All You Can Higgs

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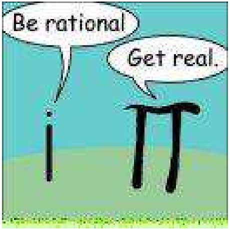
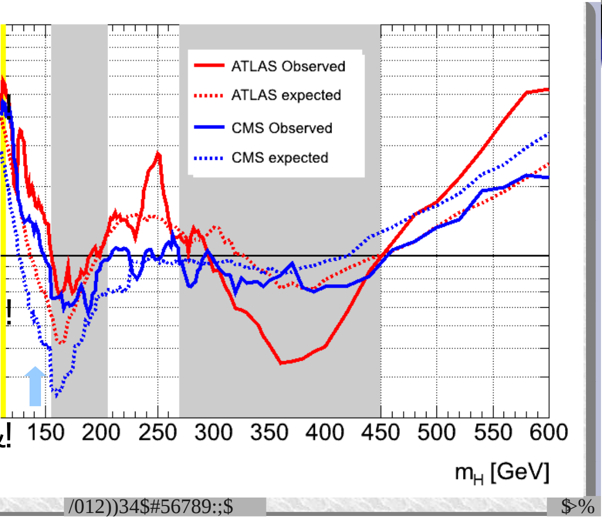
Higgs, *opinions are made to be changed or how is truth to be got at?*

(Paraphrasing George Byron)

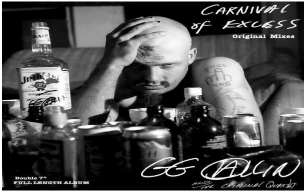




excess & exclusion

Options

- excess
- exclusion



- **If no Higgs then what?**
- *VV* -scattering

No new event at low mass, much more statistics needed.

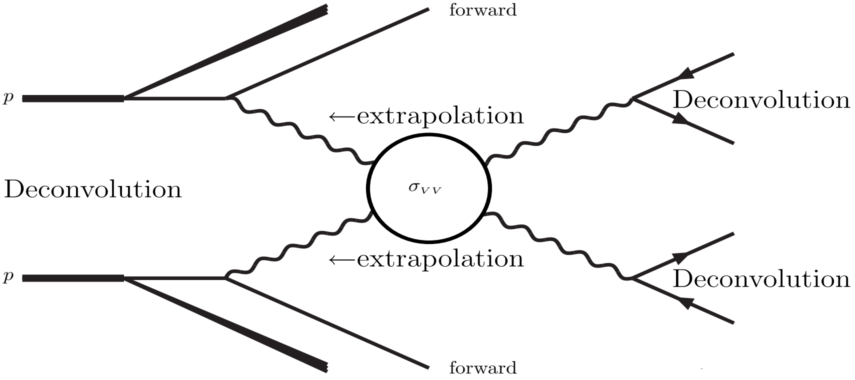
Scattering at $M_V^2 \ll s \ll M_H^2$

$$
\begin{aligned}
T^0_{1L}(\omega^+\omega^- \to \frac{1}{\sqrt{2}}zz) &= \frac{G_F^2 s^2}{256\sqrt{2}\,\pi^3}\left(\frac{20}{9}\ln\frac{m_H^2}{s} + 6\sqrt{3}\pi - \frac{905}{27} + 2\,i\pi\right) \\
T^0_B(\omega^+\omega^- \to \frac{1}{\sqrt{2}}zz) &= \frac{G_F s}{8\sqrt{2}\,\pi}
\end{aligned}
$$

$$
\begin{aligned}
T^0_{1L}(\frac{1}{\sqrt{2}}zz \to \frac{1}{\sqrt{2}}zz) &= \frac{G_F^2 s^2}{512\,\pi^3}\left(\frac{20}{3}\ln\frac{m_H^2}{s} + 10\sqrt{3}\pi - \frac{512}{9} + 4\,i\pi\right) \\
T^0_B(\frac{1}{\sqrt{2}}zz \to \frac{1}{\sqrt{2}}zz) &= 0
\end{aligned}
$$

- The **coupled j** $= 0$ **partial-wave** has two eigenvalues
- In the tree approximation $\lambda_1 < 0$, corresponding to a **repulsive interaction**
- while λ_2 gives an **attractive interaction**.
- Including one-loop corrections changes the situation. One eigen-channel (corresponding to λ_2) is always attractive, the other stays repulsive with λ_1 becoming more and more negative till some threshold, after which the behavior is reversed.

VV -scattering



Theorem

- *The scale dependence has no physical meaning, i.e. its correlation to anything else has no meaning as well. There is no correlation that can be quantified between the uncertainty band from higher orders to something.*

- *Once you try to set up something like this, you screw up the spirit of taking the scale as conservatively quantifying missing corrections.*

The μ_R problem

QED

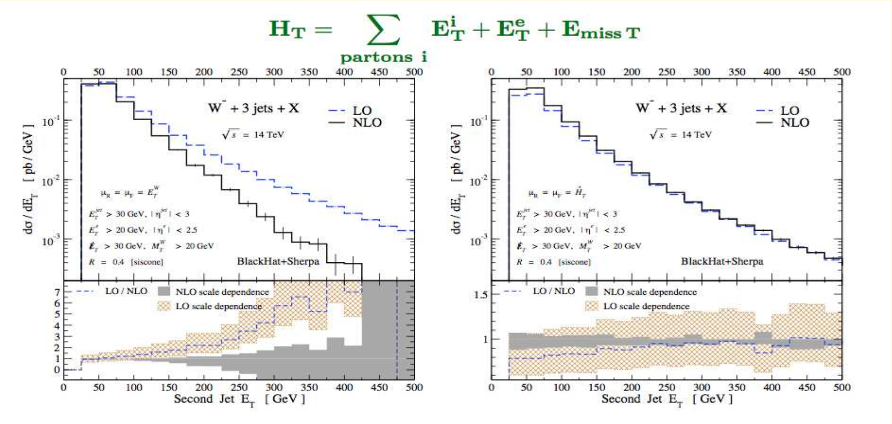
- Is there a μ_R in QED? **Yes**
- Is it a problem? **No**, $q^2 = 0$ is physical!

EW

- Is there a μ_R in EW? **Yes**
- Is it a problem? **No**!
- Are there large **logs**? **Yes**
- Use G_F - scheme and not $\alpha(0)$, i.e. **resum**

QCD one(multi)-scale? Once again, **resum** or, at least **minimize**!

Example


$H_T = \sum_{\text{partons } i} E_T^i + E_T^e + E_{\text{miss } T}$
W⁻ + 3 jets + X
LO
NLO
√s = 14 TeV
BlackHat+Sherpa
dσ / dE_T [pb / GeV]
LO / NLO
NLO scale dependence
LO scale dependence
Second Jet E_T [GeV]

The scale variation problem

Warning TH stupidity has No statistical meaning

ggF

- Fixed order $\rightsquigarrow$ scale = $M_H/2$
- Fully justified by NNL re-summation!

Multi - scale

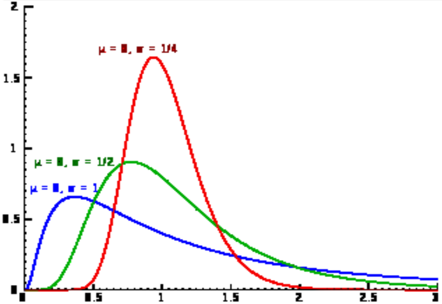
- $\mu =$ dynamical scale,
- $\mu_{\rm min} \leq \mu \leq \mu_{\rm max}$,
- are selected to (reasonably) minimize large logs

Nevertheless, the main question:

- How to deal with all independent sources of errors, of which Exps have $\mathcal{O}(200)$?

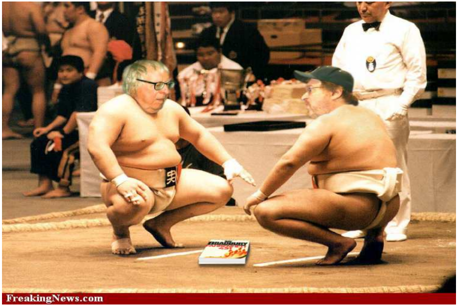
Consequent criticism:

- Priors with sharp edges are very nasty as they tend to result in computational instabilities due to discontinuities in derivatives.



Consequence:

- Hcombo decided to choose the log-normal form of priors over the flat one;
- therefore, the linear sum (LHC Higgs XS WG) and quadratic sum (LHC Higgs Combination WG) methods are reciprocally contradictory.



About interference

Hot @ High *mass*

$$A_T = A^S_{\rm LO} + \exp\left(i\,\theta_s\right) A^S_{\rm NLO} + \exp\left(i\,\theta_b\right) A^B_{\rm LO}$$

- LO = lowest (non zero) order
- S= signal, B= background, $\theta_{s,b}$ = phases.

What's available?

- $\left|A^S_{\rm LO}\right|^2, \quad \left|A^S_{\rm NLO}\right|^2 + \cdots, \quad \left|A^B_{\rm LO}\right|^2$

? $\left|A^S_{\rm LO} + \exp\left(i\,\theta_b\right) A^B_{\rm LO}\right|^2 \quad \rightsquigarrow \quad$ LO interference

! $\sigma_{\rm NLO} = {\rm K}\,\sigma_{\rm LO}$ does not imply ${\rm interference}_{\rm NLO} = {\rm K}\,{\rm interference}_{\rm LO}$

About interference II

For

$\sqrt{s} = 14\,\text{TeV}\ M_H = 600\,\text{GeV}$

$$\begin{aligned}
\sigma(gg \to l\nu l'\nu') &= 60\,\text{fb} \\
\sigma_c(gg \to l\nu l'\nu') &= 1.4\,\text{fb} \\
\sigma(gg \to H) &= 2.4\,\text{pb} \\
\text{BR}(H \to l\nu l'\nu') &= 7\,10^{-2}
\end{aligned}$$

- Cut dependence? $\Longrightarrow$
- T. Binoth et al. $\Longrightarrow$

- $I = \pm 90\,|\cos\theta|\,\%$
- $I_c = \pm 20\,|\cos\theta|\,\%$
- θ = B/S (unknow) phase $\rightsquigarrow$ Action needed
- Exact $I(I_c) = -0.7\%(10.6\,\%)$ at 200 GeV.
- Exact $I(I_c) = -5.2\%(-3.8\,\%)$ at 140 GeV.

Example

$$\sigma\left(gg(\rightarrow H)\rightarrow WW\rightarrow l\bar{\nu}\bar{l}'\nu'\right)$$

arXiv:hep-ph/0611170v1 14 TeV

sel.	$\sigma(S)$ [fb]	$\sigma(B_{gg})$ [fb]	$\sigma(S+B_{gg})$ [fb]	$\approx \theta_b$
tot	75.4	60.0	134.5	90.4^o
bkg	1.67	1.74	3.41	84.5^o

About interference III

Message

For I we need amplitudes A (interfacing different codes?) but codes have $\left|A\right|^2$ and $I = 2\,\mathrm{Re}\,(A_S\,A_B^*)$

$M_H < 2\,M_t$

- A_S from EFT $\smile$
- A_B $\smile$
- assembling A_{S+B} $\frown$

$M_H > 2\,M_t$

- A_S $\frown$
- finite width effect $\frown$

consistency

S known at NLO, B at LO $\rightsquigarrow$ $I = I_{\mathrm{app}}$ at NLO

- Gluon-gluon contributions to W^+W^- production and Higgs interference effects
  John M. Campbell, R.Keith Ellis, Ciaran Williams, FERMILAB-PUB-11-340-T, Jul 2011

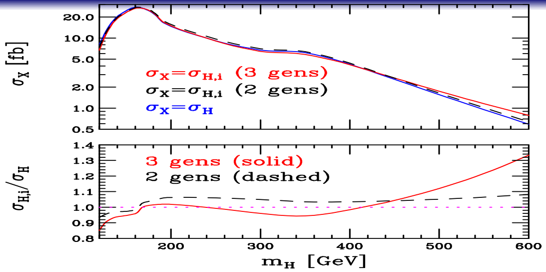


Figure 6: Upper panel: The cross sections for $gg \to H \to W^+(\to \nu_e e^+)W^-(\to \mu^- \bar{\nu}_\mu)$ in femtobarns, with $(\sigma_{H,i})$ and without (σ_H) the interference with SM $gg \to WW$ production. The dashed line represents the calculation of $\sigma_{H,i}$ including only the first two generations of quarks. Lower panel: The ratio of the cross sections with and without the interference terms. The dotted magenta line highlights the boundary between constructive and destructive interference.

M_H [GeV]	120	140	170	200	400
σ_H	7.90(1)	20.29(1)	26.13(2)	14.69(1)	4.23(1)
$\sigma_{H,i}$	6.73(1)	19.04(1)	26.25(2)	14.96(1)	4.16(1)
$\frac{\sigma_{H,i}}{\sigma_H}$	0.852	0.938	1.005	1.018	0.983

Table 5: Cross sections for $gg \to H \to W^+(\to \nu_e e^+)W^-(\to \mu^- \bar{\nu}_\mu)$ in femtobarns at $\sqrt{s} = 7$ TeV with no cuts applied, computed at leading order and either excluding (σ_H) or including $(\sigma_{H,i})$ the effect of interference with the gluon-initiated background process.

Numerical values of these cross section are shown in Table 5 for a selection of benchmark Higgs masses. We observe that the relative size of the interference is strongly dependent on the Higgs mass and that the interference changes sign at the $m_H = 2m_W$ threshold. For $m_H > 2m_W$ there are two further changes of sign, with a minimum at $m_H = 2m_t$. For very large Higgs masses the interference becomes large and positive. For reference we have also plotted in Fig. 6 the contribution to the interference from the first two generations of quarks only (i.e. setting $\mathcal{A}_{\text{massive}} = 0$ in the definitions of Eq. (4.1). The difference between

A Dissertation upon Roast Pig

Lineshape

- Consider the process

$$gg \quad \to \quad H(\to f) + X$$

  where f is a generic final state (e.g. $f = \gamma\gamma, 4\,\mathrm{f}$, etc). For the sake of simplicity we neglect folding with PDFs.
- Since the Higgs boson is a scalar resonance we can split the whole process into three parts, *production*, *propagation* and *decay*.

Propagation

The Higgs (Dyson-resummed) propagator reads as follows:

$$\Delta_H(s) = \left[s - M_H^2 + S_{HH}(s, m_t, M_H, M_W, M_Z)\right]^{-1},$$

where M_i is a renormalized mass and S_{HH} is the renormalized Higgs self-energy (to all orders but with one-particle-irreducible diagrams). We define complex poles as the (complex) solutions of

$$\begin{aligned} s_H - M_H^2 + S_{HH}(s_H, m_t, M_H, M_W, M_Z) &= 0, \\ s_W - M_W^2 + S_{WW}(s_W, m_t, M_H, M_W, M_Z) &= 0, \end{aligned}$$

etc.

Propagation

To one-loop accuracy the Higgs propagator is rewritten as

$$\Delta_H^{-1} \;=\; s - s_H\,.$$

The complex pole describing an unstable particle is parametrized as

$$s_i \;=\; \mu_i^2 - i\,\mu_i\,\gamma_i\,,$$

where μ_i is an input parameter (similar to the on-shell mass) while γ_i can be computed (as the on-shell total width).

Gauge invariance

Only the complex pole is gauge-parameter independent to all orders of perturbation theory while on-shell quantities are ill-defined beyond lowest order. Indeed, in the R_ξ gauge one has

$$\begin{aligned}\mathrm{Im}\, S_{HH\,,\,b} &= \frac{g^2}{4\,M_W^2}\, s^2 \Bigg[\left(1 - \frac{M_H^4}{s^2}\right) \left(1 - 4\,\xi_W\,\frac{M_W^2}{s}\right)^{1/2} \\ &\times \quad \theta\left(s - 4\,\xi_W M_W^2\right) + \frac{1}{2}\,(W \to Z)\Bigg],\end{aligned}$$

where ξ_V are gauge parameters.

Model independent approach

- Both μ_H and Γ_h should be kept free in order to perform a 2 dim scan of the Higgs-boson lineshape.
- For the high-mass region this remains the recommended strategy.
- Once the fits are performed it will be left to theorists to struggle with the Standard Model (SM) interpretation of the results.

Comparison with on-shell

To compute Γ_h at the same level of accuracy to which Γ_H^{OS} is known would require, at least, a three-loop calculation (the first instance where we have a four-fermion cut of the Higgs self-energy). There is a substantial difference between W, Z complex poles and the Higgs complex pole.

- In the first case W, Z decay predominantly into two (massless) fermions while
- for the Higgs boson below the WW -threshold the decay into four fermions is even larger than the decay into a $\overline{b}b$ pair.

Therefore we cannot use for the Higgs boson the well known result, valid for W, Z, i.e.

$$\mathrm{Im} S_{VV}(s) \approx \frac{\Gamma_V^{OS}}{M_V^{OS}} s .$$

Production and Decay

The complete cross section will be written as follows:

$$\sigma\left(gg\rightarrow H\rightarrow f\right) = \frac{1}{s}\int d\Phi_{gg\rightarrow f}\left[\sum_{s,c}\left|A\left(gg\rightarrow H\right)\right|^{2}\right]\frac{1}{\left|s-s_{H}\right|^{2}}$$
$$\times \left[\sum_{s,c}\left|A\left(H\rightarrow f\right)\right|^{2}\right],$$

where $\sum_{\text{spin,col}}$ is over spin and colors (averaging on the intial state). Note that the background (e.g $gg\rightarrow 4\,\text{f}$) has not been included and, strictly speaking and for reasons of gauge invariance, one should consider only the residue of the Higgs-resonant amplitude at the complex pole.

off-shell

If we decide to keep the Higgs boson off-shell also in the resonant part of the amplitude (interference S/B remains unaddressed) then we can write

$$\sum_{s,c} \left| A\,(gg \to H) \right|^2 \quad = \quad s\,F(s),$$

$$F(s) \quad = \quad \frac{\alpha_S^2}{\pi^2} \frac{G_F\, s}{288\,\sqrt{2}} \sum_q f\,(\tau_q)\;(1 + \delta_{\rm QCD})\,,$$

where $\tau_q = 4\,m_q^2/s$ and where $\delta_{\rm QCD}$ gives the QCD corrections to $gg \to H$ up to NNLO + NLL order.

Furthermore, we define

$$\Gamma_{H\to f} = \frac{1}{\sqrt{s}} \int d\Phi_{H\to f} \sum_{s,c} \left| A\left(H \to f\right) \right|^2,$$

which gives the partial decay width of a Higgs boson of virtuality s into a final state f.

$$\sigma_{gg\to H} = \frac{F(s)}{s},$$

which gives the production cross-section of a Higgs boson of virtuality s.

PO

We can write the final result in terms of pseudo-observables

$$\sigma\left(gg \to H \to f\right) = \sigma_{gg \to H} \, \frac{s^2}{\left|s - s_H\right|^2} \, \frac{\Gamma_{H \to f}}{\sqrt{s}} .$$

It is also convenient to rewrite the result as

$$\sigma\left(gg \to H \to f\right) = \sigma_{gg \to H} \, \frac{s^2}{\left|s - s_H\right|^2} \, \frac{\Gamma_H^{\mathrm{T}}}{\sqrt{s}} \, \mathrm{BR}\left(H \to f\right) ,$$

where we have introduced

$$\Gamma_H^{\mathrm{T}} = \sum_f \Gamma\left(H \to f\right) .$$

Note that we have written

the phase-space integral for $g(p_1) + g(p_2) \to f$ as

$$\begin{aligned}
\int d\Phi_{gg\to f} &= \int \prod_f d^4p_f\, \delta^+(p_f^2)\, \delta^4(p_1 + p_2 - \sum_f p_f) \\
&= \int d^4k\, \delta^4(k - p_1 - p_2) \\
&\times \int \prod_f d^4p_f\, \delta^+(p_f^2)\, \delta^4(k - \sum_f p_f),
\end{aligned}$$

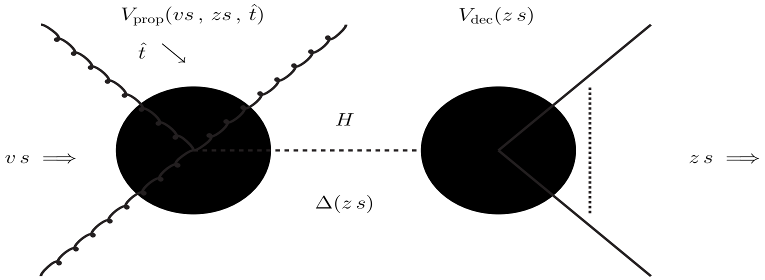
where we assume that all final states are massless.

We define an off-shell production cross-section

(for all channels) as follows:

$$\sigma^{\text{prop}}_{(\text{all channels})} = \sigma_{gg\to H}\,\frac{s^2}{\left|s - s_H\right|^2}\,\frac{\Gamma^{\mathrm{T}}_H}{\sqrt{s}}.$$

When the cross-section $gg \to H$ refers to an off-shell Higgs boson the choice of the QCD scales should be made according to the virtuality of the produced state and not to fixed value.



$$\Longrightarrow\;\; \sigma_{gg\to H+X}(vs\,,\;\hat{t}\,,\,zs)\,\frac{(zs)^2}{\Big|zs\,-\,s_H\Big|^2}\,\frac{\Gamma_{H\to f}(zs)}{(zs)^{1/2}}$$

Therefore

- for the PDFs and $\sigma_{gg\to H+X}$ one should select $\mu_F^2 = \mu_R^2 = v\,s/2$ ($v\,s$ being the invariant mass of the incoming gluons).

The off-shell Higgs production

is currently computed according to

$$\sigma_{OS}(\mu_H^2)\,\delta(z\,\hat{s}-\mu_H^2) \quad\Longrightarrow\quad \sigma_{OFS}(z\,\hat{s})\,\mathrm{BW}(z\,\hat{s}),$$

at least at lowest QCD order, where the *so-called* modified Breit–Wigner distributions is defined by

$$\mathrm{BW}(s) \quad = \quad \frac{1}{\pi}\,\frac{s\,\Gamma_H^{OS}/\mu_H^2}{\left[s-\mu_H^2\right]^2+(s\,\Gamma_H^{OS}/\mu_H)^2},$$

where now $\mu_H = M_H^{OS}$.

This ad-hoc Breit–Wigner

- cannot be derived from QFT and also is not normalizable in $[0\,,\,+\infty]$.
- Its practical purpose is to enforce a *physical* behavior for low virtualities of the Higgs boson but the usage cannot be justified.
- This modified Breit–Wigner cannot be derived from QFT and also is not normalizable in $[0\,,\,+\infty]$.
- Note that this Breit–Wigner for a running width comes from the substitution of $\Gamma \to \Gamma_{(}s) = \Gamma\, s/M^2$ in the Breit–Wigner for a fixed width Γ. This substitution is not justifiable.

Comment:

- Another important issue is that Γ_h which appears in the imaginary part of the inverse Dyson-resummed propagator is not the on-shell width since they differ by higher-order terms and their relations becomes non-perturbative when the on-shell width becomes of the same order of the on-shell mass (for on-shell masses above 800 GeV).

- The use of the complex pole is recommended even if the accuracy at which its imaginary part can be computed is not of the same quality as the NLO accuracy of the on-shell width.

Heavy Higgs cross section and line shape

- C. Anastasiou, S. Buehler, F. Herzog, A. Lazopoulos, arXiv:1107.0683

- **Default option** in iHixs

$$\hat{\sigma}_{ij\to\{H_{\rm final}\}+X}(\hat{s},\mu_f) = \int_{Q_a^2}^{Q_b^2} dQ^2 \frac{Q\Gamma_H(Q)}{\pi} \frac{\hat{\sigma}_{ij\to H}(\hat{s},Q^2,\mu_f)\mathrm{Br}_{H\to\{H_{\rm final}\}}(Q)}{(Q^2-m_H^2)^2+m_H^2\Gamma_H^2(m_H)}. \quad (4.6)$$

- **Seymour option** in iHixs
  - Resummation of VV!VV scattering.
  - Improved s-ch approximation

$$\frac{i}{\hat{s}-m_H^2} \to \frac{i\frac{m_H^2}{\hat{s}}}{\hat{s}-m_H^2+i\Gamma_H(m_H^2)\frac{\hat{s}}{m_H}}. \quad (4.9)$$

$$\hat{\sigma}_{ij\to\{H_{\rm final}\}+X}(\hat{s},\mu_f) = \int_{y_a}^{y_b} dy \frac{Q\Gamma_H(Q)}{m_H\Gamma(m_H)} \hat{\sigma}_{ij\to H}(\hat{s},Q^2,\mu_f)\mathrm{Br}_{H\to\{H_{\rm final}\}}(Q) \times f_{\rm seym}(Q,m_H), \quad (4.10)$$

$$f_{seym}(Q^2,m_H^2) \equiv \frac{m_H^4}{Q^4} \frac{\left(1-\frac{Q^2}{m_H^2}\right)^2+\delta^2}{\left(1-\frac{Q^2}{m_H^2}\right)^2+\delta^2\frac{Q^4}{m_H^4}} \quad (4.11)$$

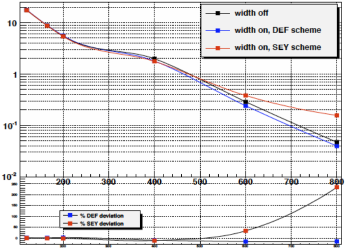


Figure 6: Comparison of the total cross section in the zero width approximation, σ^{ZWA}, with a finite width in the default scheme, σ^{DEF} and in the Seymour scheme, σ^S. In the lower panel we show the relative error one makes when adopting the ZWA, defined as $\frac{\sigma-\sigma^{ZWA}}{\sigma^{ZWA}}\cdot 100\%$

m_H	Γ_H	σ^{ZWA}	σ^{DEF}	σ^S
120	0.0038	17.57	17.66	17.57
165	0.2432	8.78	8.874	8.735
200	1.43	5.45	5.566	5.390
400	29.5	1.988	1.799	1.766
600	122	0.287	0.2409	0.3819
800	301	0.04708	0.03982	0.15683

Table 1: Total cross section for LHC at $\sqrt{s}=7$TeV with MSTW PDFs with a finite width, σ, and in the zero width approximation denoted by σ^{ZWA}.

Deviations wrt zero-width approximation (ZWA) are +30% ~ -20% difference in XS for M_H<600GeV

C. Anastasiou, S. Buehler, F. Herzog, A. Lazopoulos, arXiv:1107.0683

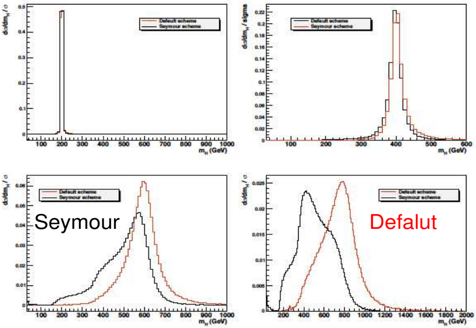


Figure 7: The invariant mass distribution of the Higgs boson with m_H = 200,400,600,800 GeV, in the default and the Seymour scheme.

Proposal:

uncertainty = 150 x M_H^3[%] (M_H in TeV)

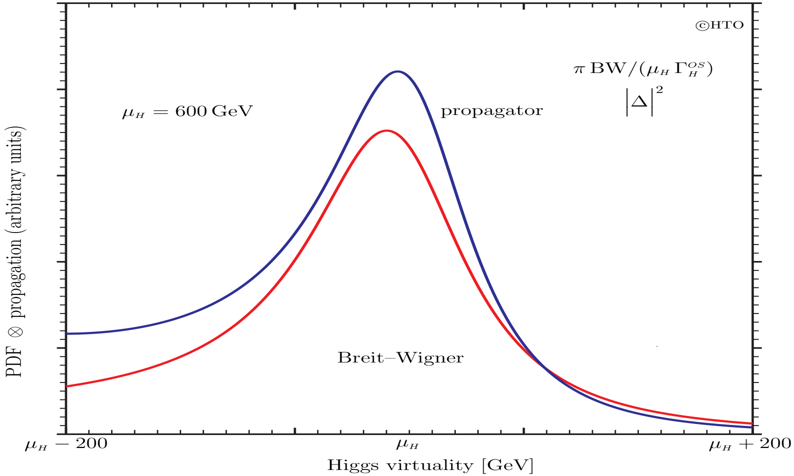
M_H [GeV]	150xM_H^3 [%]
200	1%
400	10%
600	32%
800	77%

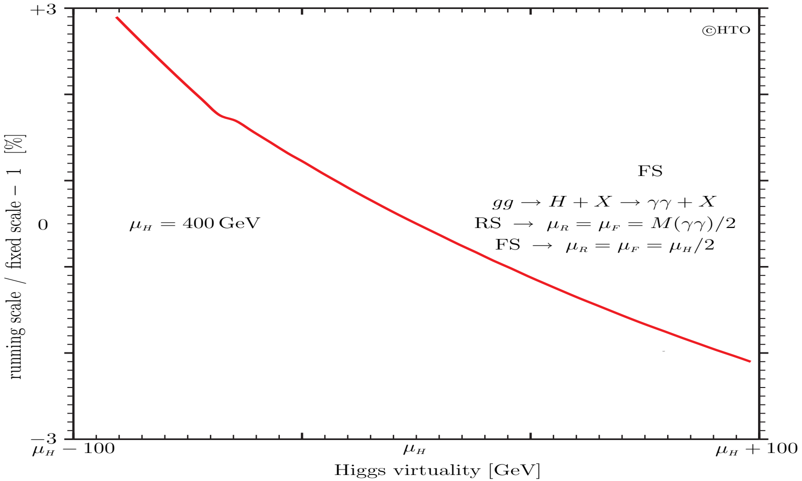
- Large distortion in Higgs invariant mass for heavy Higgs.
  - Seymour scheme tries to simulate the effects of signal-background interference off the resonant peak. Default scheme for purely Higgs signal cross-section.

Legenda

Abb.

FW	Breit–Wigner Fixed Width
RW	Breit–Wigner Running Width
OS	parameters in On-Shell scheme
Bar	parameters in Bar-scheme
FS	Ren (fact) scales fixed
RS	Ren (fact) scales running (virtuality)


©HTO
$\pi\,\mathrm{BW}/(\mu_H\,\Gamma_H^{OS})$
$|\Delta|^2$
$\mu_H = 600\,\mathrm{GeV}$
propagator
Breit–Wigner
PDF ⊗ propagation (arbitrary units)
$\mu_H - 200$
μ_H
$\mu_H + 200$
Higgs virtuality [GeV]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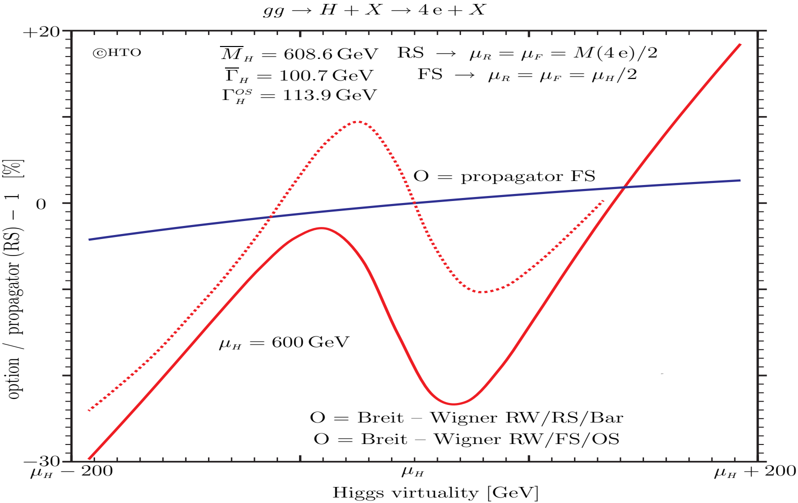
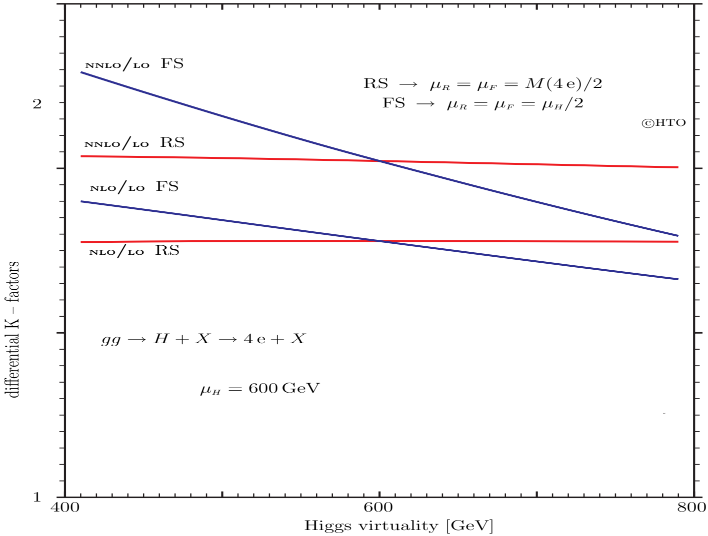
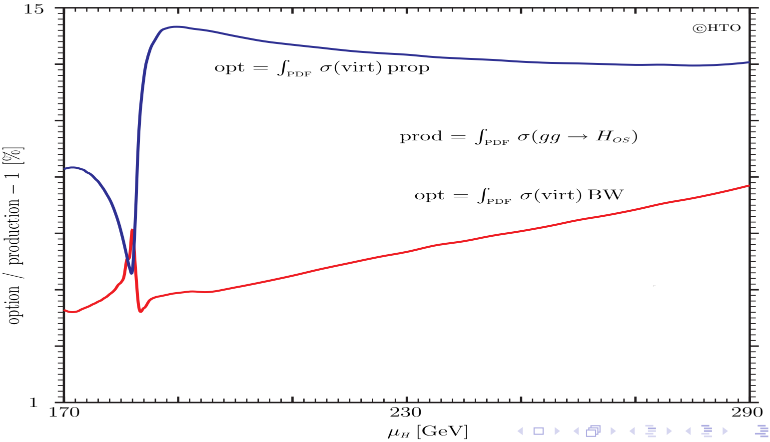
FS
$gg \to H + X \to \gamma\gamma + X$
RS $\to$ $\mu_R = \mu_F = M(\gamma\gamma)/2$
FS $\to$ $\mu_R = \mu_F = \mu_H/2$
$\mu_H = 400$ GeV
+3
0
−3
running scale / fixed scale − 1 [%]
$\mu_H - 100$
μ_H
$\mu_H + 100$
Higgs virtuality [GeV]


$gg \to H + X \to 4\,\mathrm{e} + X$
©HTO
$\overline{M}_H = 608.6$ GeV
$\overline{\Gamma}_H = 100.7$ GeV
$\Gamma_H^{OS} = 113.9$ GeV
RS $\to$ $\mu_R = \mu_F = M(4\,\mathrm{e})/2$
FS $\to$ $\mu_R = \mu_F = \mu_H/2$
O = propagator FS
$\mu_H = 600$ GeV
O = Breit − Wigner RW/RS/Bar
O = Breit − Wigner RW/FS/OS
+20
0
−30
option / propagator (RS) − 1 [%]
$\mu_H - 200$
μ_H
$\mu_H + 200$
Higgs virtuality [GeV]


NNLO/LO FS
NNLO/LO RS
NLO/LO FS
NLO/LO RS
RS $\longrightarrow$ $\mu_R = \mu_F = M(4\,e)/2$
FS $\longrightarrow$ $\mu_R = \mu_F = \mu_H/2$
©HTO
$gg \to H + X \to 4\,e + X$
$\mu_H = 600\,\mathrm{GeV}$
differential K − factors
Higgs virtuality [GeV]
2
1
400
600
800

Backup


15
1
170
230
290
μ_H [GeV]
option / production − 1 [%]
©HTO
$\text{opt} = \int_{\text{PDF}} \sigma(\text{virt})\,\text{prop}$
$\text{prod} = \int_{\text{PDF}} \sigma(gg \to H_{OS})$
$\text{opt} = \int_{\text{PDF}} \sigma(\text{virt})\,\text{BW}$

Conclusions

+

- LHC excludes
  - 155 − 206 GeV 295 − 450 GeV
- LHC combination will exclude much more
- 144 GeV $\sim 2.9\,\sigma$ in both experiments

−

- No definite prescription on Higgs lineshape exists

that all true believers break their eggs at the convenient end.

Jonathan Swift's Travels into Several Remote Nations of the World

Inevitable Surprises