.90/0.80	0.39/0.50	0.08	-0.67	-0.74	-0.73	-0.8	0.57	-0.29
ZZ	0.67	0.52/0.69	1	0.97	0.54/0.73	0.62	0.78/0.87	-0.09	-0.36	-0.34	-0.17	-0.81	0.9	-0.23
WW	0.70	0.60/0.71	0.97	1	0.63/0.75	0.69	0.80/0.86	-0.02	-0.34	-0.33	-0.20	-0.33	0.94	-0.08
WZ	0.95	0.88/1.0	0.54/0.73	0.63/0.75	1	0.9	0.55	0.1	-0.64	-0.71	-0.71	-0.73	0.61	-0.34
Wγ	0.9	0.90/0.80	0.62	0.69	0.9	1	0.63/0.53	0.32	-0.44	-0.54	-0.68	0.61	0.61	0
WQQ	0.43/0.53	0.39/0.50	0.78/0.87	0.80/0.86	0.55	0.63/0.53	1	0.08	-0.12	-0.12	-0.05	-0.15	0.64	-0.32
ZQQ	0.08	0.08	-0.09	-0.02	0.1	0.32	0.08	1	0.54	0.36	-0.26	-0.05	-0.03	0.59
ggWW	-0.67	-0.67	-0.36	-0.34	-0.64	-0.44	-0.12	0.54	1	0.98	0.65	0.81	-0.28	0.63
ggZZ	-0.75	-0.74	-0.34	-0.33	-0.71	-0.54	-0.12	0.36	0.98	1	0.79	0.91	-0.27	0.55
ttbar	-0.74	-0.73	-0.17	-0.20	-0.71	-0.68	-0.05	-0.26	0.65	0.79	1	0.97	-0.12	0.17
tW	-0.81	-0.8	-0.81	-0.33	-0.73	0.61	-0.15	-0.05	0.65	0.91	0.97	1	-0.25	0.31
tb	0.59	0.57	0.9	0.94	0.61	0.61	0.64	-0.03	-0.28	-0.27	-0.12	-0.25	1	0.04
tbq	-0.29	-0.29	-0.23	-0.08	-0.34	0	-0.32	0.59	0.63	0.55	0.17	0.31	0.04	1

…calculated with CTEQ6.6 and using MCFM

the use of correlations allows for PDF uncertainties to be reduced

processes treated as strongly correlated in blue

Higgs correlations (using CTEQ6.6)

m_H=120

	ggH	VBF	WH	ZH	ttH	Z	W+/W-	ZZ	WW	WZ	Wγ	WQQ	ZQQ	ggWW	ggZZ	ttbar	tW	tb	tbq
ggH	1	-0.57	-0.23	-0.14	-0.6	0.01	0.03	0.02	-0.20	0.04	0.23	-0.14	0.95	0.47	0.28	-0.35	-0.12	-0.24	0.52
VBF	-0.57	1	0.63/0.73	0.76	0.09	0.43	0.26/0.41	0.79	0.72	0.28/0.43	0.28/0.37	0.52/0.71	-0.41	-0.47	-0.4	-0.10	-0.28	0.65	-0.25
WH	-0.23	0.63/0.73	1	0.93	0	0.62	0.52/0.64	0.92	0.93	0.65/058	0.65/0.56	0.79/0.95	-0.02	-0.29	-0.28	-0.15	-0.28	0.99/0.77	0.05/-0.30
ZH	-0.14	0.76	0.93	1	0.03	0.64	0.53/0.66	0.99	0.99	0.55/0.71	0.63	0.83	-0.07	-0.31	-0.3	-0.14	-0.28	0.93	-0.14
ttH	-0.6	0.09	0	0.03	1	-0.61	-0.6	0	-0.05	-0.58	-0.64	0.04	-0.5	0.03	0.56	0.94	0.84	0.02	-0.07

m_H=160

	ggH	VBF	WH	ZH	ttH	Z	W+/W-	ZZ	WW	WZ	Wγ	WQQ	ZQQ	ggWW	ggZZ	ttbar	tW	tb	tbq
ggH	1	-0.61	-0.29	-0.35	-0.24	-0.32	-0.32	-0.35	-0.29	-0.29	-0.06	-0.12	0.9	0.82	0.68	0.1	0.33	-0.27	0.67
VBF	-0.61	1	0.62	0.74	0.2	0.35	0.19/0.34	0.75	0.66	0.20/0.36	0.19/0.28	0.46/0.70	-0.47	-0.46	-0.37	-0.03	-0.22	0.6	-0.29
WH	-0.29	0.62	1	0.93	0.1	0.55	0.52	0.9	0.93	0.56	0.56	0.93	-0.07	-0.26	-0.23	-0.07	-0.21	1	0.03
ZH	-0.35	0.74	0.93	1	0.16	0.54	0.43/0.58	0.98	0.97	0.45/0.63	0.52	0.93	-0.14	-0.29	-0.25	-0.04	-0.2	0.91	-0.16
ttH	-0.24	0.2	0.1	0.16	1	-0.59	-0.58	0.03	-0.03	-0.56	-0.62	-0.05	-0.54	0.33	0.51	0.92	0.8	0.04	-0.12

m_H=200

	ggH	VBF	WH	ZH	ttH	Z	W+/W-	ZZ	WW	WZ	Wγ	WQQ	ZQQ	ggWW	ggZZ	ttbar	tW	tb	tbq
ggH	1	-0.5	-0.26	-0.3	0.13	-0.59	-0.59	-0.36	-0.32	-0.55	-0.33	-0.11	0.68	0.98	0.93	0.5	0.69	-0.27	0.67
VBF	-0.5	1	0.60/0.73	0.72	0.26	0.28	0.13/0.28	0.7	0.62	0.15/0.30	0.12/0.20	0.40/0.69	-0.52	-0.44	-0.34	0.02	-0.17	0.55	-0.32
WH	-0.26	0.60/0.73	1	0.92	0.2	0.44	0.44/0.38	0.89	0.86	0.48/0.41	0.47/0.36	0.78/0.74	-0.15	-0.24	-0.2	0	-0.15	0.98/0.69	0
ZH	-0.3	0.72	0.92	1	0.24	0.46	0.34/0.51	0.95	0.93	0.37/0.56	0.43	0.74/0.85	-0.19	-0.3	-0.22	0.02	-0.14	0.88	-0.2
ttH	0.13	0.26	0.2	0.24	1	-0.57	-0.57	0.03	-0.03	-0.55	-0.63	0.03	-0.56	0.29	0.48	0.9	0.78	0.03	-0.15

Higgs correlations

m_H=300

	ggH	VBF	WH	ZH	ttH	Z	W+/W-	ZZ	WW	WZ	Wγ	WQQ	ZQQ	ggWW	ggZZ	ttbar	tW	tb	tbq
ggH	1	-0.16	-0.08	-0.09	0.66	-0.8	-0.79	-0.31	-0.31	-0.76	-0.64	-0.11	0.12	0.9	0.97	0.92	0.98	-0.23	0.43
VBF	-0.16	1	0.53/0.72	0.68	0.29	0.16	0.04/0.19	0.6	0.51	0.05/0.20	0.03	0.27/0.65	-0.57	-0.42	-0.31	0.09	-0.11	0.44	-0.39
WH	-0.08	0.53/0.72	1	0.92	0.23	0.32	0.20/0.36	0.82	0.80/0.71	0.34/0.37	0.30/0.20	0.68/0.64	-0.24	-0.22	-0.16	0.1	-0.06	0.89	-0.06
ZH	-0.09	0.68	0.92	1	0.27	0.32	0.20/0.38	0.87	0.82	0.21/0.44	0.26	0.61/0.81	-0.29	-0.25	-0.18	0.11	-0.07	0.79	-0.28
ttH	0.66	0.29	0.23	0.27	1	-0.6	-0.59	-0.05	-0.12	-0.58	-0.65	-0.04	-0.58	0.28	0.47	0.9	0.78	-0.04	-0.17

m_H=500

	ggH	VBF	WH	ZH	ttH	Z	W+/W-	ZZ	WW	WZ	Wγ	WQQ	ZQQ	ggWW	ggZZ	ttbar	tW	tb	tbq
ggH	1	0.09	0.05	0.05	0.91	-0.78	-0.76	-0.25	-0.28	-0.75	-0.73	-0.13	-0.3	0.63	0.78	0.99	0.97	-0.2	0.15
VBF	0.09	1	0.38/0.70	0.6	0.24	0.073	0.0/0.12	0.47	0.37	0/0.12	-0.08	0.11/0.59	-0.58	-0.4	-0.29	0.1	-0.08	0.29	-0.48
WH	0.05	0.38/0.70	1	0.9	0.16	0.19	0.09/0.26	0.69	0.64	0.20/0.20	0.14/0.09	0.55/0.53	-0.3	-0.21	-0.14	0.14	-0.02	0.73	-0.12
ZH	0.05	0.6	0.9	1	0.16	0.22	0.09/0.29	0.77	0.68	0.10/0.34	0.12	0.44/0.74	-0.35	-0.27	-0.19	0.13	-0.05	0.65	-0.37
ttH	0.91	0.24	0.16	0.16	1	-0.63	-0.61	-0.18	-0.23	-0.61	-0.69	-0.14	-0.57	0.3	0.48	0.89	0.79	-0.15	-0.14

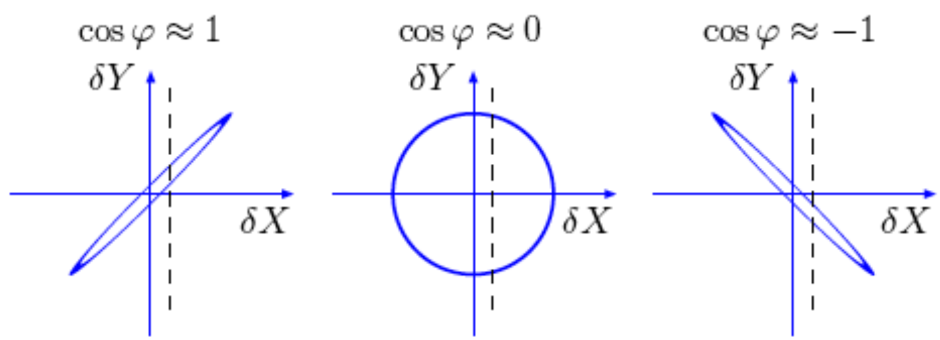
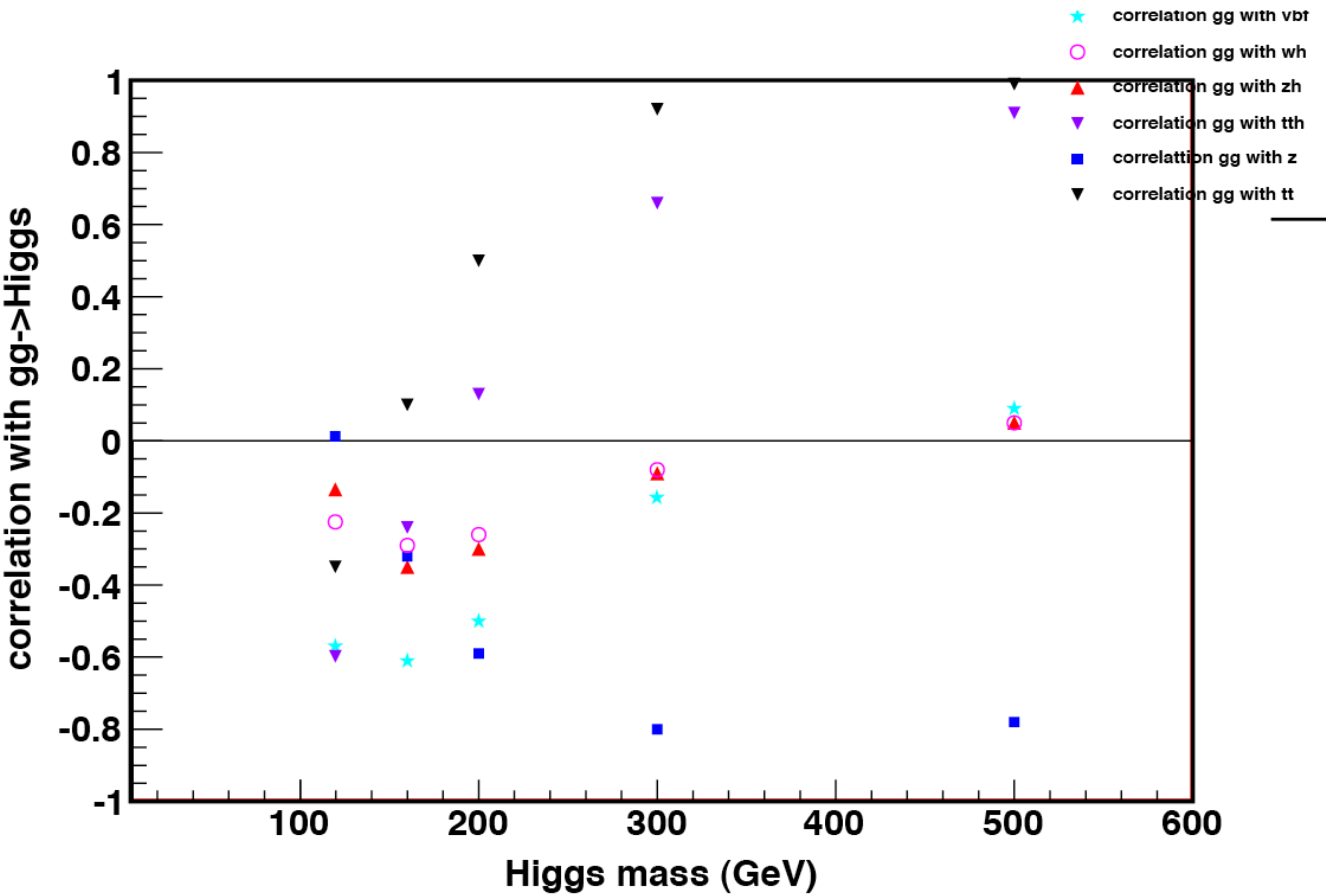


Figure 1: Dependence on the correlation ellipse formed in the $\Delta X - \Delta Y$ plane on the value of the correlation cosine $\cos\varphi$.

Some results



Extension

- The correlations should be similar for all NLO PDFs, but we would like to include correlation information from the different PDFs in the next Higgs CERN Yellow Report, as well as in future updates for the PDF4LHC working group documents
- Information (MCFM input files) available for all PDF groups
- I've done the same thing for the Tevatron; it would also be interesting to look at the correlations between the cross sections at the two accelerators

MCFM input files

```
[Flags to specify the mode in which MCFM is run]
.false.          [evtgen]
.true.           [creatent]
.false.          [skipnt]
.false.          [dswhisto]

[General options to specify the process and execution]
51               [nproc]
'tota'           [part 'lord','real' or 'virt','tota']
'z_bB'   ['runstring'] ->change runstring for each job
7000d0           [sqrts in GeV]
+1               [ih1 =1 for proton and -1 for antiproton]
+1               [ih2 =1 for proton and -1 for antiproton]
120d0            [hmass]
90d0             [scale:QCD scale choice] ->this is for HT^hat/2
90d0             [facscale:QCD fac_scale choice]
.false.          [dynamicscale] ->allows use of dynamic scale
.false.          [zerowidth]
.true.           [removebr]
10               [itmx1, number of iterations for pre-conditioning]
50000            [ncall1]
10               [itmx2, number of iterations for final run]
50000            [ncall2]
1089             [ij]    ->random number seed
.false.          [dryrun]
.true.           [Qflag]
.false.          [Gflag]

[Heavy quark masses]
172.5d0          [top mass]
4.75d0           [bottom mass]
1.5d0            [charm mass]

[Pdf selection]
'cteq66m'        [pdlabel]
4                [NGROUP, see PDFLIB]
46               [NSET - see PDFLIB]
cteq66.LHgrid    [LHAPDF group]
-1               [LHAPDF set]
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[Jet definition and event cuts]
60d0             [m34min]
120d0            [m34max]
60d0             [m56min]
120d0            [m56max]
.true.           [inclusive]
'ankt'           [algorithm] ->antikT algorithm
20d0             [ptjet_min] ->jet cut of 30 GeV; can lower it if you want
0d0              [|etajet|_min]
5.0d0            [|etajet|_max] ->max eta cut of 4.4
0.4d0            [Rcut_jet]  ->jet size of 0.4
.false.          [makecuts]  ->I generally leave this as false and make all cu
20d0             [ptlepton_min]
2.5d0            [|etalepton|_max]
25d0             [ptmin_missing]
0d0              [ptlepton(2nd+)_min]
10d0             [|etalepton(2nd+)|_max]
0.7d0            [R(jet,lept)_min]
0.7d0            [R(lept,lept)_min]
0d0              [Delta_eta(jet,jet)_min]
.false.          [jets_opphem]
0                [lepbtwnjets_scheme]
15d0             [ptmin_bjet]
2d0              [etamax_bjet]
20.d0            [ptmin_photon]
2.5d0            [etamax_photon]
0.7d0            [cone_photon]
4d0              [cone_ptcut]

[Anomalous couplings of the W and Z]
0.0d0            [Delta_g1(Z)]
0.0d0            [Delta_K(Z)]
0.0d0            [Delta_K(gamma)]
0.0d0            [Lambda(Z)]
0.0d0            [Lambda(gamma)]
2.0d0            [Form-factor scale, in TeV]

[How to resume/save a run]
.false.          [readin]
.false.          [writeout]
''               [ingridfile]
''               [outgridfile]

[Technical parameters that should not normally be changed]
.false.          [debug]
.true.           [verbose]
.false.          [new_pspace]
.false.          [virtonly]
.false.          [realonly]
.true.           [spira]
.false.          [noglue]
.false.          [ggonly]
.false.          [gqonly]
.false.          [vanillafiles]
1                [nmin]
2                [nmax]
.true.           [clustering]
.false.          [realwt]
0                [colourchoice]
1d-2             [rtsmin]
1d-4             [cutoff]
1d0              [aii]
1d0              [aif]
1d0              [afi]
1d0              [aff]
```

Other benchmarking exercises

- We have benchmarked evolution codes against each other
- It would also be useful to benchmark fit results/predictions for datasets in common use in current global fits
- If we (CTEQ) try to use other group's PDFs, and what we think is their procedure (heavy quark scheme, etc), we often get χ^2 much worse than quoted, so there are details missing
- For datasets like HERA 1 combined
  - what χ^2 each group gets
  - what normalization
  - systematic error shifts, χ^2 contribution from systematic error shifts
- Will an experimentalist using a generic code get the same result?
- Study: what experiments (primarily) determine which PDF parameters
- See Pavel's slides for more details

LHC data

- Comparisons to 2010 LHC data (I)
  - W/Z cross sections
  - tT cross sections
- Comparisons to 2010 data (II)
  - W/Z rapidity, asymmetry distributions
  - inclusive jet, dijet cross sections
    - ATLAS will make available all correlated systematic error information for jet cross sections
    - now CMS will as well
  - χ^2 and systematic error shifts before and after inclusion in global fit

NNLO

- Previous published benchmarking exercise was at NLO
- Now all groups have NNLO as well, so the study should be extended to that level
- No NNLO predictions in MCFM, so have to choose other codes for W/Z, Higgs, but otherwise try to standardize the input
  - no NNLO code for tT