RESUMMATIONS IN QCD

- A PROGRESS REPORT -

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Outline

- Introducing resummations
- Soft gluons and large logs
- Recent developments: from theory ...
- ... to phenomenology
- Outlook

A BIT OF HISTORY

A subject with a long history ...

JULY 15, 1937 PHYSICAL REVIEW VOLUME 52

Note on the Radiation Field of the Electron

F. BLOCH AND A. NORDSIECK*
Stanford University, California

(Received May 14, 1937)

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A FIRST LOOK

Large logs

- **Multi-scale** problems in **renormalizable** quantum field theories have perturbative corrections of the general form $\alpha_s^n \log^k \left(Q_i^2/Q_j^2\right)$, which may **spoil** the reliability of the perturbative expansion.
  - **Renormalization** and **factorization** logs: $\alpha_s^n \log^n \left(Q^2/\mu^2\right)$
  - **High-energy logs**: $\alpha_s^n \log^{n-1} (s/t)$
  - **Sudakov** logs: $\alpha_s^n \log^{2n-1} (1-z)$, $\quad 1-z = W^2/Q^2,\, 1-M^2/\hat{s}\,,\, Q_\perp^2/Q^2,\, \ldots$

- **Sudakov** logs are **universal**: they originate from **infrared and collinear singularities**: they **exponentiate** and can be **resummed**

$$\underbrace{\frac{1}{\epsilon}}_{\text{virtual}} + \underbrace{(Q^2)^\epsilon \int_0^{m^2} \frac{dk^2}{(k^2)^{1+\epsilon}}}_{\text{real}} \qquad \Longrightarrow \quad \ln(m^2/Q^2)$$

  - For **inclusive** observables: **analytic** resummation to high logarithmic accuracy.
  - For **exclusive** final states: **parton shower** event generators, (N)LL accuracy.

- **Resummation** probes the **all-order structure** of perturbation theory.
  - **Power-suppressed** corrections to QCD cross sections can be studied.
  - Links to the **strong coupling** regime can be established for SUSY gauge theories.

The perturbative exponent

A classic way to organize Sudakov logarithms is in terms of the Mellin (Laplace) transform of the momentum space cross section (Catani et al. 93),

$$
\begin{aligned}
d\sigma(\alpha_s, N) &= \sum_{n=0}^{\infty} \left(\frac{\alpha_s}{\pi}\right)^n \sum_{k=0}^{2n} c_{nk} \log^k N + \mathcal{O}(1/N) \\
&= H(\alpha_s) \exp\Big[\log N \, g_1(\alpha_s \log N) + g_2(\alpha_s \log N) + \alpha_s \, g_3(\alpha_s \log N) + \ldots\Big] + \mathcal{O}(1/N)
\end{aligned}
$$

This displays the main features of Sudakov resummation

- Predictive: a k-loop calculation determines g_k and thus a whole tower of logarithms to all orders in perturbation theory.
- Effective:
  - the range of applicability of perturbation theory is extended (finite order: $\alpha_s \log^2 N$ small. NLL resummed: α_s small);
  - the renormalization scale dependence is naturally reduced.
- Theoretically interesting: resummation ambiguities related to the Landau pole give access to non-perturbative power-suppressed corrections.
- Well understood:
  - NLL Sudakov resummations exist for most inclusive observables at hadron colliders, NNLL and approximate N^3LL in simple cases.
  - Different `schools' (USA, Italian, SCET ...) compete, complacency is not an option, active and lively debate.

Color singlet hard scattering

A well-established formalism exists for distributions in processes that are electroweak at tree level (Gardi, Grunberg 07). For an observable r vanishing in the two-jet limit

$$\frac{d\sigma}{dr} = \delta(r)\left[1 + \mathcal{O}(\alpha_s)\right] + C_R \frac{\alpha_s}{\pi} \left\{ \left[-\frac{\log r}{r} + \frac{b_1 - d_1}{r} \right]_+ + \mathcal{O}(r^0) \right\} + \mathcal{O}(\alpha_s^2)$$

The Mellin (Laplace) transform, $\sigma(N) = \int_0^1 dr\, (1-r)^{N-1}\, \frac{d\sigma}{dr}$

exhibits log N singularities that can be organized in exponential form

$$\sigma\left(\alpha_s, N, Q^2\right) = H(\alpha_s)\, \mathcal{S}\left(\alpha_s, N, Q^2\right) + \mathcal{O}\left(1/N\right)$$

where the exponent of the `Sudakov factor' is in turn a Mellin transform

$$\mathcal{S}\left(\alpha_s, N, Q^2\right) = \exp\left\{ \int_0^1 \frac{dr}{r} \left[(1-r)^{N-1} - 1 \right] \mathcal{E}\left(\alpha_s, r, Q^2\right) \right\}$$

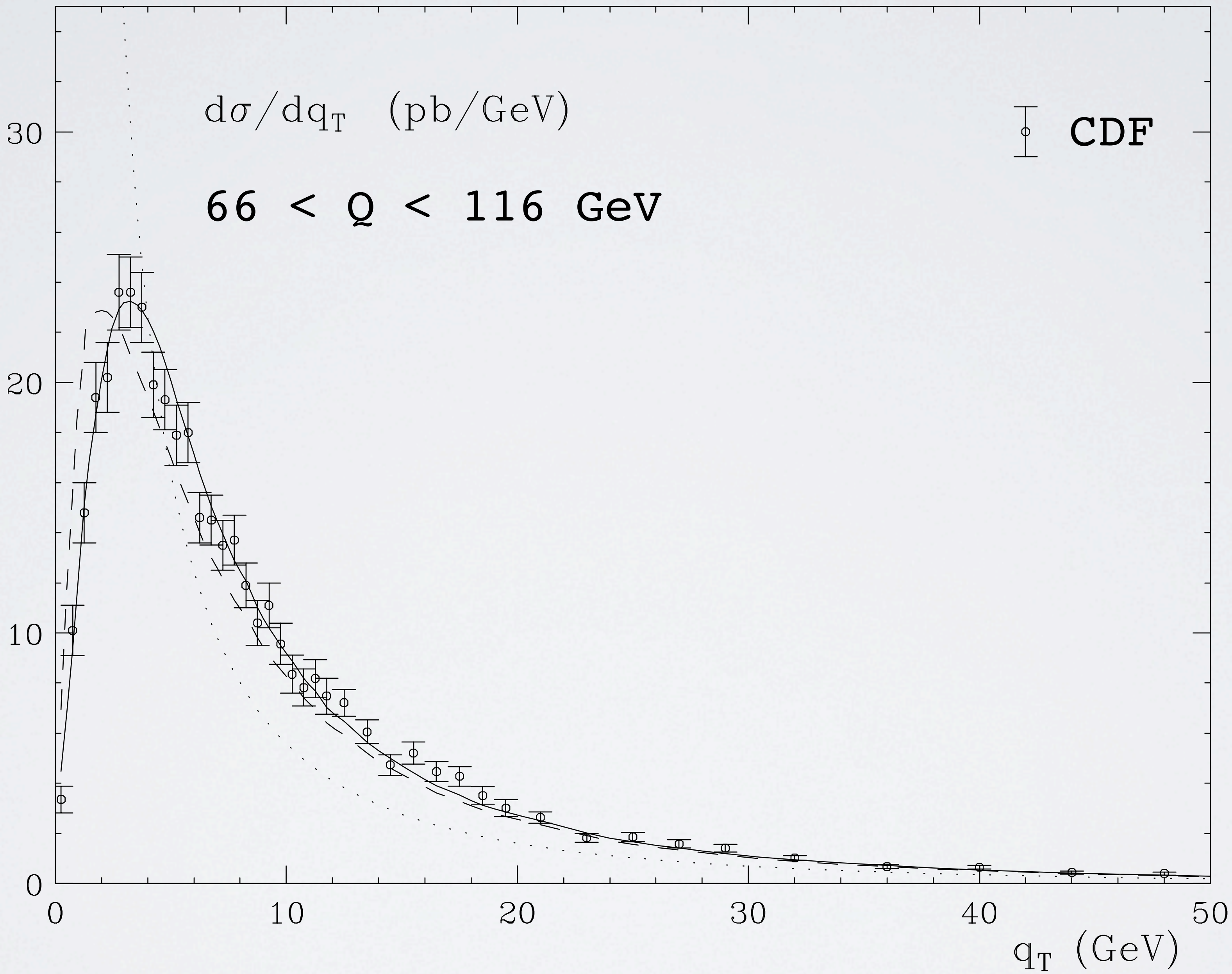
and the general form of the kernel is

$$\mathcal{E}\left(\alpha_s, r, Q^2\right) = \int_{r^2 Q^2}^{r Q^2} \frac{d\xi^2}{\xi^2}\, A\left(\alpha_s(\xi^2)\right) + B\left(\alpha_s(r Q^2)\right) + D\left(\alpha_s(r^2 Q^2)\right)$$

where A is the cusp anomalous dimension, and B and D have distinct physical characters.

Impact of resummation

Z-boson q_T spectrum at Tevatron (Kulesza et al. 03)



CDF data on Z production compared with QCD predictions at fixed order (dotted), with joint resummation (dashed), and with the inclusion of power corrections (solid).

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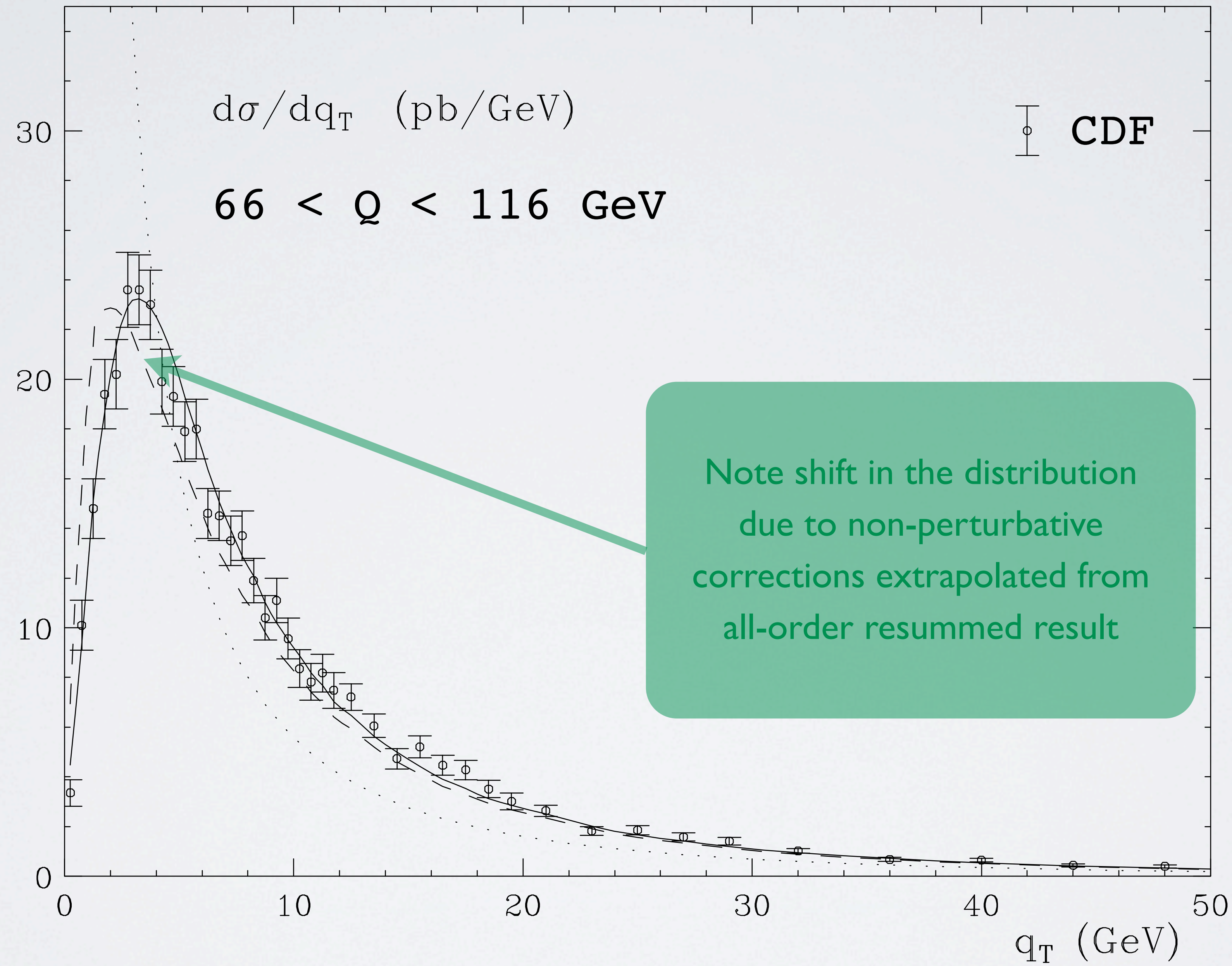

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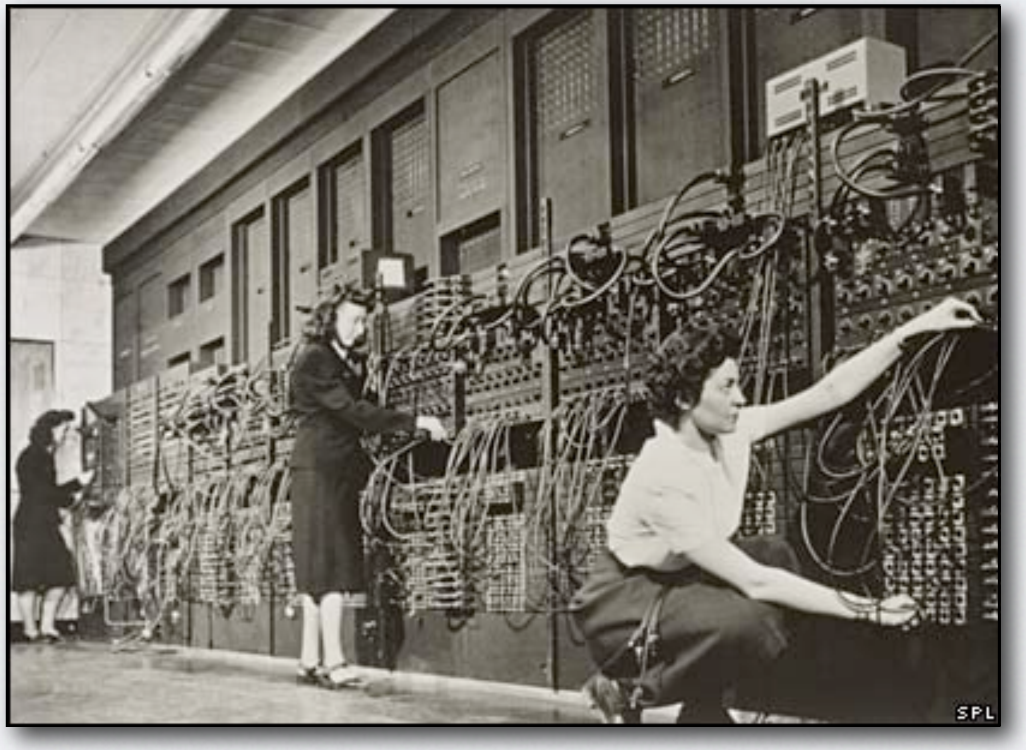
A SECOND LOOK



Factorization

All factorizations separating dynamics at different energy scales lead to resummation of logarithms of the ratio of scales.

Renormalization is a textbook example.

- Renormalization factorizes cutoff dependence.

$$G_0^{(n)}\left(p_i, \Lambda, g_0\right) = \prod_{i=1}^{n} Z_i^{1/2}\left(\Lambda/\mu, g(\mu)\right)\; G_R^{(n)}\left(p_i, \mu, g(\mu)\right)$$

- Factorization requires the introduction of an arbitrarily chosen scale μ.

- Results must be independent of the arbitrary choice of μ.

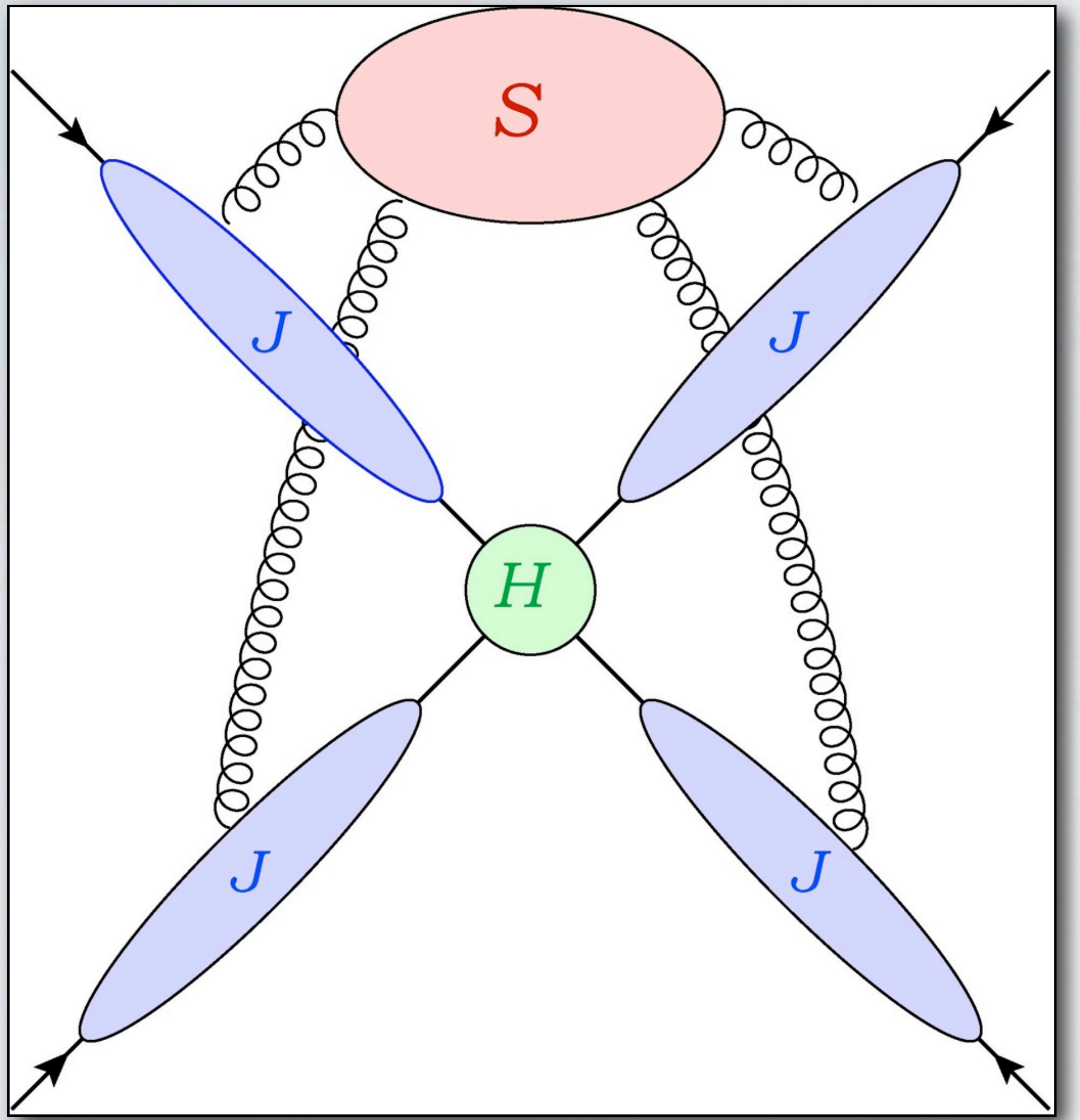
$$\frac{dG_0^{(n)}}{d\mu} = 0 \quad \rightarrow \quad \frac{d\log G_R^{(n)}}{d\log\mu} = -\sum_{i=1}^{n} \gamma_i\left(g(\mu)\right) \ .$$

- The simple functional dependence of the factors is dictated by separation of variables.

- Proving factorization is the difficult step: it requires all-order diagrammatic analyses. Evolution equations follow automatically.

- Solving RG evolution resums logarithms of Q^2/μ^2 into $\alpha_s(\mu^2)$.

Sudakov Factorization



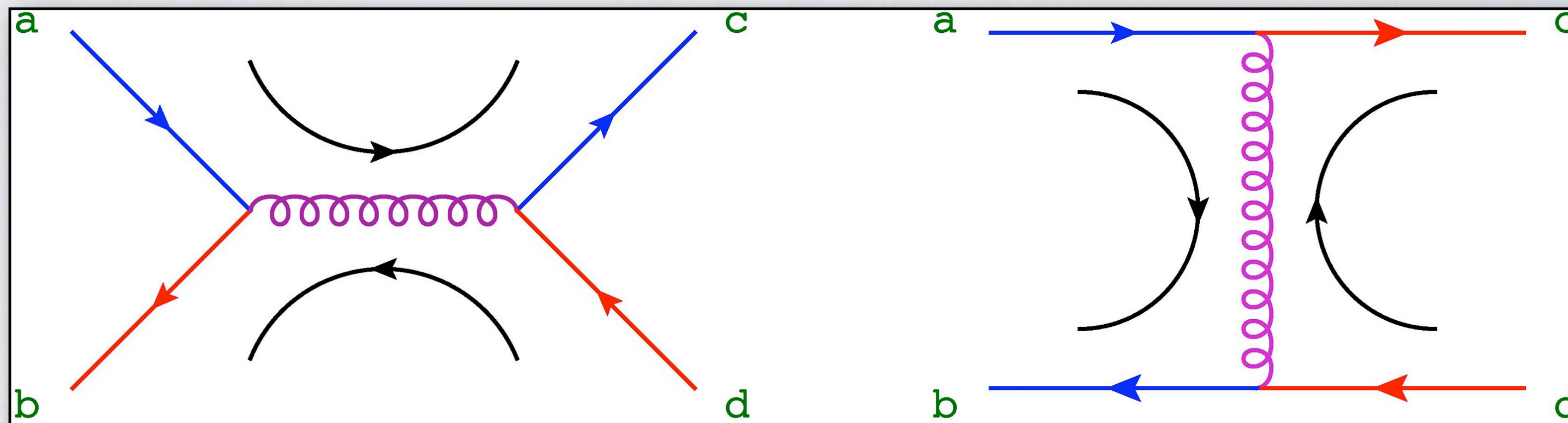
Leading integration regions in loop momentum space
for Sudakov factorization

- Sudakov logarithms are remainders of infrared and collinear divergences.
- Divergences arise in scattering amplitudes from leading regions in loop momentum space.
- Soft gluons factorize both form hard (easy) and from collinear (intricate) virtual exchanges.
- Jet functions J represent color singlet evolution of external hard partons.
- The soft function S is a matrix mixing the available color representations.
- In the planar limit soft exchanges are confined to wedges: S is proportional to the identity.
- In the planar limit S can be reabsorbed defining jets as square roots of elementary form factors.
- Beyond the planar limit S is determined by an anomalous dimension matrix Γ_S.

Color flow

In order to understand the matrix structure of the soft function it is sufficient to consider the simple case of quark-antiquark scattering.

At tree level



Tree-level diagrams and color flows for quark-antiquark scattering

For this process only two color structures are possible. A basis in the space of available color tensors is

$$c^{(1)}_{abcd} = \delta_{ab}\delta_{cd}\,, \qquad c^{(2)}_{abcd} = \delta_{ac}\delta_{bd}$$

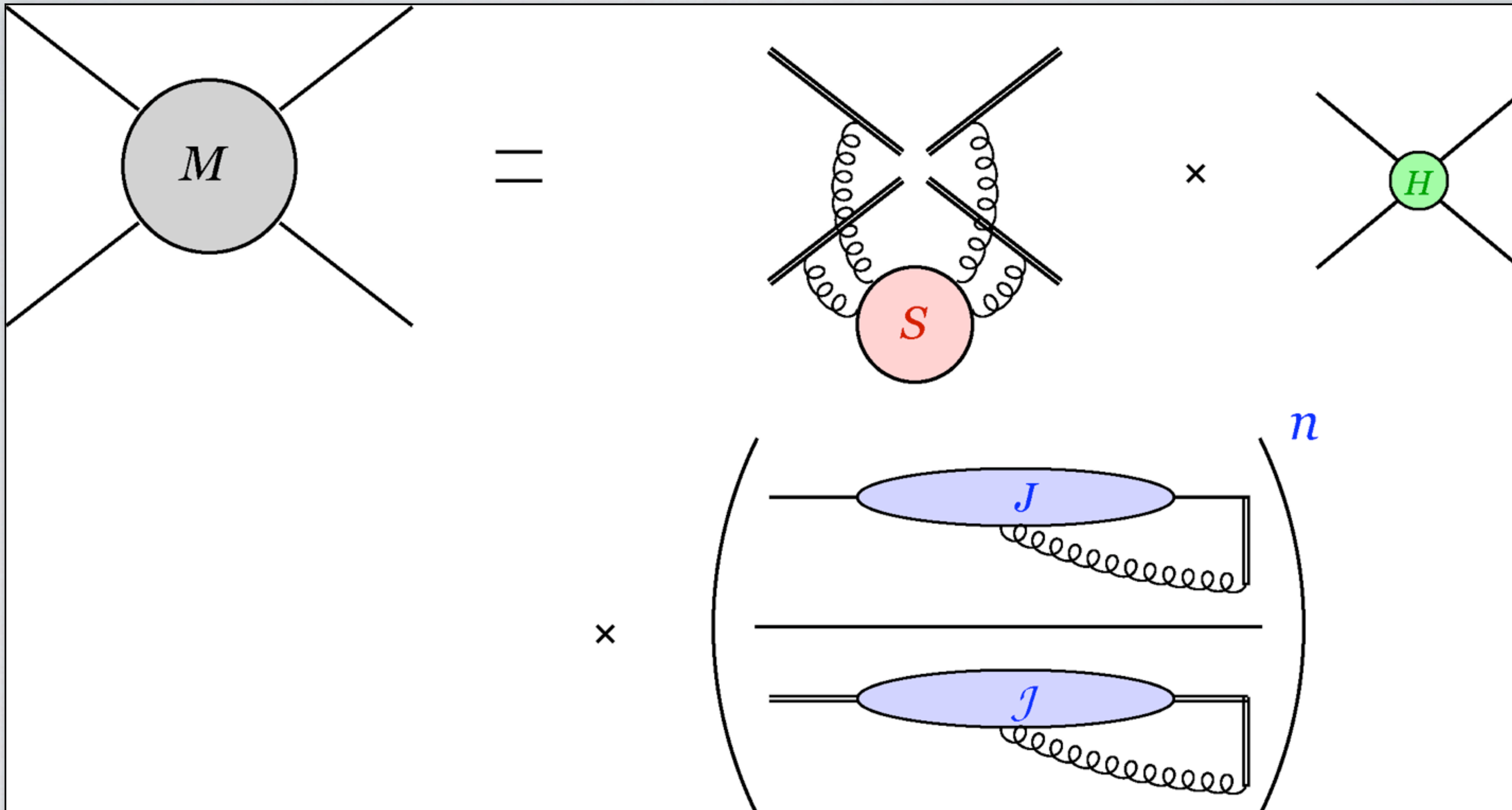
The matrix element is a vector in this space, and the Born cross section is

$$\mathcal{M}_{abcd} = \mathcal{M}_1\, c^{(1)}_{abcd} + \mathcal{M}_2\, c^{(2)}_{abcd} \longrightarrow \sum_{\text{color}} |\mathcal{M}|^2 = \sum_{J,L} \mathcal{M}_J\, \mathcal{M}^*_L\ \mathrm{tr}\left[c^{(J)}_{abcd}\left(c^{(L)}_{abcd}\right)^\dagger\right] \equiv \mathrm{Tr}\Big[HS\Big]_0$$

A virtual soft gluon will reshuffle color and mix the components of this vector

$$\text{QED}: \quad \mathcal{M}_{\text{div}} = S_{\text{div}}\mathcal{M}_{\text{Born}}\,; \qquad \text{QCD}: \quad [\mathcal{M}_{\text{div}}]_J = [S_{\text{div}}]_{JL}\,[\mathcal{M}_{\text{Born}}]_L$$

Sudakov factorization: pictorial



A pictorial representation of Sudakov factorization for fixed-angle scattering amplitudes

Soft Matrices

The soft function S is a matrix, mixing the available color tensors. It is defined by a correlator of Wilson lines.

$$\left(c_L\right)_{\{\alpha_k\}} \mathcal{S}_{LK}\left(\beta_i \cdot \beta_j, \alpha_s(\mu^2), \epsilon\right) = \sum_{\{\eta_k\}} \langle 0 | \prod_{i=1}^{n} \left[\Phi_{\beta_i}(\infty, 0)_{\alpha_k, \eta_k}\right] |0\rangle \left(c_K\right)_{\{\eta_k\}} ,$$

The soft function S obeys a matrix RG evolution equation

$$\mu \frac{d}{d\mu} \mathcal{S}_{IK}\left(\beta_i \cdot \beta_j, \alpha_s(\mu^2), \epsilon\right) = - \mathcal{S}_{IJ}\left(\beta_i \cdot \beta_j, \alpha_s(\mu^2), \epsilon\right) \Gamma^{\mathcal{S}}_{JK}\left(\beta_i \cdot \beta_j, \alpha_s(\mu^2), \epsilon\right)$$

- Γ^S is singular due to overlapping UV and collinear poles.

S is a pure counterterm. In dimensional regularization, using $\alpha_s(\mu^2 = 0, \epsilon < 0) = 0$,

$$\mathcal{S}\left(\beta_i \cdot \beta_j, \alpha_s(\mu^2), \epsilon\right) = P \exp\left[-\frac{1}{2}\int_0^{\mu^2} \frac{d\xi^2}{\xi^2} \Gamma^{\mathcal{S}}\left(\beta_i \cdot \beta_j, \alpha_s(\xi^2, \epsilon), \epsilon\right)\right] .$$

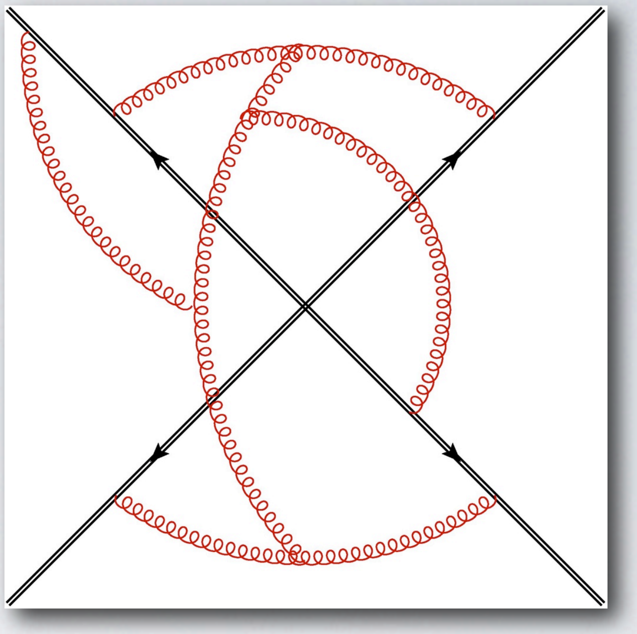
The determination of the soft anomalous dimension matrix Γ^S is the keystone of the resummation program for multiparton amplitudes and cross sections.

- It governs the interplay of color exchange with kinematics in multiparton processes.
- It is the only source of multiparton correlations for singular contributions.
- Collinear effects are `color singlet' and can be extracted from two-parton scatterings.

RECENT DEVELOPMENTS

Surprising Simplicity

- The matrix Γ_S can be computed from the UV poles of S.
- Computations can be performed directly for the exponent: the relevant diagrams are called "webs".
- Γ_S appears highly complex at high orders.
- g-loop webs directly correlate color and kinematics of up to g+1 Wilson lines.



A web contributing to the soft anomalous dimension matrix

The two-loop calculation (Aybat, Dixon, Sterman 06) leads to a surprising result: for any number of external massless partons

$$\Gamma_{\mathcal{S}}^{(2)} = \frac{\kappa}{2}\,\Gamma_{\mathcal{S}}^{(1)} \qquad \kappa = \left(\frac{67}{18} - \zeta(2)\right) C_A - \frac{10}{9} T_F C_F \,.$$

➡ No new kinematic dependence; no new matrix structure.

➡ κ is the two-loop coefficient of $\gamma_K(\alpha_s)$, rescaled by the appropriate quadratic Casimir,

$$\gamma_K^{(i)}(\alpha_s) = C^{(i)}\left[2\,\frac{\alpha_s}{\pi} + \kappa\left(\frac{\alpha_s}{\pi}\right)^2 + \mathcal{O}\left(\alpha_s^3\right)\right] .$$

The Dipole Formula

The two-loop result led to an all-order understanding. For massless partons, the soft matrix obeys a set of exact equations that correlate color exchange with kinematics.

The simplest solution to these equations is a sum over color dipoles (Becher, Neubert; Gardi, LM, 09). It leads to an ansatz for the all-order singularity structure of all multiparton fixed-angle massless scattering amplitudes: the dipole formula.

- All soft and collinear singularities can be collected in a multiplicative operator Z

$$\mathcal{M}\left(\frac{p_i}{\mu}, \alpha_s(\mu^2), \epsilon\right) = Z\left(\frac{p_i}{\mu_f}, \alpha_s(\mu_f^2), \epsilon\right) \, \mathcal{H}\left(\frac{p_i}{\mu}, \frac{\mu_f}{\mu}, \alpha_s(\mu^2), \epsilon\right) \, ,$$

- Z contains both soft singularities from S, and collinear ones from the jet functions. It must satisfy its own matrix RG equation

$$\frac{d}{d \ln \mu} Z\left(\frac{p_i}{\mu}, \alpha_s(\mu^2), \epsilon\right) \, = \, - \, Z\left(\frac{p_i}{\mu}, \alpha_s(\mu^2), \epsilon\right) \, \Gamma\left(\frac{p_i}{\mu}, \alpha_s(\mu^2)\right) \, .$$

The matrix Γ inherits the dipole structure from the soft matrix. It reads

$$\boxed{\Gamma_{\rm dip}\left(\frac{p_i}{\mu}, \alpha_s(\mu^2)\right) = -\frac{1}{4} \, \widehat{\gamma}_K\left(\alpha_s(\mu^2)\right) \sum_{j \neq i} \ln\left(\frac{-2 \, p_i \cdot p_j}{\mu^2}\right) \mathbf{T}_i \cdot \mathbf{T}_j \, + \sum_{i=1}^{n} \gamma_{J_i}\left(\alpha_s(\mu^2)\right) \, .}$$

Note that all singularities are generated by integration over the scale of the coupling.

Features of the dipole formula

- All known results for IR divergences of massless gauge theory amplitudes are recovered.
- The absence of multiparton correlations implies remarkable diagrammatic cancellations.
- The color matrix structure is fixed at one loop: path-ordering is not needed.
- All divergences are determined by a handful of anomalous dimensions.
- The cusp anomalous dimension plays a very special role: a universal IR coupling.

Can this be the definitive answer for IR divergences in massless non-abelian gauge theories?

- There are precisely two sources of possible corrections.
  - Quadrupole correlations may enter starting at three loops: they must be tightly constrained functions of conformal cross ratios of parton momenta.

$$\Gamma\left(\frac{p_i}{\mu}, \alpha_s(\mu^2)\right) = \Gamma_{\rm dip}\left(\frac{p_i}{\mu}, \alpha_s(\mu^2)\right) + \Delta\left(\rho_{ijkl}, \alpha_s(\mu^2)\right) , \qquad \rho_{ijkl} = \frac{p_i \cdot p_j \, p_k \cdot p_l}{p_i \cdot p_k \, p_j \cdot p_l}$$

  - The cusp anomalous dimension may violate Casimir scaling beyond three loops.

$$\gamma_K^{(i)}(\alpha_s) = C_i \, \widehat{\gamma}_K(\alpha_s) + \widetilde{\gamma}_K^{(i)}(\alpha_s)$$

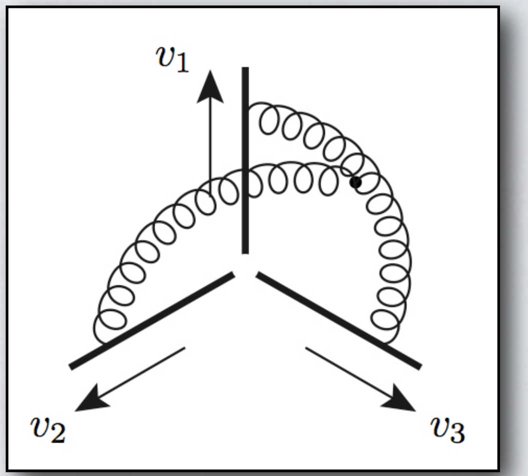
  - The functional form of Δ is further constrained by: collinear limits, Bose symmetry and transcendentality bounds (Becher, Neubert; Dixon, Gardi, LM, 09).
  - A four-loop analysis indicates that Casimir scaling holds (Vernazza, EPS 2011).

Massive particles

- The striking simplicity of the massless result does not carry over to massive partons.
  - The g-loop exponent will generally involve (g+1)-parton correlations.
  - An analytic calculation at two loops was carried out (Becher, Neubert; Ferroglia et al.; Mitov et al.; Kidonakis, 09) with interesting results.

$$\Gamma\left(\frac{p_i}{\mu}, M_i, \alpha_s(\mu^2)\right) = \Gamma_{\rm dip}\left(\frac{p_i}{\mu}, M_i, \alpha_s(\mu^2)\right) + {\rm i} \sum_{i,j,k} f_{abc}\, \mathbf{T}_i^a\, \mathbf{T}_j^b\, \mathbf{T}_k^c\, F_1\left(\beta_{ij}, \beta_{jk}, \beta_{ik}\right) + \ldots$$

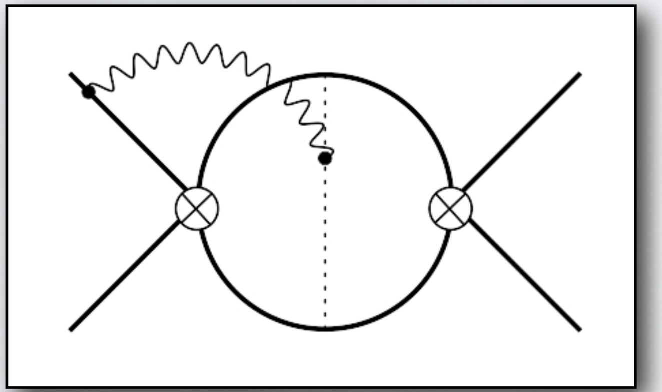
$$F_1^{(2)}\left(\beta_{ij}, \beta_{jk}, \beta_{ik}\right) = \frac{4}{3} \sum_{i,j,k} \epsilon_{ijk}\; g\left(\beta_{ij}\right) \beta_{ki} \coth \beta_{ki}$$



Three-parton correlations at two loops

  - The result still displays unexpected structure and simplicity.

- Another class of singularities of massive amplitudes is understood and resummed.
  - When massive particles are pair-produced near threshold, Coulomb singularities $\log^p \beta/\beta^k$ arise.
  - They can be organized using effective field theory (NRQCD).
  - A novel factorization theorem has been derived and applied to heavy colored particle production (Beneke et al., 09).


Soft and Coulomb gluons at two loops

$$\hat{\sigma}_{pp'}\left(\hat{s}, \mu\right) = \sum_{KL} H_{KL}\left(M, \mu\right) \int dw \sum_{R_\alpha} J_{R_\alpha}\left(E - \frac{w}{2}\right) S_{KL}^{R_\alpha}\left(w, \mu\right)$$

The high-energy limit

The high-energy limit

The dipole formula has recently been applied to the `Regge limit', s >> t, providing a novel viewpoint on the phenomenon of Reggeization (Del Duca et al. 11).

- Reggeization: logarithms log(s/t) are generated by the Reggeized t-channel propagator

$$\frac{1}{t} \longrightarrow \frac{1}{t}\left(\frac{s}{-t}\right)^{\alpha(t)} \qquad \alpha(t) = \frac{\alpha_s(-t,\epsilon)}{4\pi}\,\alpha^{(1)} + \left(\frac{\alpha_s(-t,\epsilon)}{4\pi}\right)^2 \alpha^{(2)} + \mathcal{O}\left(\alpha_s^3\right)$$

- The infrared matrix Z factorizes and acts as a Reggeization operator

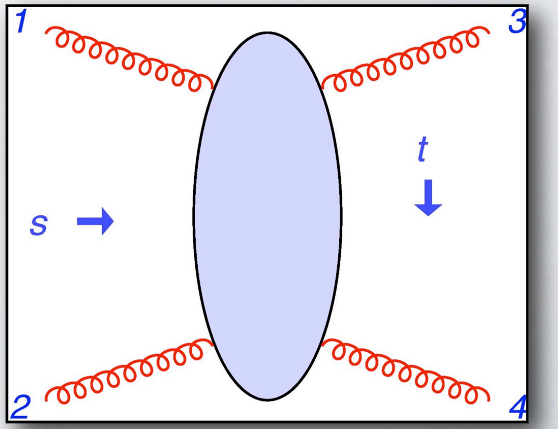
$$Z\left(p_i,\alpha_s,\epsilon\right) = \widetilde{Z}\left(\frac{s}{t},\alpha_s,\epsilon\right) Z_{\mathbf{1}}\left(t,\alpha_s,\epsilon\right) \qquad \widetilde{Z}\left(\frac{s}{t},\alpha_s,\epsilon\right) = \exp\left\{K\left(\alpha_s,\epsilon\right)\left[\ln\left(\frac{s}{-t}\right)\mathbf{T}_t^2 + \mathrm{i}\pi\mathbf{T}_s^2\right]\right\},$$

where we defined the s- and t-channel color operators $\mathbf{T}_s = \mathbf{T}_1 + \mathbf{T}_2\,, \qquad \mathbf{T}_t = \mathbf{T}_1 + \mathbf{T}_3\;.$

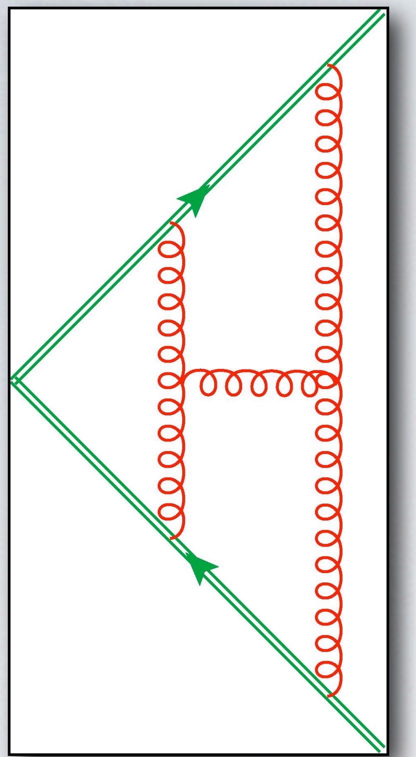
- Leading logarithmic Reggeization immediately follows for generic representations.

$$\mathcal{M}|_{\mathrm{LL}} = \left(\frac{s}{-t}\right)^{K\,\mathbf{T}_t^2} Z_{\mathbf{1}}\,\mathcal{H} \qquad K\left(\alpha_s,\epsilon\right) \equiv -\frac{1}{4}\int_0^{\mu^2}\frac{d\lambda^2}{\lambda^2}\,\widehat{\gamma}_K\left(\alpha_s(\lambda^2,\epsilon)\right)$$

- Reggeization (of singular contributions) can be studied for subleading logs and generic color configurations.
  - Results for Regge trajectories at LL and NLL are recovered.
  - Reggeization is seen to generically break down at NNLL.
  - The formalism applies in Multi-Regge kinematics.



Multiparton webs



A web

- Infrared divergences of gauge scattering amplitudes exponentiate.
- The exponent can be computed directly in terms of a subset of the original diagrams with modified color factors, called `webs'. (Gatheral 83; Frenkel, Taylor 84)
- For amplitudes with two hard partons (color singlet), webs have a precise topological characterization and special properties.
  - Webs are `two-eikonal-irreducible' diagrams.
  - Webs have modified color factors that can be computed recursively.
  - Webs have no nested UV subdivergences.

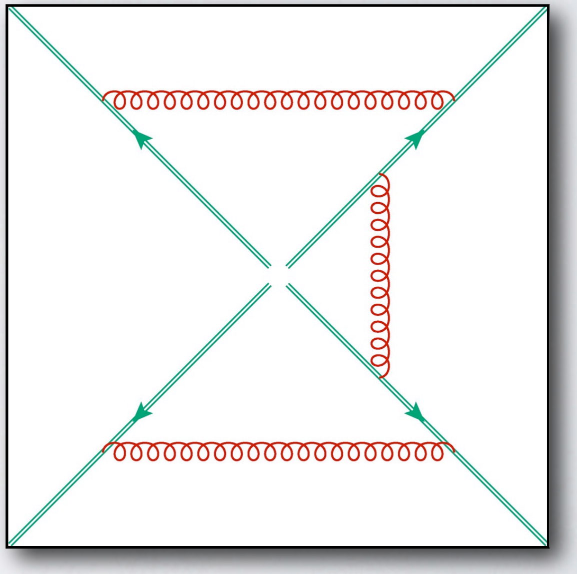
- We are now understanding the structure of the multileg exponent. (Gardi et al. '10-'11; Mitov, Sterman, Sung '10)

$$\mathcal{Z} \equiv \int [\mathcal{D}A_s^\mu]\, \mathrm{e}^{\mathrm{i}S(A_s^\mu)} \left[\Phi^{(1)} \otimes \cdots \otimes \Phi^{(L)}\right] = \exp\left[\sum_D \tilde{C}(D)\, \mathcal{F}(D)\right] .$$

- Multiparton webs are sets of diagrams whose kinematic and color structures mix. They are not all irreducible.
- Modified color factors are given by web mixing matrices

$$\widetilde{C}(D) = \sum_{D'} R(D, D')\, C(D')$$

- All subleading poles are determined by lower-order webs.

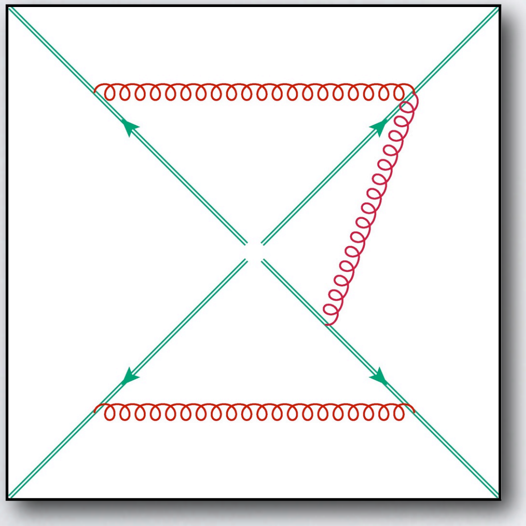


Is this a web?

Beyond the eikonal

- Hadronic cross sections near partonic threshold receive non-singular logarithmic corrections $\alpha_s^p \log^k(1 - z)$, or $\alpha_s^p \log^k N/N$, which may be relevant for phenomenology. Can they also be organized and resummed? (Kraemer et al.; Vogt et al.; Grunberg, ...)
  - For two-parton processes, $O(N^0)$ contributions exponentiate (Laenen, LM, 03).
  - Phenomenological evidence indicates that also `sub-eikonal' logs partly exponentiate.
  - An ansatz summarizes the resummable for Drell-Yan (and DIS) (Laenen et al., 06).

$$\ln\left[\widehat{\omega}(N)\right] = \mathcal{F}_{\rm DY}\left(\alpha_s(Q^2)\right) + \int_0^1 dz\, z^{N-1}\left\{\frac{1}{1-z}\, D\left[\alpha_s\left(\frac{(1-z)^2Q^2}{z}\right)\right] + 2\int_{Q^2}^{(1-z)^2Q^2/z}\frac{dq^2}{q^2}\, P_s\left[z, \alpha_s(q^2)\right]\right\}_+$$



Is THIS a web?

- A systematic study of soft-gluon dynamics beyond the eikonal approximation has been undertaken (Laenen et al. '08, 10).
  - A class of factorizable contributions exponentiate via NE webs

$$\mathcal{M} = \mathcal{M}_0 \exp\left[\sum_{D_{\rm eik}} \tilde{C}(D_{\rm eik})\, \mathcal{F}(D_{\rm eik}) + \sum_{D_{\rm NE}} \tilde{C}(D_{\rm NE})\, \mathcal{F}(D_{\rm NE})\right] .$$

  - "Feynman rules" for the NE exponent, including "seagull" vertices.
  - Non-factorizable contribution can be studied using Low's theorem.

PHENOMENOLOGY

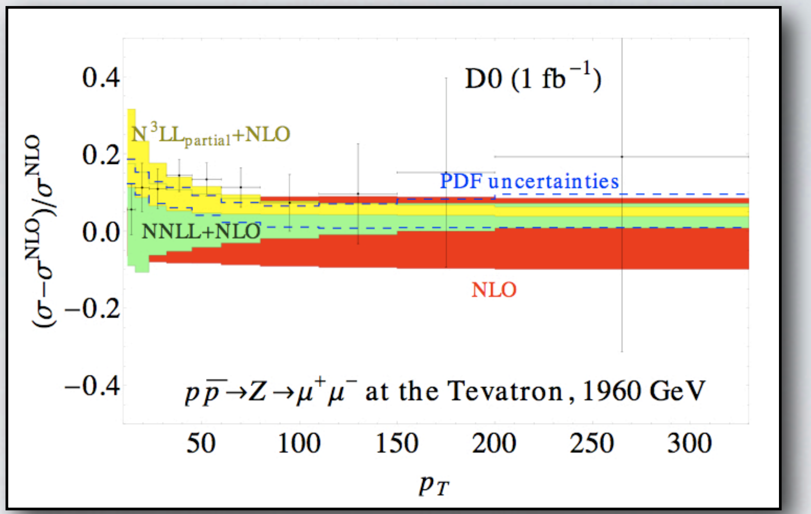
PHENOMENOLOGY

No. 28—"Three Log" Load of Sugar Pine at the Mill Pond.

Electroweak annihilation

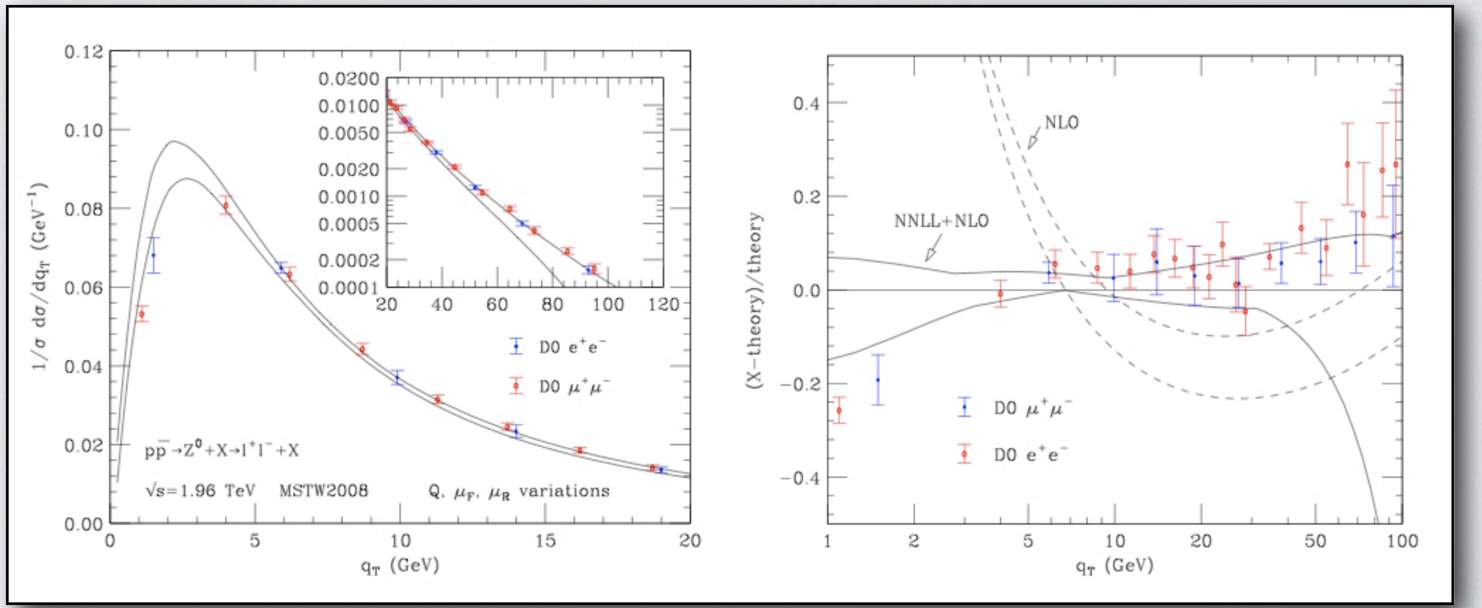
- Classic threshold resummation (for σ_{TOT}) is possible to `well-approximated' N^3LL.

- At large measured transverse momentum one is again close to partonic threshold.
  - p_T-threshold resummation now performed to approximate N^3LL (Becher, Schwartz, 11).



Large p_T D0 data vs. `threshold' resummation

- Transverse momentum resummation is available at NNLL (Bozzi et al., 10).
  - Favorable comparison to Tevatron data.
  - Small theoretical uncertainty.
  - Awaiting LHC data comparison.

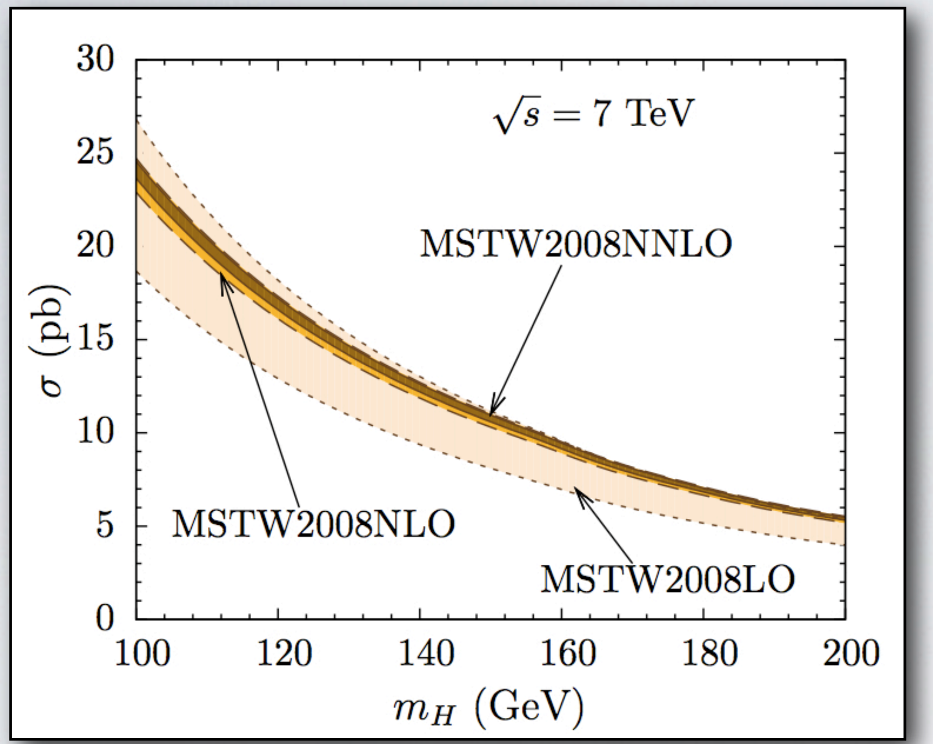


q_T spectrum of Z bosons at Tevatron (D0) compared to NNLL resummation

- Caveat: detailed SCET analysis (Becher and Neubert, 11) indicates (large) modification of 3-loop coefficient!
  - Theoretically interesting `collinear anomaly', transverse momentum pdf issues.

- NLL predictions for p_T spectrum also from SCET (Mantry, Petriello, 10)

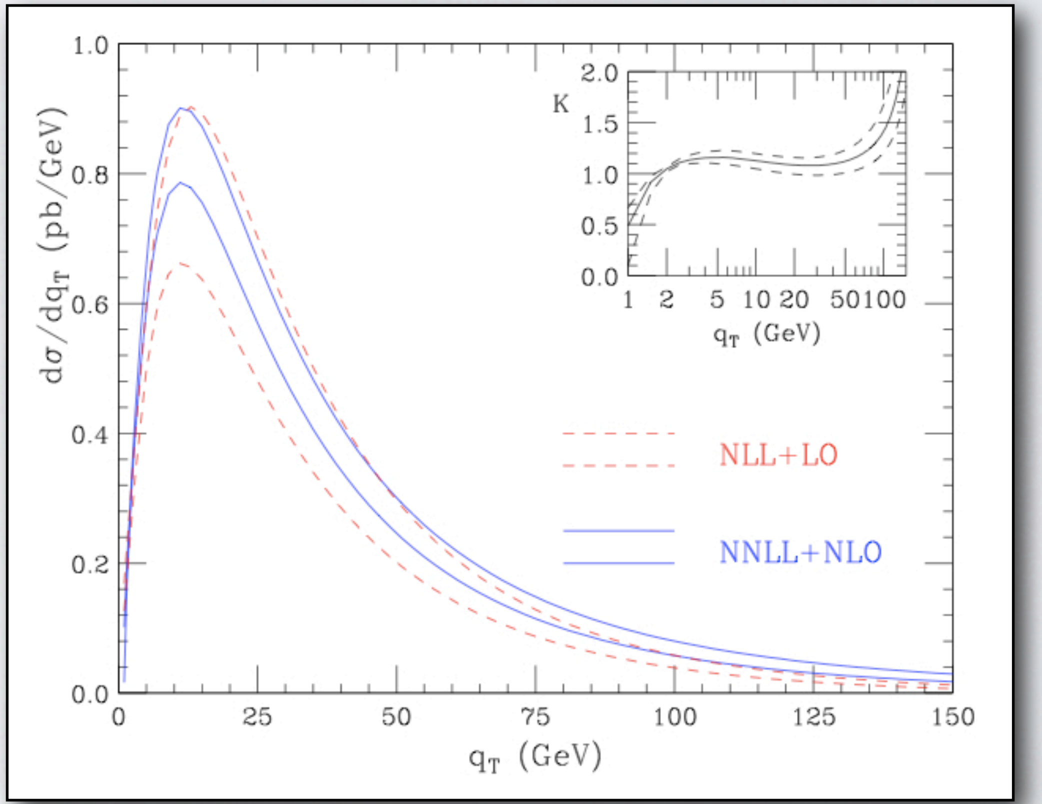
Higgs production



N³LL resummed cross section for Higgs production via gluon fusion at LHC

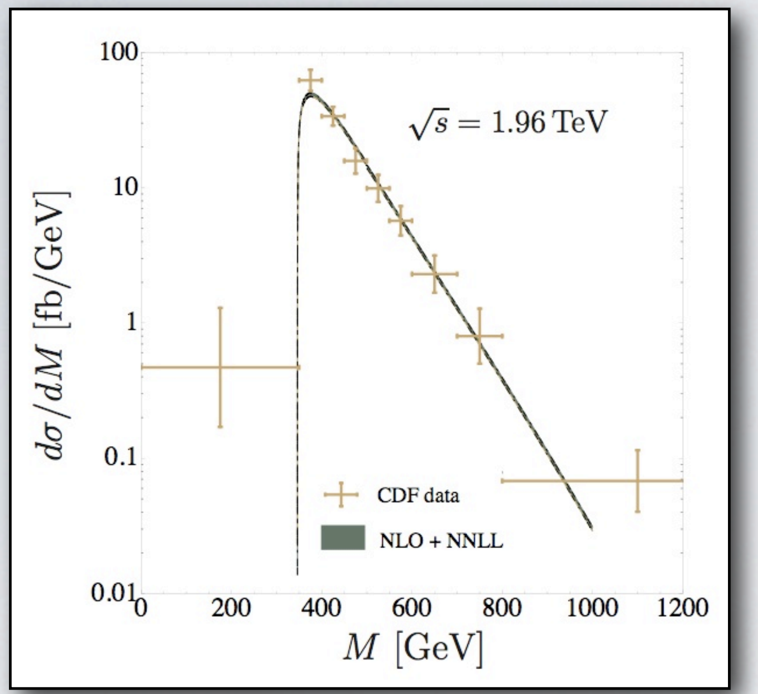
- The total cross section for gg->H is known to N^3LL and NNLO, with NLO EW corrections.
  - One of the best-known observables in the SM.
  - A combined analysis (Ahrens et al. 11) gives a 3% (th) + 8% (pdf) + 1% (mq) uncertainty.
  - Ongoing debate on theoretical and pdf uncertainty (Baglio et al. 11).

- The p_T distribution for gg->H is known to NNLL and NNLO (M. Grazzini et al. '07, '10)
  - Resummation reduces scale uncertainty
  - A subtle polarization effect uncovered but not implemented yet (Catani, Grazzini, 10)
  - Impact of revised three-loop coefficient must be gauged



NNLL resummed pT distribution for Higgs production via gluon fusion at LHC

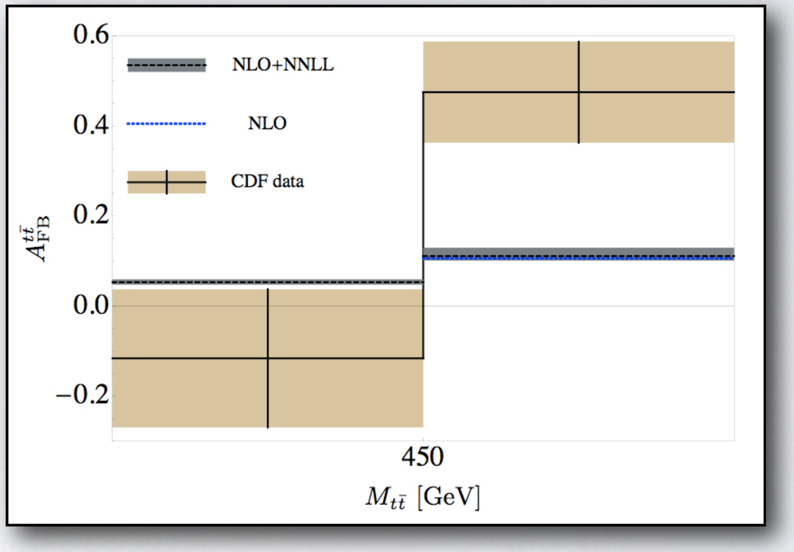
Top distributions



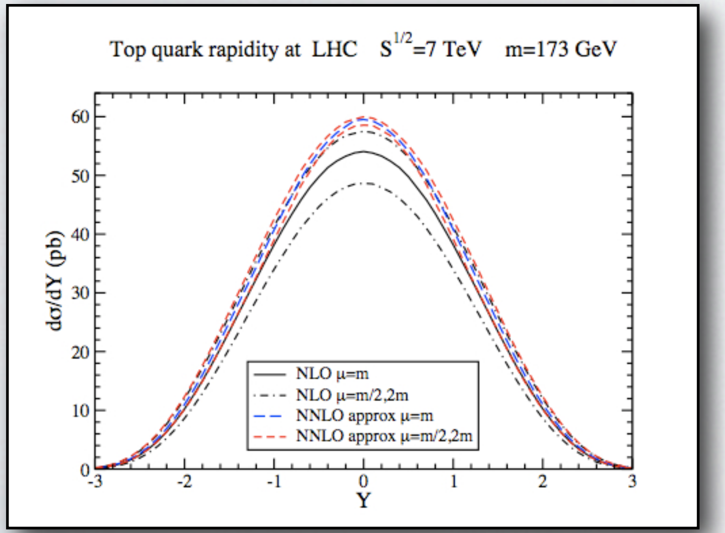
NNLL top-antitop invariant mass spectrum compared to CDF data

- The calculation of the two-loop massive anomalous dimension matrix makes it possible to perform NNLL resummation for generic distributions (Ahrens et al., 09).
  - Invariant pair mass distribution shows remarkable agreement with CDF data (LHC awaited).
  - Negligible theoretical uncertainty.
  - Different choices of kinematics and frame possible, vast menu of distributions available.

- The Tevatron top-antitop FB asymmetry can be computed in QCD at NNLL+NLO (Ahrens et al., 09).
  - Negligible impact on NLO result: the solution to the Tevatron puzzle is not QCD higher orders.



NNLL top-antitop FB asymmetry compared to CDF data



Top rapidity distribution at approximate NNLO

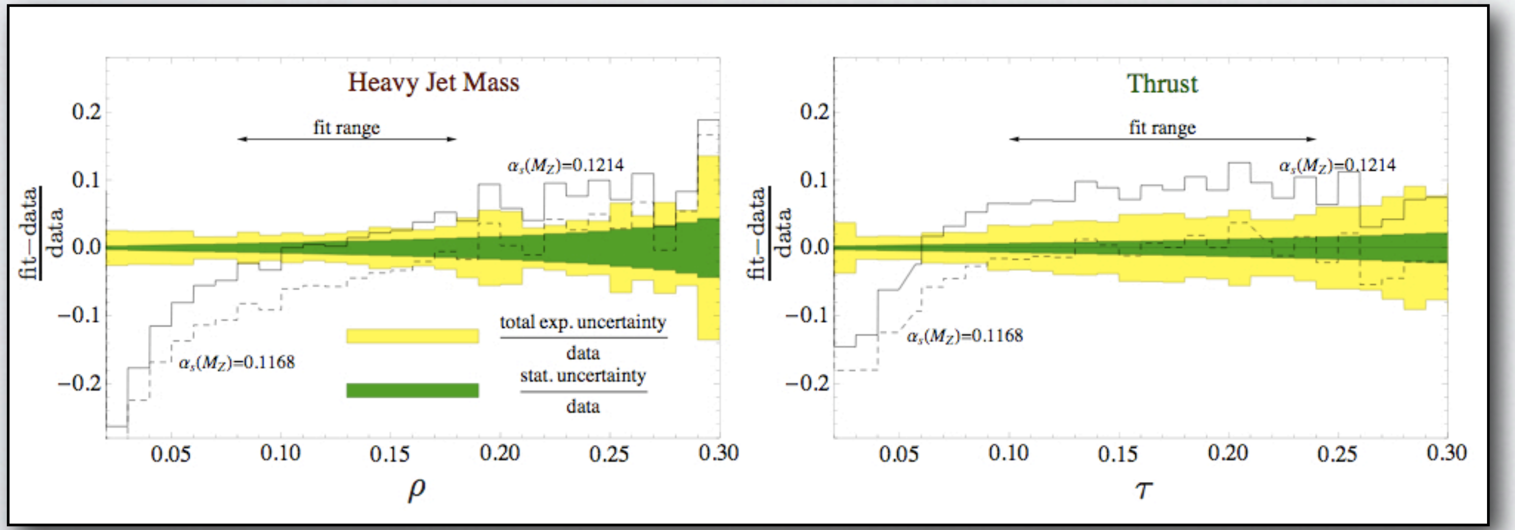
- Resummation can also be used in a simplified way to compute approximate higher order corrections and distributions (Kidonakis, 10-11)
  - Some conceptual and technical issues avoided; partial reduction in scale uncertainties.

Event shapes

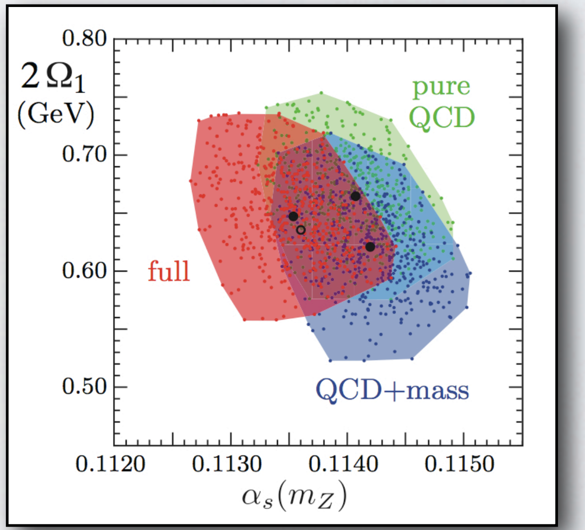
- First studies of event shapes with exact NNLO information and (well) approximated N^{3}LL resummation have appeared (Becher, Schwartz, 08; Schwartz, Cien; Abbate et al. 10).
- The studies deploy neat tricks (Padé approximants, numerical determination of 2-loop soft coefficients) and great care (hadronization, b-mass, QED corrections).
- Perturbative agreement between SCET and standard resummation (Gehrmann et al., 11).
- Significant differences remain in the final results for the strong coupling.

$$
\begin{aligned}
\alpha_s(M_Z^2) &= 0.1172 \pm 0.0022 \qquad \text{thrust (BS)}\\
\alpha_s(M_Z^2) &= 0.1220 \pm 0.0031 \qquad \text{jet mass (SC)}\\
\alpha_s(M_Z^2) &= 0.1135 \pm 0.0010 \qquad \text{thrust (AFHMS)}
\end{aligned}
$$

- Many possible sources of discrepancy, the main suspect remains hadronization/MC.
- The problem is still not fully understood: do we really know α_s to percent accuracy?



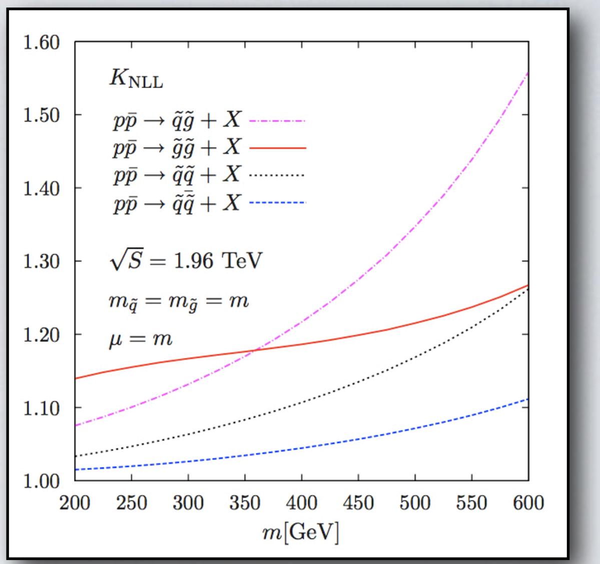
Comparing the α_s fit quality for thrust and heavy jet mass at N^{3}LL (SC)



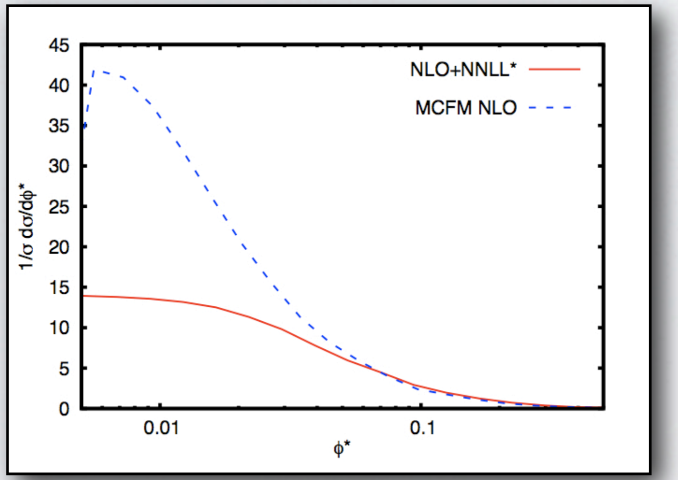
Joint fit of α_s and hadronization parameter Ω_1 from N^{3}LL thrust (AFHMS)

Miscellanea

- Soft gluon resummations are being applied to SUSY particles.
  - SUSY particles are heavy (and getting heavier ...), close to threshold: corrections useful for exclusion limits.
  - Gaugino and slepton production (singlets) (Klasen 06-11).
  - Colored sparticle production (requires soft matrices) (Kulesza et al. 09-11; Beneke et al., general color, 10).



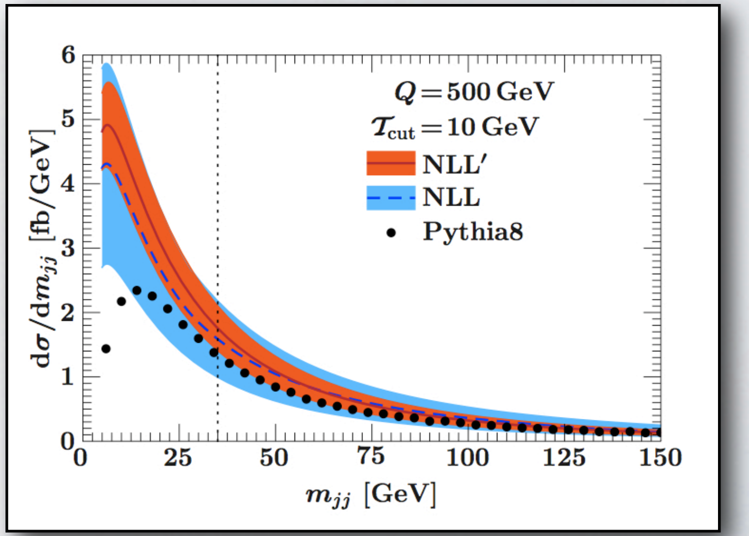
NLL K-factors for squarks and gluinos



NNLL vs. NLO φ* distribution

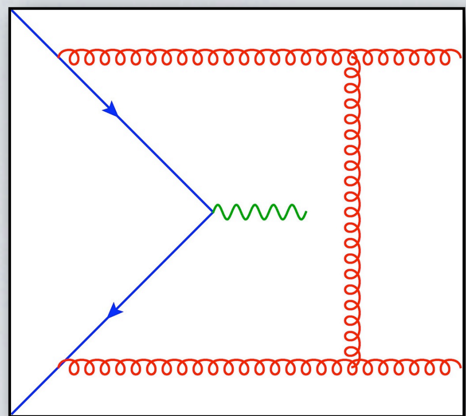
- New observables, designed by experiments, require (and get) soft gluon resummation (Banfi et al. 09-11).
  - Variables related to the angle between leptons are more accurately measured than the p_T of the lepton pair.
  - Resummation is crucial close to Born configuration.

- Complex jet observables are designed and resummed.
  - Jet shapes to study internal structure of jets, useful for boosted heavy particle production (Ellis et al. 09-11).
  - Dijet mass distribution with fixed \`background' event shape (\`N-jettiness'), extension of SCET (Bauer et al. 11).

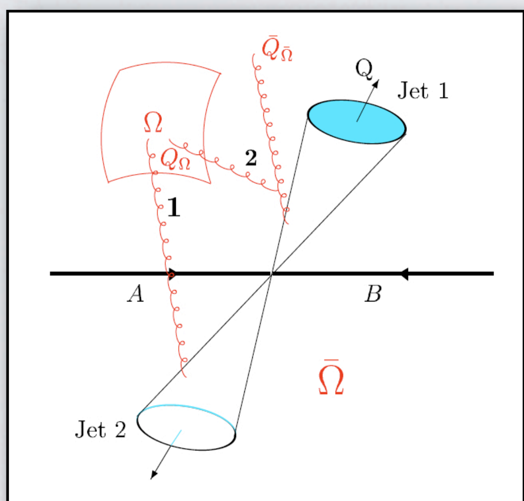


NLL dijet mass distribution for fixed N-jettiness

Jactuum caveat emptor



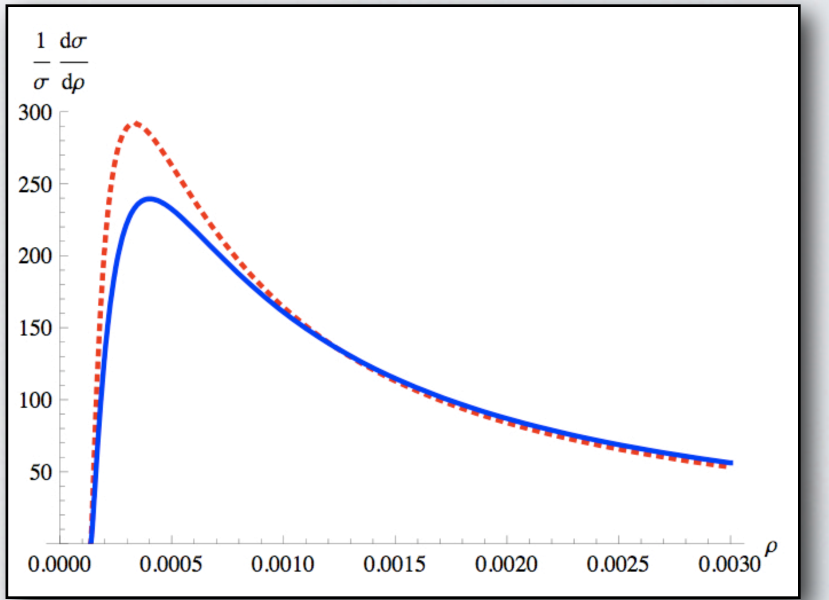
Spectator interaction via Glauber gluon



Non-global logarithms for energy flow

- As the jet observables proliferate and are resummed, several caveats must be kept in mind.
  - Glauber gluons: they cancel in inclusive jet cross sections (Aybat, Sterman 09), but no proof if jets are opened up. They are not in SCET, might be added (Bauer et al., 10).
  - Non-global logarithms: arise whenever gluon emission phase space is cut up (Dasgupta, Salam, 01); affect observables at single logarithmic level; resummable only at large N_C.
  - Jet algorithms: the choice of jet algorithm affects both non-global and ordinary Sudakov logarithms. Clustering correlates `independent' gluons, except for anti-kT (Banfi et al., 05-06).

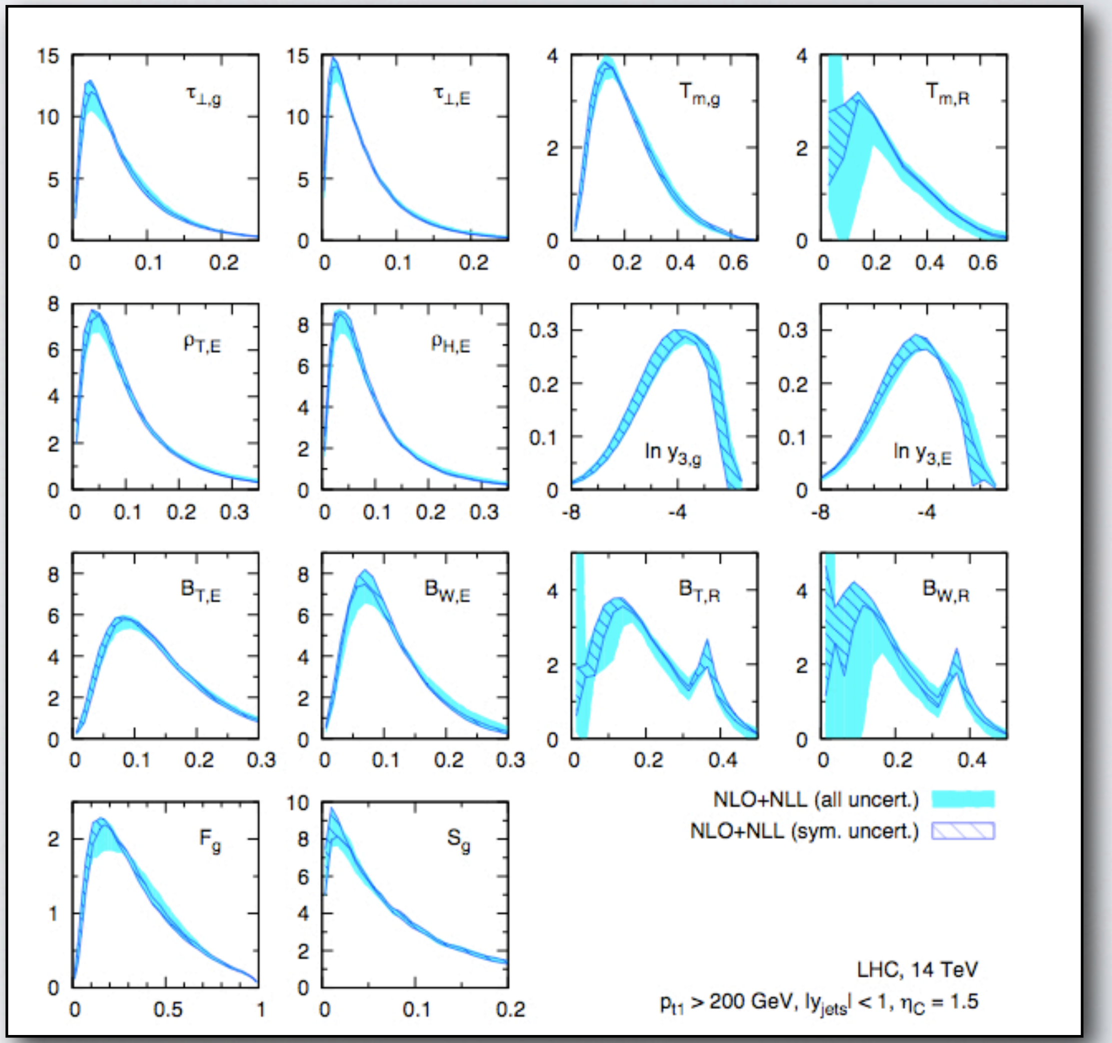
- A striking example of the impact of NG logs on jet shapes.
  - Jet shapes measure properties of a single jet in a multijet event. They are generically affected by NG logs.
  - For a typical jet shape (`in-jet angularity') NG logs change the height of the (formally NLL) distribution by 15-20% in the small-R limit (Banfi et al., 10).



The impact of non-global logarithms on jet shapes

Hadronic event shapes

- An interesting alternative to the use of jets, which bypasses the need for an algorithm, is to introduce global event shapes, in analogy to those used in e^+e^- annihilation.
  - The hadronic environment requires suppressing the beam region.
- NLL+NLO resummation can be performed numerically with the program `Caesar` recently generalized to hadron collisions (Banfi, Salam, Zanderighi, 10).
- Numerically resummable event shapes are carefully characterized:
  - Functional constraints.
  - Continuous globalness.
  - Recursive IR safety.
- A vast variety of event shapes is introduced, categorized and resummed.
  - Simple example: transverse thrust.
- Relevant issues for NLL+NLO resummation of event shapes are dealt with in detail.
  - Control of non-global logs.
  - Transition particle-jet (algorithm issues).
  - Possible superleading logs.
  - Matching to NLOJET++.
  - Power corrections (analytic and MC).
  - Impact of underlying event.



A menu of NLL-resummed hadronic event shapes

OUTLOOK

Summary

- Resummations are a powerful tool both for theory and for phenomenology.
  - ✓ Explore the boundary between perturbative and non-perturbative physics.
  - ✓ Are necessary for precision phenomenology.
- Resummations have a long history, but
  - ✓ past few years have seen very intense LHC-motivated activity and theoretical progress.
- Factorization theorems $\Rightarrow$ Evolution equations $\Rightarrow$ Exponentiation.
  - ✓ Sudakov factorization $\Rightarrow$ soft-gluon resummation (also formalized by SCET).
  - ✓ Multiparton processes require anomalous dimension matrices.
- Remarkable progress on the theory side.
  - ✓ We are understanding the all-order structure of the perturbative exponent.
  - ✓ For massless partons the dipole formula may give the definitive answer.
  - ✓ For massive partons the general two-loop anomalous dimension matrix is known.
  - ✓ SCET provides new insights: momentum space resummation, `collinear anomaly'.
  - ✓ Ongoing efforts to go beyond the eikonal approximation.
- A vast array of phenomenological applications, many vital for LHC precision physics.
  - ✓ Electroweak annihilation processes are known to high logarithmic accuracy.
  - ✓ Top distributions can be computed with unprecedented theoretical precision.
  - ✓ The strong coupling can be precisely determined from resummed event shapes.
  - ✓ Hadronic event shapes provide a flexible alternative tool for hadron collisions.
- We have come a long way, but each step forward brings new insight and new questions ...

"And if you enjoyed my summation you're going to love my new CD,
'The Very Best Summations of Walter J. Prescott'!"

THANK YOU