# Soft gluon resummation in $t\bar{t}$ production at hadron colliders

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# Outline

- soft gluon resummation in $t\bar{t}$ production
- comparison of different NNLL calculations for total cross section
- FB asymmetry and differential distributions

*Based on series of papers on resummation at NNLL in SCET with*
**V. Ahrens, A. Ferroglia, M. Neubert, and L. L. Yang**

arXiv:1106.6051 [hep-ph]
arXiv:1105. 5824 [hep-ph] +computer program for total cross section
arXiv:1103.0550 [hep-ph]
JHEP 1009 (2010) 097 (arXiv:1003.5827 [hep-ph])
Phys.Lett. B687 (2010) 331-337 (arXiv:0912.3375 [hep-ph])

Two routes:

1) NNLO in fixed order

- goal: calculate all $\mathcal{O}(\alpha_s^2)$ corrections to Born processes

2) soft gluon resummation

Berger, Contopanagos; Bonciani, Catani, Mangano, Nason; Kidonakis, Laenen, Sterman; Kidonakis, Laenen, Moch, Vogt; Langenfeld, Moch, Uwer '08, '09; Czakon, Mitov, Sterman '09; Kidonakis '10; Beneke et. al. '09; Ahrens, Ferroglia, Neubert, BP, Yang ('10, '11)

- goal: calculate (presumably dominant) logarithmic corrections to all orders
- extended from NLL to NNLL in last few years

# Idea of resummation

1) a given $d\hat{\sigma}$ has double logs from real gluon emission in soft limit $\lambda \to 0$

2) using counting $L \equiv \ln\lambda \sim 1/\alpha_s$, can re-organize the perturbative series as

$$
\begin{aligned}
d\hat{\sigma} \quad &\sim \quad \overbrace{\overbrace{1+\alpha_s(L^2+L+1+\mathcal{O}(\lambda))}^{NLO}+\alpha_s^2(L^4+L^3+L^2+L+1+\mathcal{O}(\lambda))}^{approx.NNLO}+\mathcal{O}(\alpha_s^3) \\
&\sim \quad \exp\Big(\underbrace{\underbrace{Lg_1(\alpha_s L)+g_2(\alpha_s L)}_{NLL}+\alpha_s g_3(\alpha_s L)}_{NNLL}+\cdots\Big)\underbrace{C(\alpha_s)}_{\text{constants}}+\mathcal{O}(\lambda)
\end{aligned}
$$

- resummed formulas (NLL, NNLL, etc.) exponentiate large logs
- approximate NNLO formulas keep logarithmic parts of full correction

# Resummation in three soft limits

| Name | Observable | Soft limit |
|---|---|---|
| pair-invariant-mass (PIM) | $d\sigma/dM_{t\bar{t}}dy$ | $(1-z) = 1 - M_{t\bar{t}}^2/\hat{s} \to 0$ |
| single-particle-inclusive (1PI) | $d\sigma/dp_T dy$ | $s_4 = \hat{s} + \hat{t}_1 + \hat{u}_1 \to 0$ |
| production threshold | $\sigma$ | $\beta = \sqrt{1 - 4m_t^2/\hat{s}} \to 0$ |

$$d\sigma = PDFs \otimes [d\hat{\sigma}_{\rm NNLL}(\ln\lambda) + \mathcal{O}(\lambda)]\,; \quad \lambda \in \{\beta,\, s_4,\, 1-z\}$$

- partonic resummation useful for hadronic cross sections only if singular terms in partonic cross sections are enhanced compared to $\mathcal{O}(\lambda)$ terms after convolution with PDFs ("dynamical threshold enhancement")

# NNLL calculations

1) production threshold ($\beta \to 0$, total cross section only)

- Langenfeld, Moch, Uwer '08, '09; Czakon, Mitov, Sterman '09; Beneke, Czakon, Falgari, Mitov, Schwinn '09
- method for joint resummation of soft gluon and Coulomb terms ($\sim \ln^m \beta/\beta^n$) developed in Beneke, Falgari, Schwinn '09

2) PIM kinematics ($M_{t\bar{t}}$ and $y$ distributions in $t\bar{t}$ frame)

- in SCET Ahrens, Ferroglia, Neubert, BP, Yang '10

3) 1PI kinematics ($p_T$ and $y$ distributions in lab frame)

- Kidonakis '10
- in SCET Ahrens, Ferroglia, Neubert, BP, Yang '11

# Applications

- total cross section
  - comparison of results in production threshold, PIM, and 1PI limits using "approximate NNLO formulas"
- forward-backward asymmetry
- differential cross sections

# Approximate NNLO formulas

$$\hat{\sigma}^{(2)\,\text{approx p.t.}}_{ij=q\bar{q},\,gg} = \sum_{m=1}^{4} d_{2m}^{\text{p.t.}} \ln^m \beta + \frac{1}{\beta}\left(c_{22}^{\text{p.t.}} \ln^2 \beta + c_{11}^{\text{p.t.}} \ln \beta + c_{10}^{\text{p.t.}}\right) + \frac{c_{20}^{\text{p.t.}}}{\beta^2} + \hat{R}'^{\,\text{p.t.}}(\beta)$$

$$\hat{\sigma}^{(2)\,\text{approx. 1PI}}_{ij=q\bar{q},gg} = \int d\hat{t}_1 ds_4 \left\{ \sum_{m=0}^{3} d_m^{\text{1PI}} \left[\frac{\ln^m(2E_s(s_4)/\mu_f)}{s_4}\right]_+ + c^{\text{1PI}}\delta(s_4) + \hat{R}'^{\,\text{1PI}}(s_4) \right\}$$

- pieces in blue determined exactly from NNLL (or Coulomb res. for p.t.)
- two schemes appear in literature for 1PI (and PIM) kinematics:

| | scheme | extra terms in $R$ | "damping factor" |
|---|---|---|---|
| Ahrens et al | $1\text{PI}_{\text{SCET}}$: $2E_s(s_4) = \frac{s_4}{\sqrt{m_t^2+s_4}}$ | $\frac{1}{s_4}\ln^n(1+s_4/m_t^2)$ | —— |
| Kidonakis | 1PI: $2E_s(s_4) \approx s_4/m_t$ | —— | $\hat{\sigma}^{(2)} \to \hat{\sigma}^{(2)} \times 2m_t/\sqrt{\hat{s}} = \hat{\sigma}^{(2)} \times \sqrt{1-\beta^2}$ |

- different groups also include different structure of $\ln^n m_t/\mu$ terms (HATHOR includes all, Kidonakis includes $\delta(s_4)$, SCET includes some of $\delta(s_4)$ but not all)

# Results from approximate NNLO formulas

$m_t = 173.1$ GeV, $m_t/2 < \mu_f = \mu_r < 2m_t$, MSTW2008 90% CL

1) production threshold (result from HATHOR Aliev et. al. '10)

| | Tevatron | LHC (7 TeV) |
|---|---|---|
| $\sigma_{\rm NLO}$ (pb) | $6.72^{+0.36\,+0.37}_{-0.76\,-0.24}$ | $159^{+20\,+8}_{-21\,-9}$ |
| $\sigma_{\rm NNLO\,approx}$ | $7.11^{+0.30\,+0.4}_{-0.40\,-0.3}$ | $164^{+3\,+9}_{-9\,-9}$ |

2) 1PI threshold ($m_t$=173 GeV) Kidonakis '10

| | | |
|---|---|---|
| $\sigma_{\rm NNLO\,approx,\,1PI}$ | $7.08^{+0.00\,+0.36}_{-0.24\,-0.24}$ | $163^{+7\,+9}_{-5\,-9}$ |

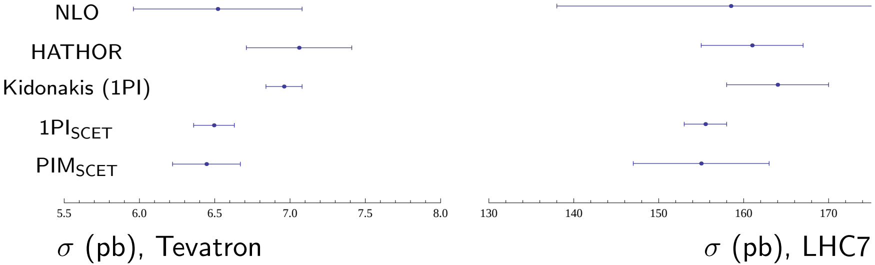
3) PIM and 1PI threshold in SCET Ahrens et.al. '10, '11

| | | |
|---|---|---|
| $\sigma_{\rm NNLO\ approx,\ 1PI_{SCET}}$ | $6.63^{+0.00\,+0.33}_{-0.27\,-0.24}$ | $155^{+3\,+8}_{-2\,-9}$ |
| $\sigma_{\rm NNLO\ approx,\ PIM_{SCET}}$ | $6.62^{+0.05\,+0.33}_{-0.40\,-0.24}$ | $155^{+8\,+8}_{-8\,-9}$ |

Note: large discrepancy between PIM and 1PI kinematics observed in Kidonakis et. al. '01 not present in SCET calculation.

# NNLO approximations at Tevatron and LHC7

$m_t = 173.1$ GeV, $m_t/2 < \mu_f = \mu_r < 2m_t$, MSTW2008




- scale variation not necessarily good indication of uncertainties from subleading terms in soft limits
- can either use NLO, take envelope of the three NNLO approximations, or make arguments for a particular NNLO approximation

# Comparing $\beta \to 0$ (HATHOR) vs. 1PI or PIM

Phase space for $\beta \to 0$ is special case of 1PI and PIM thresholds

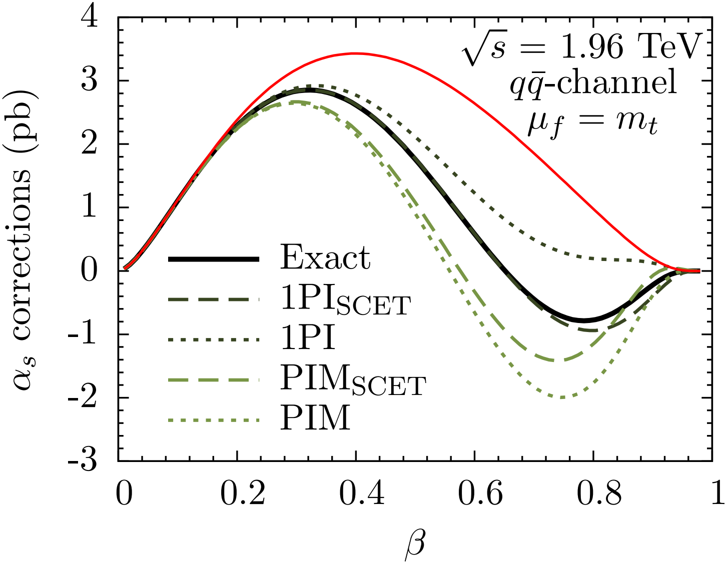
PIM/1PI contain extra subleading terms in $\beta$, but (at NNLL) miss some $\ln^n \beta/\beta^m$ related to Coulomb resummation. Numerically ($\mu = m_t$):

$$
\begin{array}{lccc}
 & \hat{\sigma}^{1\text{PI}_\text{SCET},\text{PIM}_\text{SCET}}(\beta) & -\hat{\sigma}^{\text{HATHOR}}(\beta) = & \mathcal{O}(\ln^n \beta/\beta^m) + \mathcal{O}(1,\beta) \\
\text{Tevatron [pb]} & 6.63 & -7.11 = & +0.07 - 0.55
\end{array}
$$

- subleading terms in $\beta$ not generically small

$\Rightarrow$ to use HATHOR must argue that approximation in $\beta \to 0$ limit does not receive power corrections of generic size, and that subleading terms in $\text{PIM}_\text{SCET}/1\text{PI}_\text{SCET}$ are unphysical

# Approximate vs. exact NLO (red = $\beta \to 0$)



# Forward-backward asymmetry

$$A^i_{\mathrm{FB}} = \frac{N_t(y^i > 0) - N_t(y^i < 0)}{N_t(y^i > 0) + N_t(y^i < 0)} = \frac{\sigma_A}{\sigma_S}$$

Tevatron:

- total asymmetry measured in $i = p\bar{p}$ or $t\bar{t}$ rest frame
- also with cuts on $M_{t\bar{t}}$ and $\Delta y = y_t - y_{\bar{t}}$ in $t\bar{t}$ frame

LHC:

- $A_{\mathrm{FB}} = 0$, because initial state is symmetric
- can define non-vanishing (differential) charge asymmetries

# The forward-backward asymmetry in the Standard Model

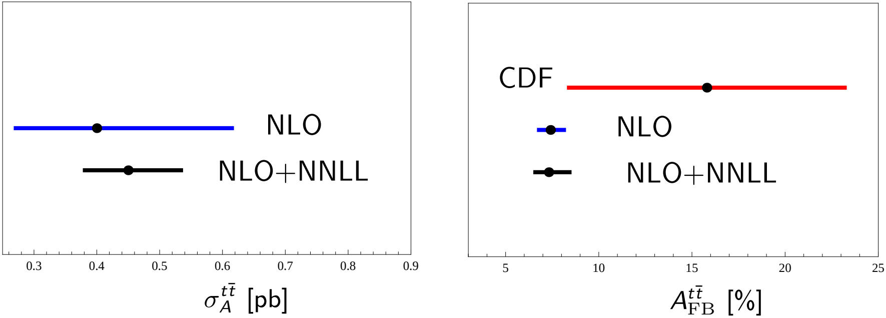
$$A_{\rm FB} = \frac{\sigma_A}{\sigma_S} = \frac{\alpha_s^3 \sigma^{(0)}_{A,q\bar{q}} + \alpha_s^4 \sigma^{(1)}_{A,q\bar{q}} + \dots}{\alpha_s^2 \sigma^{(0)}_S + \alpha_s^3 \sigma^{(1)}_S + \dots} = \alpha_s A^{(0)}_{\rm FB} + \dots$$

Status of Standard Model calculations for $A_{\rm FB}$

- $A^{(0)}_{\rm FB}$ in [Kuhn,Rodrigo 1998] (will call "NLO" since uses $d\sigma$ at NLO)
- (mixed QCD)-electroweak corrections
  [Kuhn, Rodrigo 1998],[Bernreuther, Si 2010], [Hollik, Pagani 2011]
- $A_{\rm FB}$ to NLO + NLL in $t\bar{t}$ frame with cuts on $M_{t\bar{t}}$
  [Almeida, Sterman, Vogelsang 2008]
- $A_{\rm FB}$ to NLO + NNLL with cuts on $\Delta y$, $M_{t\bar{t}}$ in $t\bar{t}$ frame, cuts on $y$ in lab frame, also differential charge asymmetry at LHC
  [Ahrens, Ferroglia, Neubert, BP, Yang 2011]

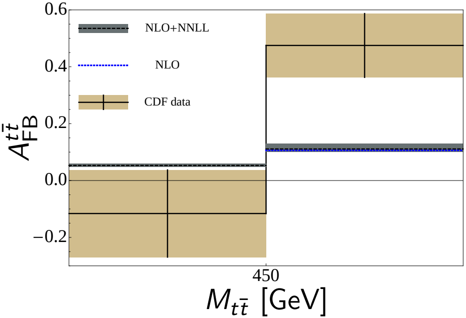
# Total asymmetry in $t\bar{t}$ frame

$m_t = 173.1$ GeV, $m_t/2 < \mu_f = \mu_r < 2m_t$ , MSTW2008 90% CL




- resummation roughly halves scale dependence in $\sigma_A$ and $\sigma$ compared to NLO, but scale dependence of $A_{\mathrm{FB}}$ somewhat larger
- theory and experiment agree at about $1\sigma$

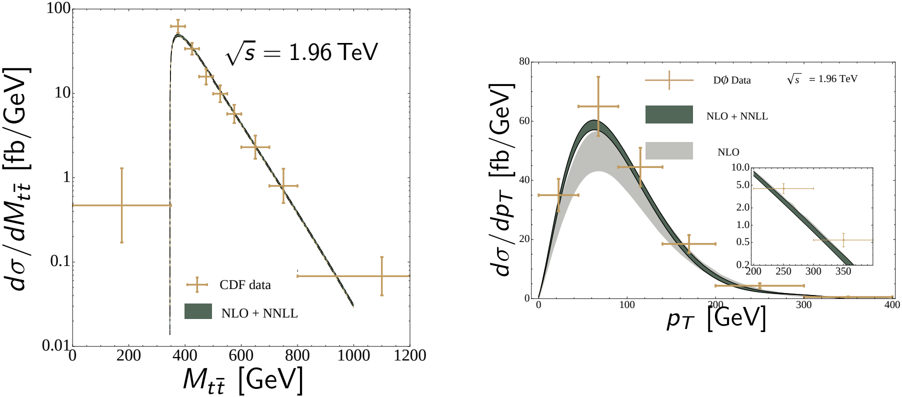
# $A_{\text{FB}}$ WITH CUTS ON $M_{t\bar{t}}$




| | $M_{t\bar{t}} \leq 450$ GeV | | $M_{t\bar{t}} > 450$ GeV | |
|---|---|---|---|---|
| | $\sigma_A^{tt}$ [pb] | $A_{\text{FB}}^{tt}$[%] | $\sigma_A^{tt}$ [pb] | $A_{\text{FB}}^{tt}$[%] |
| CDF | | $-11.6^{+15.3}_{-15.3}$ | | $47.5^{+11.2}_{-11.2}$ |
| NLO | $0.17^{+0.08+0.02}_{-0.05-0.00}$ | $5.3^{+0.3+0.1}_{-0.4-0.1}$ | $0.21^{+0.12+0.02}_{-0.07-0.00}$ | $10.6^{+1.1+0.3}_{-0.8-0.1}$ |
| NLO+NNLL | $0.21^{+0.04+0.02}_{-0.03-0.00}$ | $5.2^{+0.7+0.1}_{-0.5-0.0}$ | $0.24^{+0.05+0.02}_{-0.04-0.00}$ | $11.1^{+1.9+0.3}_{-1.0-0.0}$ |

Resummation at NNLL is mild effect [Ahrens,Ferroglia,Neubert,BP,Yang]

# Invariant mass and $p_T$ distributions at NLO+NNLL vs. Tevatron data



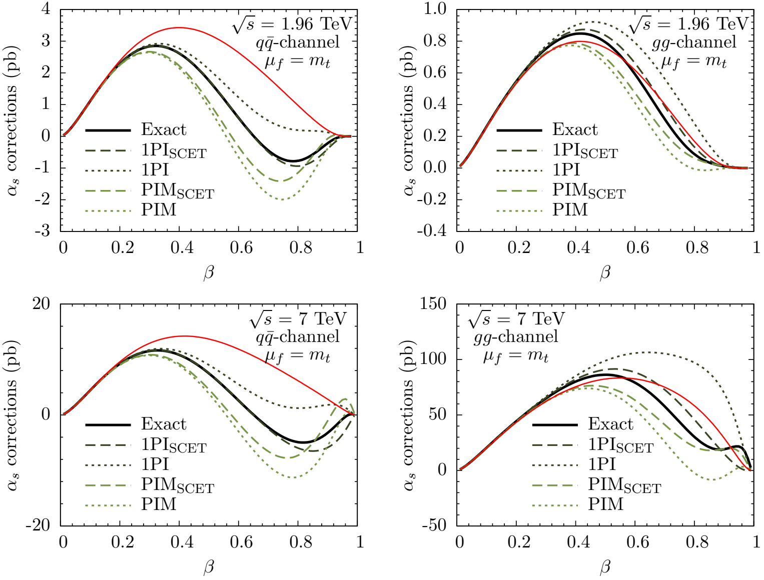

- $M_{t\bar{t}}$ and $p_T$ distributions at NLO+NNLL from [Ahrens et. al.]
- normalization and shape of distributions consistent with data
- can also study rapidity distributions (but no measurements so far)

# Summary

- Two routes beyond NLO
  - NNLO in fixed order (in progress)
  - soft gluon resummation to NLO+NNLL $\leftrightarrow$ approximate NNLO (known in three different soft limits: production threshold, 1PI, PIM)
- NLO+NNLL calculations have reduced scale uncertainties compared to NLO, but can question whether these reliably estimate uncalculated contributions in different limits
- Higher-order QCD corrections from soft gluon resummation do not explain current discrepancies in $A_{\mathrm{FB}}$ at the Tevatron

backup slides

# Approximate vs. exact NLO (red = $\beta \to 0$)

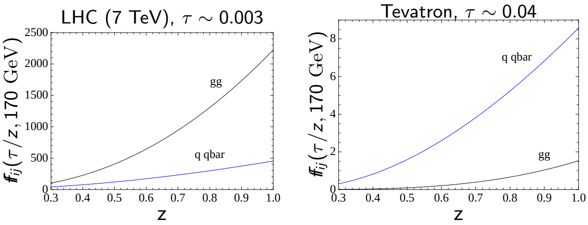


# Dynamical threshold enhancement

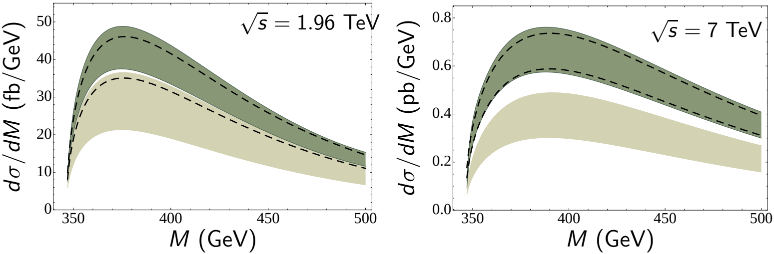
$$\frac{d\sigma}{dM_{t\bar{t}}} \sim \sum_{i,j} \int_\tau^1 \frac{dz}{z}\, ff_{ij}(\tau/z, \mu_f) \left[ \delta(1-z) C_0^{ij} + \sum_m \sum_{n \le 2m-1} \alpha_s^m d_{mn}^{ij} \left[ \frac{\ln^n(1-z)}{1-z} \right]_+ + \dots \right]$$

Leading terms in $z \longrightarrow 1$ limit dominant if:

- $\tau = M_{t\bar{t}}^2/s \longrightarrow 1$ (high invariant mass)
- $ff_{ij}(\tau/z, \mu)$ largest as $z \longrightarrow 1$, even if $\tau$ not close to 1 ("dynamical threshold enhancement")



# Dominance of soft gluon corrections at NLO




- green band = exact fixed order at NLO ($\mu_f = 200$, 800 GeV)
- dashed lines = leading terms for $z \to 1$ at NLO ($\mu_f = 200$, 800 GeV)

Soft gluon corrections dominate cross section even at low $M_{t\bar{t}}$