Higher Order Calculations

Zoltan Kunszt, ETH, Zurich

QCD at the LHC

St Andrews, Scotland, August 23, 2011

The predictions of the SM for LHC are given in perturbation theory in the QCD improved standard model

$$d\hat{\sigma}_n^{(0)} \approx |M_n^{(0)}|^2 d\Phi_{n-2}$$

n=3,...12..

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Tree level: Fully automated cross section calculations for LHC in case of SM and BSM

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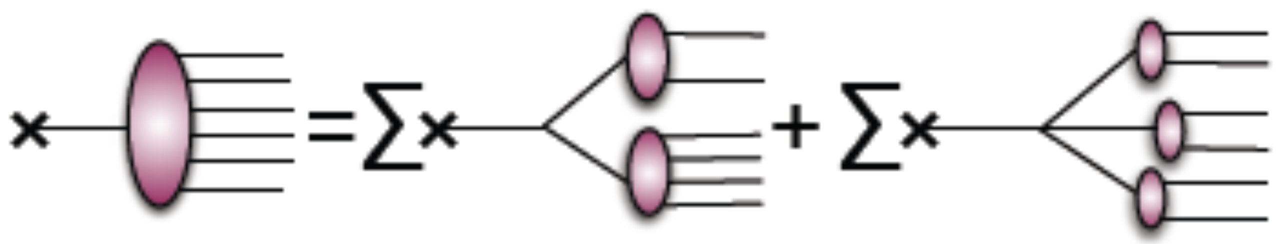

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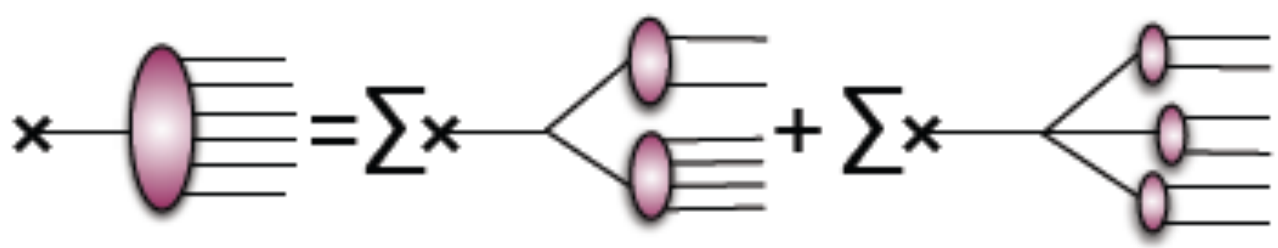

Factorial vs. polymomial (exponential) growths of the evaluation time with the number of the external legs

DATA CALL FOR HIGHER ORDER CORRECTIONS

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Improvements at NLO

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Improvements at NLO

- ☑ Reduced theoretical uncertainties due to more meaningful scale dependence and more precisely predicted rates and shapes
- ☑ Data at Tevatron and LEP fully validate the improvements of the agreement between theory and experiment if NLO corrections are included
- ☑ Smaller uncertainties in extrapolating measured background cross-sections into signal regions
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- Traditional Feynman-diagram approach with Passerine-Veltman reduction have factorial growth with the number of the legs

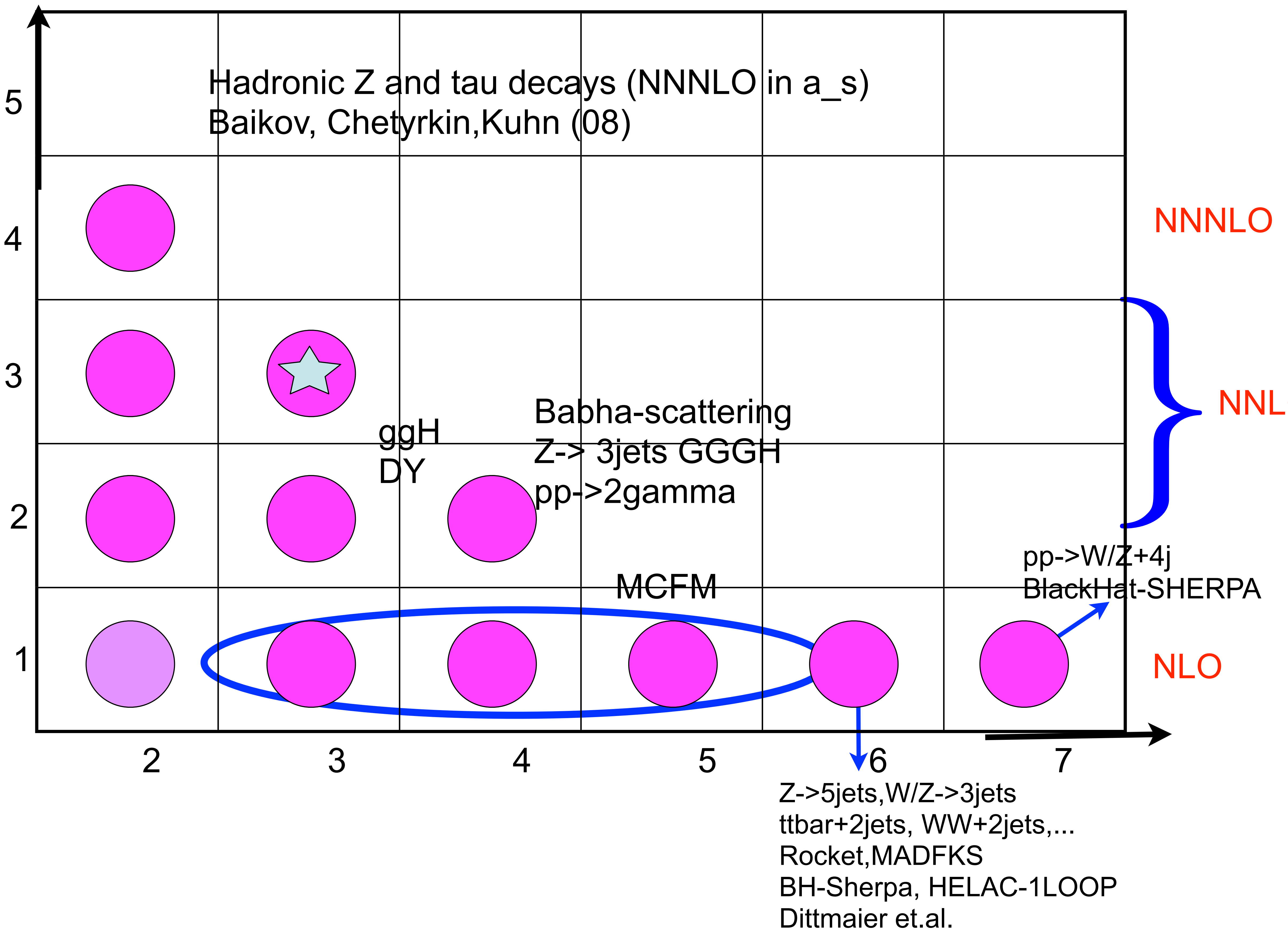
- Major technical improvements in the last 15 years

  recursion relations, double box loop integral, IBP identities and Lorenz invariance identities for loop integrals, Laporta algorithm, understanding the structure of soft and collinear singularities, unitarity method, OPP reduction....

- NNLO and multi-leg NLO revolution

- NNLO evolution of patron densities

What is available?



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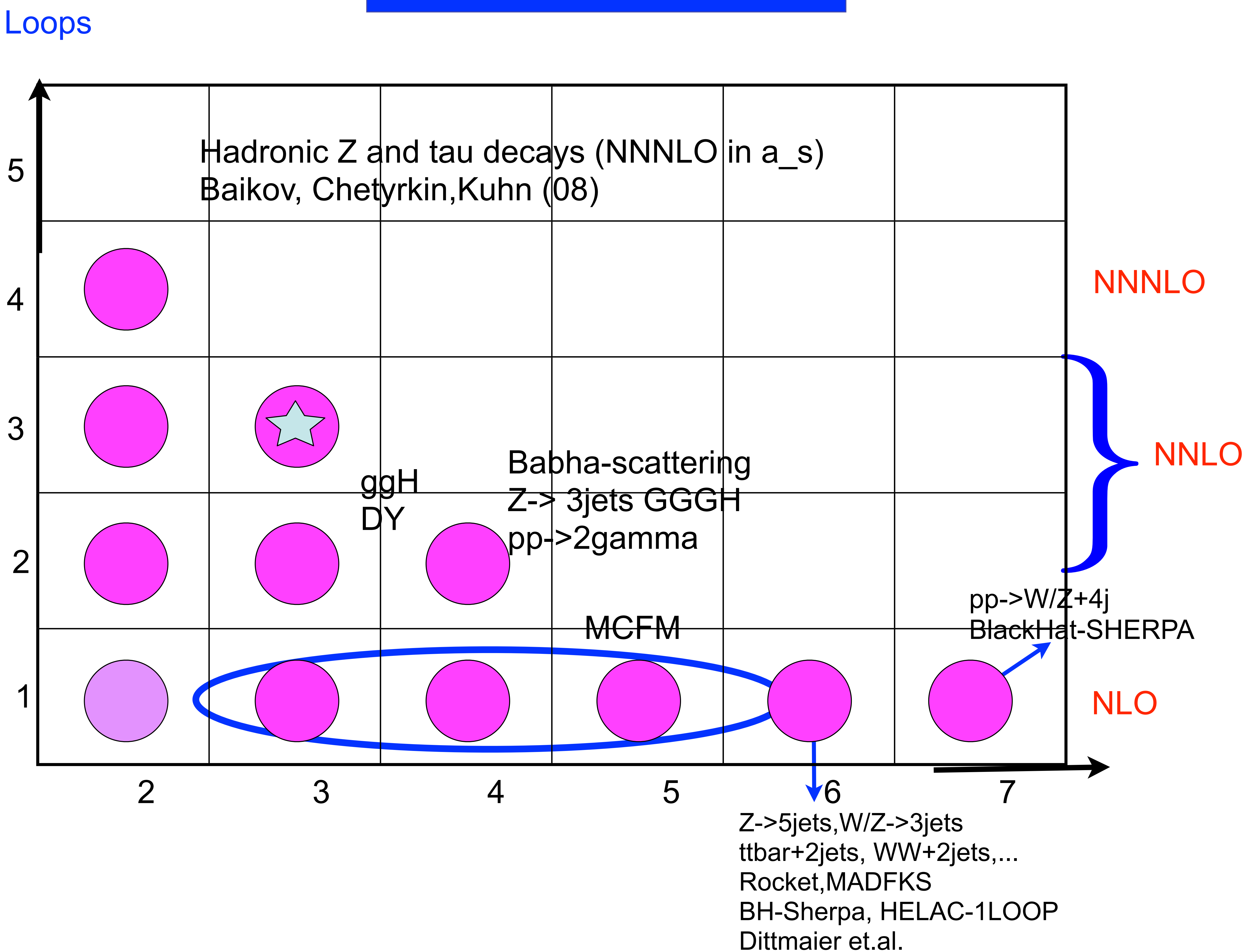

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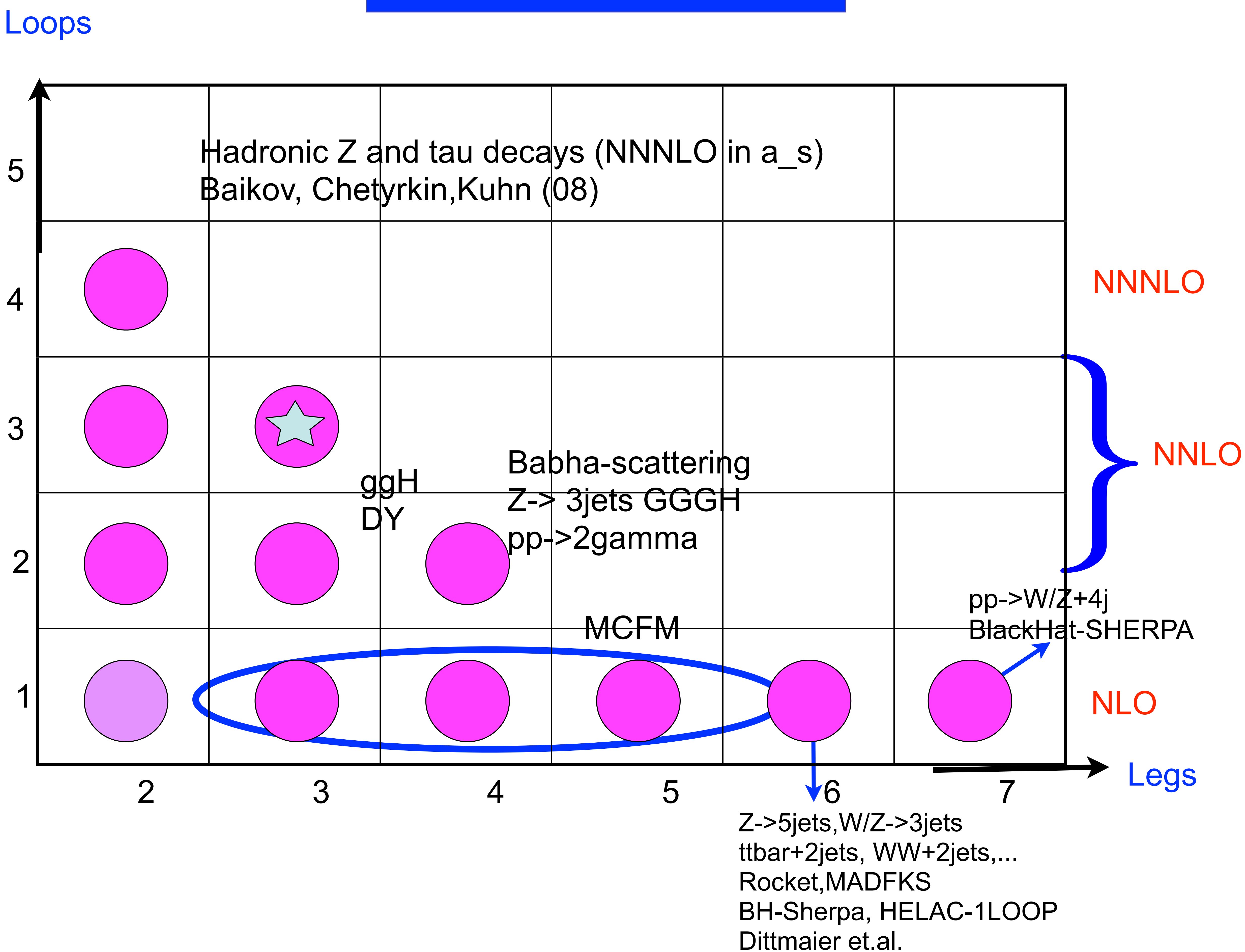

5-loop NNLO calculation, 2-loop NNLO, 1-loop NLO multi-leg

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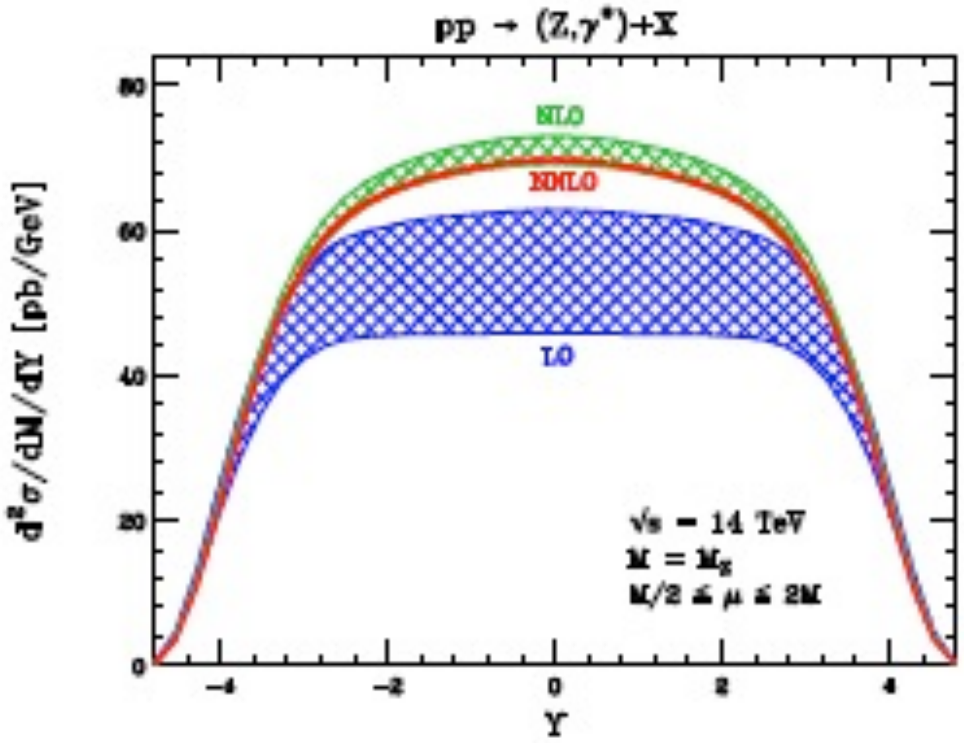

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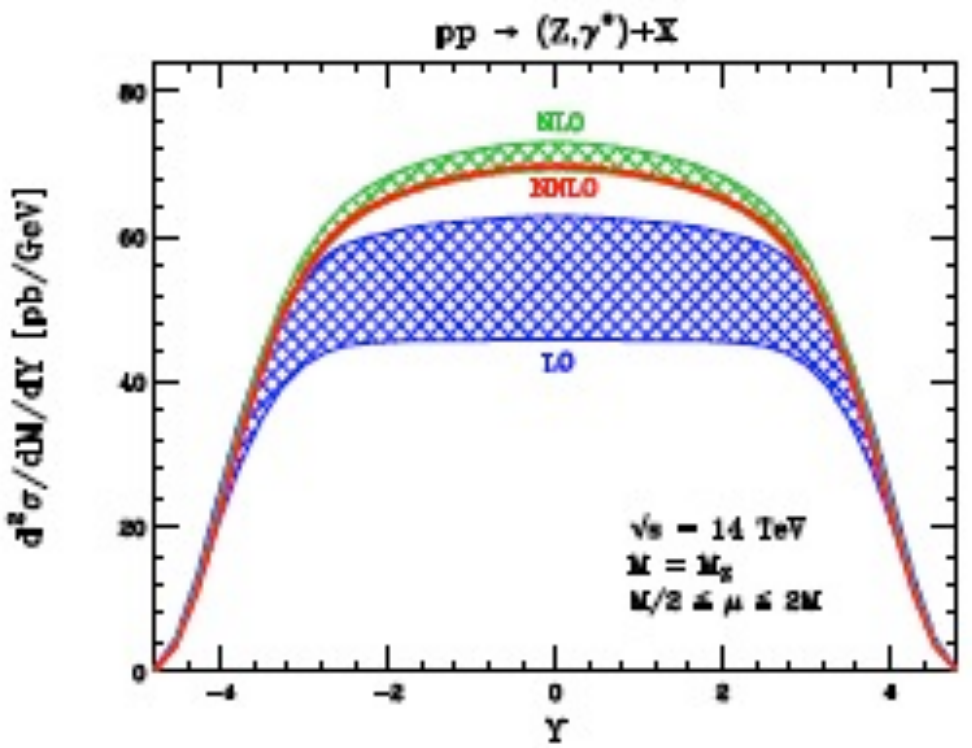
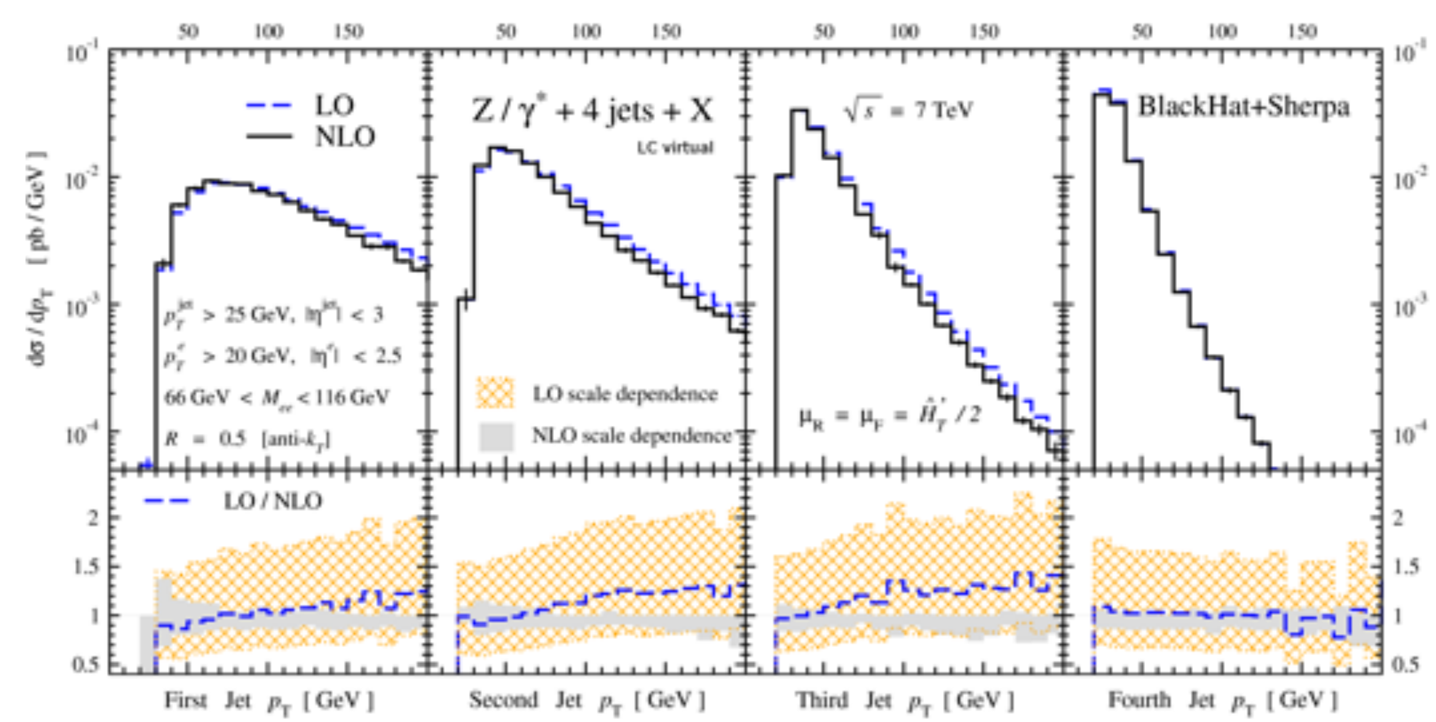


$Z/\gamma^* + 4$jet production
BlackHat (2011)



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Automatic calculation up to n=8...10...

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Bern, Dixon, Kosower (BDK) ; Ellis, Giele, ZK, Melnikov (EGKM); Britto, Cachazo, Feng (BCF)

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$$\mathcal{A}_N(p_1,p_2,\ldots,p_N) = \sum_{1\le i_1<i_2<i_3<i_4\le N} d_{i_1i_2i_3i_4}(p_1,p_2,\ldots,p_N) I_{i_1i_2i_3i_4}$$

$$+ \sum_{1\le i_1<i_2<i_3\le N} c_{i_1i_2i_3}(p_1,p_2,\ldots,p_N) I_{i_1i_2i_3}$$

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In 4D kinematics, the integrand of any one-loop Feynman amplitude with arbitrary number of external legs can be written in the standard form of linear combination of quadro-,triple-,double-,single-pole terms

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The numerator functions are polynomial in the transverse component of the loop momentum

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Box (rank four): $D_P = 3, D_T = 1, \quad l_T^2 = s_1^2 \sim 1$ $\qquad l_T^2 = l^2 - l_P^2\,, \quad l_P^\mu = (l q_i) v_i^\mu$

Triangle (rank three): $D_P = 2, D_T = 2, \quad l_T^2 = s_1^2 + s_2^2 \sim 1$

Bubble (rank two): $D_P = 1, D_T = 3, \quad l_T^2 = s_1^2 + s_2^2 + s_3^2 \sim 1$

$$\int [d\,l]\, \frac{\bar{d}_{ijkl}(l)}{d_i d_j d_k d_l} = \int [d\,l]\, \frac{d_{ijkl} + \tilde{d}_{ijkl}\, n_1 \cdot l}{d_i d_j d_k d_l} = d_{ijkl} \int [d\,l]\, \frac{1}{d_i d_j d_k d_l} = d_{ijkl} I_{ijkl} \;,$$

Numerator functions are fixed in terms of tree amplitudes

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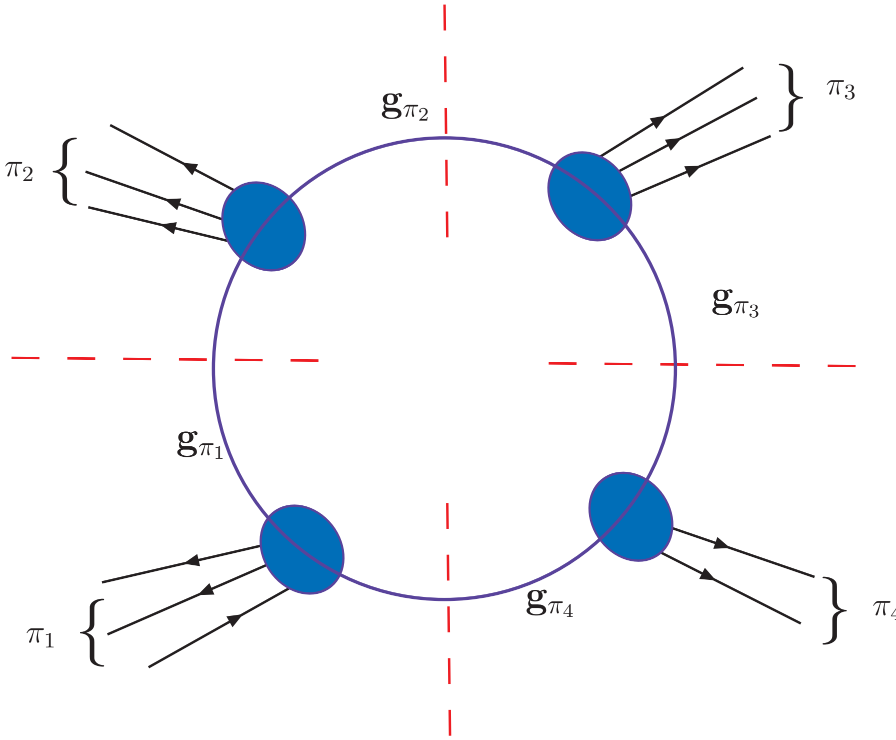
Two key points:

A) parameters are independent from the loop momenta: they can be calculated from the value of the residua of the amplitude in the loop momentum

B) The residua factorize into the product of tree amplitudes



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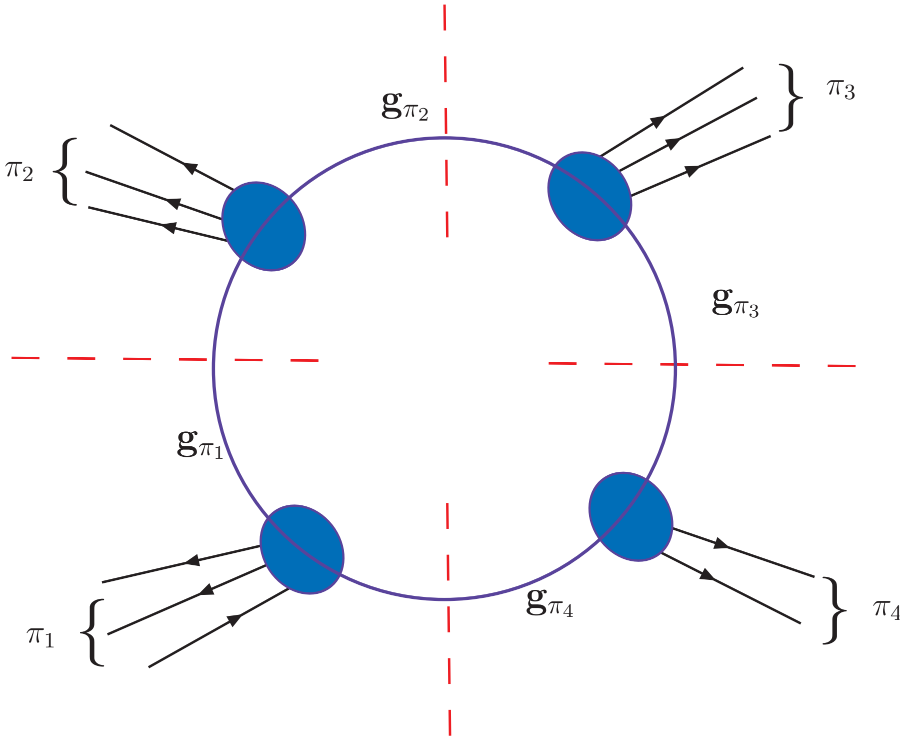

$$\overline{d}_{ijkl}(l) = \text{Res}_{ijkl}\left(\mathcal{A}_N(l)\right)$$

$d_i=d_j=d_k=d_l=0$

two solutions, 4-cut

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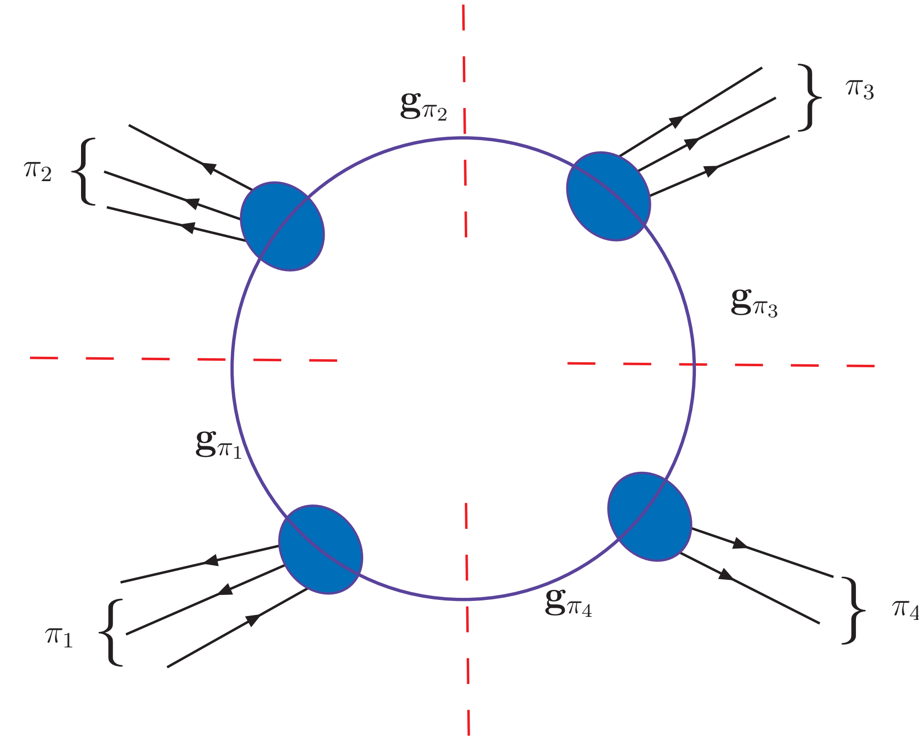

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$$\bar{d}_{ijkl}(l) = \text{Res}_{ijkl}\left(\mathcal{A}_N(l)\right)$$

$d_i=d_j=d_k=d_l=0$ two solutions, 4-cut

$$\bar{c}_{ijk}(l) = \text{Res}_{ijk}\left(\mathcal{A}_N(l) - \sum_{l\neq i,j,k} \frac{\bar{d}_{ijkl}(l)}{d_i d_j d_k d_l}\right)$$

$d_i=d_j=d_k=0$ infinite # of solutions

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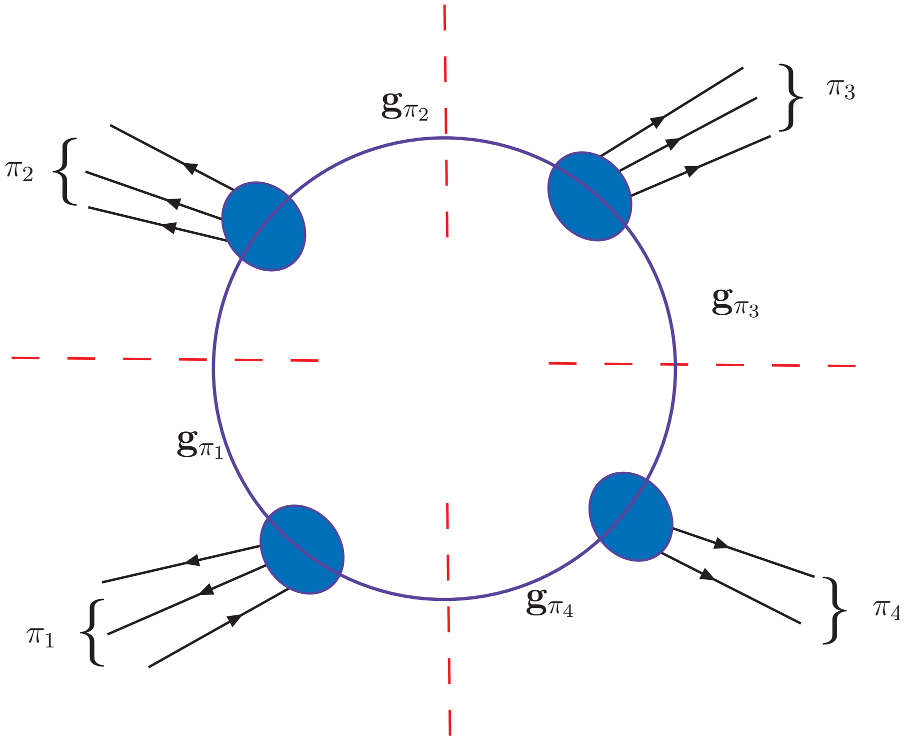

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$d_i=d_j=d_k=0$ infinite # of solutions

$d_i=d_j=0$

$$\bar{b}_{ij}(l) = \text{Res}_{ij}\left(\mathcal{A}_N(l) - \sum_{k\neq i,j} \frac{\bar{c}_{ijk}(l)}{d_i d_j d_k} - \frac{1}{2!}\sum_{k,l\neq i,j} \frac{\bar{d}_{ijkl}(l)}{d_i d_j d_k d_l}\right)$$

infinite # of solutions

New ideas led to many new results for multi-leg processes

A number of 2->4 calculations have been performed with the new methods

- Rocket, Black Hat + Sherpa, Rocket + MadFKS have been used for the processes

$$pp \to W/Z + 3\text{jets}\,, WW + 2\text{jets}\,,\; e^+e^- \to 5\text{jets}$$

Berger,Bern,Dixon, Febres-Cordero, Forde, Gleisberg, Ita, Kosower,
Ellis,Frixione, Frederix, Giele, ZK, Melia, Melnikov, Rontsch, Zanderighi

- HELAC-1Loop, Feynman diagrams based method have been applied for

$$pp \to WW + b\bar{b}\,, t\bar{t} + 2\text{jets}\,, t\bar{t} + b\bar{b}\,, b\bar{b}b\bar{b}$$

Bevilaqua, Czakon, Van Hameren, Papadopoulos, Pittau, Worek
Bredenstein, Denner, Ditmaier, Kallweit, Pozzorini
Binoth, Greiner, Guffanti, Guillet, Reiter, Reuter

Recent developments for NLO codes

- BlackHat + Sherpa: W/Z+ 4jets
  - improvements: First use of N = 4 derived expressions (Dixon, Henn, Plefka, & Schuster)

    Six quark subprocesses are included

- CutTools+HELAC: ttbar+2jets (Bevilacqua, Czakon, Papadopoulos, Worek)
- Rocket: W^+W^- + 2jets (Melia, Melnikov, Rontsch,Zanderighi)

  ttbar+1photon with top decay (Melnikov, Schultze, Scharf)

- New implementations
  - Samurai: Mastrolia, Ossola, Reiter, & Tramontano (OPP)
  - NGluon: Badger, Biedermann, & Uwer (D-dim. unitarity)
  - MadLoop: Hirschi, Frederix, Frixione, Garzelli, Maltoni, & Pittau
  - GPU implementation: Giele, Stavenga, Winter
  - Unordered colordressed amplitudes Giele, ZK, Winter

- Analytic work
  - Badger, Campbell, Ellis (pp->Wbbar); Badger, Sattler, Yundin (pp->ttbar)
  - Almeida, Britto, Feng & Mirabella

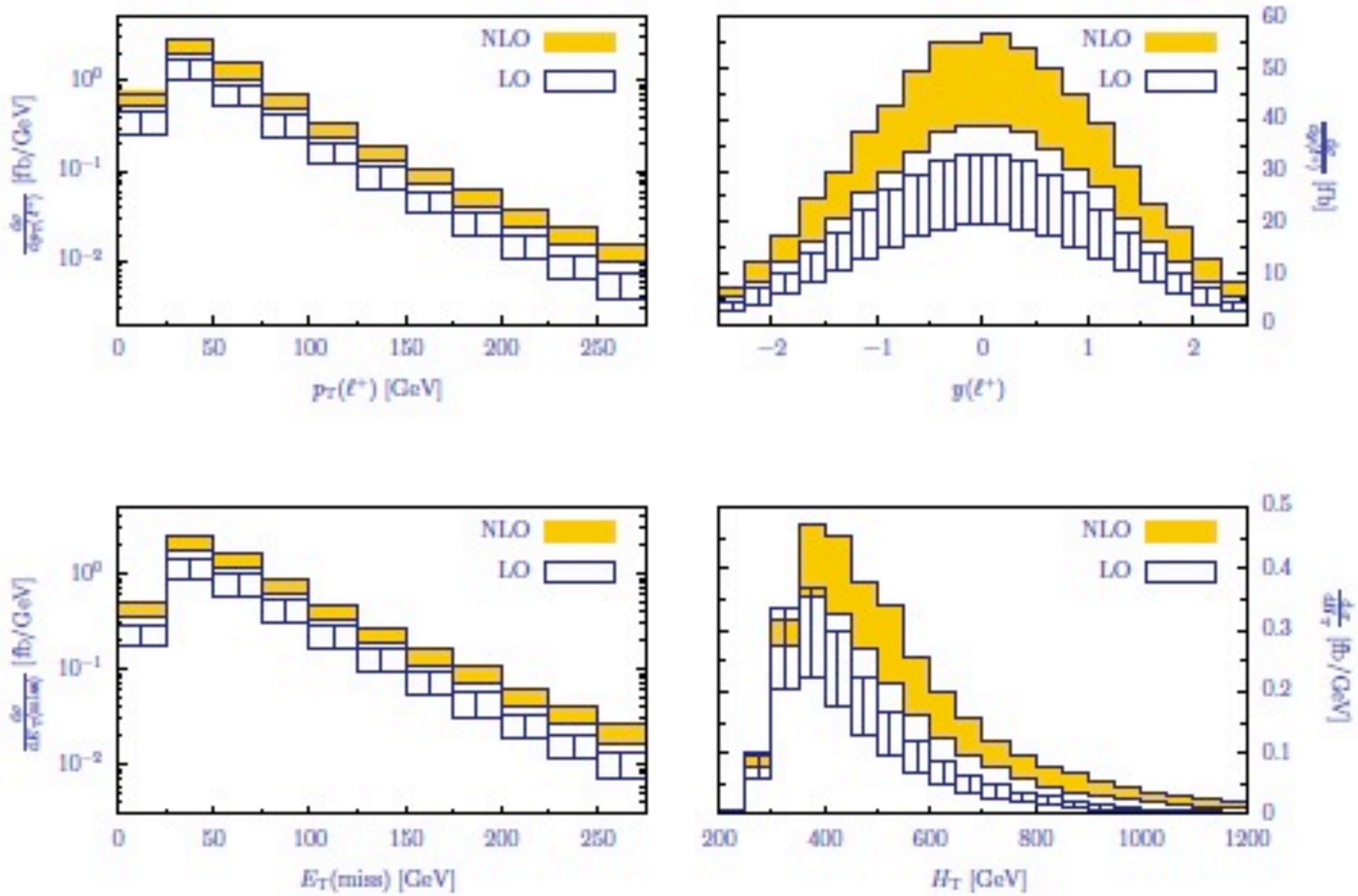
Recent contributions

For example Melnikov, Schulze and Scharf

The process $t\bar{t} + \gamma$ is an interesting SM signal

- We calculated $pp \to t\bar{t} + \gamma \to b\bar{b}\ \ell\nu\ jj + \gamma$ at NLO QCD
  - realistic and flexible setup
  - include top decays and account for all spin correlations
  - allow photon radiation off decay products
- Tevatron: good agreement with CDF measurement
- LHC: possibility to measure electromagnetic couplings of the top quark
- Large contribution from radiative top decays

Schulze, loopfest ’11



$$\sigma^{\mathrm{LO}}_{t\bar{t}\gamma} = 74.5^{+24.0}_{-16.9}\ \mathrm{fb}$$

$$\sigma^{\mathrm{NLO}}_{t\bar{t}\gamma} = 138^{+30}_{-23}\ \mathrm{fb}$$

$$\sigma^{\mathrm{decay}}_{t\bar{t}\gamma} = 56\%\ \sigma^{\mathrm{tot}}_{t\bar{t}\gamma}$$

- large K-factor $\Rightarrow$ extra phase space for additional jet
- no reduction of scale dependence $\Rightarrow$ opening up of q-g channel at NLO

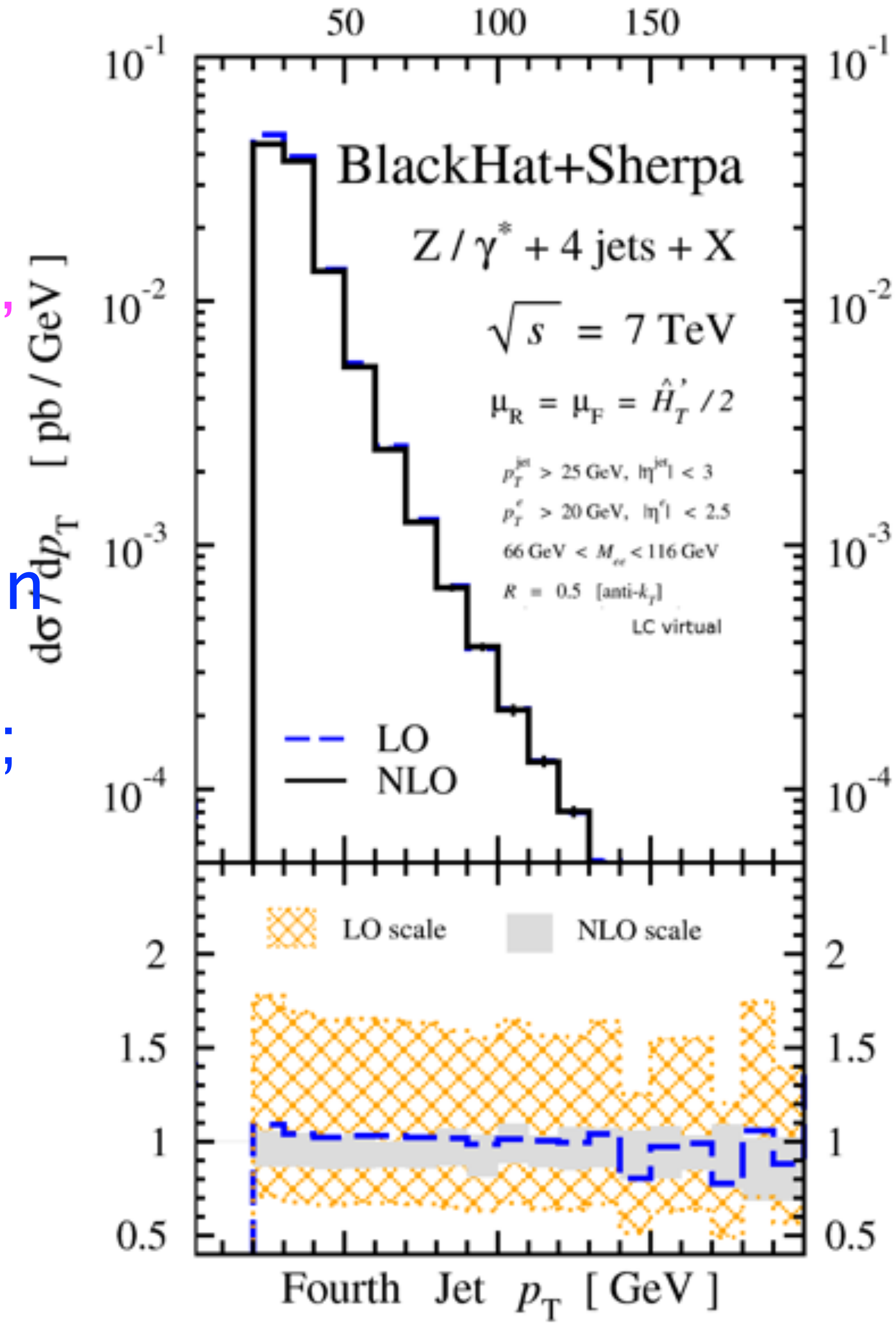
(similar features as in $t\bar{t}$ production)

Z+4 Jets

BLACK HAT arXiv:1108.2229

H. Ita, Z. Bern, L. J. Dixon, F. Febres Cordero,
D. A. Kosower, D. Maître

- Improvement in scale dependence
- Fourth jet p_T: little LO→NLO change in shape
- Leading three jet p_Ts: shape changes; each successive jet falls faster
- Leading color



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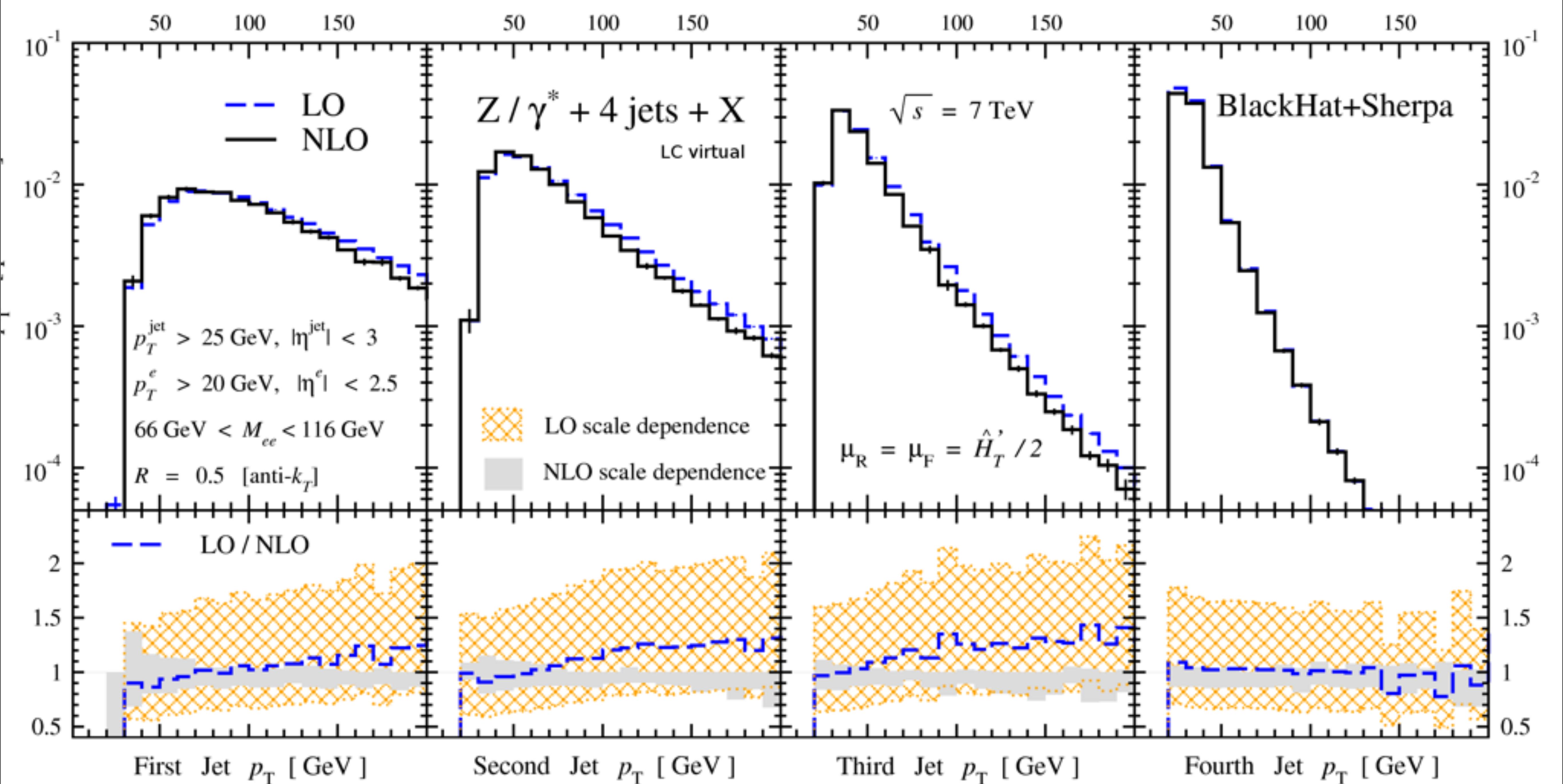
LO
NLO
Z / γ* + 4 jets + X
LC virtual
√s = 7 TeV
BlackHat+Sherpa
p_T^jet > 25 GeV, |η^jet| < 3
p_T^e > 20 GeV, |η^e| < 2.5
66 GeV < M_ee < 116 GeV
R = 0.5 [anti-k_T]
LO scale dependence
NLO scale dependence
μ_R = μ_F = Ĥ'_T / 2
LO / NLO
First Jet p_T [GeV]
Second Jet p_T [GeV]
Third Jet p_T [GeV]
Fourth Jet p_T [GeV]

Public NLO codes?

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☑ No flexible one-loop programs are available

- Rocket and Black Hat are private codes for multi-leg processes
- Version of Rocket when massive particles appear
- Golem is not in shape for plugin
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☑ MadLoop: Madgraph version of Helac One-Loop

Hirschi,Frederix,Grazelli,Pittau ('11)

Code for calculating physics signals at the Tevatron and the LHC at NLO accuracy

MCFM

Code for calculating physics signals at the Tevatron and the LHC at NLO accuracy

MCFM

Final state	Notes	Reference
W/Z		
diboson (W/Z/γ)	photon fragmentation, anomalous couplings	hep-ph/9905386, arXiv:1105.0020
Wbb	massless b-quark massive b quark	hep-ph/9810489 arXiv:1011.6647
Zbb	massless b-quark	hep-ph/0006304
W/Z+1 jet		
W/Z+2 jets		hep-ph/0202176, hep-ph/0308195
Wc	massive c-quark	hep-ph/0506289
Zb	5-flavour scheme	hep-ph/0312024
Zb+jet	5-flavour scheme	hep-ph/0510362

Final state	Notes	Reference
H (gluon fusion)		
H+1 jet (g.f.)	effective coupling	
H+2 jets (g.f.)	effective coupling	hep-ph/0608194, arXiv:1001.4495
WH/ZH		
H (WBF)		hep-ph/0403194
Hb	5-flavour scheme	hep-ph/0204093
t	s- and t-channel (5F), top decay included	hep-ph/0408158
t	t-channel (4F)	arXiv:0903.0005, arXiv:0907.3933
Wt	5-flavour scheme	hep-ph/0506289
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- ☆ **Monumental work of Ellis+Campell and collaborators over 13 years**
- ☆ **MCFM v6.0 30-50 processes !**
- ☆ **General frame work. Each process is implemented by separate consideration**
- ☆ **By now widely used and provides standard reference**

Towards automated MCFM?   MADLOOP

Towards automated MCFM? MADLOOP

INTEGRATED RESULTS

- Errors are the MC integration uncertainty only
- Cuts on jets, γ^*/Z decay products and photons, but no cuts on b quarks (their mass regulates the IR singularities)
- Efficient handling of exceptional phase-space points: their uncertainty always at least two orders of magnitude smaller than the integration uncertainty
- Running time: two weeks on ~150 node cluster leading to rather small integration uncertainties
- MadFKS+MadLoop results are fully differential in the final states (but only parton-level)

	Process	μ	n_{lf}	Cross section (pb) LO	Cross section (pb) NLO
a.1	$pp \to t\bar{t}$	m_{top}	5	123.76 ± 0.05	162.08 ± 0.12
a.2	$pp \to tj$	m_{top}	5	34.78 ± 0.03	41.03 ± 0.07
a.3	$pp \to tjj$	m_{top}	5	11.851 ± 0.006	13.71 ± 0.02
a.4	$pp \to t\bar{b}j$	$m_{top}/4$	4	25.62 ± 0.01	30.96 ± 0.06
a.5	$pp \to t\bar{b}jj$	$m_{top}/4$	4	8.195 ± 0.002	8.91 ± 0.01
b.1	$pp \to (W^+ \to)e^+\nu_e$	m_W	5	5072.5 ± 2.9	6146.2 ± 9.8
b.2	$pp \to (W^+ \to)e^+\nu_e j$	m_W	5	828.4 ± 0.8	1065.3 ± 1.8
b.3	$pp \to (W^+ \to)e^+\nu_e jj$	m_W	5	298.8 ± 0.4	300.3 ± 0.6
b.4	$pp \to (\gamma^*/Z \to)e^+e^-$	m_Z	5	1007.0 ± 0.1	1170.0 ± 2.4
b.5	$pp \to (\gamma^*/Z \to)e^+e^- j$	m_Z	5	156.11 ± 0.03	203.0 ± 0.2
b.6	$pp \to (\gamma^*/Z \to)e^+e^- jj$	m_Z	5	54.24 ± 0.02	56.69 ± 0.07
c.1	$pp \to (W^+ \to)e^+\nu_e b\bar{b}$	$m_W + 2m_b$	4	11.557 ± 0.005	22.95 ± 0.07
c.2	$pp \to (W^+ \to)e^+\nu_e t\bar{t}$	$m_W + 2m_{top}$	5	0.009415 ± 0.000003	0.01159 ± 0.00001
c.3	$pp \to (\gamma^*/Z \to)e^+e^- b\bar{b}$	$m_Z + 2m_b$	4	9.459 ± 0.004	15.31 ± 0.03
c.4	$pp \to (\gamma^*/Z \to)e^+e^- t\bar{t}$	$m_Z + 2m_{top}$	5	0.0035131 ± 0.0000004	0.004876 ± 0.000002
c.5	$pp \to \gamma t\bar{t}$	$2m_{top}$	5	0.2906 ± 0.0001	0.4169 ± 0.0003
d.1	$pp \to W^+W^-$	$2m_W$	4	29.976 ± 0.004	43.92 ± 0.03
d.2	$pp \to W^+W^- j$	$2m_W$	4	11.613 ± 0.002	15.174 ± 0.008
d.3	$pp \to W^+W^+ jj$	$2m_W$	4	0.07048 ± 0.00004	0.1377 ± 0.0005
e.1	$pp \to HW^+$	$m_W + m_H$	5	0.3428 ± 0.0003	0.4455 ± 0.0003
e.2	$pp \to HW^+ j$	$m_W + m_H$	5	0.1223 ± 0.0001	0.1501 ± 0.0002
e.3	$pp \to HZ$	$m_Z + m_H$	5	0.2781 ± 0.0001	0.3659 ± 0.0002
e.4	$pp \to HZ j$	$m_Z + m_H$	5	0.0988 ± 0.0001	0.1237 ± 0.0001
e.5	$pp \to Ht\bar{t}$	$m_{top} + m_H$	5	0.08896 ± 0.00001	0.09869 ± 0.00003
e.6	$pp \to Hb\bar{b}$	$m_b + m_H$	4	0.16510 ± 0.00009	0.2099 ± 0.0006
e.7	$pp \to Hjj$	m_H	5	1.104 ± 0.002	1.036 ± 0.002

Rikkert Frederix, Aug 4, 2011

Virtual NLO matrix elements as plugins into NLO Parton Shower MC's

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MC@NLO :

- Well tested for many processes
- Matches NLO to HERWIG and HERWIG++
- Angular ordered Parton Shower
- One may have negative weights
- Available also for PHYTIA

Frixione, Webber (03)

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Frixione, Webber (03)

POWHEG: - Parton Showers can be interfaced

- HERWIG, SHERPA, PHYTIA
- Only positive weights, resumming subleading non-logarithmic corrections
- Modular, POWHEGBOX can use existing NLO calculations

  WW,WZ,ZZ (Melia et.al.)

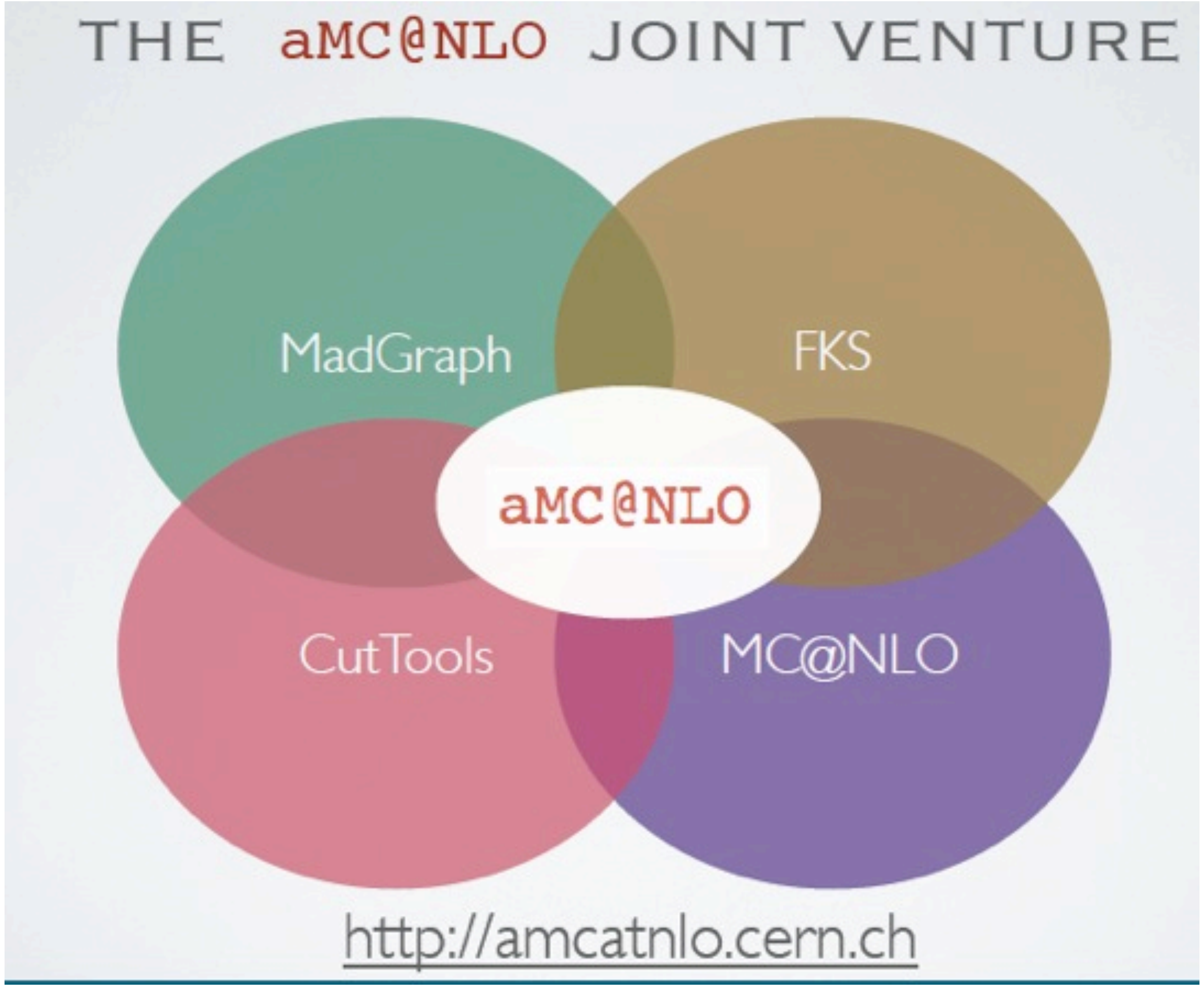
  NLO results HELAC (Kardos et.al.),

  HERWIG++, SHERPA (Hoeche et.al)

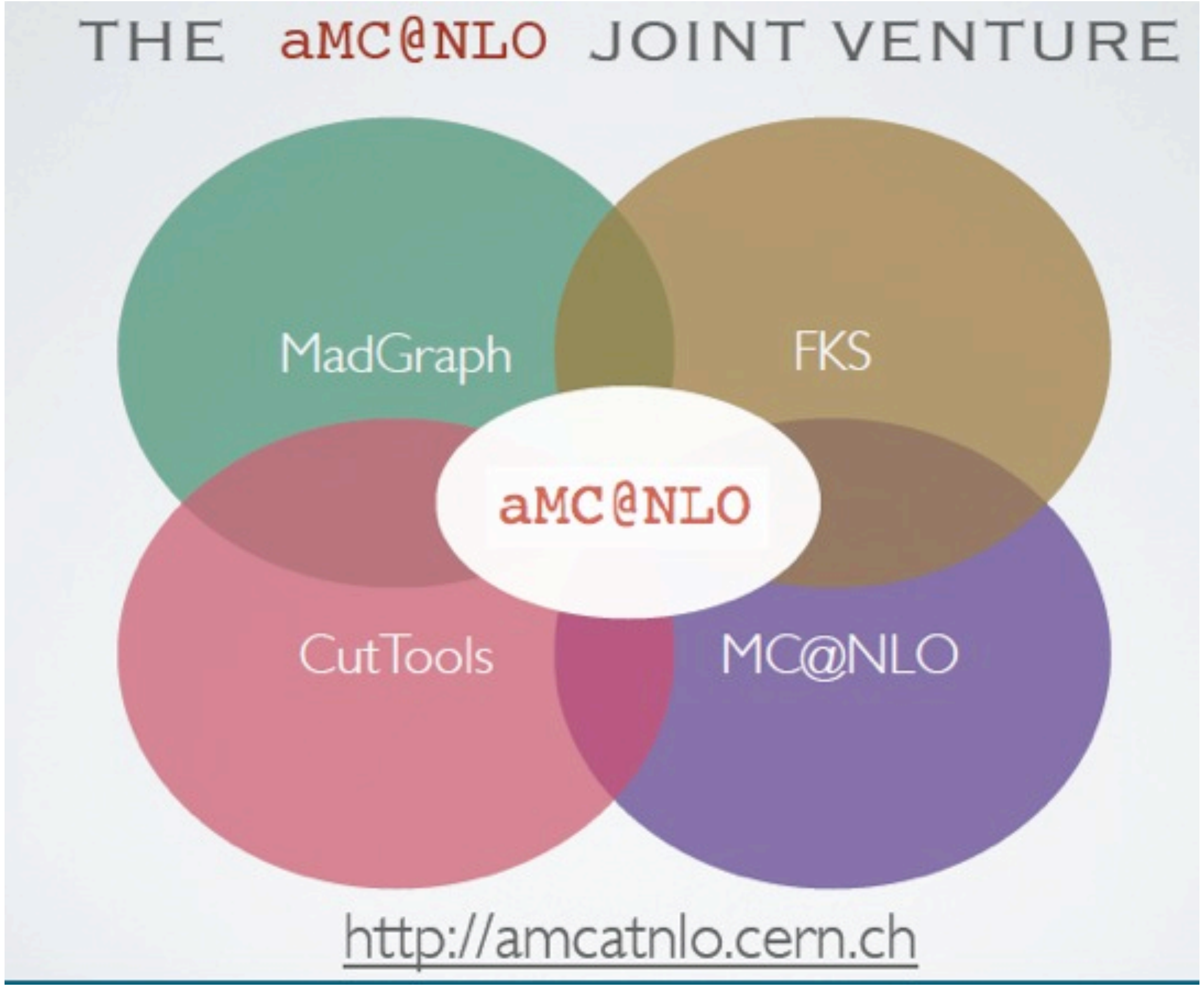
Nason (04)
Frixione, Nason, Oleari (07)

Toward automated MC@NLO

Toward automated MC@NLO

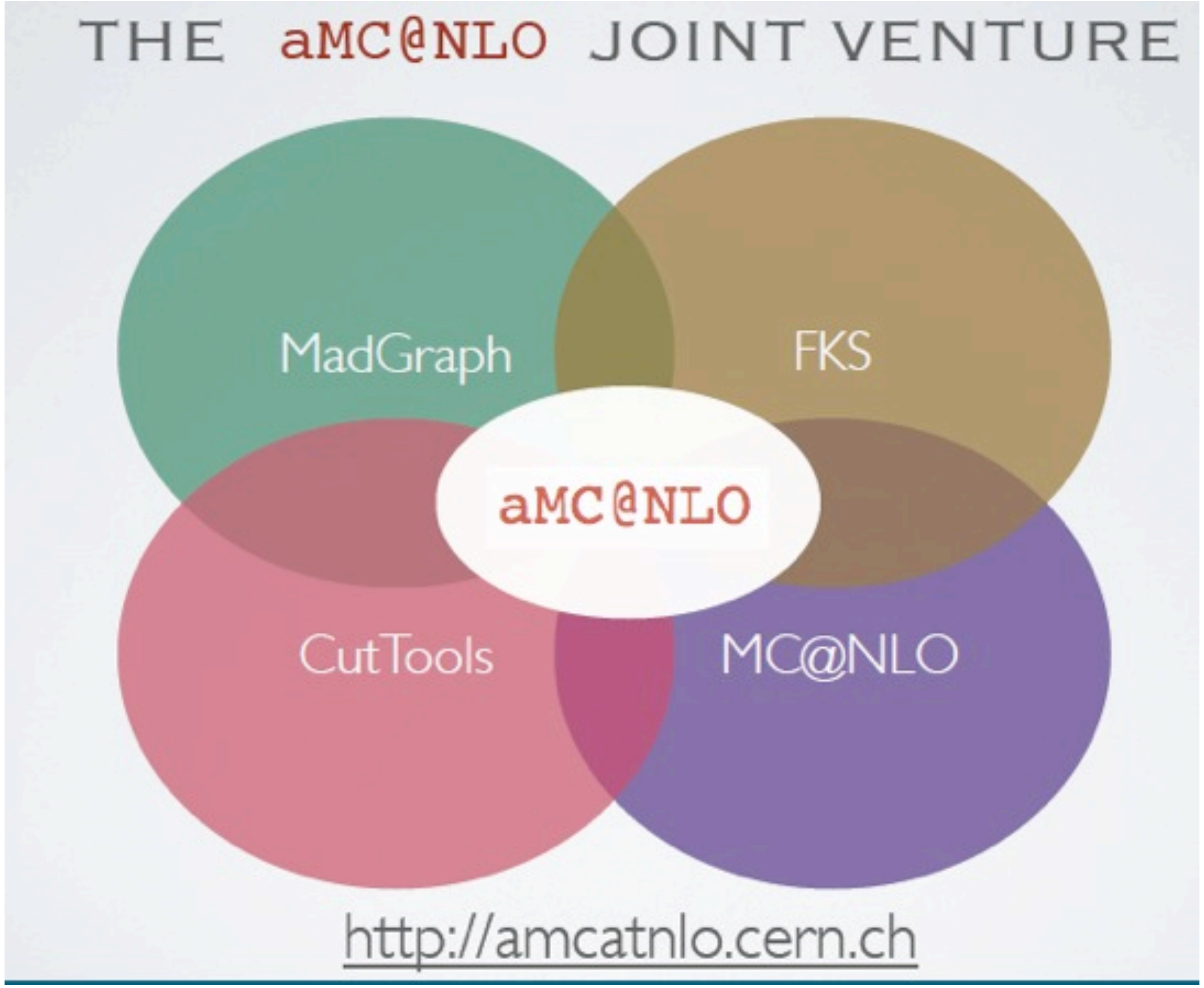


Toward automated MC@NLO



Automatic implementation of the MC counter term for Herwig6, Herwig++ and Phytia

Toward automated MC@NLO



Automatic implementation of the MC counter term for Herwig6, Herwig++ and Phytia

Automated NLO SHERPA...

What is used in the data analysis ?

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Higgs-search at the LHC

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Search for the Higgs-boson decaying into W^+W^- in fully leptonic final states

CMS PAS HIG-11-003

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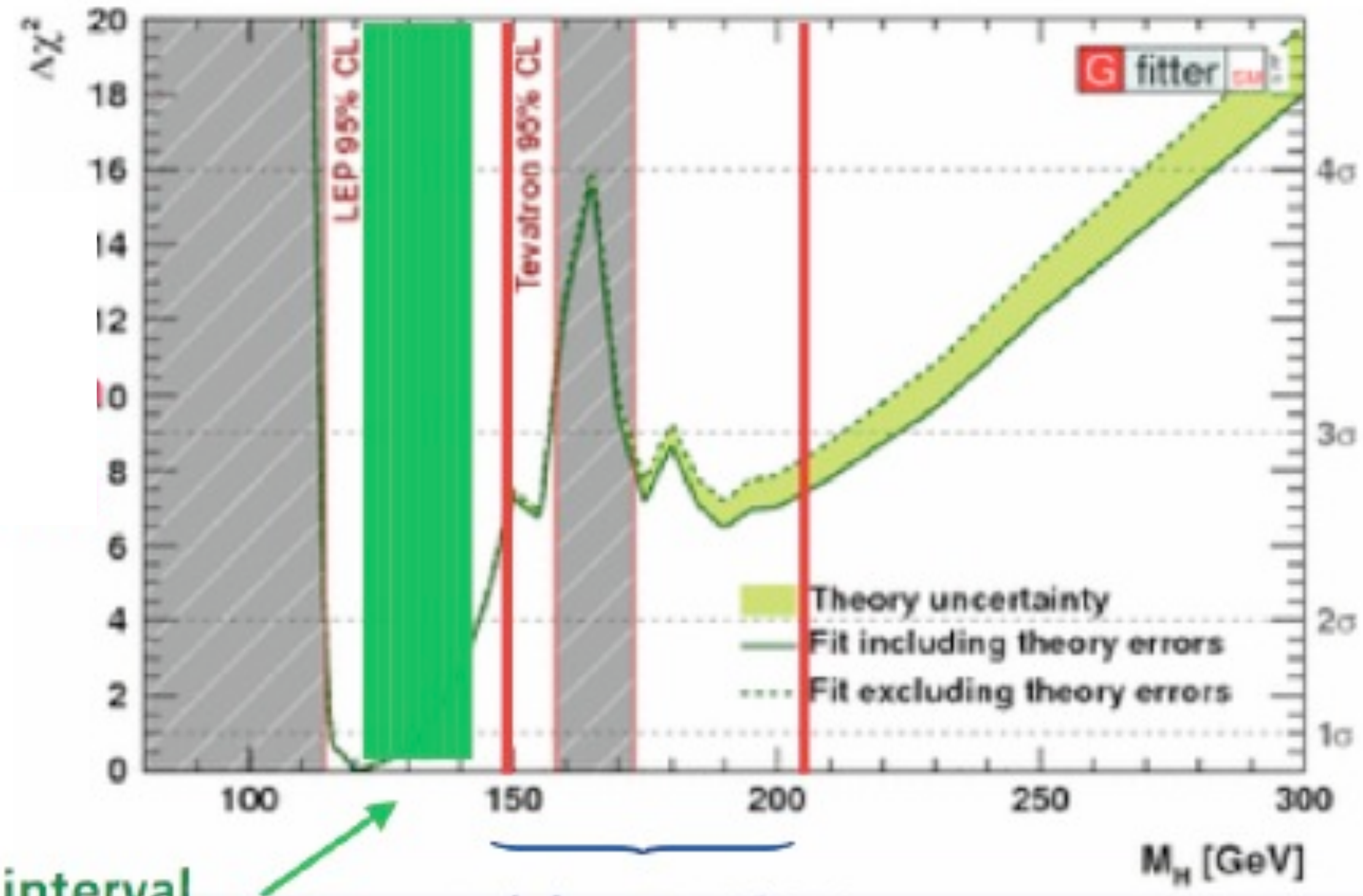
Search for the Higgs-boson produced in association with W/Z boson and decaying into a b-quark pair

ATLAS-CONF-2011-103

Precision calculations for Higgs-search

The SM Higgs is close to be observed or excluded



interval with excess ~2.5 σ

Excl. by ATLAS/CMS

also 300 < m_H < 450 GeV is excluded

slide taken from Altarelli

Search for the Higgs-boson decaying into W^+W^- in fully leptonic final states

Search for the Higgs-boson decaying into W^+W^- in fully leptonic final states

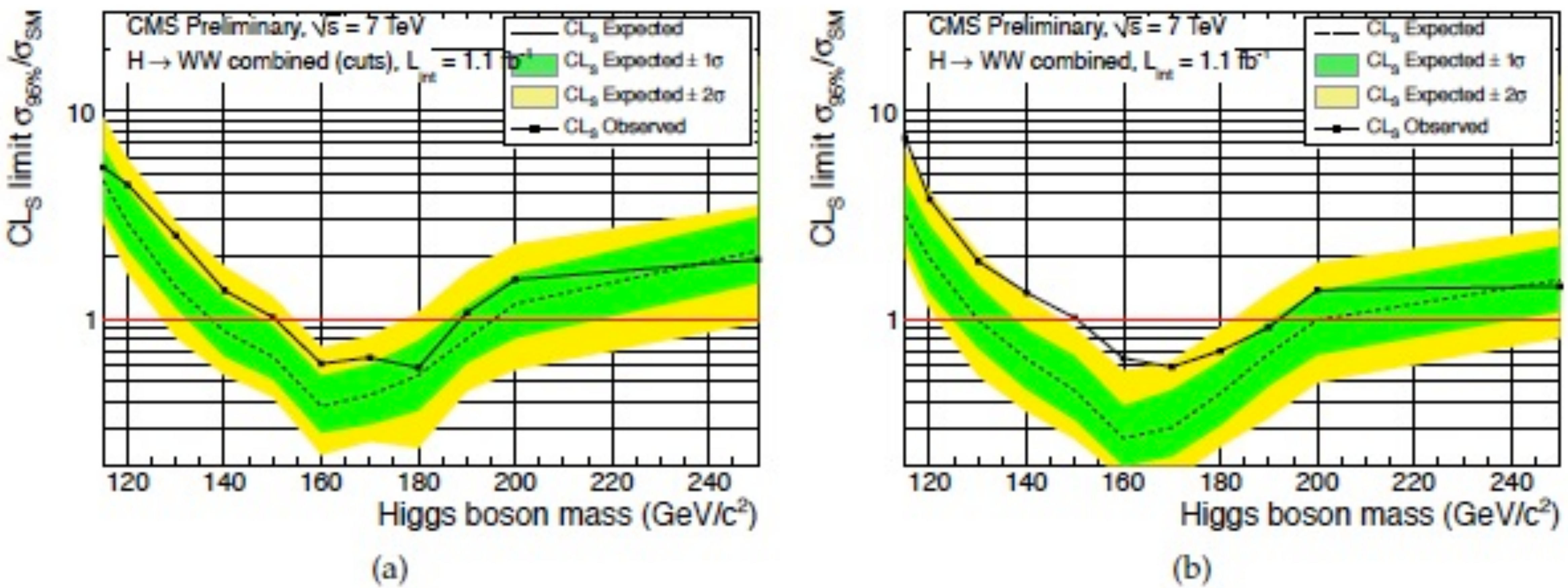


Figure 3: 95% expected and observed C.L. upper limits on the cross section times branching ratio $\sigma_H \times BR(H \to W^+W^- \to 2\ell 2\nu)$, relative to the SM value using (a) cut-based and (b) multivariate BDT event selections. Results are obtained using the CL_s approach.

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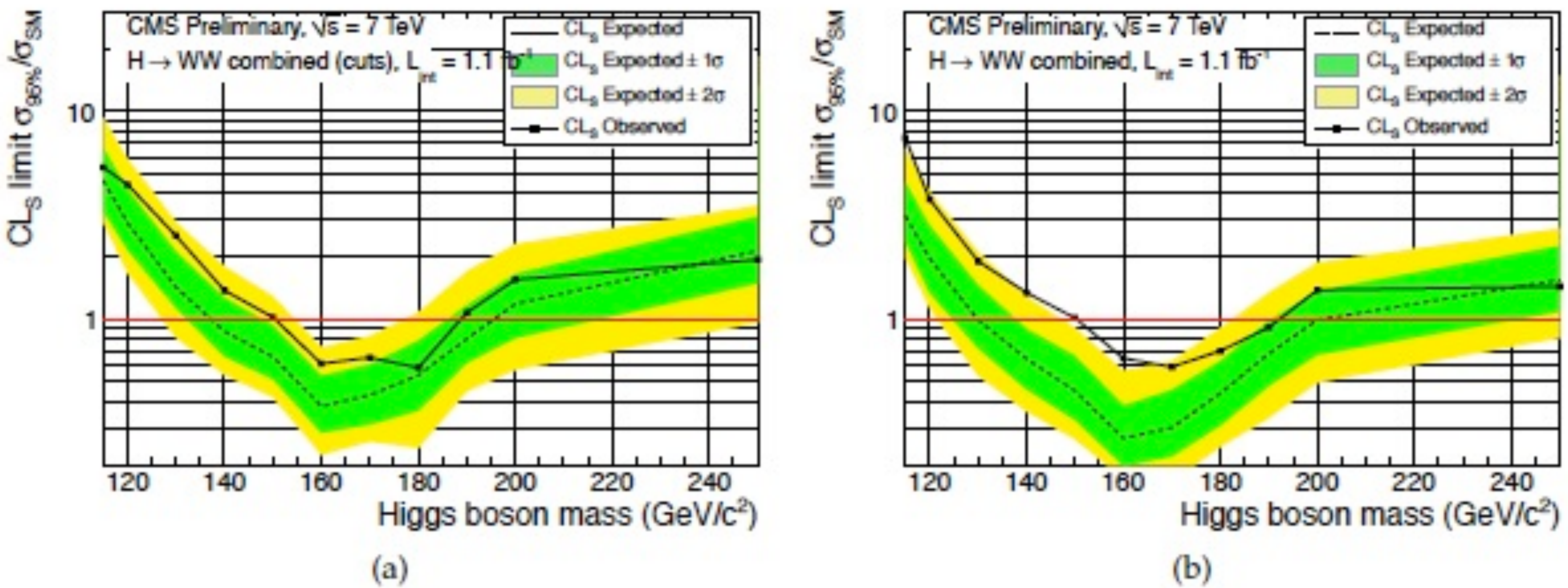
CMS PAS HIG-11-003



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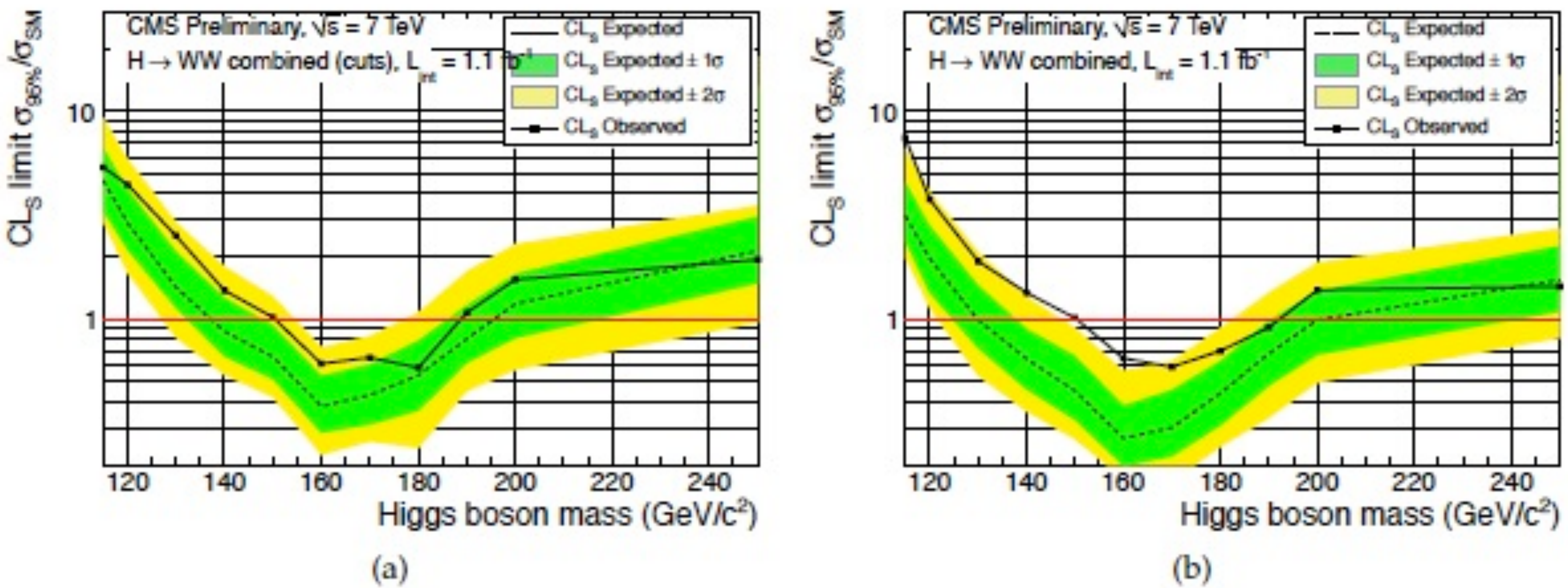
Search for the Higgs-boson decaying into W^+W^- in fully leptonic final states

CMS PAS HIG-11-003

In this analysis we used the POWHEG program [11] to generate Monte Carlo event samples for the $H \rightarrow W^+W^-$ and Drell-Yan processes. The $q\bar{q} \rightarrow W^+W^-$, W + jets, $t\bar{t}$, and tW processes are generated using the MADGRAPH [12] event generator, the $gg \rightarrow W^+W^-$ process using the GG2WW event generator [13], and the remaining processes using PYTHIA [14]. The default set of parton distribution functions (PDF) used to generate these samples is CTEQ6L [15]. Cross section calculations at Next-to-the-Next-to-Leading Order (NNLO) are used for the $H \rightarrow W^+W^-$ process, while NLO calculations are used for background cross sections. The detector response is simulated for all processes using a detailed description of the CMS detector, based on the GEANT4 package [16].



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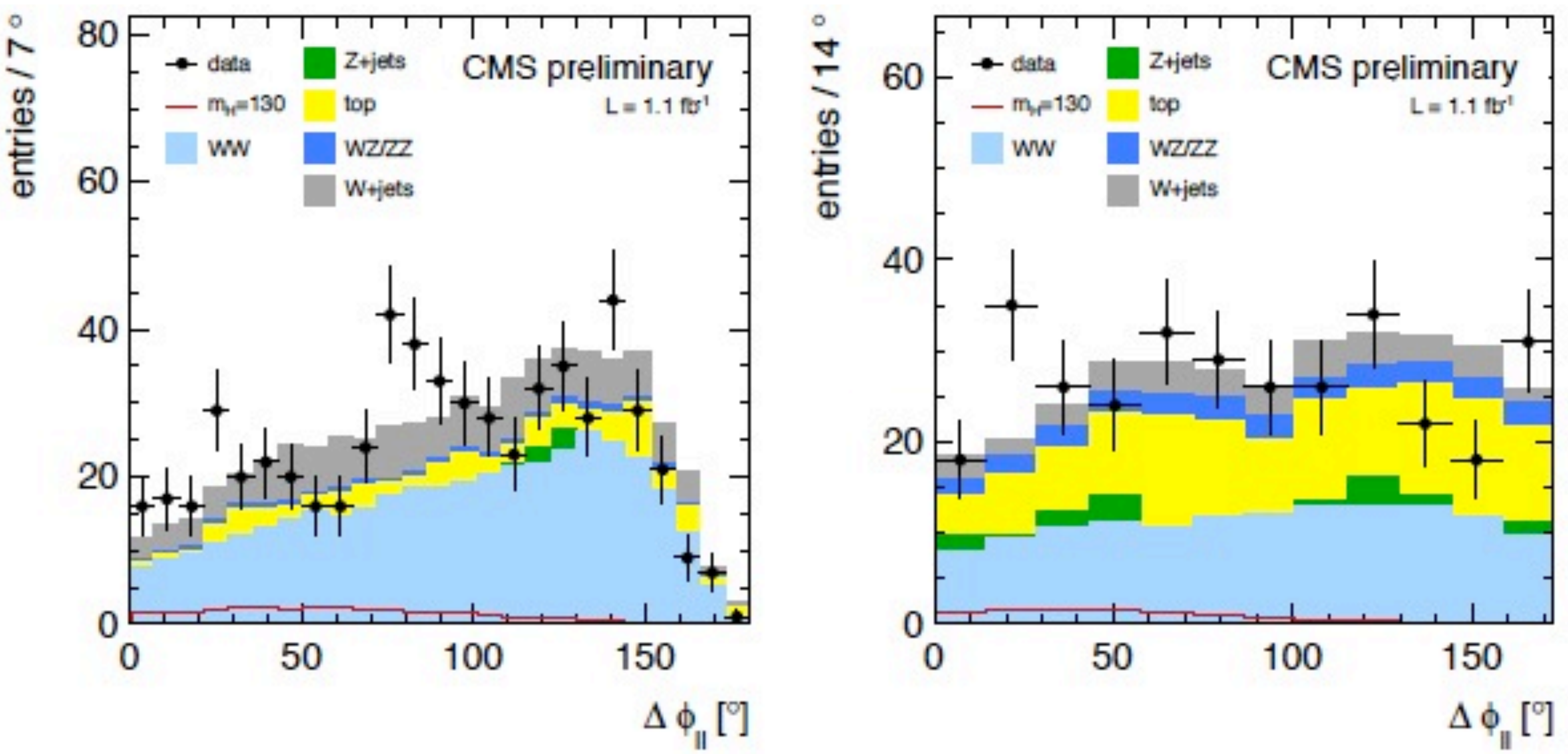
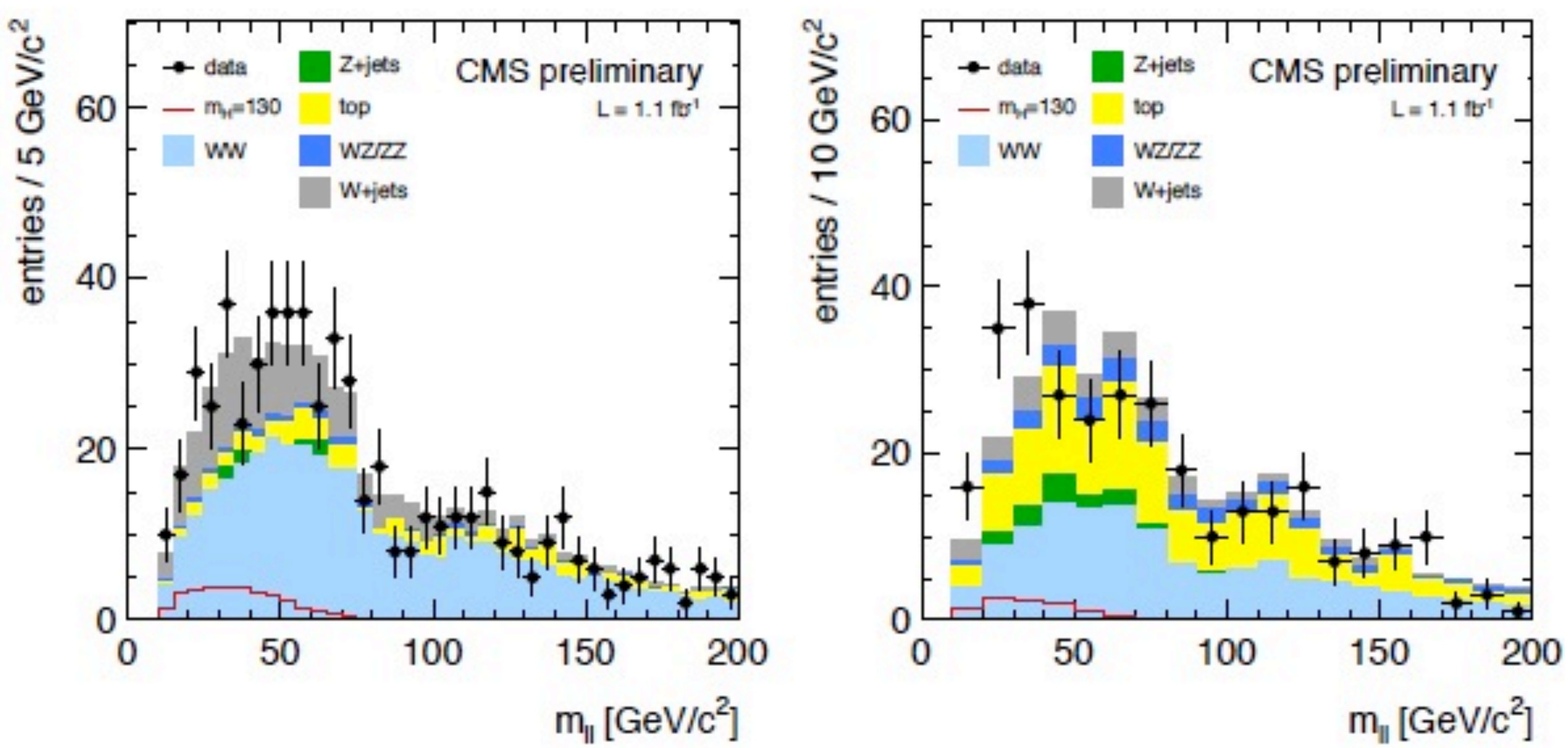
Inclusive and exclusive cross-sections for $\mathbf{p + p(\bar{p}) \rightarrow W^+W^-, W^\pm Z,\ ZZ}$



(a) (b)



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ATLAS-CONF-2011-103

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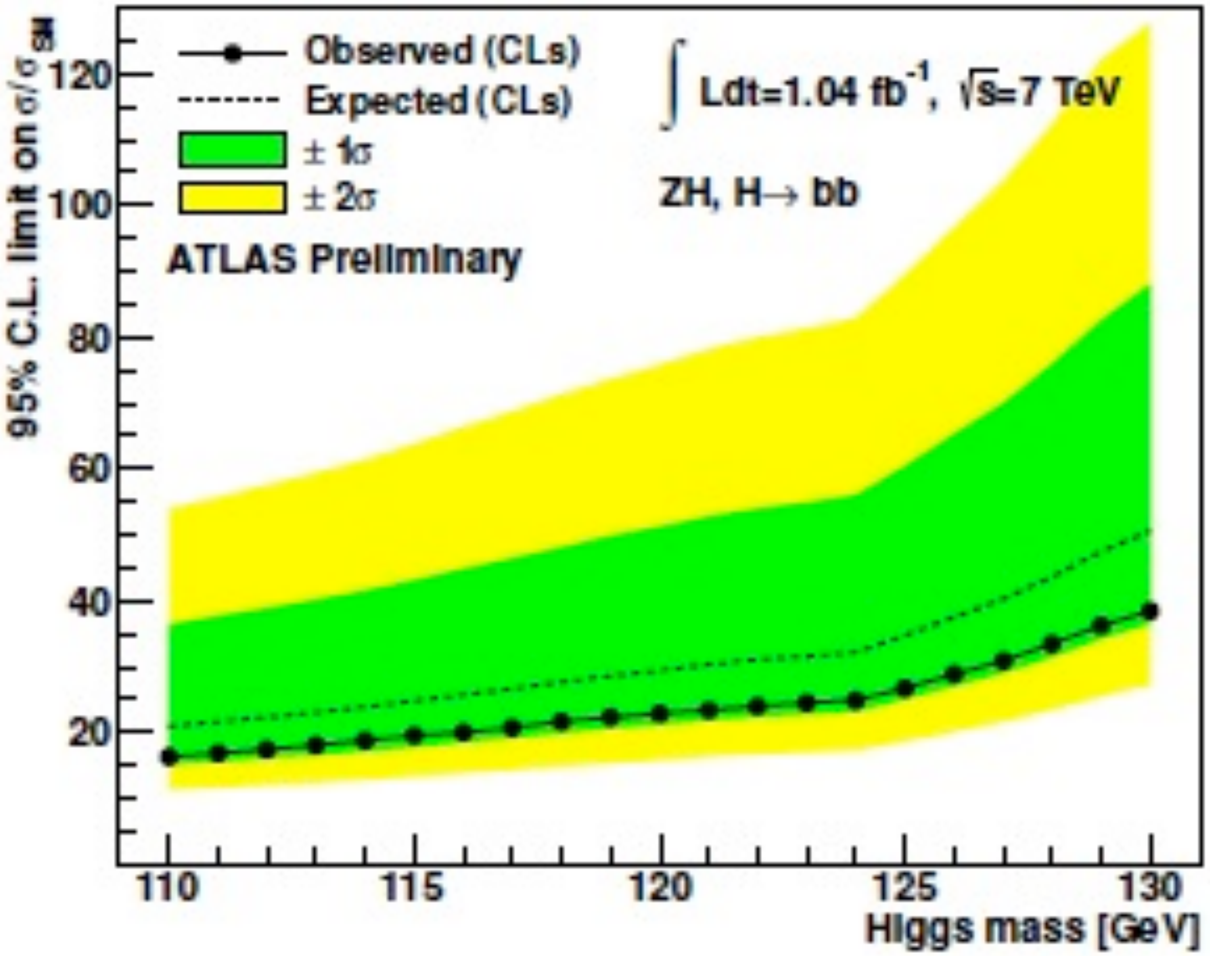
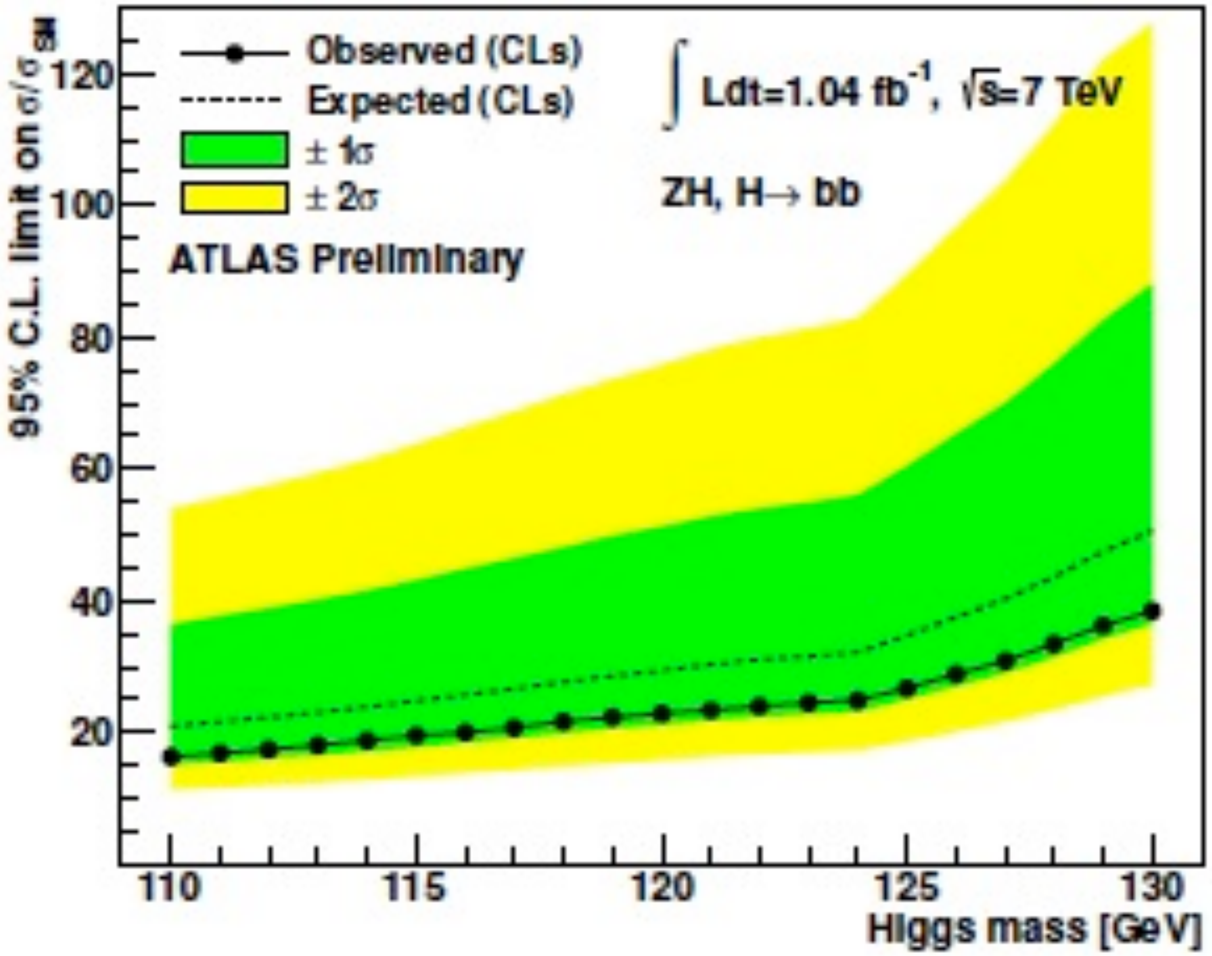
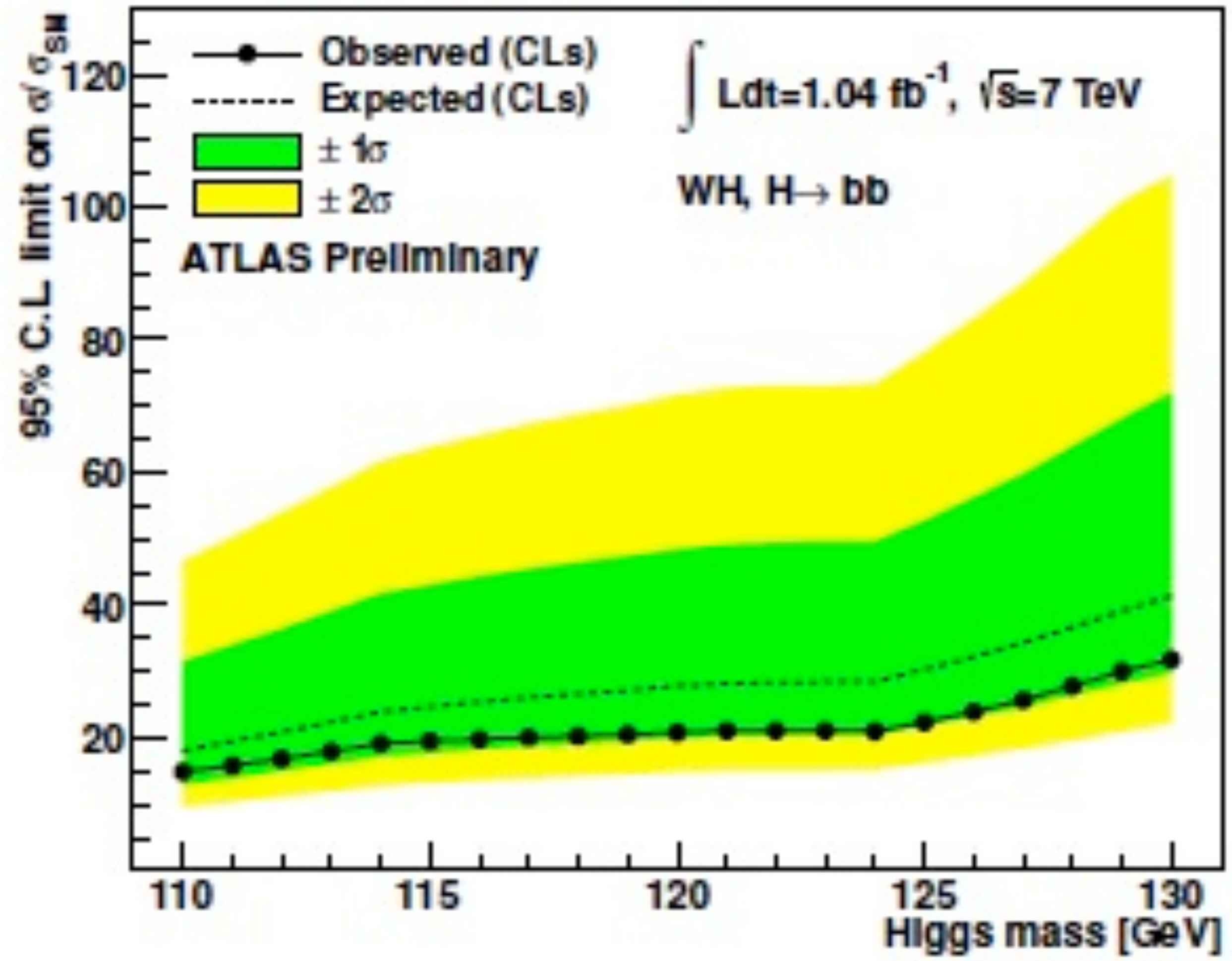


Search for the Higgs-boson produced in association with W/Z boson and decaying into a b-quark pair

ATLAS-CONF-2011-103





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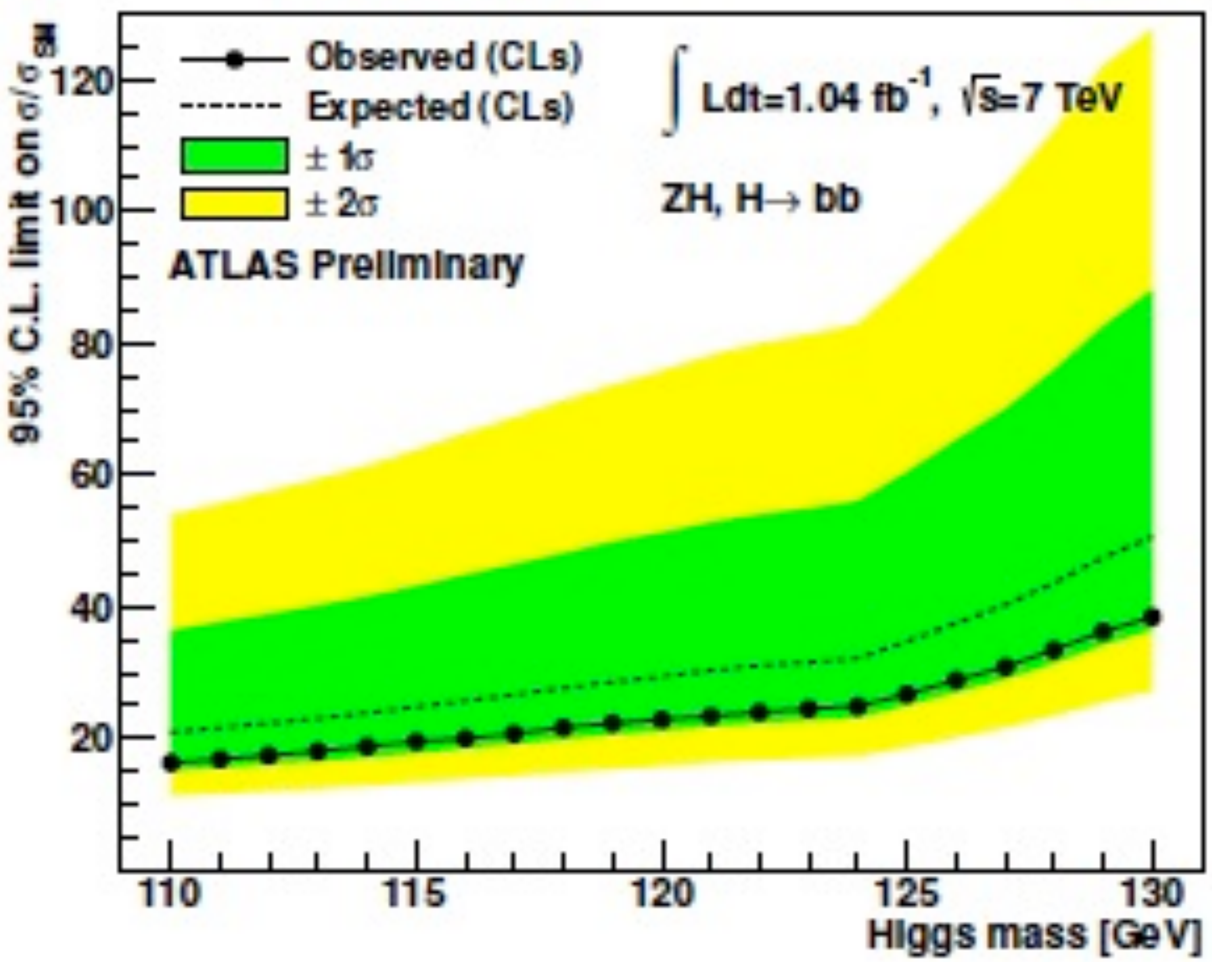

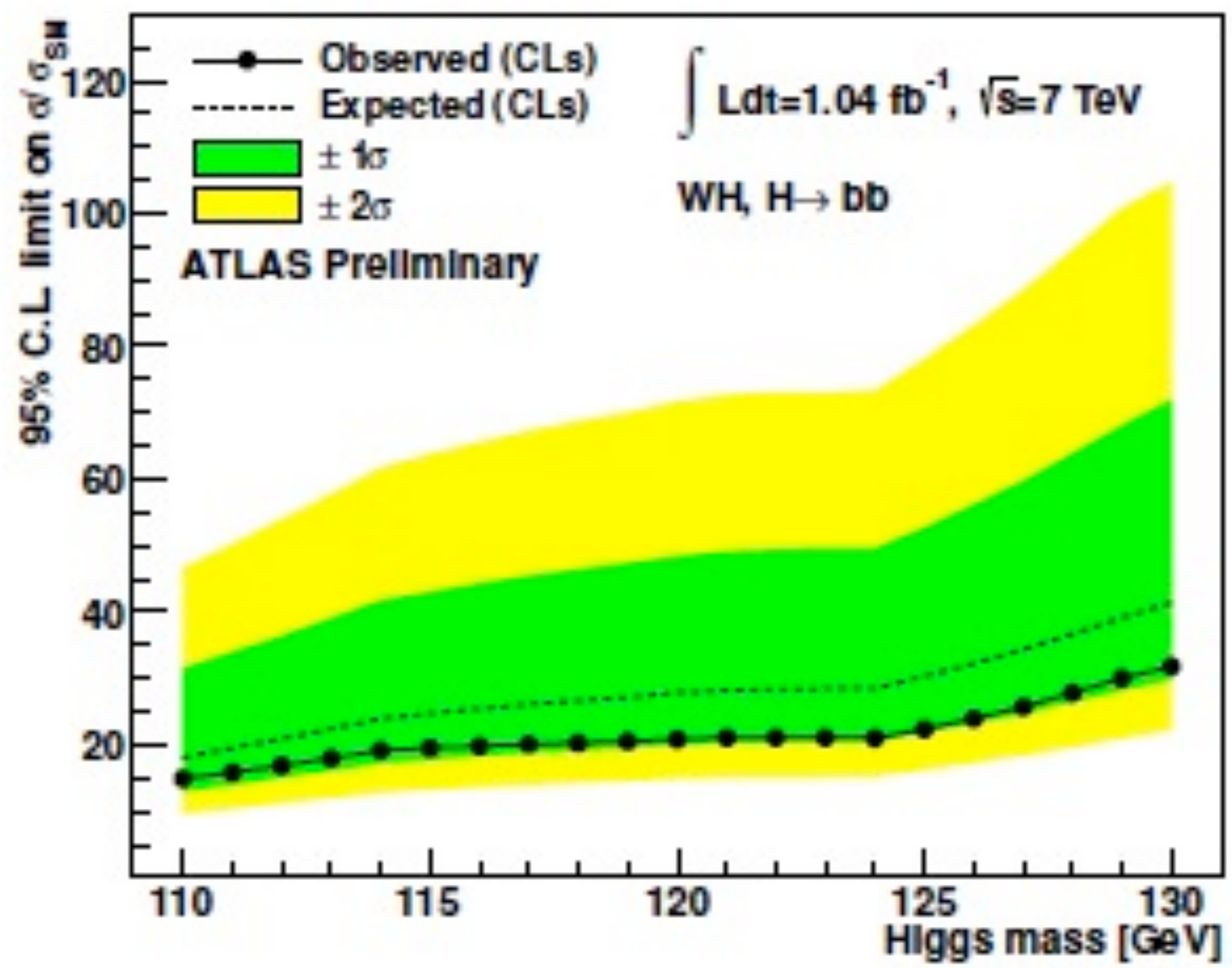

Figure 10: Expected (dashed) and observed (solid line) exclusion limits for the $ZH \to \ell\ell b\bar{b}$ (top) and $WH \to \ell\nu b\bar{b}$ (bottom) channels expressed as the ratio to the Standard Model cross-section, using the profile-likelihood method with CL_s. The green and yellow areas represent the 1σ and 2σ ranges of the expectation in the absence of a signal.

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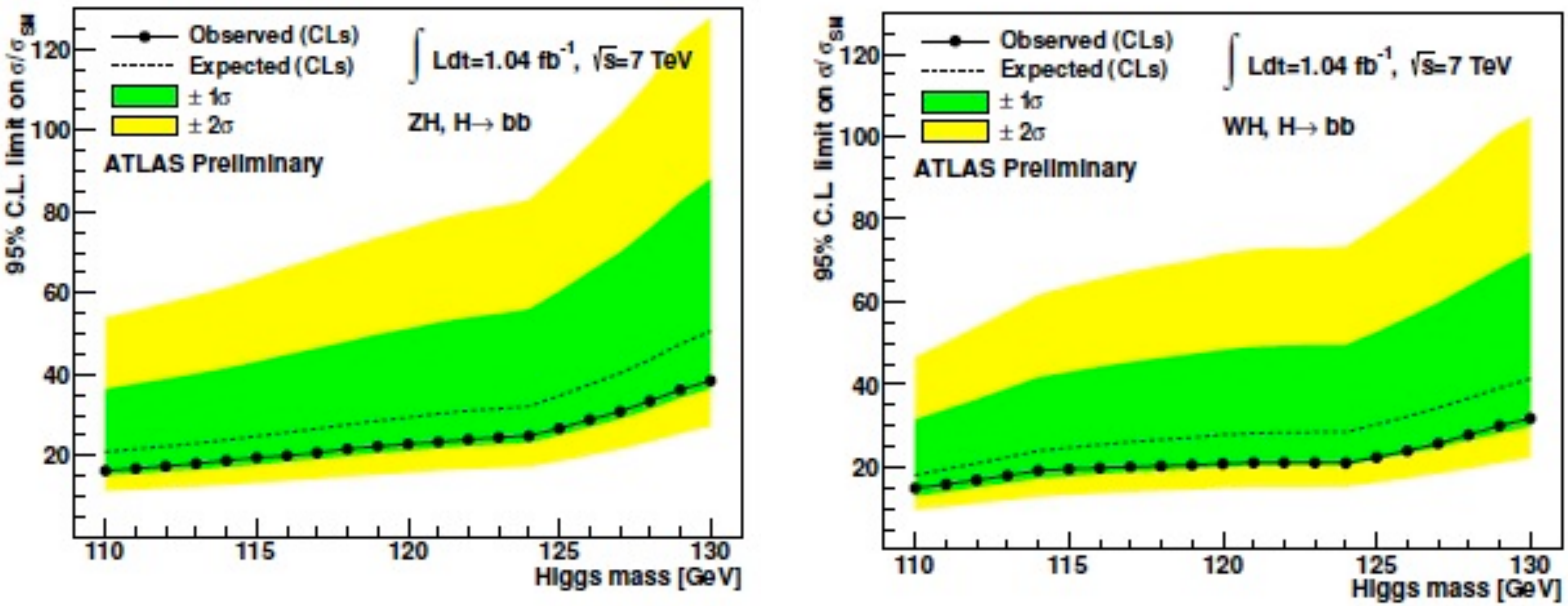
The background processes are modelled with several different event generators. The ALPGEN generator [31] interfaced with the HERWIG program [32] for parton showers and hadronization is used to simulate W/Z+jets events. The MC@NLO generator [33], interfaced to HERWIG and JIMMY [34] for the simulation of underlying events, is used for the production of top-quarks and the diboson (ZZ, WZ and WW) MC events. For the WW diboson samples, an additional contribution from gluon-initiated diagrams is modelled using gg2WW [35]. The HERWIG generator is used to simulate additional diboson WW samples.



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$p + p \to l\nu b\bar{b} + X$ with Higgs-search cuts

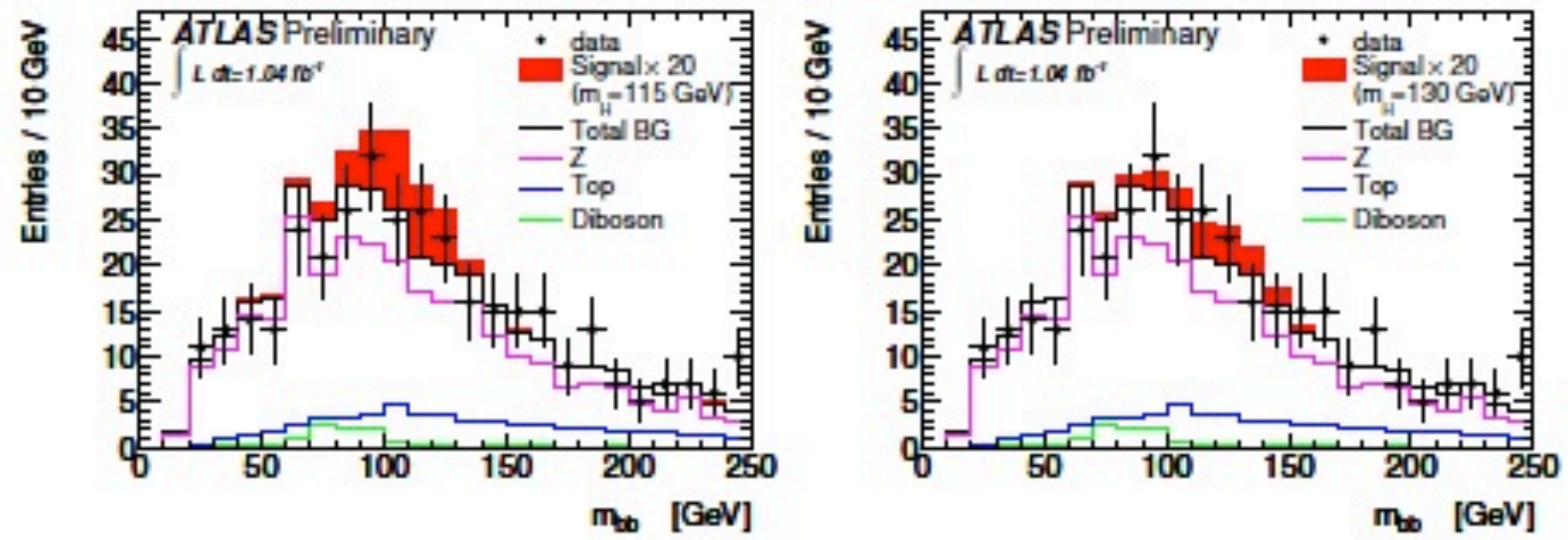

Figure 8: The invariant mass, $m_{b\bar{b}}$, for $ZH \to \ell\ell b\bar{b}$ for $m_H = 115$ (left) and 130 GeV (right). The signal distribution is enhanced by a factor of 20 for visibility.

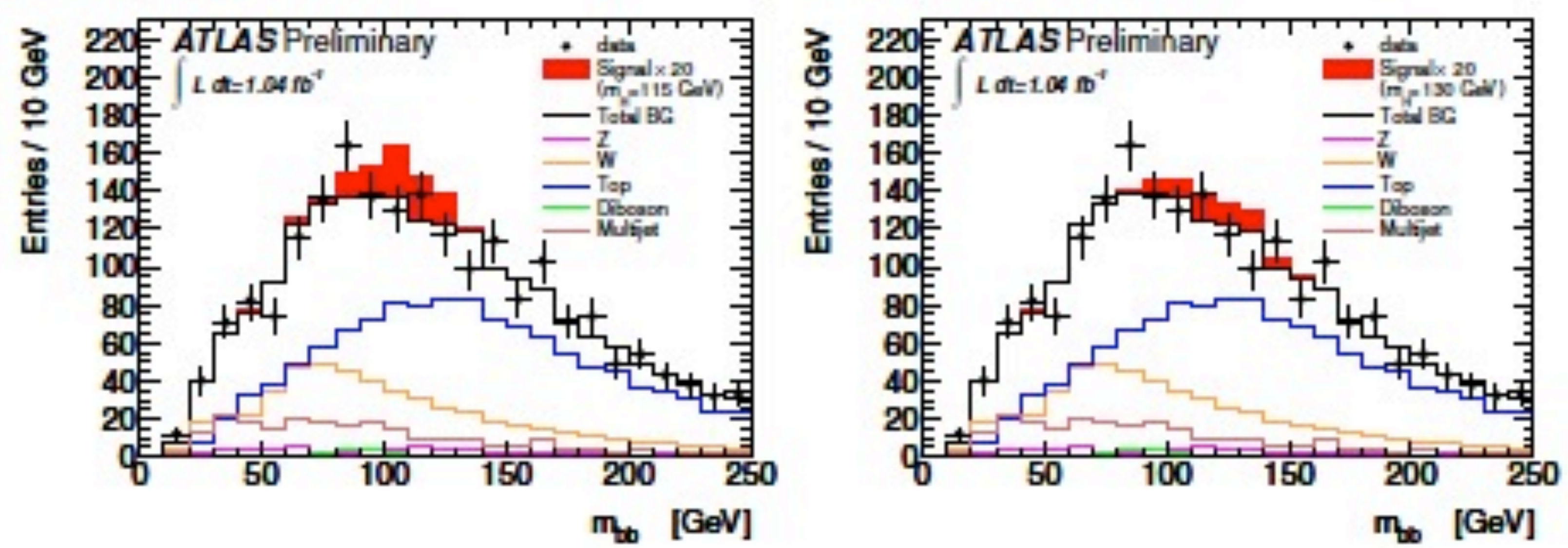

Figure 9: The invariant mass, $m_{b\bar{b}}$, for $WH \to \ell\nu b\bar{b}$ for $m_H = 115$ (left) and 130 GeV (right). The signal distribution is enhanced by a factor of 20 for visibility.

$p + p \longrightarrow l\nu b\bar{b} + X$ with Higgs-search cuts

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SM Higgs production
x-sections at NNLO +
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Handbook of LHC Higgs
cross-sections (Ed. Mariotti,
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m_H (GeV)	$\sigma(WH)$ (pb)	$\sigma(ZH)$ (pb)	Branching Ratio $H \rightarrow b\bar{b}$
110	0.875	0.472	0.745
115	0.755	0.360	0.705
120	0.656	0.316	0.649
125	0.573	0.278	0.578
130	0.501	0.245	0.494

SM Higgs production x-sections at NNLO + NLO EW in pp collisions at 7 TeV for W/X+H+X with H->bbar decay

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Process	Generator	$\sigma \times BR$
WH	PYTHIA	See Tab. 1
ZH	PYTHIA	See Tab. 1
$W \to \ell\nu$	ALPGEN	10.46 nb [38, 39]
$Z/\gamma * \to \ell\ell$	ALPGEN, PYTHIA	
$m_{\ell\ell} > 40$ GeV		1.07nb [38, 40]
$m_{\ell\ell} > 60$ GeV		0.989 nb [38, 40]
WW	MC@NLO+gg2WW	46.23 pb [35, 36]
$WW \to l\nu qq$	HERWIG	46.23 pb [35, 36]
WZ	MC@NLO	
$66 < m_{\ell\ell} < 116$ GeV		18.0 pb [36]
ZZ	MC@NLO, PYTHIA	
$66 < m_{\ell\ell} < 116$ GeV		5.96 pb [36]
Top-quark		
$t\bar{t}$	MC@NLO	164.6 pb [41]
t–channel	MC@NLO	58.7 pb [36]
s–channel	MC@NLO	3.94 pb [36]
Wt–channel	MC@NLO	13.1 pb [36]
$b\bar{b} \to \mu\mu$	PYTHIA	73.9 nb
$c\bar{c} \to \mu\mu$	PYTHIA	28.4 nb

Table 2: Monte Carlo programs used for modelling signal and background processes and the cross-sections times branching ratio (BR) used to normalize the different processes. Branching ratios correspond to the decays shown. Where two generators are given the second is used to estimate systematic uncertainties.

Fully differential NNLO calculation for 4-leg processes ?

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see talk by Nigel Glover

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$e^+e^- \to \mathrm{3jets}$ (2002-2009)

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pp->2jets at NNLO
pp->ttbar at NNLO
pp->V+jet at NNLO
pp-> VV at two loop

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$$e^+e^- \longrightarrow 3\text{jets} \quad \text{(2002-2009)}$$

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pp->V+jet at NNLO
pp-> VV at two loop

It is hard but it is feasible for up to 4 leg processes

$e^+ + e^- \rightarrow$ 3 jets at NNLO

Gehrmann-De Ridder, Gehrmann, Heinrich, Glover (07)
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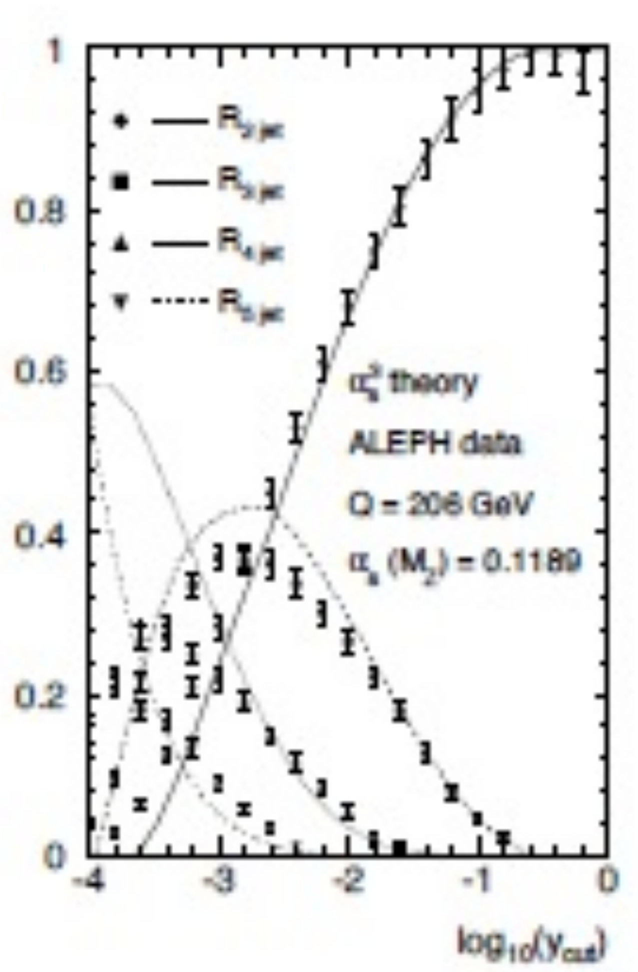

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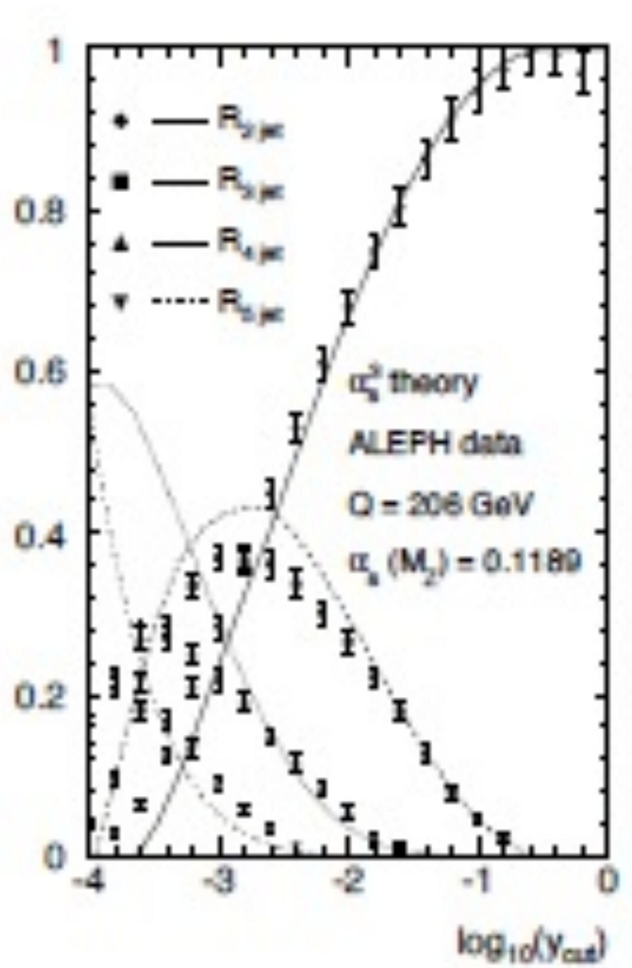

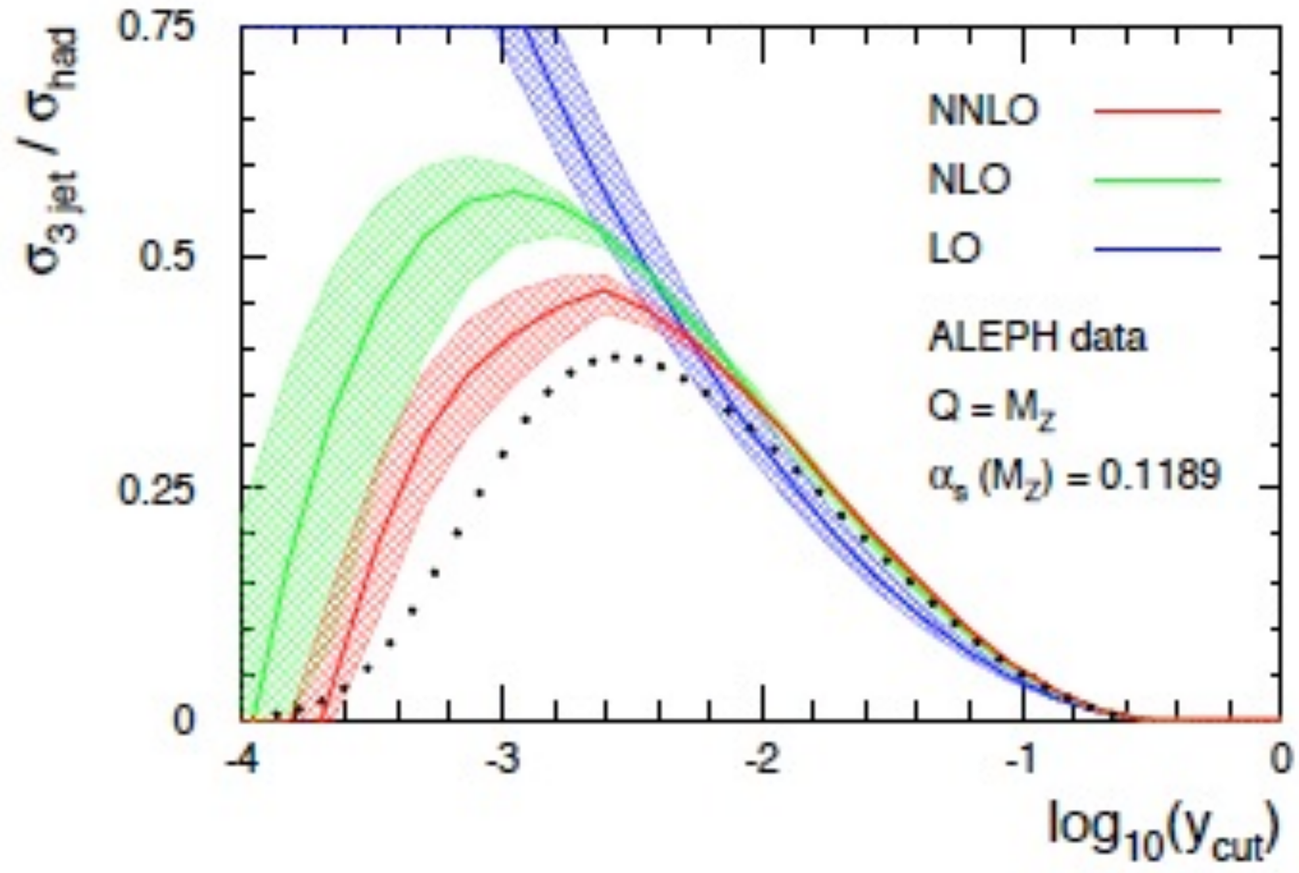

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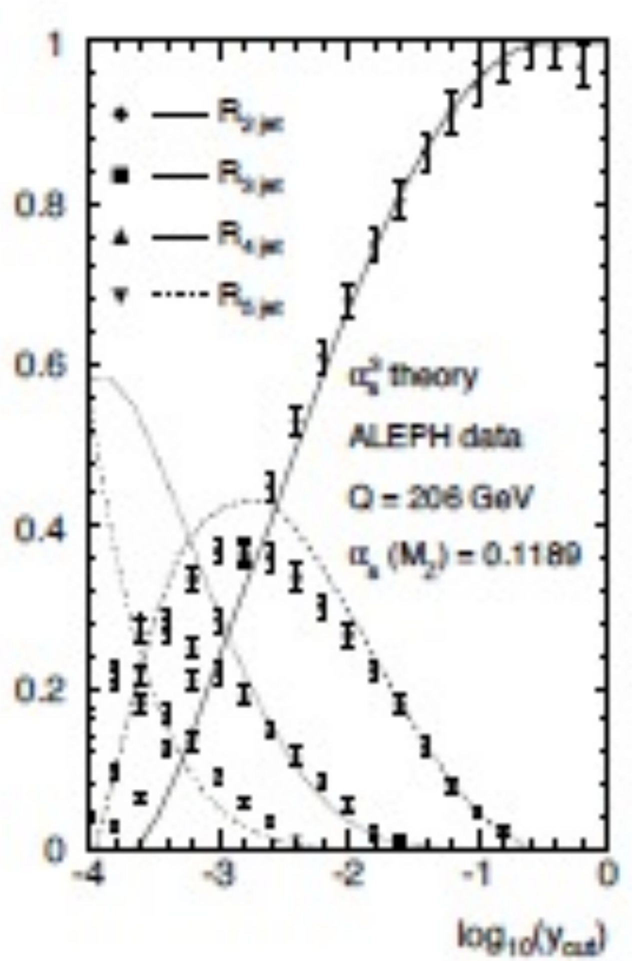

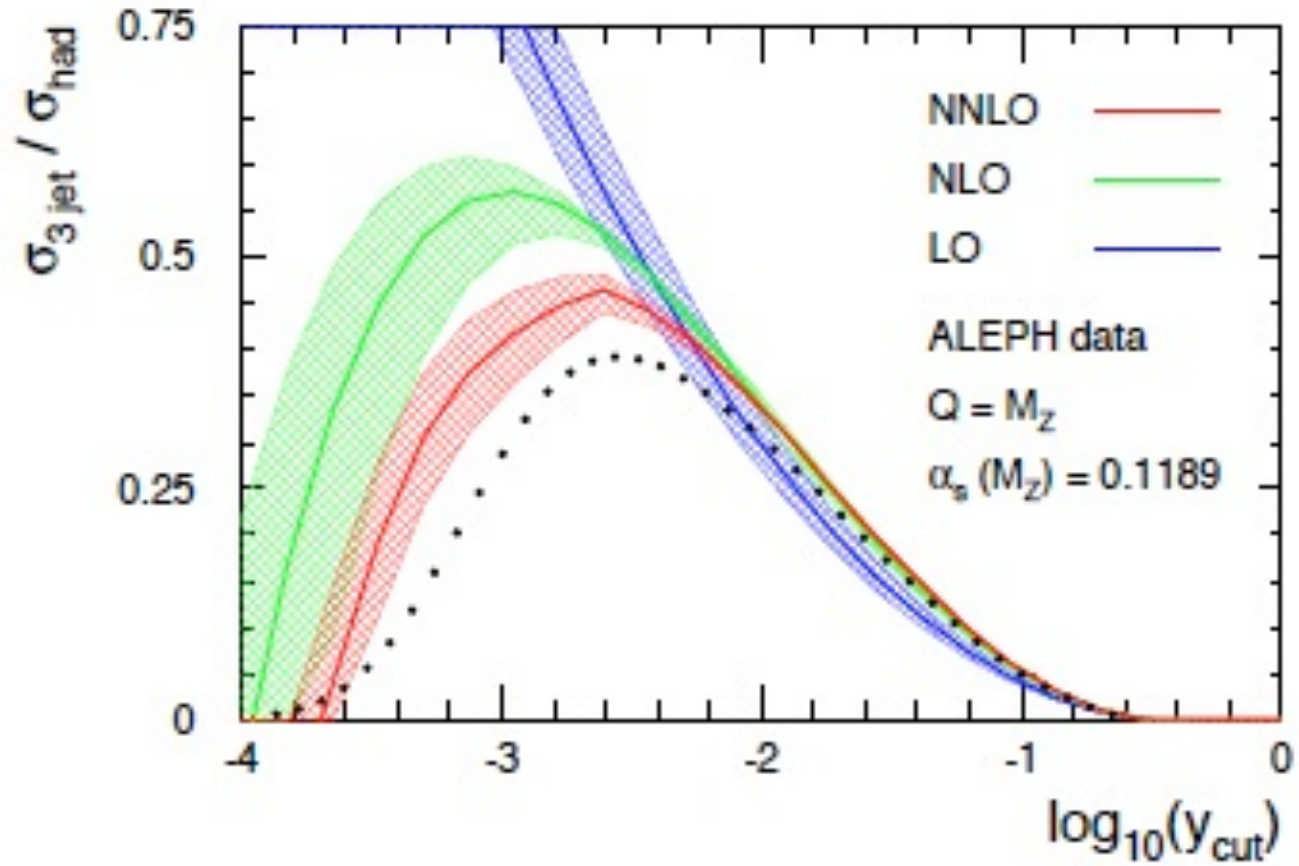

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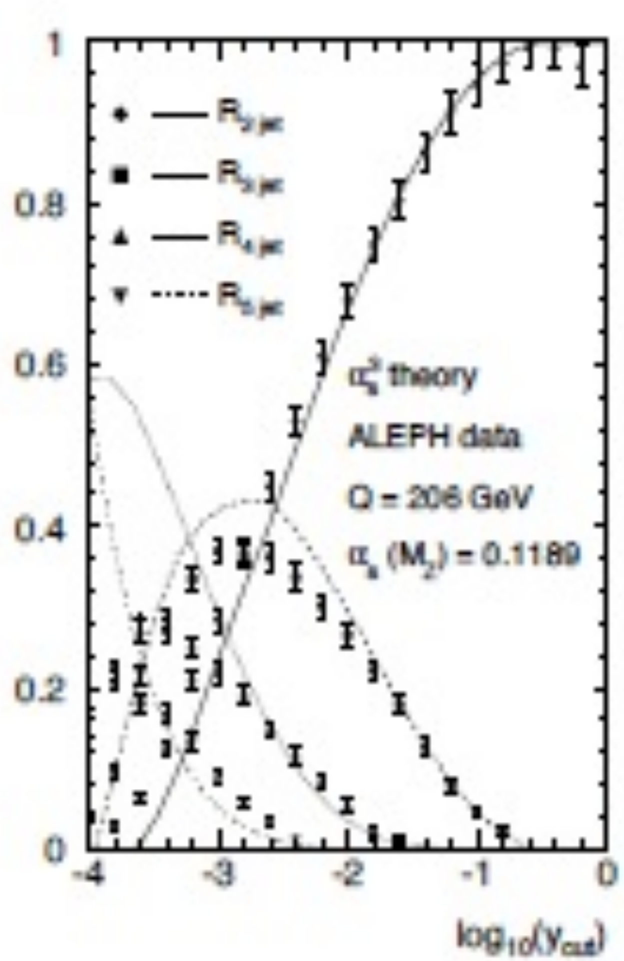

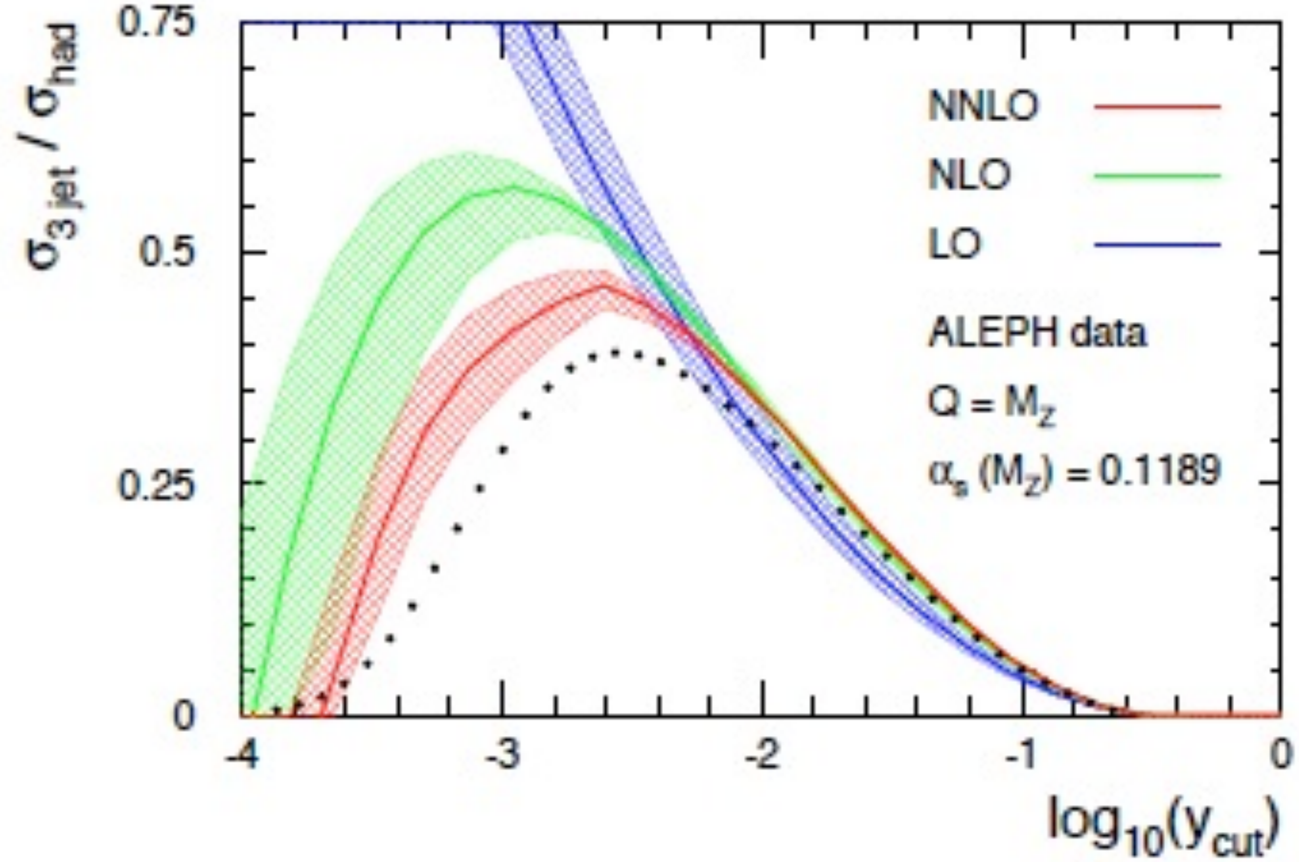

$$\alpha_s(M_Z) = 0.1175 \pm 0.0020(exp) \pm 0.0015(th)$$

pp → γγ at NNLO

S. Catani, L. Cieri, D. de Florian, G.Ferrera, M. Grazzini

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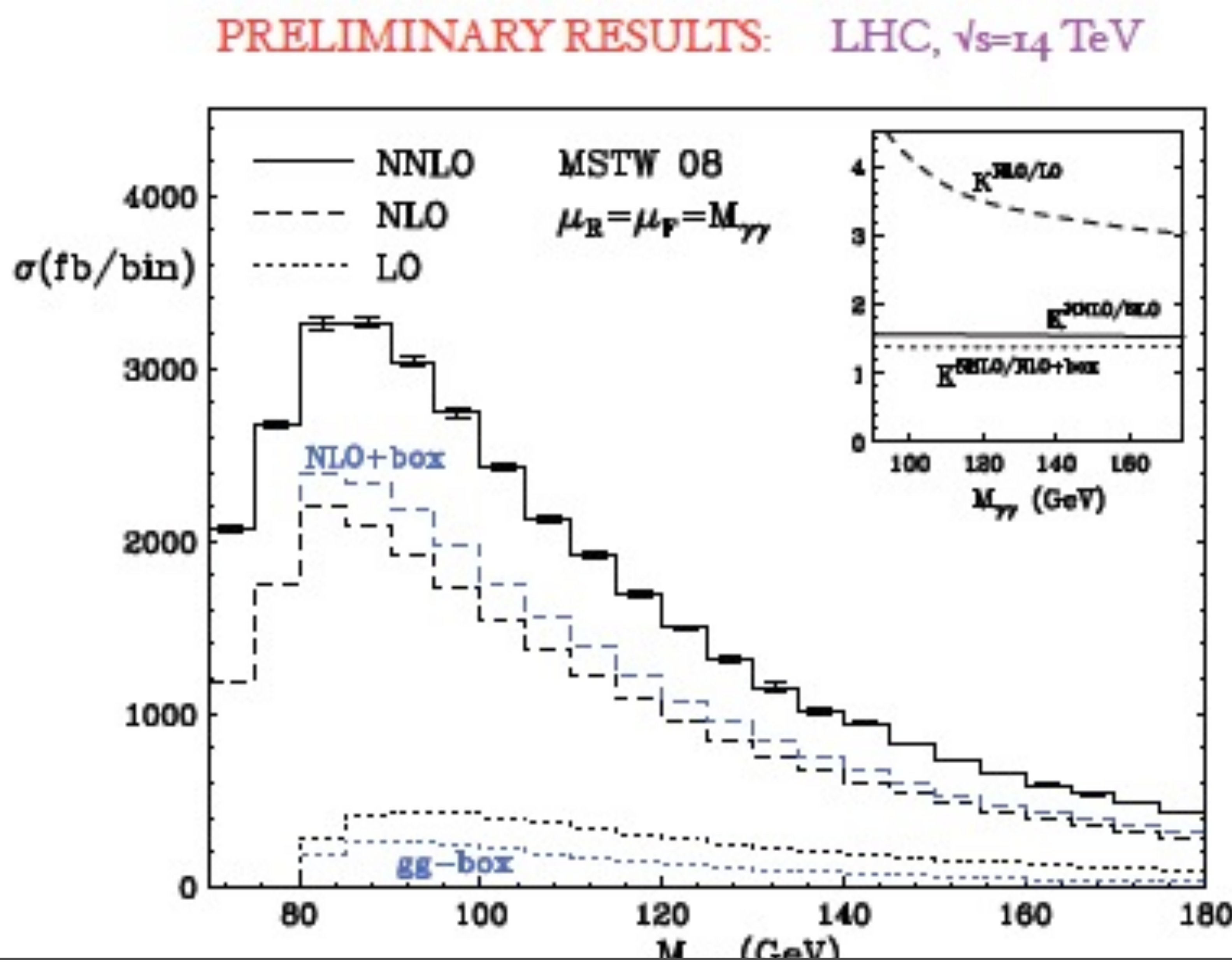

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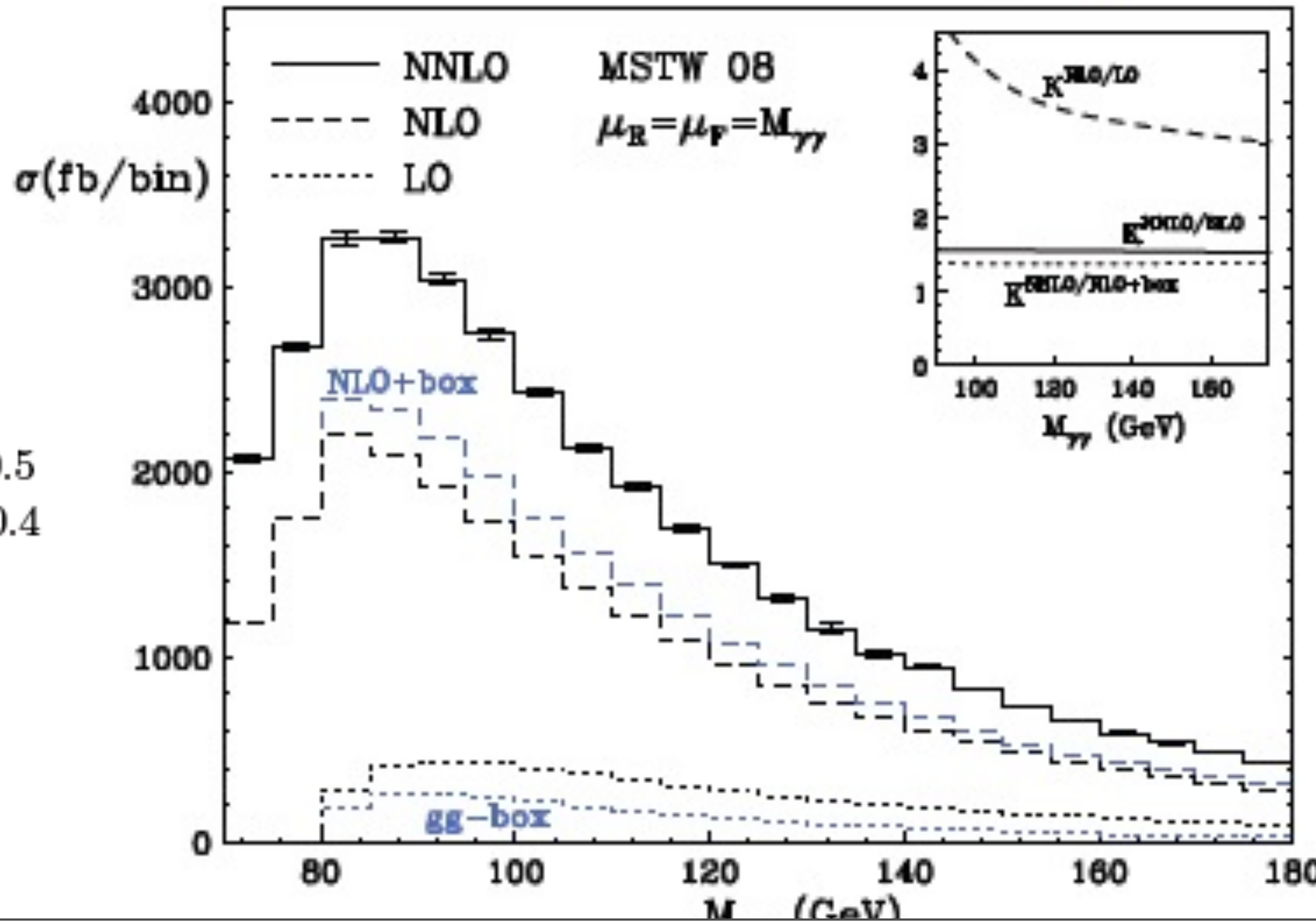
$$E_T^{had}(\delta) \leq \chi(\delta)$$

$$\chi(\delta) = \epsilon_\gamma E_T^\gamma \left(\frac{1-\cos(\delta)}{1-\cos(R_0)} \right)^n$$

$n = 1$
$\epsilon_\gamma = 0.5$
$R_0 = 0.4$

PRELIMINARY RESULTS: LHC, √s=14 TeV



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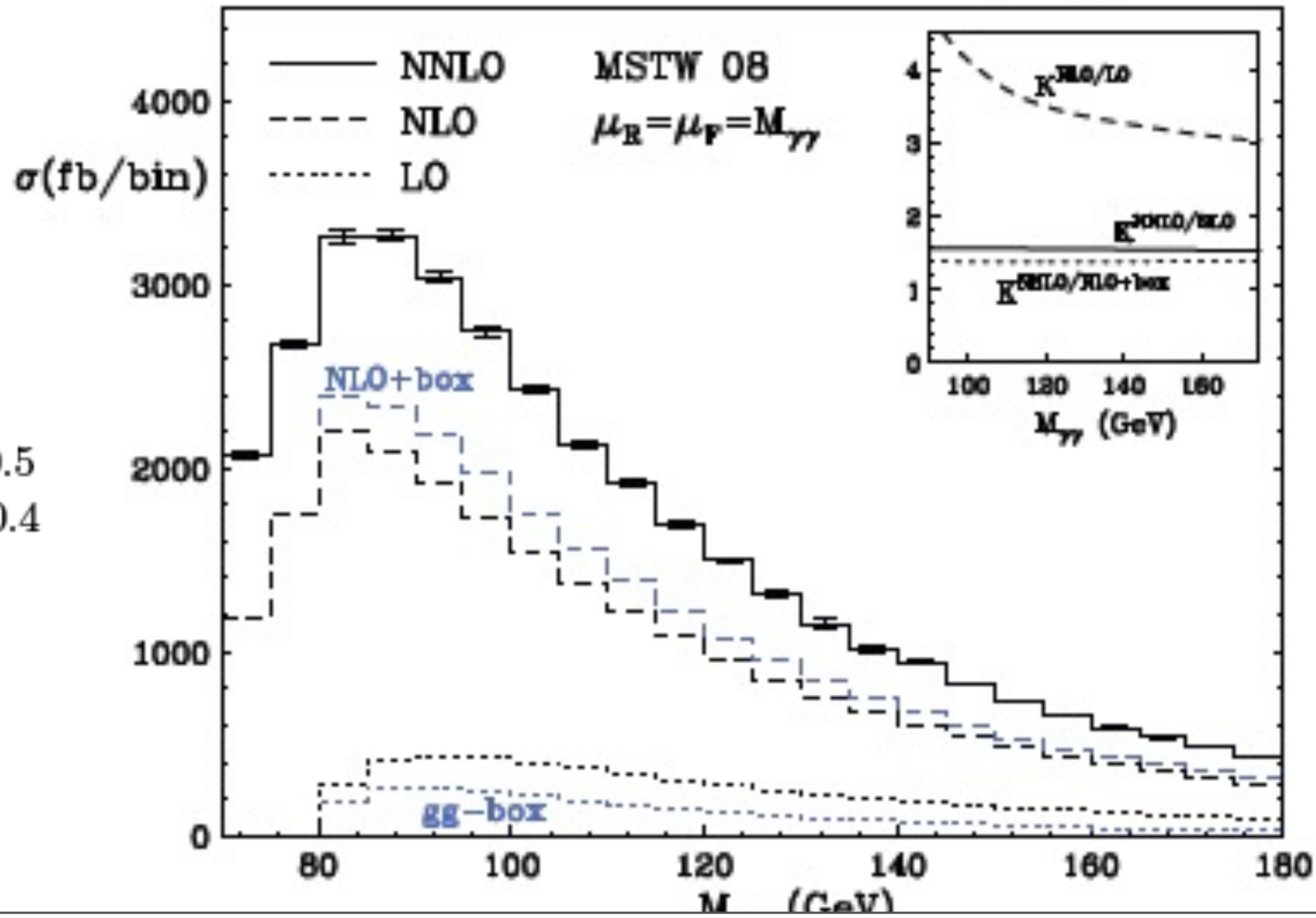
$p^\gamma_T \geq 40$ GeV $|\eta^\gamma| \leq 2.5$

60 GeV ≤ $M_{\gamma\gamma}$ ≤ 180 GeV

PRELIMINARY RESULTS: LHC, √s=14 TeV



Room and need for new ideas to perform NNLO calculations more efficiently

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with arbitrary cuts and experimental observables

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Anastasiou, Herzog, Lazopoulos arXiv:1011.4867

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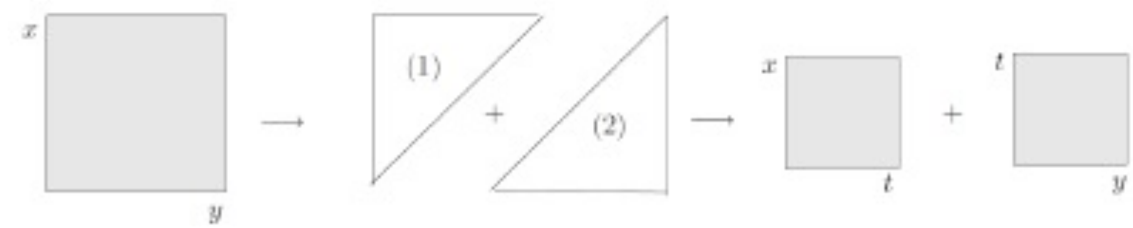
Talk by Franz Herzog at CERN THLPCC1 Workshop

Toy example for sector decomposition

$$I = \int_0^1 dxdy \frac{x^\epsilon}{x(ax+y)}$$



$$dxdy = dxdy\left[\Theta(x \geq y) + \Theta(y \geq x)\right]$$

y=tx x=ty

$$I = \int_0^1 dxdt \frac{(x)^\epsilon}{x(a+t)} + \int_0^1 dtdy \frac{(yt)^\epsilon}{yt(at+1)}$$

$y = tx$ $x = ty$

Proliferation of integrals

Toy example for non-linear mapping

$$I = \int_0^1 dx dy \frac{x^\epsilon}{x(ax+y)}$$

$$x \to xy$$

$$\mapsto \int_0^1 dy \int_0^{\frac{1}{y}} dx \frac{(xy)^\epsilon}{xy(ax+1)}$$

$$x \mapsto \frac{x(y/a)}{1-x+(y/a)} \qquad \mapsto \int_0^1 dx dy \frac{(xy)^\epsilon}{xy} (a(1-x)+y)^{-\epsilon}$$

factorizes the singularity and preserves integration boundaries

Successfull application for V-V, V-R and RR overlapping integrals

Fully differential NNLO calculation for 2->3 processes

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Will it be needed?

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We need a second NNLO revolution to beat the factorial growth

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Recent papers by

Mastorlia and Ossala **arXiv:1107.6041**
Kosower and Larsen **arXiv:1108.1180**

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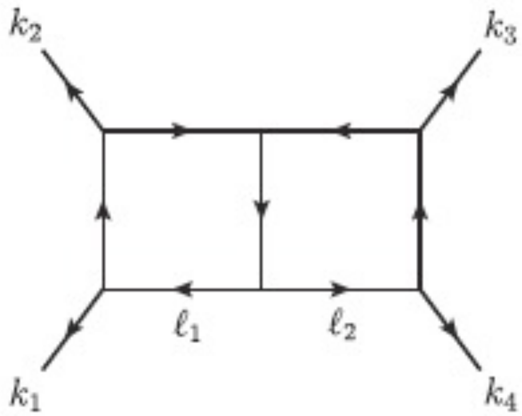
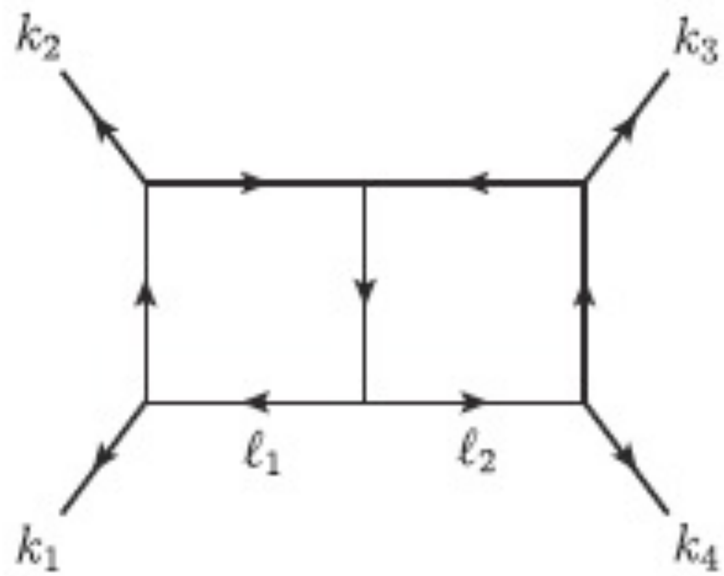
$$P_{2,2}^{**}[f(\ell_1,\ell_2)] = \int \frac{d^D\ell_1}{(2\pi)^D}\frac{d^D\ell_2}{(2\pi)^D}\frac{f(\ell_1,\ell_2)}{\ell_1^2(\ell_1-k_1)^2(\ell_1-K_{12})^2(\ell_1+\ell_2)^2\ell_2^2(\ell_2-k_4)^2(\ell_2-K_{34})^2}$$



OPP-like analysis for double box diagrams with maximal cuts

$$P_{2,2}^{**}[f(\ell_1,\ell_2)] = \int \frac{d^D\ell_1}{(2\pi)^D}\frac{d^D\ell_2}{(2\pi)^D}\frac{f(\ell_1,\ell_2)}{\ell_1^2(\ell_1-k_1)^2(\ell_1-K_{12})^2(\ell_1+\ell_2)^2\ell_2^2(\ell_2-k_4)^2(\ell_2-K_{34})^2}$$

$$D_1 = D_2 = D_3 = D_4 = d_5 = D_6 = D_7 = 0$$



What is the most general irreducible parametrization of $f(l_1, l_2)$?

How many terms lead to non-vanishing integral ?

How to project out non-vanishing coefficients?

Conclusions

Conclusions

- Consolidation afte NLO “revolution” towards automated codes
  aMC@NLO
- Room for more ideas yet to get more efficient algorithms
- Great challenge and motivation by the
  CMS and ATLAS new data sets
- More precision and so more detailed studies of higher order
  corrections are needed
- Fully differential NNLO calculations for 4-leg processes
  will be available in the near future
- Truly exiting new era in particle physics phenomenology