Prospects for $2 \to 2$ NNLO processes at the LHC

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based on work of/with Radja Boughezal, James Currie, Pedro Jimenez Delgado, Aude Gehrmann-De Ridder, Thomas Gehrmann, Gudrun Heinrich, Gionata Luisoni, Pier Francesco Monni, Joao Pires, Mathias Ritzmann, Steven Wells

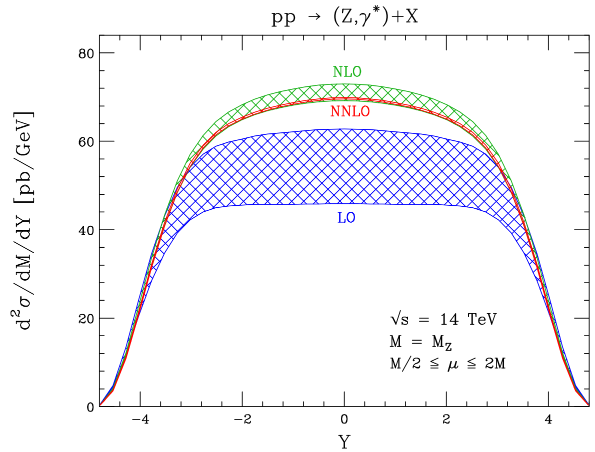
NNLO calculations for $2 \to 2$ processes

$$d\sigma = \sum_{i,j} \int \frac{d\xi_1}{\xi_1} \frac{d\xi_2}{\xi_2} f_i(\xi_1, \mu_F^2) f_j(\xi_2, \mu_F^2) d\hat{\sigma}_{ij}(\alpha_s(\mu_R), \mu_R, \mu_F)$$

$$d\hat{\sigma}_{ij} = d\hat{\sigma}_{ij}^{LO} + \left(\frac{\alpha_s(\mu_R)}{2\pi}\right) d\hat{\sigma}_{ij}^{NLO} + \left(\frac{\alpha_s(\mu_R)}{2\pi}\right)^2 d\hat{\sigma}_{ij}^{NNLO} + \mathcal{O}(\alpha_s^3)$$

Processes of interest

- ✓ $pp \to 2$ jets
- ✓ $pp \to \gamma$+jets
- ✓ $pp \to \gamma\gamma$
- ✓ $pp \to V$+jet
- ✓ $pp \to t\bar{t}$
- ✓ $pp \to VV$
- ✓ $pp \to H$+jet
- ✓ $\ldots$

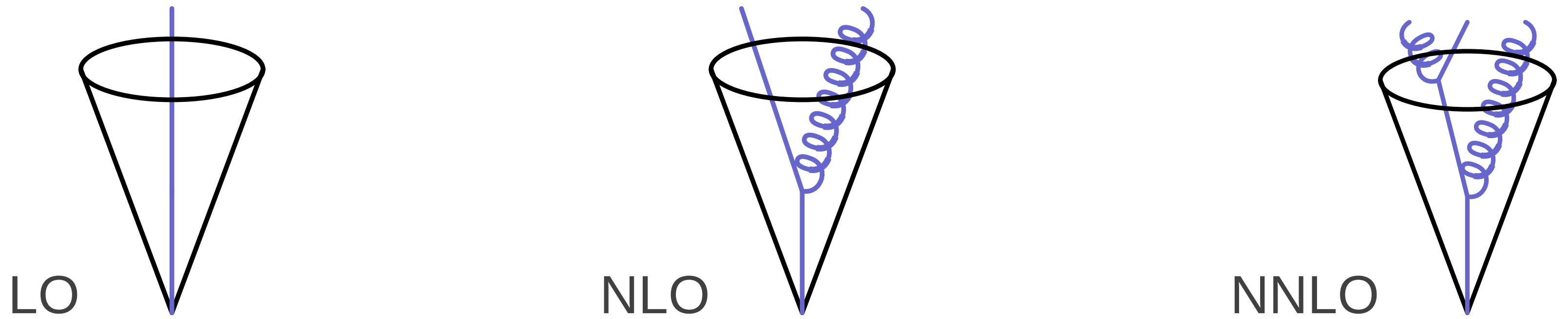


Massively reduced theoretical error

Anastasiou, Dixon, Melnikov, Petriello (04)

Arguments in favour of NNLO

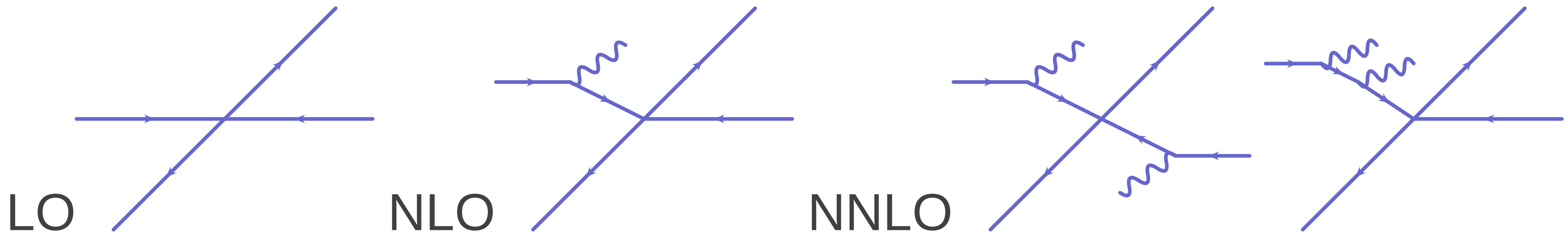
- ✓ Reduced renormalisation scale dependence
- ✓ Event has more partons in the final state so perturbation theory can start to reconstruct the shower
  $\Rightarrow$ better matching of jet algorithm between theory and experiment



- ✓ Reduced power correction as higher perturbative powers of $1/\ln(Q/\Lambda)$ mimic genuine power corrections like $1/Q$

Arguments in favour of NNLO

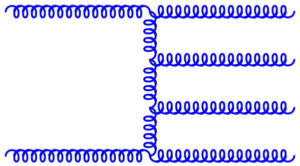
- ✓ Better description of transverse momentum of final state due to double radiation off initial state



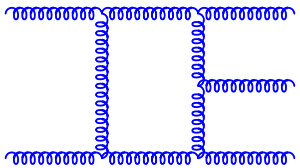
  - ✓ At LO, final state has no transverse momentum
  - ✓ Single hard radiation gives final state transverse momentum, even if no additional jet
  - ✓ Double radiation on one side, or single radiation of each incoming particle gives more complicated transverse momentum to final state
- ✓ NNLO is the first serious estimate of the error

Anatomy of a NNLO calculation e.g. $pp \to 2j$

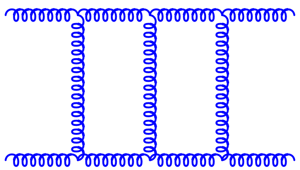
- ✓ double real radiation matrix elements $\mathrm{d}\hat{\sigma}^{RR}_{NNLO}$
  - ✓ implicit poles from double unresolved emission



- ✓ single radiation one-loop matrix elements $\mathrm{d}\hat{\sigma}^{RV}_{NNLO}$
  - ✓ explicit infrared poles from loop integral
  - ✓ implicit poles from soft/collinear emission



- ✓ two-loop matrix elements $\mathrm{d}\hat{\sigma}^{VV}_{NNLO}$
  - ✓ explicit infrared poles from loop integral
  - ✓ including square of one-loop amplitude



$$\mathrm{d}\hat{\sigma}_{NNLO} \sim \int_{\mathrm{d}\Phi_{m+2}} \mathrm{d}\hat{\sigma}^{RR}_{NNLO} + \int_{\mathrm{d}\Phi_{m+1}} \mathrm{d}\hat{\sigma}^{RV}_{NNLO} + \int_{\mathrm{d}\Phi_{m}} \mathrm{d}\hat{\sigma}^{VV}_{NNLO}$$

- ✓ Antenna method to extract implicit poles developed for $e^+e^- \to 3$ jets

Basics of subtraction method - I

✓ General form of (renormalised) cross section

$$
\begin{aligned}
\mathrm{d}\hat{\sigma}_{NNLO} \equiv & \int_{\mathrm{d}\Phi_{m+2}} \left(\mathrm{d}\hat{\sigma}^{RR}_{NNLO} - \mathrm{d}\hat{\sigma}^{S}_{NNLO}\right) + \int_{\mathrm{d}\Phi_{m+2}} \mathrm{d}\hat{\sigma}^{S}_{NNLO} \\
+ & \int_{\mathrm{d}\Phi_{m+1}} \left(\mathrm{d}\hat{\sigma}^{RV}_{NNLO} - \mathrm{d}\hat{\sigma}^{VS,1}_{NNLO}\right) + \int_{\mathrm{d}\Phi_{m+1}} \mathrm{d}\hat{\sigma}^{VS,1}_{NNLO} + \int_{\mathrm{d}\Phi_{m+1}} \mathrm{d}\hat{\sigma}^{MF,1}_{NNLO} \\
+ & \int_{\mathrm{d}\Phi_{m}} \mathrm{d}\hat{\sigma}^{VV}_{NNLO} + \int_{\mathrm{d}\Phi_{m}} \mathrm{d}\hat{\sigma}^{MF,2}_{NNLO}
\end{aligned}
$$

✓ $\mathrm{d}\hat{\sigma}^{S}_{NNLO}$ is the double real radiation subtraction term - subtracted and added back in in integrated form

✓ $\mathrm{d}\hat{\sigma}^{V,S,1}_{NNLO}$ is the real-virtual radiation subtraction term - subtracted and added back in in integrated form

✓ $\mathrm{d}\hat{\sigma}^{MF,1}_{NNLO}$ and $\mathrm{d}\hat{\sigma}^{MF,2}_{NNLO}$ are the mass factorisation counter terms

Basics of subtraction method - II

✓ The aim is to recast the NNLO cross section in the form

$$\begin{aligned}
\mathrm{d}\hat{\sigma}_{NNLO} &= \int_{\mathrm{d}\Phi_{m+2}} \left[\mathrm{d}\hat{\sigma}^{RR}_{NNLO} - \mathrm{d}\hat{\sigma}^{S}_{NNLO}\right] \\
&+ \int_{\mathrm{d}\Phi_{m+1}} \left[\mathrm{d}\hat{\sigma}^{RV}_{NNLO} - \mathrm{d}\hat{\sigma}^{T}_{NNLO}\right] \\
&+ \int_{\mathrm{d}\Phi_{m}} \left[\mathrm{d}\hat{\sigma}^{VV}_{NNLO} - \mathrm{d}\hat{\sigma}^{U}_{NNLO}\right],
\end{aligned}$$

where the terms in each of the square brackets is finite, well behaved in the infrared singular regions and can be evaluated numerically.

$$\begin{aligned}
\mathrm{d}\hat{\sigma}^{T}_{NNLO} &= \mathrm{d}\hat{\sigma}^{VS,1}_{NNLO} - \int_1 \mathrm{d}\hat{\sigma}^{S,1}_{NNLO} - \mathrm{d}\hat{\sigma}^{MF,1}_{NNLO}, \\
\mathrm{d}\hat{\sigma}^{U}_{NNLO} &= -\int_1 \mathrm{d}\hat{\sigma}^{VS,1}_{NNLO} - \int_2 \mathrm{d}\hat{\sigma}^{S,2}_{NNLO} - \mathrm{d}\hat{\sigma}^{MF,2}_{NNLO}.
\end{aligned}$$

Integrated three-parton tree antennae

$\mathcal{X}_3^0$	Final-Final	Initial-Final	Initial-Initial
A	✓ [1]	✓ [2]	✓ [2]
D	✓ [1]	✓ [2]	✓ [2]
E	✓ [1]	✓ [2]	✓ [2]
F	✓ [1]	✓ [2]	✓ [2]
G	✓ [1]	✓ [2]	✓ [2]

[1] Gehrmann-De Ridder, Gehrmann, NG, (05)

[2] Daleo, Gehrmann, Maitre, (06)

$\mathcal{S}$	Final-Final	Initial-Final	Initial-Initial
S	✓ [1]	✓ [2]	

[1] Gehrmann-De Ridder, Gehrmann, NG, Heinrich, (07)

[2] Daleo, Gehrmann-De Ridder, Gehrmann, Luisoni, (09)

Integrated three-parton one-loop antennae

$\mathcal{X}_3^1$	Final-Final	Initial-Final	Initial-Initial
A, $\tilde{A}$, $\hat{A}$	✓ [1]	✓ [2]	✓ [3]
D, $\hat{D}$	✓ [1]	✓ [2]	✓ [3]
F, $\hat{F}$	✓ [1]	✓ [2]	✓ [3]
E, $\tilde{E}$, $\hat{E}$	✓ [1]	✓ [2]	✓ [3]
G, $\tilde{G}$, $\hat{G}$	✓ [1]	✓ [2]	✓ [3]

[1] Gehrmann-De Ridder, Gehrmann, NG, (05)

[2] Daleo, Gehrmann-De Ridder, Gehrmann, Luisoni, (09)

[3] Gehrmann, Monni, (11)

Integrated four-parton tree antennae

$\mathcal{X}_4^0$	Final-Final	Initial-Final	Initial-Initial
A, $\tilde{A}$	✓ [1]	✓ [2]	
B	✓ [1]	✓ [2]	✓ [3]
C	✓ [1]	✓ [2]	
D	✓ [1]	✓ [2]	
E, $\tilde{E}$	✓ [1]	✓ [2]	✓ [3]
F	✓ [1]	✓ [2]	
G, $\tilde{G}$	✓ [1]	✓ [2]	
H	✓ [1]	✓ [2]	✓ [3]

[1] Gehrmann-De Ridder, Gehrmann, NG, (05)

[2] Daleo, Gehrmann-De Ridder, Gehrmann, Luisoni, (09)

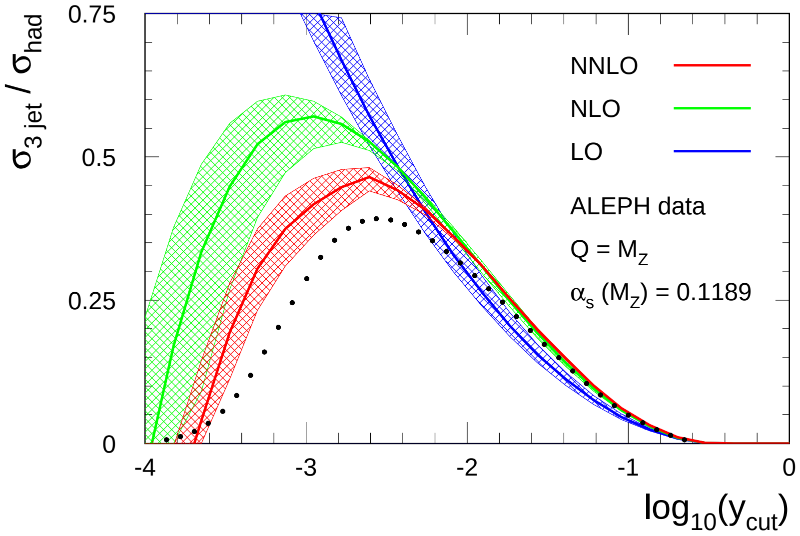
[3] Boughezal, Gehrmann-De Ridder, Ritzmann, (10)

Remaining Initial-Initial functions depend on further 20 master integrals

$e^+e^- \rightarrow 3$ jets at NNLO

Method thoroughly tried and tested for partons only in the final state

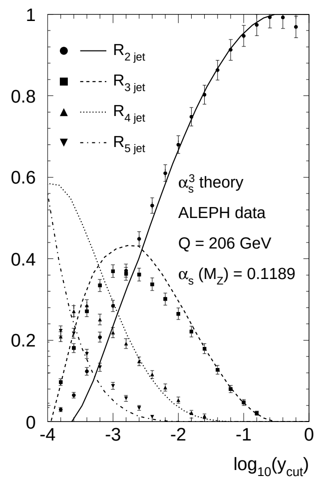
Gehrmann-De Ridder, Gehrmann, Heinrich, NG (07)



- ✓ NNLO corrections to jet rate small
  - ✓ stable perturbative prediction
  - ✓ resummation not needed
  - ✓ theory error below 2%
  - ✓ small hadronisation corrections
- ✓ α_s extraction from jet rates

  Dissertori, Gehrmann-De Ridder, Gehrmann, Heinrich, Stenzel, NG (09)

  - ✓ fit at $y_{cut} = 0.02$
  - ✓ consistent results at other y_{cut}



$$\alpha_s(M_Z) = 0.1175 \pm 0.0020(exp) \pm 0.0015(th)$$

Applications to LHC processes

- ✓ All relevant matrix elements for $pp \to 2$ jet and $pp \to V + 1$ jet processes available for some time
- ✓ Can expect to have parton-level NNLO predictions for $pp \to 2$ jet and $pp \to V + 1$ jet in next couple of years
- ✓ Hope for significant reduction in theory (renormalisation scale/factorisation scale) dependence
- ✓ LHC already has increased dynamic range for jet studies - rapidity, transverse energy.
- ✓ Combined with excellent experimental jet energy scale uncertainty, there is the opportunity for improved measurements of
  - ✓ Parton distributions
  - ✓ Strong coupling
  - ✓ Internal structure of the jet
  - ✓ Rapidity gaps between the jets

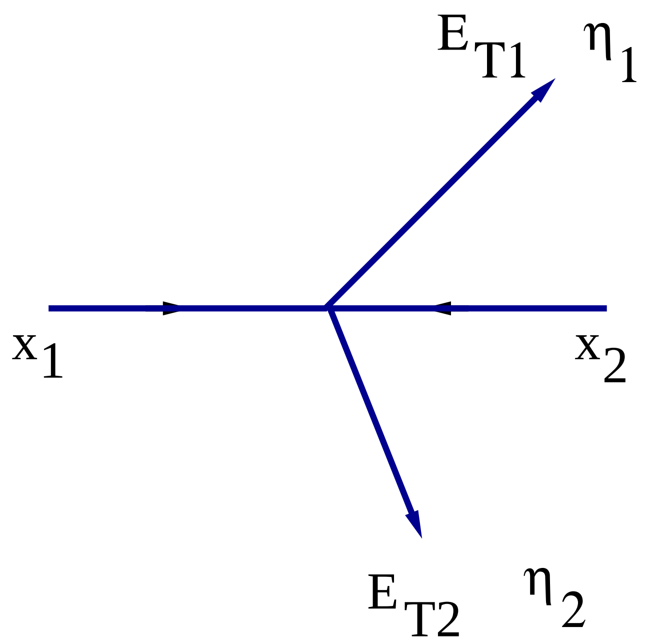
Maximising the impact of NNLO calculations

Triple differential form of the cross section

$$\frac{d^3\sigma}{dE_T d\eta_1 d\eta_2} = \frac{1}{8\pi}\sum_{ij} x_1 f_i(x_1,\mu_F)\, x_2 f_j(x_2,\mu_F)\, \frac{\alpha_s^2(\mu_R)}{E_T^3}\, \frac{|\mathcal{M}_{ij}(\eta^*)|^2}{\cosh^4\eta^*}$$

✓ Direct link between observables E_T, η_1, η_2 and momentum fractions/parton luminosities

$$x_1 = \frac{E_T}{\sqrt{s}}\left(\exp(\eta_1)+\exp(\eta_2)\right),$$
$$x_2 = \frac{E_T}{\sqrt{s}}\left(\exp(-\eta_1)+\exp(-\eta_2)\right)$$

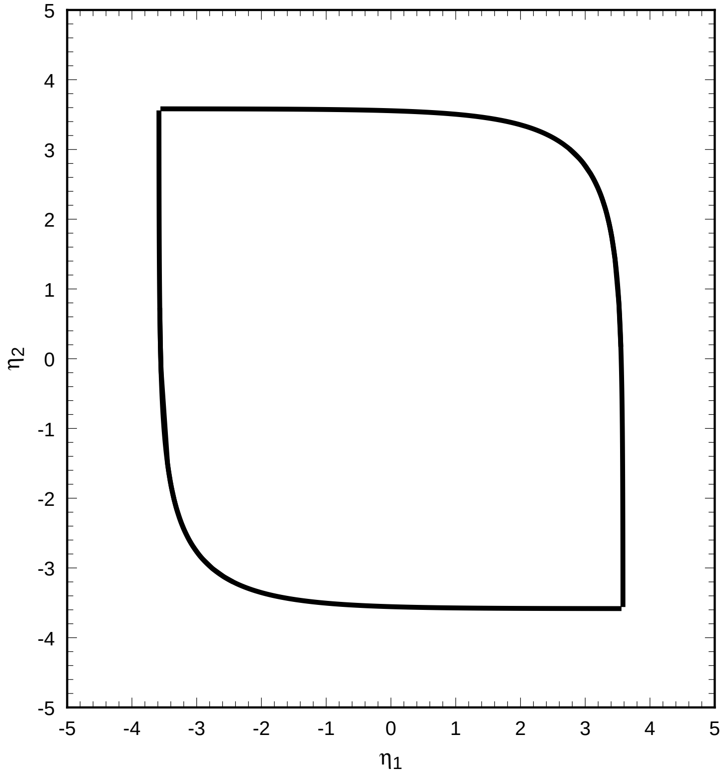


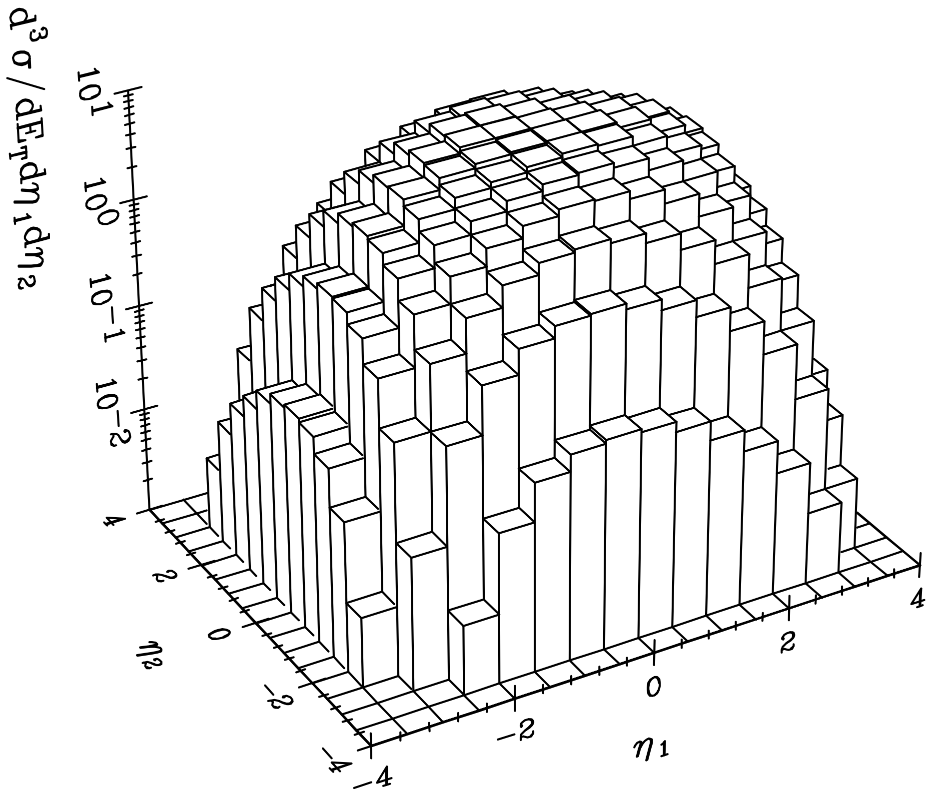
✓ and matrix elements that only depend on

$$\eta^* = \frac{1}{2}(\eta_1 - \eta_2)$$

Triple differential distribution

✓ Range of x_1 and x_2 fixed allowed LO phase space for jets
$E_T \sim 200$ GeV at $\sqrt{s} = 7$ TeV





✓ Shape of distribution can be understood by looking at parton luminosities and matrix elements (in for example the single effective subprocess approximation)

Giele, NG, Kosower, hep-ph/9412338

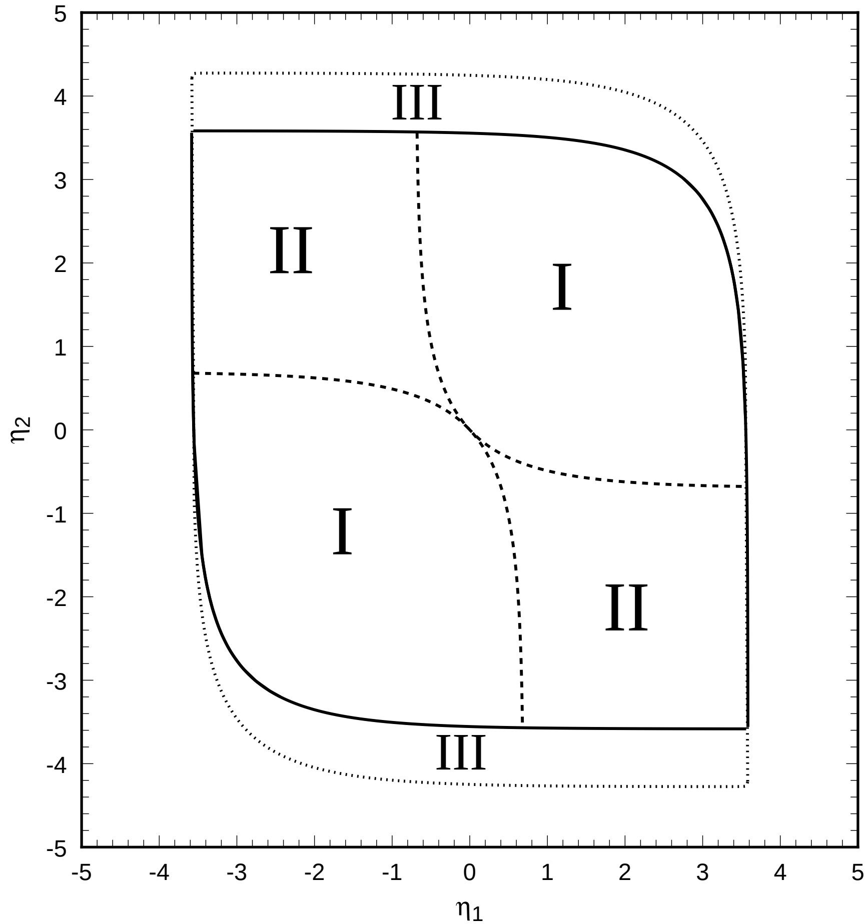
Phase space considerations

✓ Phase space boundary fixed when one or more parton fractions $\to 1$.

I $\eta_1 > 0$ and $\eta_2 > 0$ OR $\eta_1 < 0$ and $\eta_2 < 0$
$\Rightarrow$ one x_1 or x_2 is less than x_T
- small x

II $\eta_1 > 0$ and $\eta_2 < 0$ OR $\eta_1 < 0$ and $\eta_2 > 0$
$\Rightarrow$ both x_1 and x_2 are bigger than x_T
- large x

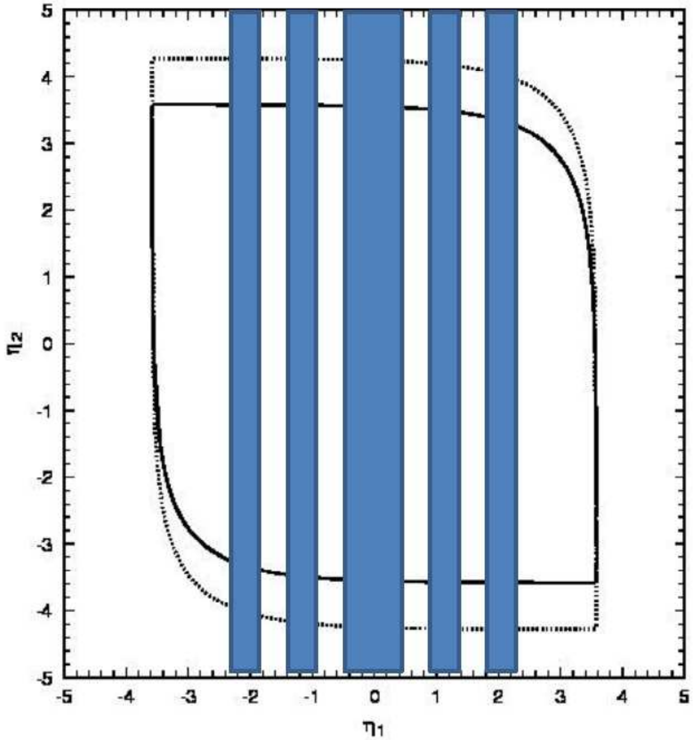
III growth of phase space at NLO (if $E_{T1} > E_{T2}$)

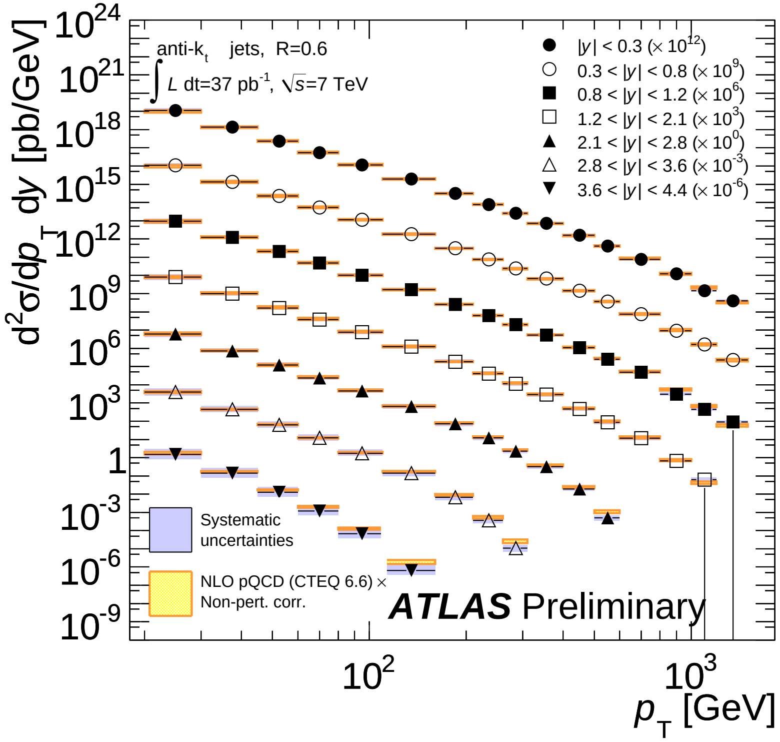


$$\left[x_T^2 < x_1 x_2 < 1 \qquad \text{and} \qquad x_T = 2E_T/\sqrt{s} \right]$$

Single Jet Inclusive Distribution

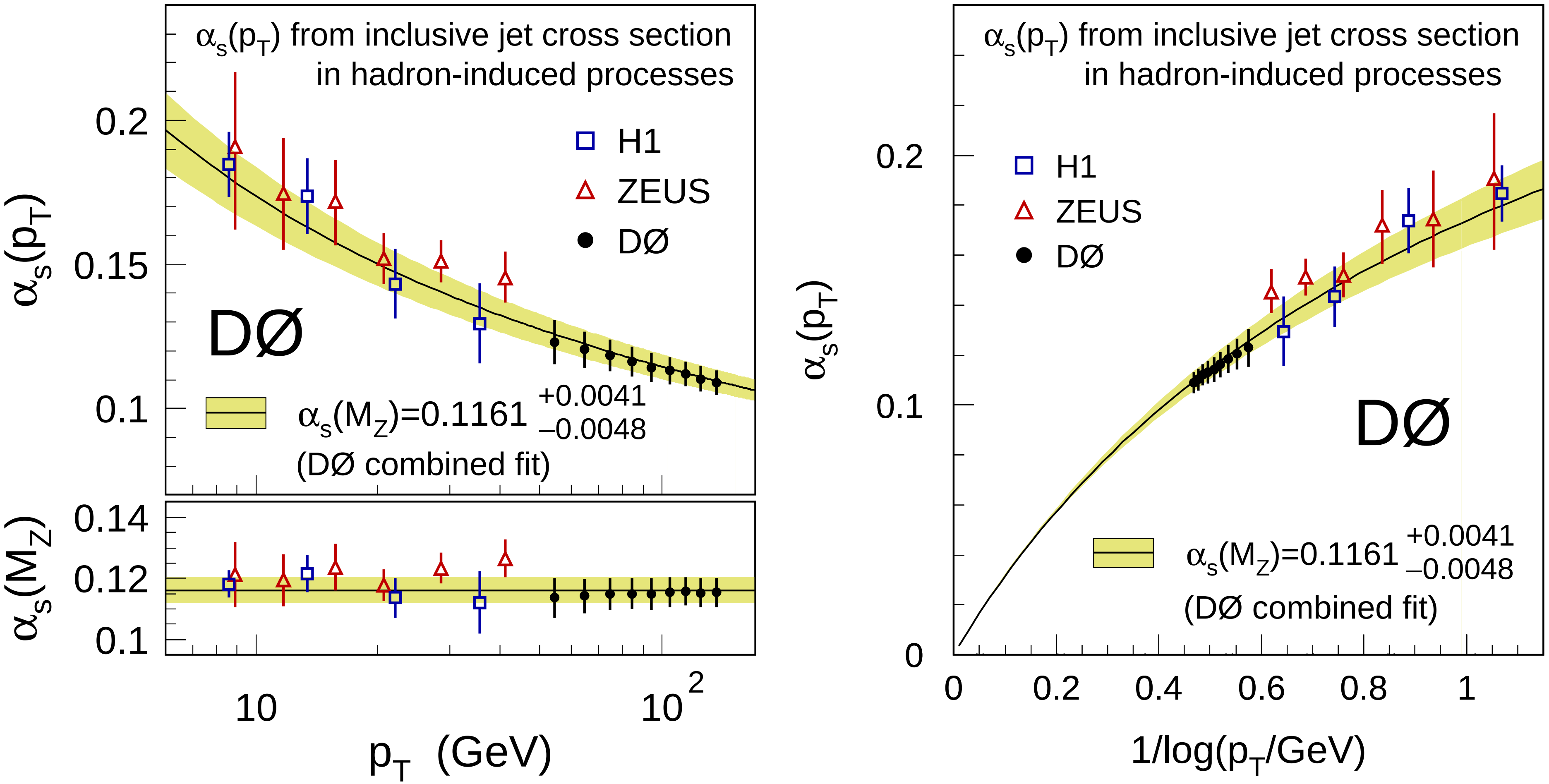
✓ Single Jet Inclusive Distribution is just a slice of the triple differential distribution, moving from $(x_1, x_2) = (1, x_T^2 \cosh^2(\eta^*))$ to $(x_T^2 \cosh^2(\eta^*), 1)$ where $\eta^* = \frac{1}{2}(\eta_1 - \eta_2)$





Measurements of strong coupling

We can extract α_s using input PDF's (with varying α_s) fixed by DIS, etc e.g.



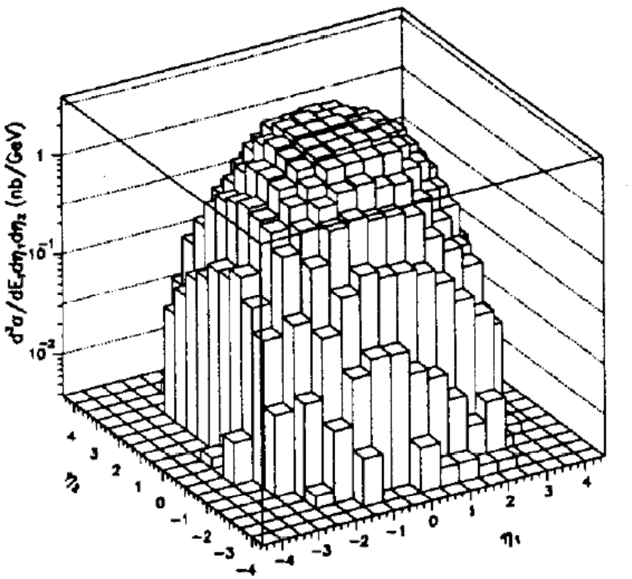
D0, arXiv:0911.2710

Measurements of strong coupling

- ✓ With incredible jet energy resolution, the LHC can do better
- ✓ and simultaneously fit the parton density functions and strong coupling
- ✓ If the systematic errors can be understood, the way to do this is via the triple differential cross section

Giele, NG, Yu, hep-ph/9506442

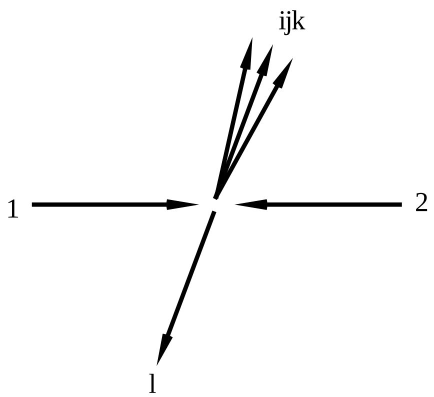
- ✓ and add NNLO $W^{\pm}$+jet, Z+jet, γ+jet calculations as they become available



D0 preliminary, 1994

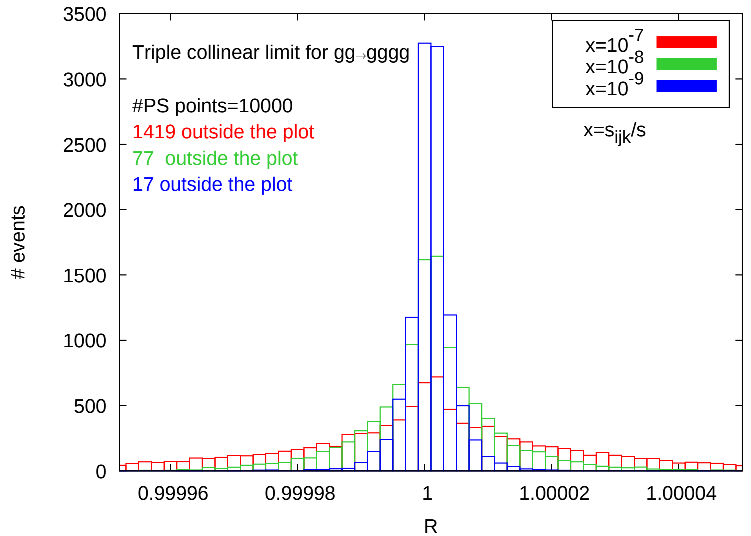
Applications to LHC processes - status

- ✓ All relevant matrix elements for $pp \to 2$ jet and $pp \to V+1$ jet processes available for some time
- ⚠ Aim to push “leading colour gluons-only" $pp \to 2$ jets all the way to the end to demonstrate proof of concept
- ✓ Double unresolved subtraction terms for leading colour six-gluon process tested



(a) Example configuration of a triple collinear event with $s_{ijk} \to 0$.

(b) Distribution of $\mathrm{d}\hat{\sigma}^R_{NNLO}/\mathrm{d}\hat{\sigma}^S_{NNLO}$ for 10000 triple collinear phase space points.



Pires, NG, (10)

Applications to LHC processes - status

- Real Virtual subtraction terms for one-loop five-gluon process almost complete Gehrmann-De Ridder, Pires, NG
- Few remaining “initial-initial" integrals necessary to complete “leading colour gluons-only" $pp \to 2$ jet (same type of integrals as already encountered)
- ✓ Same integrals are needed for all other processes
- Aim to have “leading colour gluons-only" $pp \to 2$ jet in place in next few months
- In parallel, coding of sub-leading colour contributions, quark processes and $pp \to V + 1$ jet underway