RESUMMATION IN HARD QCD PROCESSES

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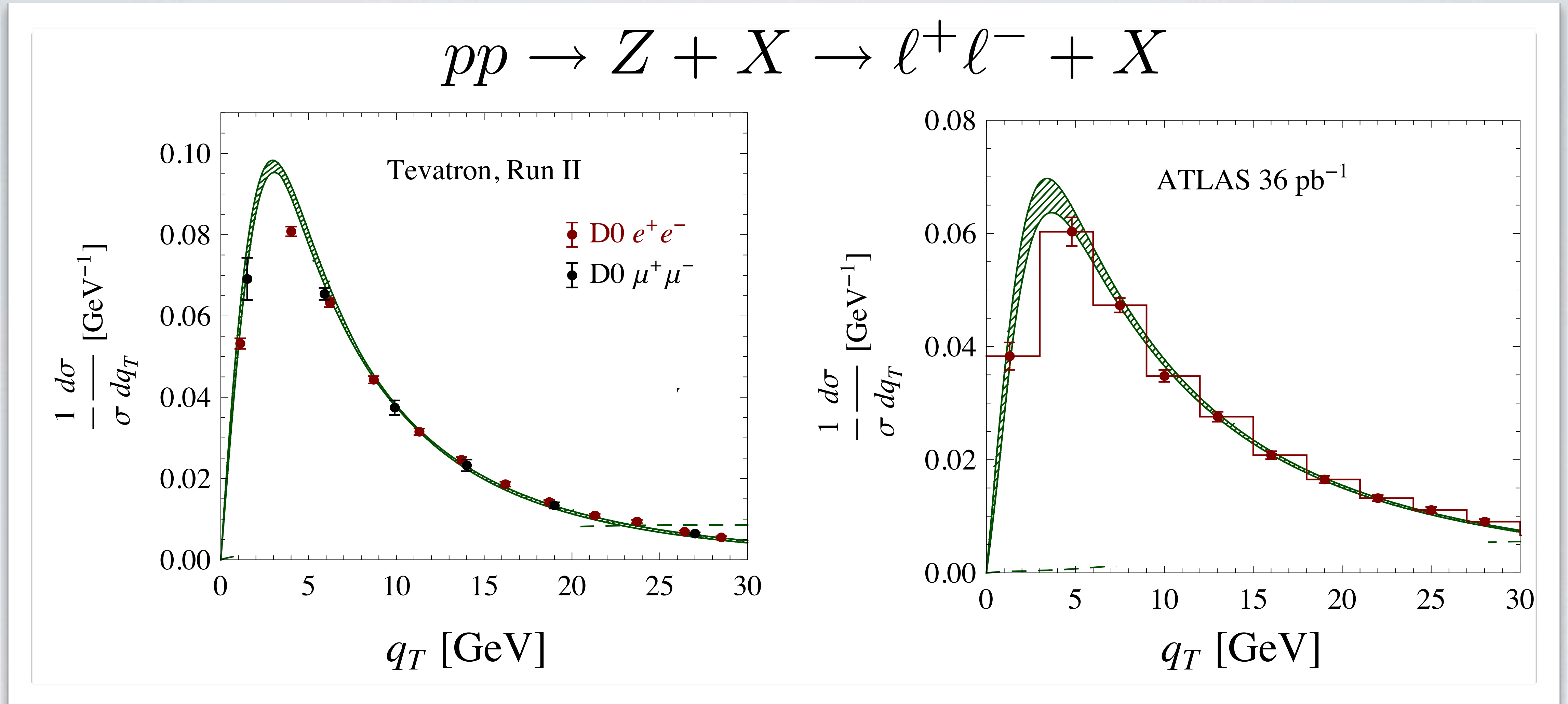
80’ revival: recent work on transverse momentum resummation

Walter Giele assigned me a quite general title “Resummation effects in hard QCD”. Since Lorenzo Magnea gave a comprehensive review of all recent work in this field, I will focus on the above topic.

TRANSVERSE MOMENTUM SPECTRUM



q_T spectrum of electroweak bosons (i.e. Z,W, γ^*, H) is one of the most basic quantities that can be measured at hadron collider.

Benchmark process, important for W-mass determination, Higgs search. New results both from Tevatron and LHC

PERTURBATIVE EXPANSION

The perturbative expansion of the q_T spectrum contains singular terms of the form (M is the invariant mass of the lepton pair)

$$\frac{d\sigma}{dq_T^2} = \frac{1}{q_T^2}\Big[A_1^{(1)}\alpha_s \ln\frac{M^2}{q_T^2} + \alpha_s A_0^{(1)} + A_3^{(2)}\alpha_s^2 \ln^3\frac{M^2}{q_T^2} + \dots$$
$$+A_{2n-1}^{(n)}\alpha_s^n \ln^{2n-1}\frac{M^2}{q_T^2} + \dots\Big] + \dots$$

which ruin the perturbative expansion at $q_T \ll M$ and must be resummed to all orders.

Classic example of an observable which needs resummation!
Achieved by Collins, Soper and Sterman (CSS) '84.

PARTY LIKE IT'S 1984

A lot of recent work on transverse momentum resummation

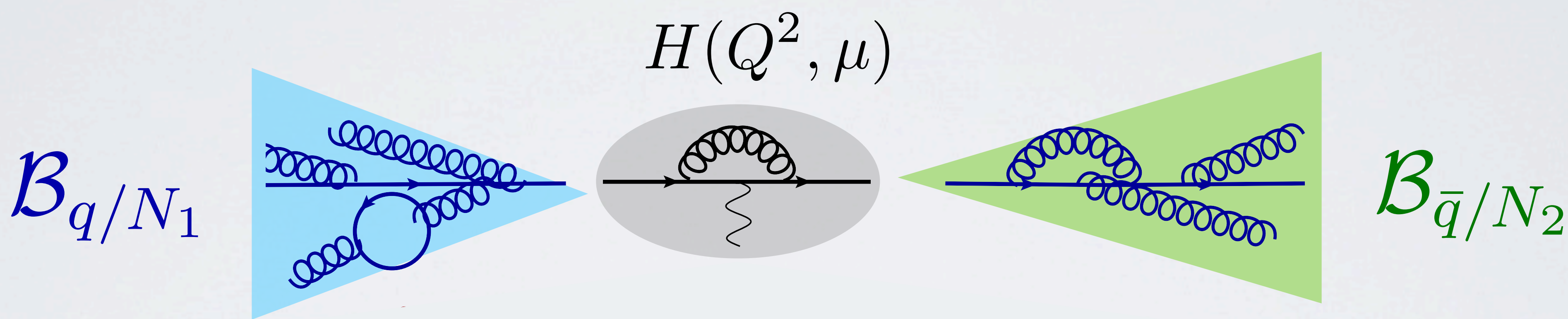
- Higher accuracy.
  - Computation of all singular terms at $O(\alpha_s^2)$ accuracy. Catani and Grazzini '09, '11
  - New NNLL codes (in addition to RESBOS) Bozzi, Catani Ferrera, de Florian, Grazzini '10; TB, Neubert, Wilhelm, in preparation
  - derivation of missing NNLL coefficient $A^{(3)}$ TB, Neubert '10
  - NNLL threshold resummation at large q_T TB, Lorentzen, Schwartz '11
- Factorization of the cross section, definition of transverse PDFs
  - using Soft-Collinear Effective Theory Mantry Petriello '09, '10; TB Neubert '10
  - traditional framework Collins '11

FACTORIZATION ANOMALY

FACTORIZATION

Factorization at low q_T proceeds in two steps

1.) Use $q_T \ll M_Z$ to factorize cross section



"hard function" × "transverse PDF" × "transverse PDF"

2.) Use $\Lambda_{QCD} \ll q_T$ to factorize

$$\mathcal{B}_{i/N}(\xi, x_T^2, \mu) = \sum_j \int_\xi^1 \frac{dz}{z} \mathcal{I}_{i\leftarrow j}(z, x_T^2, \mu)\, \phi_{j/N}(\xi/z, \mu) + \mathcal{O}(\Lambda_{\rm QCD}^2\, x_T^2)$$

"transverse PDF" = "matching coefficient" × "standard PDF"

REGULARIZATION

Well known that transverse PDF

$$\mathcal{B}_{q/N}(z, x_T^2, \mu) = \frac{1}{2\pi} \int dt\, e^{-izt\bar{n}\cdot p} \langle N(p)|\, \bar{\chi}(t\bar{n} + x_\perp)\, \frac{\bar{\not{n}}}{2}\, \chi(0)\, |N(p)\rangle$$

is not defined without additional regulators.

Different possibilites

- Use non-light-like gauge CSS '84
- Keep power suppressed small light-cone component, (i.e. use "fully unintegrated PDF") Mantry Petriello '09
- Following Smirnov '93, we use analytic regulator for collinear propagators TB, Neubert '10
- Multiply with with strategically chosen combination of light-like and time-like Wilson lines. Collins '11

FACTORIZATION ANOMALY

What God has joined together, let no man separate...

Regularization of *individual* PDFs is delicate, but the product of PDFs is well defined and the regulator can be removed.

However, regulator induces dependence on the the hard scale *M*, which remains even when the regulator is sent to zero. We prove that this anomalous *M* dependence exponentiates in the form

$$\left[\mathcal{B}_{q/N_1}(z_1, x_T^2, \mu)\, \mathcal{B}_{\bar{q}/N_2}(z_2, x_T^2, \mu)\right]_{M^2} = \left(\frac{x_T^2 M^2}{4e^{-2\gamma_E}}\right)^{-F_{q\bar{q}}(x_T^2,\mu)} B_{q/N_1}(z_1, x_T^2, \mu)\, B_{\bar{q}/N_2}(z_2, x_T^2, \mu)\,,$$

Anomaly: Classically, $\langle N_1(p)|\, \bar{\chi}_{hc}(x_+ + x_\perp) \not{\bar{n}}\, \chi_{hc}(0)\, |N_1(p)\rangle$ is invariant under a rescaling of the momentum of the other nucleon N_2. Quantum theory needs regularization. Symmetry cannot be recovered after removing regulator. Not an anomaly of QCD, but of the low energy theory (the factorization theorem).

FACTORIZATION ANOMALY

- RG invariance of the cross section implies presence *M* dependence of product of transverse PDFs. Anomaly exponent must fulfill

$$\frac{dF_{q\bar{q}}(x_T^2,\mu)}{d\ln\mu} = 2\Gamma^F_{\text{cusp}}(\alpha_s)$$

- Anomaly also affects other observables
    - Processes with small masses, e.g. EW Sudakov resummation
    - Jet-broadening. Have derived all-order form of anomaly for small broadening. TB, Bell, Neubert '11
    - Regge limits

RESUMMED RESULT FOR CROSS SECTION

$$\frac{d^3\sigma}{dM^2\,dq_T^2\,dy} = \frac{4\pi\alpha^2}{3N_c M^2 s} \sum_q e_q^2 \sum_{i=q,g} \sum_{j=\bar{q},g} \int_{\xi_1}^1 \frac{dz_1}{z_1} \int_{\xi_2}^1 \frac{dz_2}{z_2}$$

$$\times \left[C_{q\bar{q}\to ij}\left(\frac{\xi_1}{z_1}, \frac{\xi_2}{z_2}, q_T^2, M^2, \mu\right) \phi_{i/N_1}(z_1,\mu)\, \phi_{j/N_2}(z_2,\mu) + (q, i \leftrightarrow \bar{q}, j) \right]$$

The hard-scattering kernel is

$$C_{q\bar{q}\to ij}(z_1, z_2, q_T^2, M^2, \mu) = H(M^2,\mu) \frac{1}{4\pi} \int d^2x_\perp\, e^{-iq_\perp\cdot x_\perp} \left(\frac{x_T^2 M^2}{4e^{-2\gamma_E}}\right)^{-F_{q\bar{q}}(x_T^2,\mu)}$$

$$\times I_{q\leftarrow i}(z_1, x_T^2, \mu)\, I_{\bar{q}\leftarrow j}(z_2, x_T^2, \mu)$$

- Two sources of *M* dependence: hard function and anomaly
- Fourier transform can be evaluated numerically or analytically, if higher-log terms are expanded out.

RELATION TO CSS

If adopt the choice $\mu = \mu_b = 2e^{-\gamma_E}/x_\perp$ in our result reduces to CSS formula, provided we identity (see backup slide for definition of A,B,C)

$$A(\alpha_s) = \Gamma^F_{\text{cusp}}(\alpha_s) - \frac{\beta(\alpha_s)}{2}\frac{dg_1(\alpha_s)}{d\alpha_s},$$

$$B(\alpha_s) = 2\gamma^q(\alpha_s) + g_1(\alpha_s) - \frac{\beta(\alpha_s)}{2}\frac{dg_2(\alpha_s)}{d\alpha_s},$$

$$C_{ij}(z,\alpha_s(\mu_b)) = \left[H(\mu_b^2,\mu_b)\right]^{1/2} I_{i\leftarrow j}(z,0,\alpha_s(\mu_b)),$$

anomaly contribution

$$g_1(\alpha_s) = F(0,\alpha_s)$$

$$g_2(\alpha_s) = \ln H(\mu^2,\mu)$$

Use these relations to derive unknown three-loop coefficient, necessary for NNLL resummation

$$A^{(3)} = \Gamma_2^F + \beta_0\, g_1''(0) = 239.2 - 652.9 \neq \Gamma_2^F$$

Not equal to the cusp anom. dim. as was usually assumed!

DIVERGENT EXPANSIONS, AND OTHER SURPRISES

TRANSVERSE MOMENTUM SPECTRUM

The spectrum has a number of quite remarkable features which we now discuss in turn:

- Expansion in α_s : strong factorial divergence
- q_T-spectrum:
  - calculable, even near $q_T = 0$
  - expansion around $q_T = 0$: extremely divergent
- Long-distance effects associated with Λ_{QCD}
  - small, but OPE breaks down

LEADING MOMENTUM DEPENDENCE

Up to corrections suppressed by powers of α_s, the q_T-dependence of our formula result has the form

$$\frac{1}{4\pi}\int d^2x_\perp\, e^{-iq_\perp\cdot x_\perp}\, e^{-\eta L_\perp - \frac{1}{4}aL_\perp^2} \equiv \frac{e^{-2\gamma_E}}{\mu^2}\, K\Big(\eta, a, \frac{q_T^2}{\mu^2}\Big)$$

with $L_\perp = \ln\dfrac{x_T^2\mu^2}{4e^{-2\gamma_E}}$, and the two quantities

$$\eta = \frac{C_F\alpha_s}{\pi}\ln\frac{M^2}{\mu^2} = \mathcal{O}(1) \qquad\qquad a = \alpha_s(\mu)\times\mathcal{O}(1)$$

Since a is suppressed one can try to expand K in it.

FACTORIAL DIVERGENCE

Unfortunately, the series in *a* is strongly factorially divergent:

$$K(\eta, a, 1)\big|_{\exp} = \sum_{n=0}^{\infty} \boxed{\frac{(2n)!}{n!}} \left(-\frac{a}{4}\right)^n \left[\frac{1}{(1-\eta)^{2n+1}} - e^{-2\gamma_E}\right] + \ldots$$

first noted by Frixione, Nason, Ridolfi '99

Can Borel resum it, which makes the nonperturbative and highly nontrivial *a* dependence explicit

$$K(\eta, a, 1)\big|_{\text{Borel}} = \sqrt{\frac{\pi}{a}} \left\{ e^{\frac{(1-\eta)^2}{a}} \left[1 - \text{Erf}\left(\frac{1-\eta}{\sqrt{a}}\right)\right] - e^{-2\gamma_E + \frac{1}{a}} \left[1 - \text{Erf}\left(\frac{1}{\sqrt{a}}\right)\right]\right\} + \ldots$$

In practice, it is simplest, to use the exact expression and evaluate *K*-function numerically.

VERY LOW Q_T

For moderate q_T, the natural scale choice is $\mu = q_T$. However, detailed analysis shows that near $q_T \approx 0$ the Fourier integral is dominated by

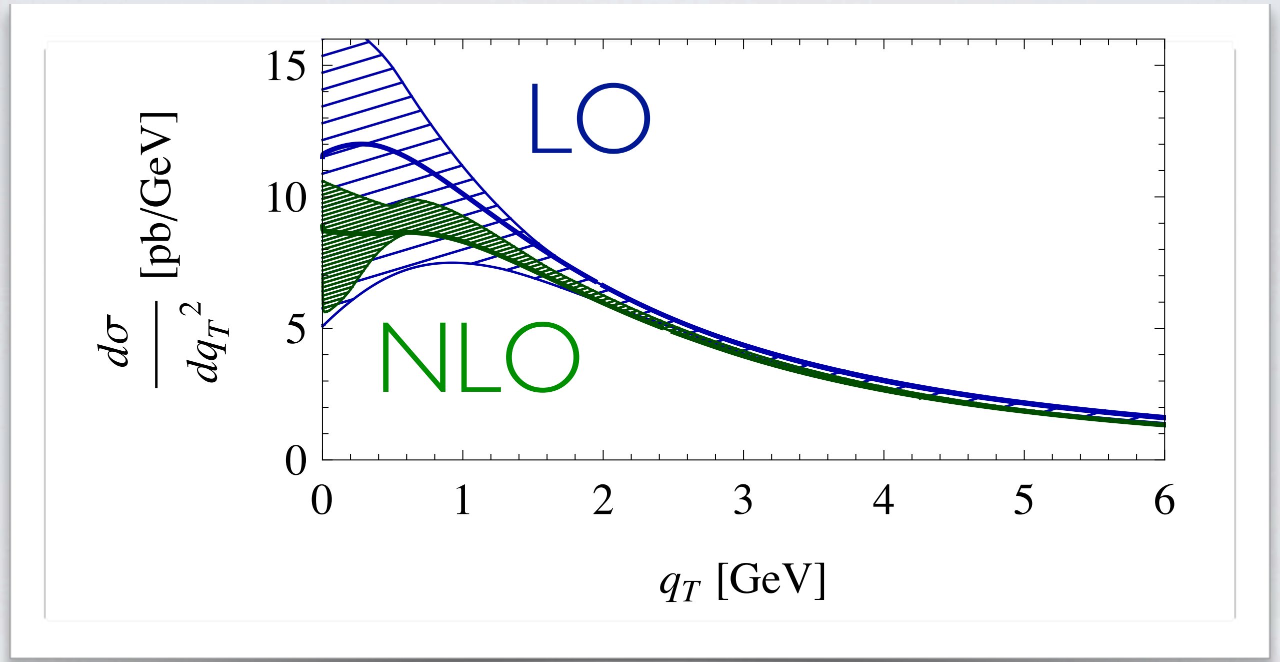
$$\langle x_T^{-1} \rangle = q_* = M \exp\left(-\frac{\pi}{2 C_F \alpha_s(q_*)} \right) = 1.9 \text{ GeV} \quad \text{for } M = M_Z$$

which corresponds to $\eta = 1$.

→ Spectrum can be computed with short-distance methods down to $q_T = 0$!

INTERCEPT AT Q_T=0

$$\frac{d\sigma}{dq_T^2}$$



bands from scale-variation by factor 2

- Dedicated analysis of $q_T \to 0$ limit yields:

$$\frac{d\sigma}{dq_T^2} \sim \frac{\mathcal{N}}{\sqrt{\alpha_s}}\, e^{-\#/\alpha_s}\,(1 + c_1\alpha_s + \dots)$$

Parisi, Petronzio 1979;
Collins, Soper, Sterman 1985; Ellis, Veseli 1998

- Essential singularity at α_s=0 ! We have computed the normalization $\mathcal{N}$ and NLO coefficient c_1 .

SLOPE AT Q_T=0?

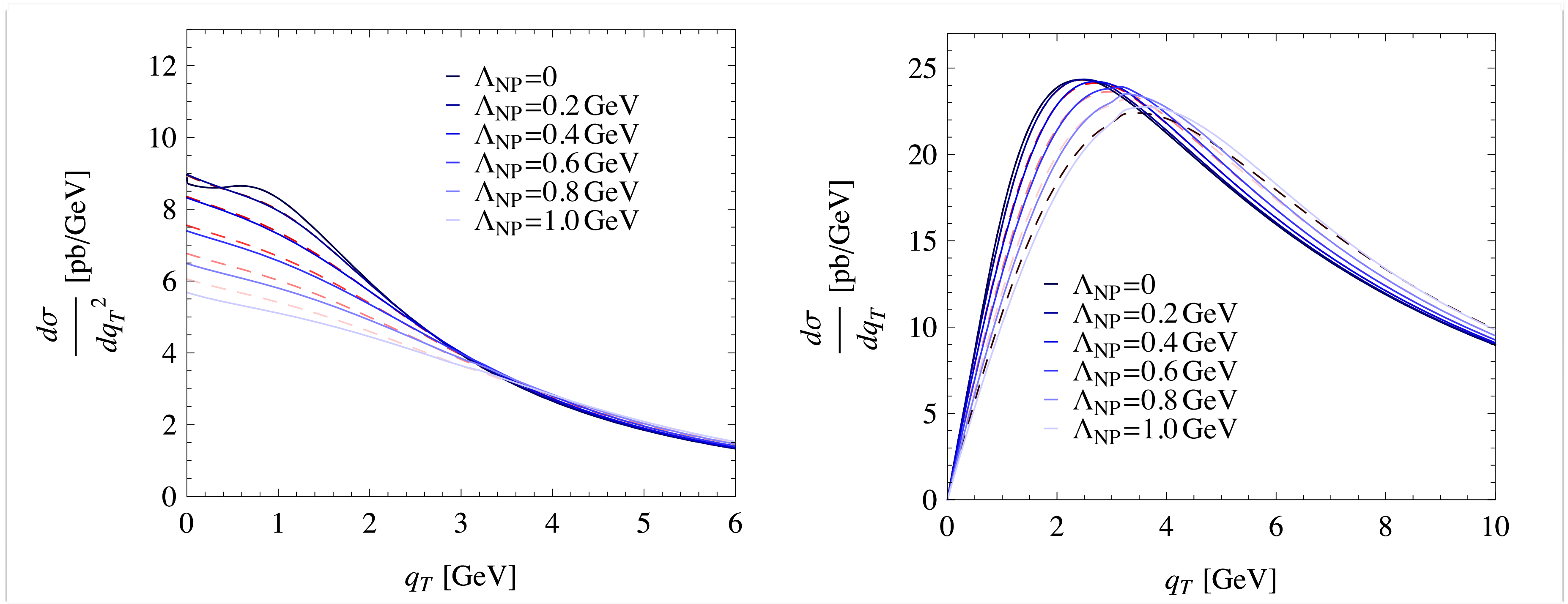
Given our result for the intercept, we can also try to obtain derivatives with respect to q_T^2. Leading term is obtained by expanding

$$\frac{1}{4\pi}\int d^2x_\perp\, e^{-iq_\perp\cdot x_\perp}\, e^{-\eta L_\perp - \frac{1}{4}aL_\perp^2} \equiv \frac{e^{-2\gamma_E}}{\mu^2}\, K\Big(\eta, a, \frac{q_T^2}{\mu^2}\Big)$$

Yields violently divergent series

$$K(\eta = 1, a, q_T)\big|_{\exp} \sim \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\sqrt{a}} e^{n^2/a} \left(\frac{q_T^2}{q_*^2}\right)^{n-1}$$

NON-PERTURBATIVE EFFECTS



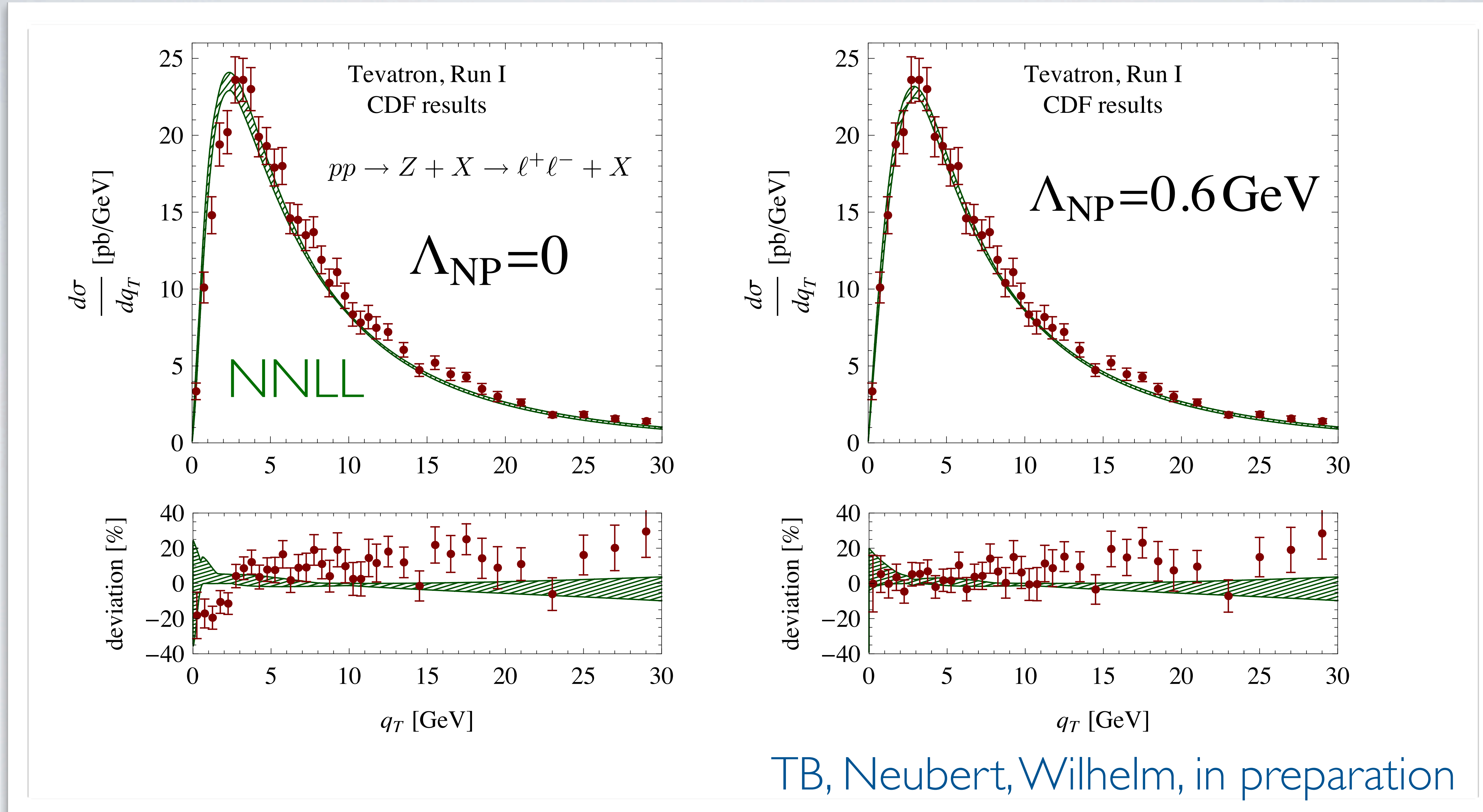
$$B_{q/N}(\xi, x_T^2, \mu) = f_{\rm hadr}(x_T \Lambda_{\rm NP})\, B_{q/N}^{\rm pert}(\xi, x_T^2, \mu)$$

- Blue curves: Gaussian cutoff, red dashed lines: dipole cutoff.

$$f_{\rm hadr}^{\rm gauss}(x_T \Lambda_{\rm NP}) = \exp\left(-\Lambda_{\rm NP}^2\, x_T^2\right), \qquad f_{\rm hadr}^{\rm pole}(x_T \Lambda_{\rm NP}) = \frac{1}{\left(1 + \frac{1}{2}\Lambda_{\rm NP}^2\, x_T^2\right)}$$

- Slight shift of the peak, largely independent of the form of the cutoff

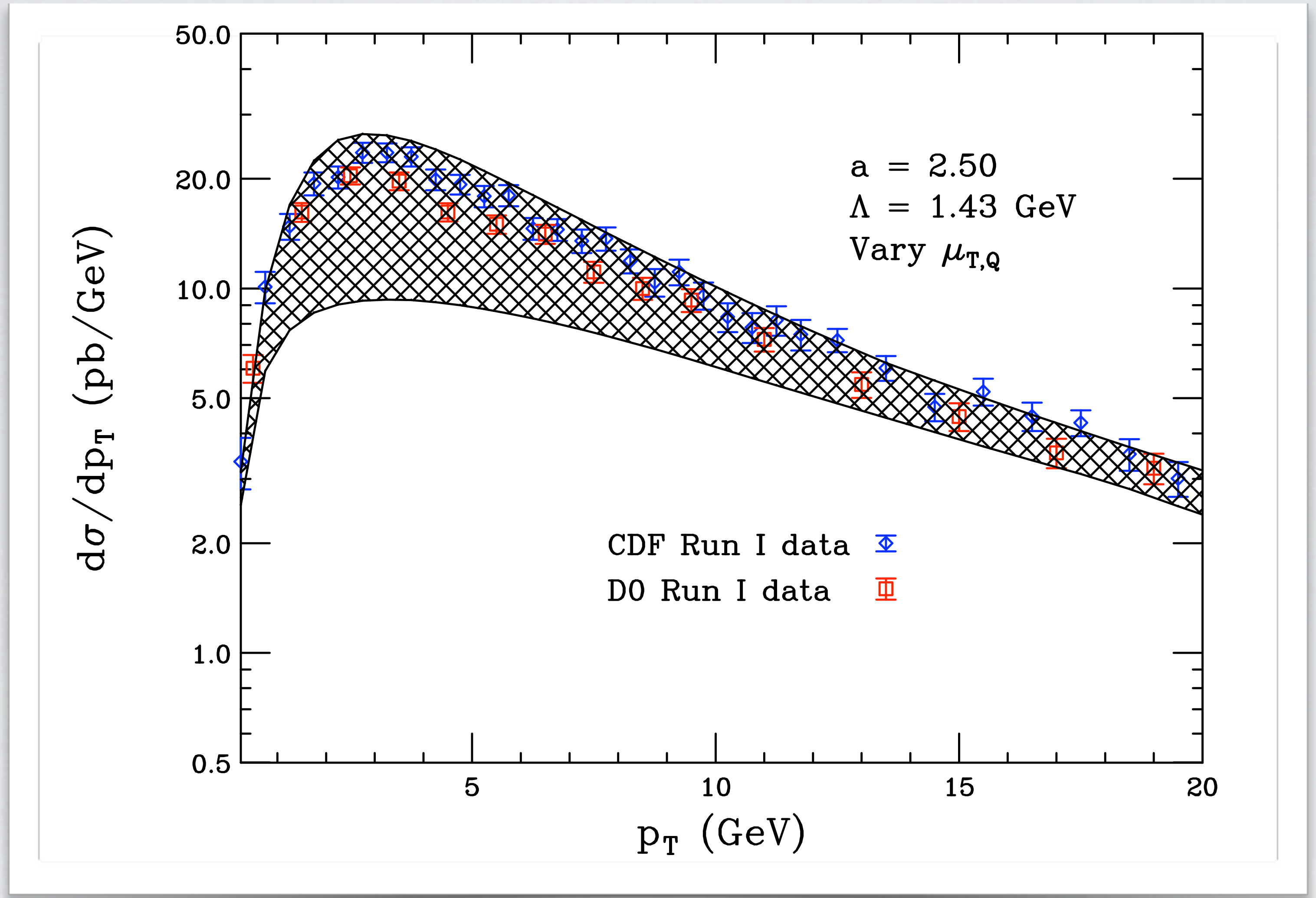
TEVATRON, RUN I



- Scale variation by factor of 2 from default $\mu = q_* + q_T$.

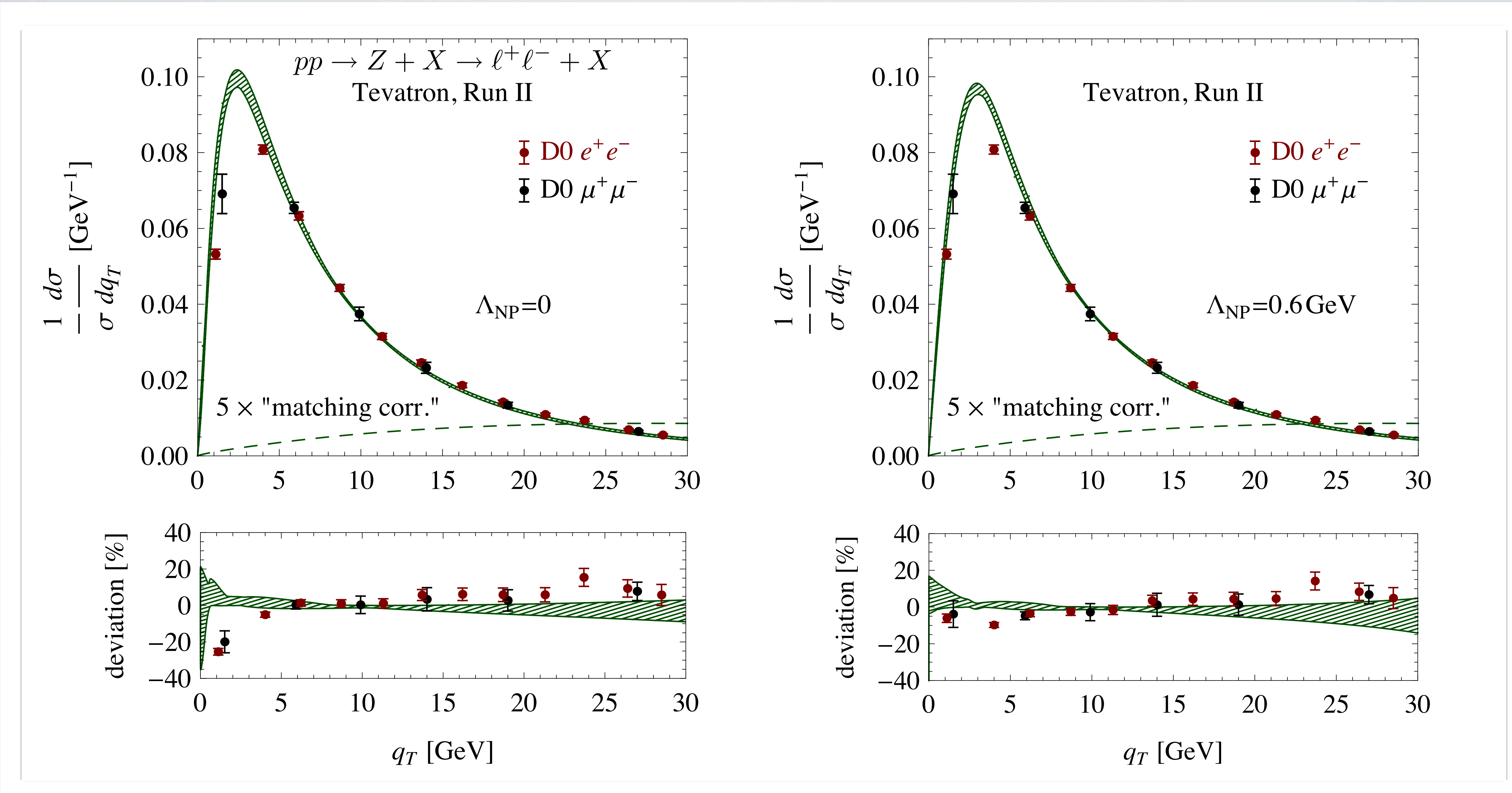
MANTRY AND PETRIELLO

0911.4135, 1007.3773, 1011.0757



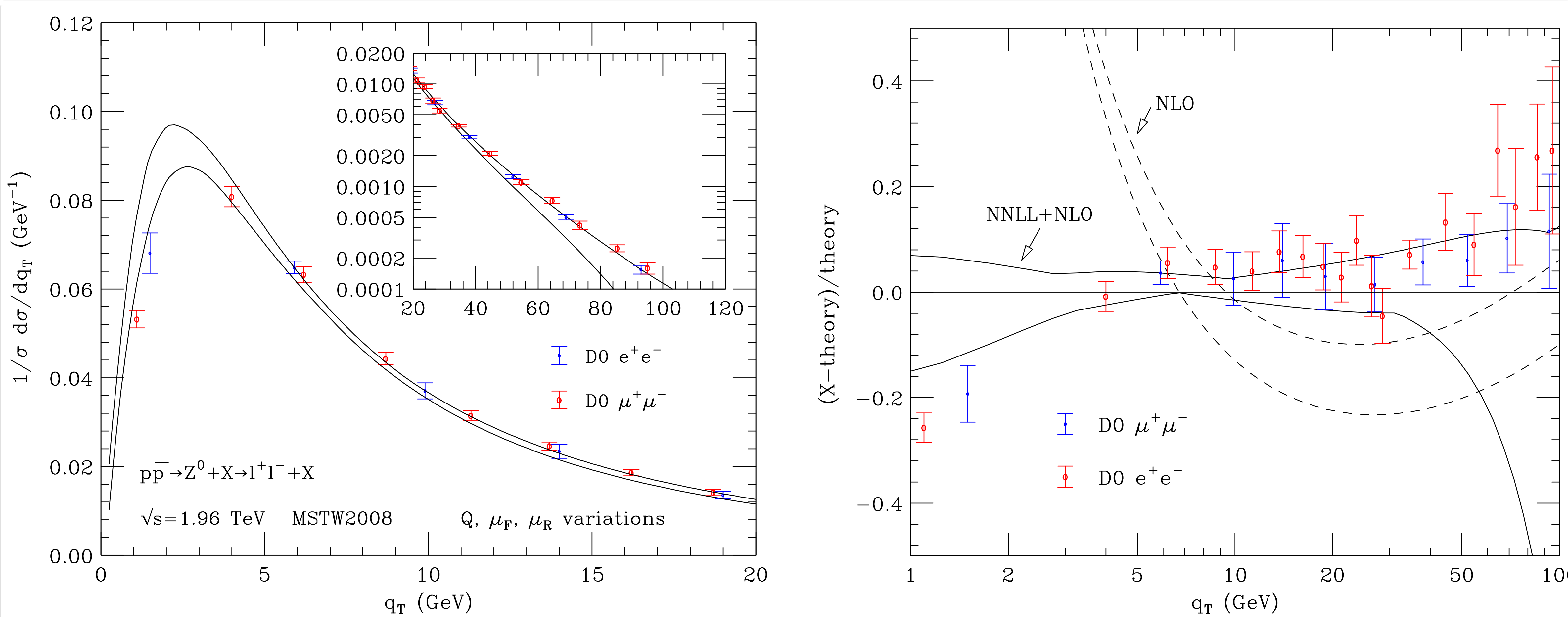
- Include NP corrections. Scales are varied by a factor $\sqrt{2}$
- Do not exponentiate anomalous log's: NLL in *amplitude*, LL in exponent.

D0, RUN II



- Correction from matching to O(α_s) fixed order result at has been multiplied by 5; is negligible in peak region.

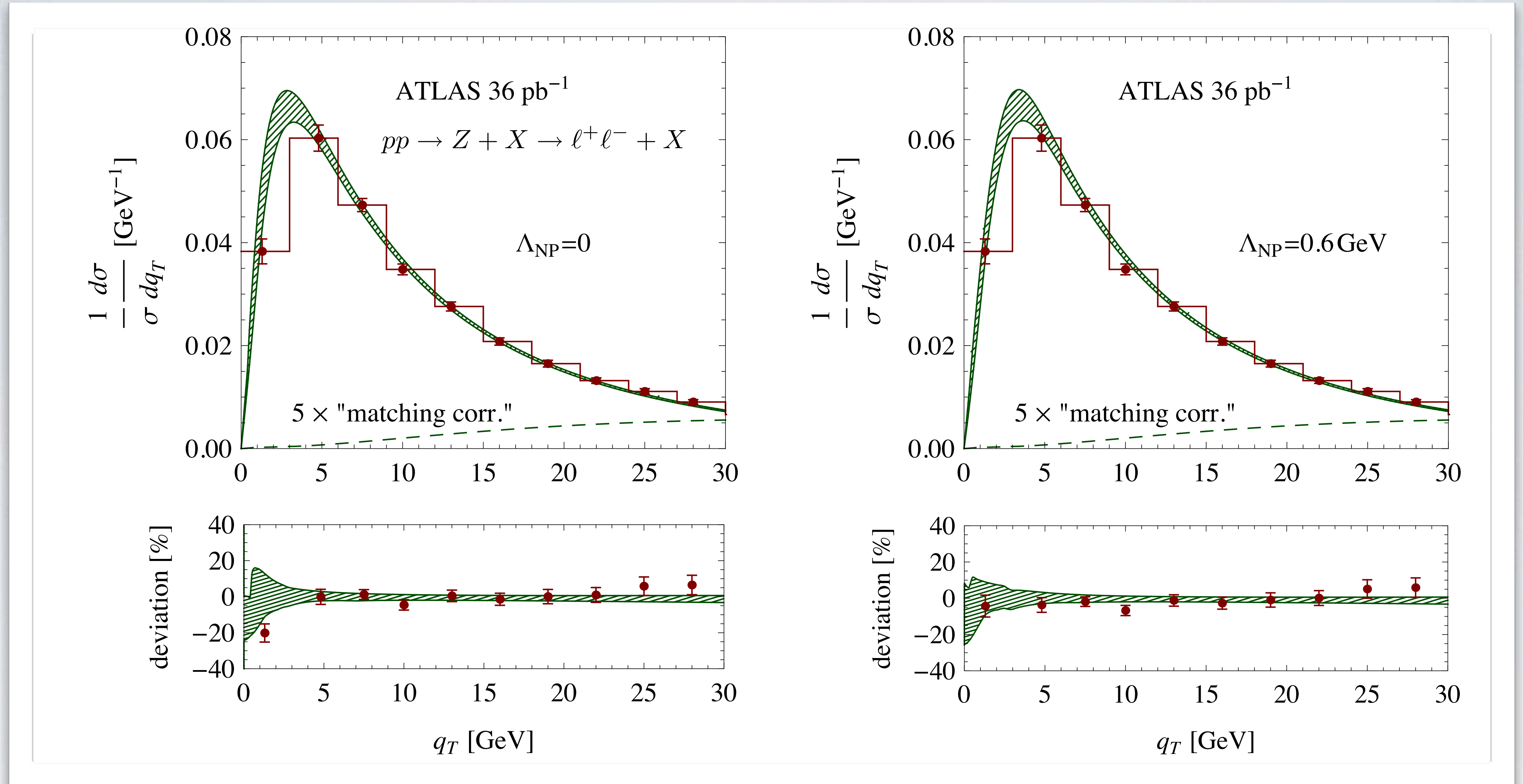
BOZZI ET AL.



Bozzi, Catani, Ferrera, de Florian, Grazzini '10

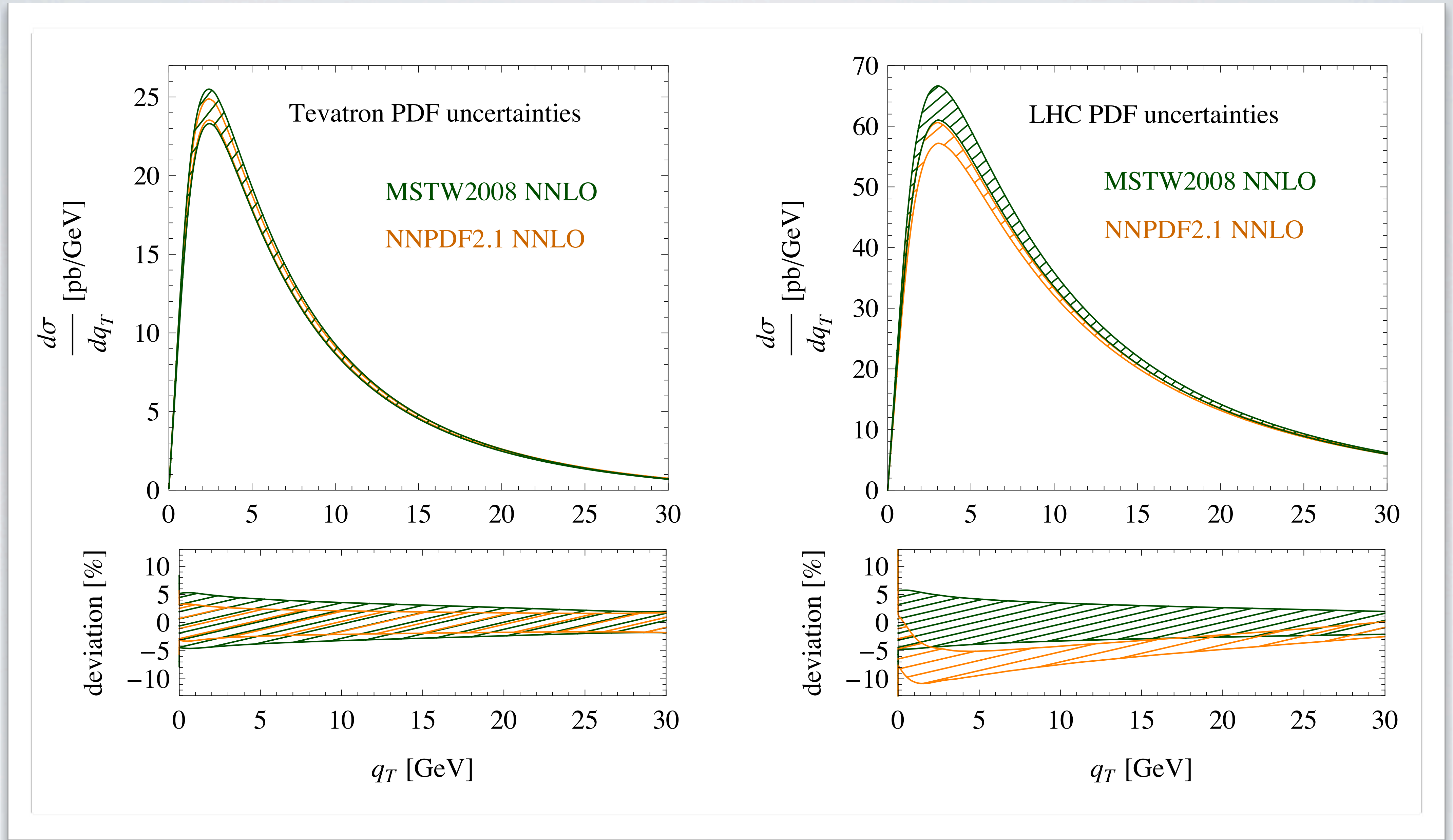
- Nice agreement with our result within uncertainties (same peak position, our peak is 6% higher, tail about 4% lower).
- Do not use non-perturbative parameter.

ATLAS RESULTS



- Same non-perturbative parameter as at the Tevatron. Need finer binning for clear evidence for non-perturbative effects.
- ATLAS data (and thus also our results) agree well with RESBOS.
- Preliminary CMS result is available as well, but only with lepton cuts.

PDF UNCERTAINTIES



- 90% C.L. for MSTW; 1σ band for NNPDF

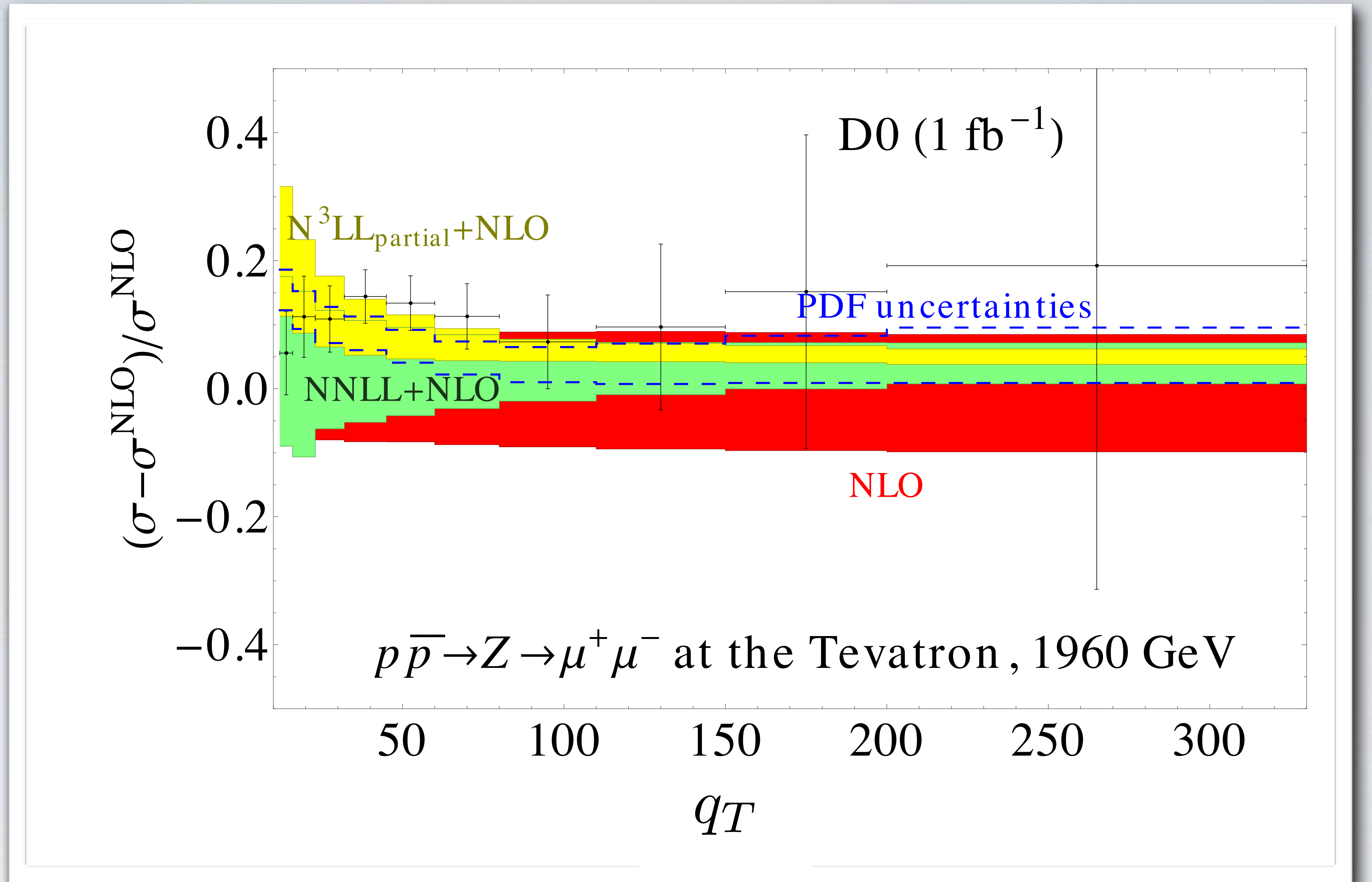
LARGE Q_T

- Focussed here on low q_T , but one can also perform threshold resummation for q_T spectrum [or joint - threshold and q_T - see Kulesza et al. '02].
- Mostly relevant at large q_T: due to the fall-off of the PDFs, very high-q_T W's and Z's are mostly produced near threshold.
- Factorization theorem:

$$\frac{\mathrm{d}\hat{\sigma}_I}{\mathrm{d}\hat{s}\,\mathrm{d}\hat{t}} = \hat{\sigma}_I^B(\hat{s},\hat{t})\, H_I(\hat{s},\hat{t},M_V,\mu) \times \int \mathrm{d}k\, J_I(m_X^2 - 2E_J k) S_I(k,\mu)\,,$$

RESUMMATION TO NNLL

- Have all the necessary input for NNLL resummation and
- almost of the input for N^3LL
  - All necessary anomalous dimensions Becher, Schwartz '09.
    - Hard anomalous dimension follows from general results of Magnea, Gardi '09 and TB, Neubert '09
  - Two-loop quark TB, Neubert '06 and gluon TB, Bell '10 jet functions
  - Logarithmic part of two-loop hard and soft functions



TB, Lorentzen, Schwartz, 1106.4310

Moderate shift of the central value, but much reduced scale dependence, below PDF uncertainty.

SUMMARY

- Renewed interest in transverse momentum spectrum. Many surprising features
  - soft-collinear factorization broken by an anomaly,
  - product of two transverse PDFs can be defined without additional regulator, but has anomalous dependence on hard momentum transfer
  - emergence of nonperturbative scale q_*~2GeV: spectrum is short-distance dominated, even at very low q_T
  - strongly divergent expansions in α_s, q_T/q_*, Λ_{QCD}/q_*.
- Three-loop coefficient $A^{(3)}$, the last missing piece needed for NNLL accuracy.
- NNLL results compare well with LHC data. It would be nice to have finer binning in the peak region to study non-perturbative effects.

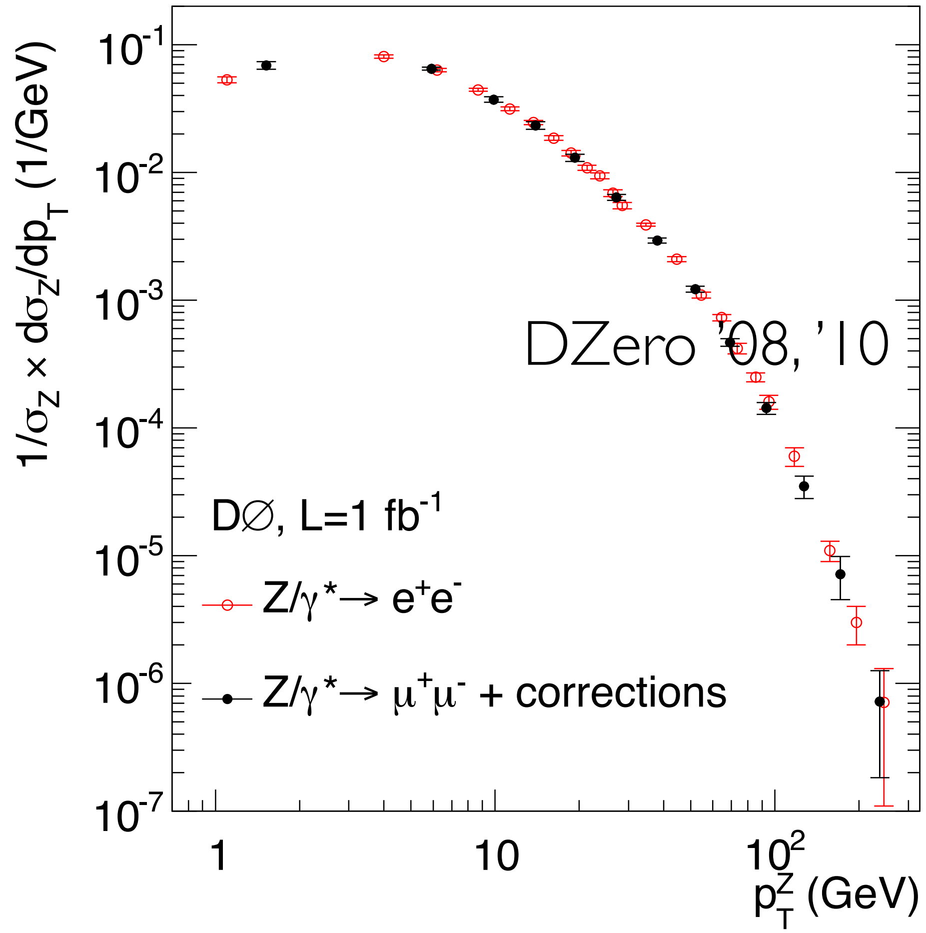
EXTRA SLIDES

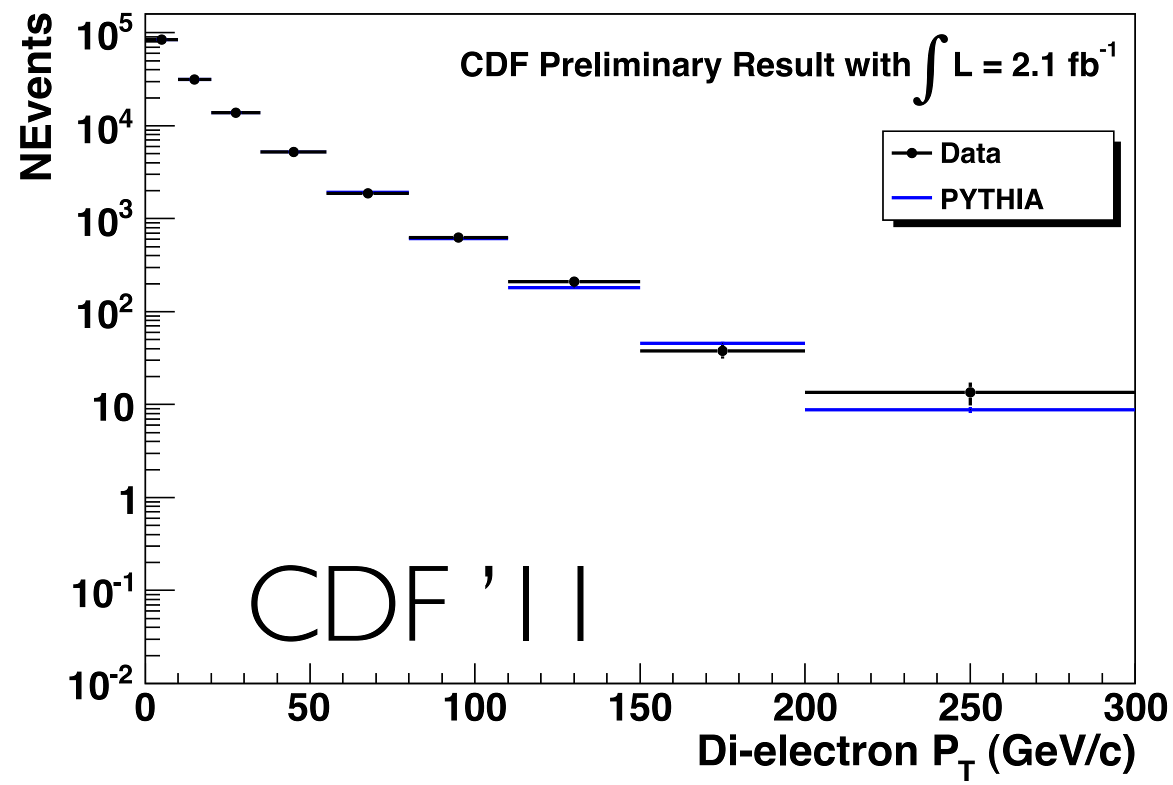
COLLINS SOPER STERMAN FORMULA

$$
\begin{aligned}
\frac{d^3\sigma}{dM^2\,dq_T^2\,dy} = \frac{4\pi\alpha^2}{3N_cM^2s}\,\frac{1}{4\pi}\int d^2x_\perp\, e^{-iq_\perp\cdot x_\perp}\sum_q e_q^2 \sum_{i=q,g}\sum_{j=\bar{q},g}\int_{\xi_1}^1\frac{dz_1}{z_1}\int_{\xi_2}^1\frac{dz_2}{z_2} \\
\times \exp\left\{-\int_{\mu_b^2}^{M^2}\frac{d\bar{\mu}^2}{\bar{\mu}^2}\left[\ln\frac{M^2}{\bar{\mu}^2}\,A\big(\alpha_s(\bar{\mu})\big)+B\big(\alpha_s(\bar{\mu})\big)\right]\right\} \\
\times \Big[C_{qi}\big(z_1,\alpha_s(\mu_b)\big)\,C_{\bar{q}j}\big(z_2,\alpha_s(\mu_b)\big)\,\phi_{i/N_1}(\xi_1/z_1,\mu_b)\,\phi_{j/N_2}(\xi_2/z_2,\mu_b)+(q,i\leftrightarrow\bar{q},j)\Big]
\end{aligned}
$$

- The low scale is $\mu_b = b_0/x_T$, and we set $b_0 = 2e^{-\gamma_E}$.
- Landau-pole singularity in the Fourier transform. To use the formula, one needs additional prescription to deal with this.

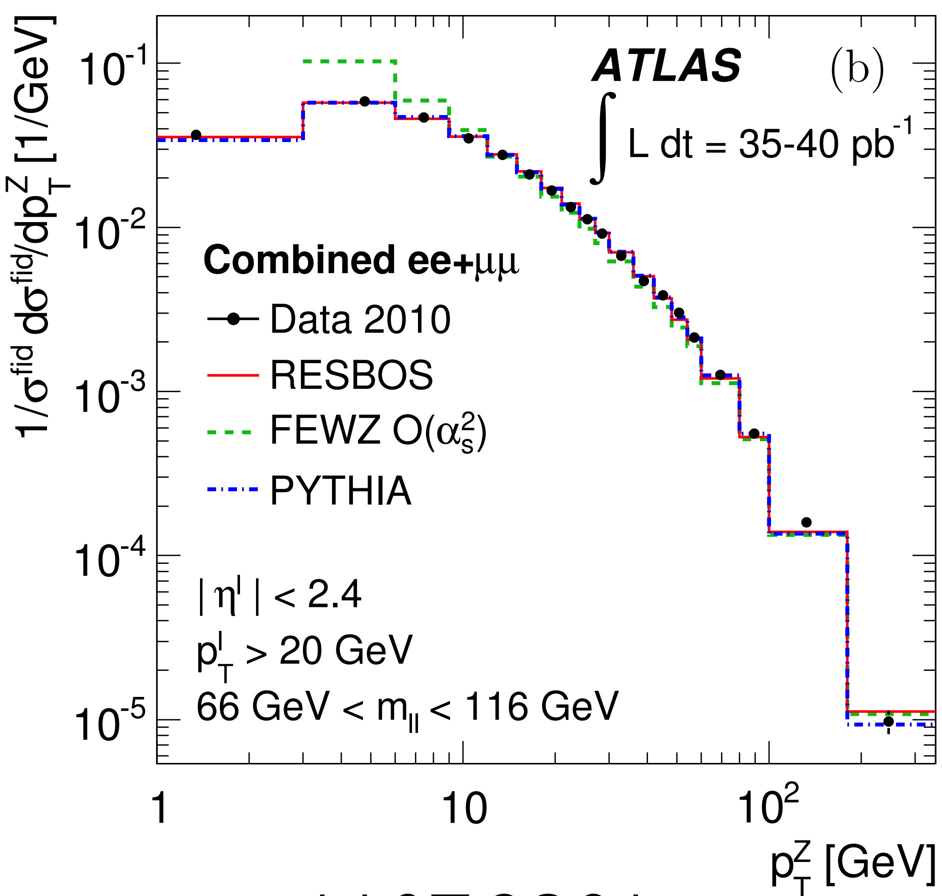
Z-PRODUCTION AT THE TEVATRON





- 80% of all events have $q_T < 16$ GeV, where resummation is necessary.

NEW LHC RESULTS FOR Z-SPECTRUM



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