Neutral-current DIS at NNLO in the S-ACOT-χ scheme

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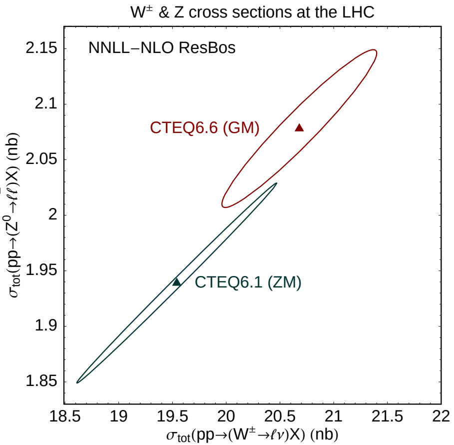
Full paper: **http://bit.ly/SACOTNNLO11**, to be submitted this week

DIS at NNLO in the S-ACOT-χ scheme

- General-mass (and not zero-mass of fixed-flavor number) treatment of c, b mass terms in DIS is essential for predicting precision W, Z cross sections at the LHC *(Tung et al., hep-ph/0611254)*

- Quark mass effects in DIS are comparable to NNLO terms
- NNLO implementation in the S-ACOT-χ scheme is ready to be released
- **What is new compared to other mass schemes** *(FFN, FONLL, TR'...)*?

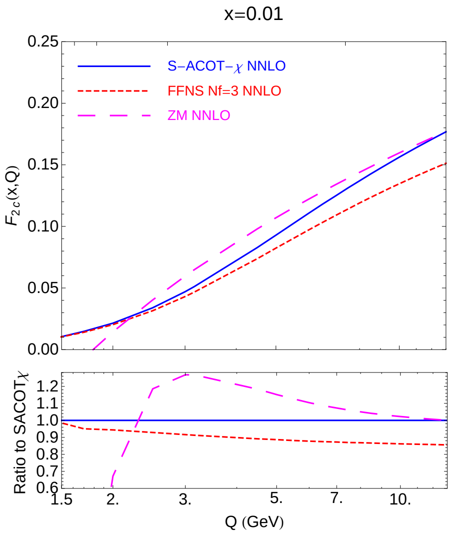


Simplified Aivazis-Collins-Olness-Tung scheme

ACOT, PRD 50 3102 (1994); Collins, PRD 58 (1998) 094002; Kramer, Olness, Soper, PRD (2000) 096007

- Is proved to all orders by the QCD factorization theorem for DIS with massive quarks *(Collins, 1998)*
- Is relatively simple
  - One value of N_f (and one PDF set) in each Q range
  - Sets $m_Q = 0$ in ME with incoming c or b
  - matching to FFN by kinematical rescaling; **implemented as a part of the QCD factorization theorem** New!
- Reduces to the ZM $\overline{MS}$ scheme at $Q^2 \gg m_Q^2$, without additional renormalization
- Reduces to the FFN scheme at $Q^2 \approx m_Q^2$
  - has reduced dependence on tunable parameters at NNLO

S-ACOT-χ scheme: merging FFN and ZM



ACOT reduces to FFNS at $Q \approx m_c$ and to ZM at $Q \gg m_c$

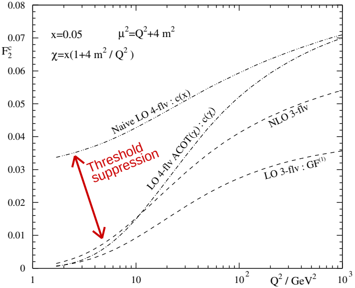
Les Houches toy PDFs, evolved at NNLO with threshold matching terms

Cancellations between subtractions and other terms at $Q \approx m_c$ and $Q \gg m_c$; details in backup slides

Energy conservation in the factorization theorem

- Phase space suppression due to energy conversation is numerically important at $Q \approx m_c$
- Collinear approximation for heavy quarks may not obey energy conservation without an additional condition

- In the new treatment, the requirement of energy conservation is included **as a part of the factorization theorem** *(cf. the paper)*
  - correct kinematics of HQ production is reproduced without constraints on derivatives, damping factors...



New! Energy conservation and factorization (cont.)

- The Z projection operator of the QCD factorization theorem is modified to obey EC in all channels
  - Other details of the proof are unchanged
- This naturally leads to χ rescaling **in Wilson coefficients** with an incoming heavy quark h:

$$\text{in } F(x,Q) = \sum_{a=g,u,d,\dots,h} \int \frac{d\xi}{\xi} C_a\left(\frac{x}{\xi}, \frac{Q}{\mu}, \frac{m_h}{Q}\right) f_{a/p}(\xi,\mu),$$

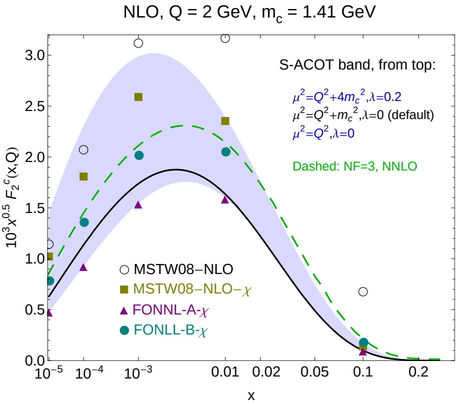
$$C_h\left(\frac{x}{\xi}, \frac{Q}{\mu}, \frac{m_h}{Q}\right) \approx C_h\left(\frac{\chi}{\xi}, \frac{Q}{\mu}, m_h = 0\right) \theta(\chi \le \xi \le 1),$$

$$\text{where } \chi \equiv x\left(1 + \frac{4m_h^2}{Q^2}\right).$$

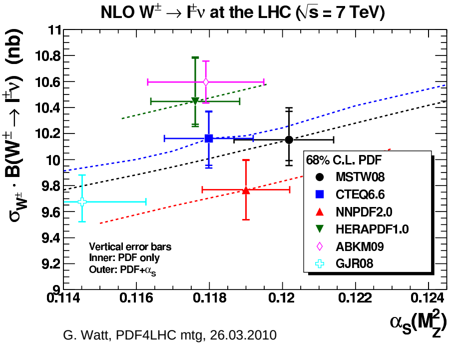
- The target (PDF) subgraphs are given by **universal** operator matrix elements

Input parameters of the S-ACOT-χ scheme

At NLO, the charm mass m_c and factorization scale μ of CTEQ PDFs are **tuned** to best describe the DIS data



2009 Les Houches HQ benchmarks with toy PDFs; default $\mu = Q$



W, Z cross sections; $m_c = 1.3$ GeV in CTEQ6.6

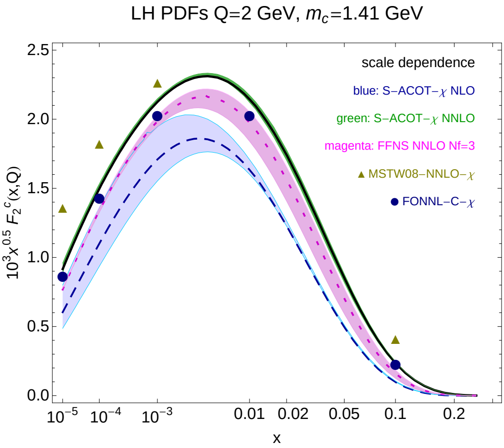
NNLO results for $F_2^{(c)}(x, Q^2)$ - Preliminary

At NNLO and $Q \approx m_c$:

S-ACOT-$\chi\,(N_f = 4) \approx$ FFN $(N_f = 3)$
without tuning

- S-ACOT is numerically close to other NNLO schemes
- NNLO expressions are close to the FONLL-C scheme

(Forte, Laenen, Nason, arXiv:1001.2312).



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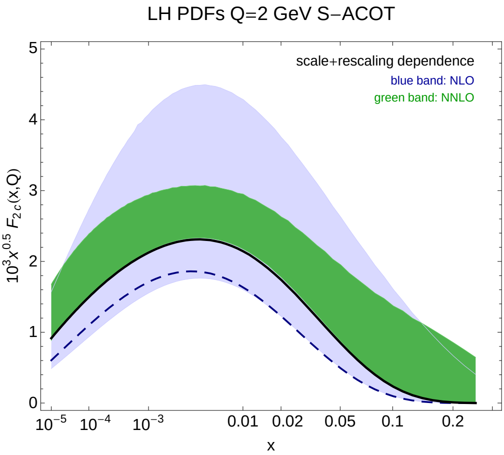


- Even without rescaling (**a wrong choice!**), NNLO cross sections are much closer to FFN at $Q \approx m_c$ than at NLO

Details of the NNLO computation

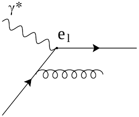
- NNLO evolution for α_s and PDFs (HOPPET)
  - matching coefficients relating the PDFs in N_f and N_{f+1} schemes *(Smith, van Neerven, et al.)*
- NNLO Wilson coefficient functions for $F_2(x, Q)$, $F_L(x, Q)$
- Pole quark masses or $\overline{MS}$ quark masses as an input
- CT10.1: pole masses $m_c = 1.3$ GeV, $m_b = 4.75$ GeV (as in CT10)

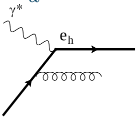
Components of inclusive $F_{2,L}(x,Q)$

Components of inclusive $F_{2,L}(x,Q^2)$ are classified according to the quark couplings to the photon

$$F = \sum_{l=1}^{N_l} F_l + F_h \quad (1)$$

$$F_l = e_l^2 \sum_a \left[C_{l,a} \otimes f_{a/p}\right](x,Q), \qquad F_h = e_h^2 \sum_a \left[C_{h,a} \otimes f_{a/p}\right](x,Q). \quad (2)$$





At $\mathcal{O}(\alpha_s^2)$:

$$F_h^{(2)} = e_h^2 \left\{ c_{h,h}^{NS,(2)} \otimes (f_{h/p} + f_{\bar{h}/p}) + C_{h,l}^{(2)} \otimes \Sigma + C_{h,g}^{(2)} \otimes f_{g/p} \right\}$$
$$F_l^{(2)} = e_l^2 \left\{ C_{l,l}^{NS,(2)} \otimes (f_{l/p} + f_{\bar{l}/p}) + c^{PS,(2)} \otimes \Sigma + c_{l,g}^{(2)} \otimes f_{g/p} \right\}. \quad (3)$$

Structure of factorized expressions is reminiscent of the ZM scheme (e.g., in MVV 2005)

Components of inclusive $F_{2,L}(x,Q)$

- Lower case $c^{(2)}_{a,b}$, $\hat{f}^{(k)}_{a,b}$ $\rightarrow$ ZM expressions
  Zijlstra and Van Neerven PLB272 (1991), NPB383 (1992)
  S. Moch, J.A.M. Vermaseren and A. Vogt, NPB724 (2005)

- Upper case $C^{(2)}_{a,b}$, $F^{(k)}_{a,b}$ $A^{(k)}_{a,b}$ $\rightarrow$ coeff. functions, structure functions and subtractions with $m_c \neq 0$,
  Buza *et al.*, NPB 472 (1996); EPJC1 (1998);
  Riemersma, *et al.* PLB 347 (1995); Laenen *et al.* NPB392 (1993)

- All building blocks are available from literature

Components of inclusive $F_{2,L}(x,Q)$

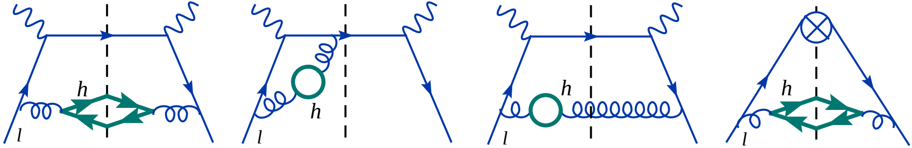
The separation into F_l and F_h (according to the quark's electric charge e_i^2) is valid at all Q

The "light-quark" F_l contains some subgraphs with heavy-quark lines, denoted by "$G_{l,l,heavy}$".

The "heavy-quark" $F_h \neq F_2^c$:

$$F_2^c = F_h + (G_{l,l,heavy})_{real},$$

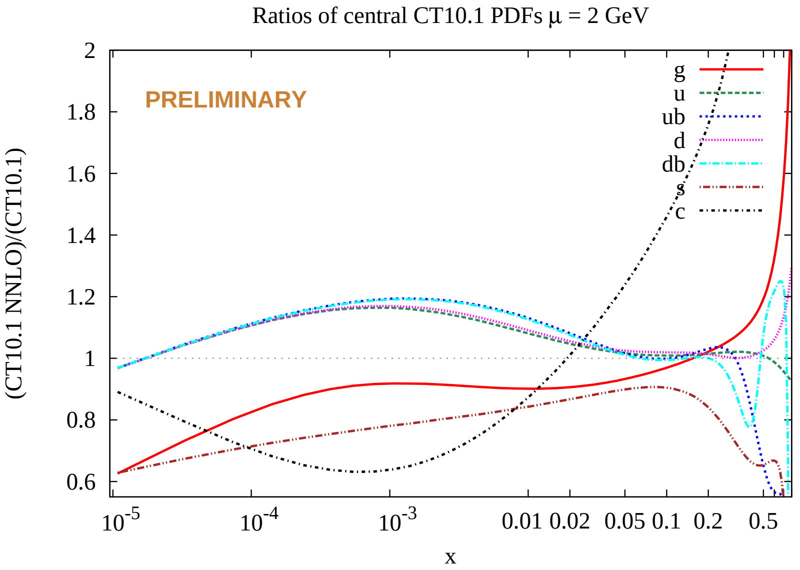
where $G_{i,j} = C_{i,j}^{(2)},\ \ F_{i,j}^{(2)}$, and $A_{i,j}^{(2)}$



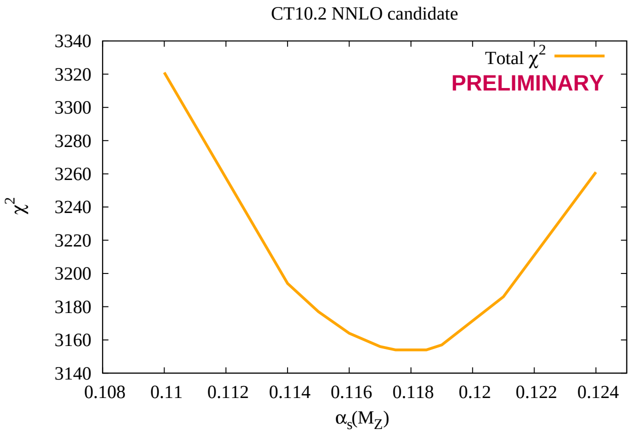
Conclusions

- In the CTEQ-TEA fit, an NNLO calculation for $F_{2,L}^{c,b}$ in the S-ACOT scheme is demonstrated to be viable.
- S-ACOT-χ formalism provides recipe-like formulas for implementing NNLO
- Energy conservation is realized as a part of the QCD factorization theorem
- It leads to rescaling of Wilson coefficient functions with incoming heavy quarks. The PDFs are given by universal operator matrix elements.
- First NNLO fits are being currently investigated

Candidate NNLO fit (compared to CT10.1 NLO)



Dependence on α_s in the CT10.1 fit



α_s decreases slightly at NNLO, has about the same PDF uncertainty as at NLO

- NLO: $\alpha_s(M_Z) = 0.11964 \pm 0.0064$ at 90% c.l.
- NNLO: $\alpha_s(M_Z) = 0.118 \pm 0.005$

Backup slides
1. Details on S-ACOT-χ scheme at NNLO

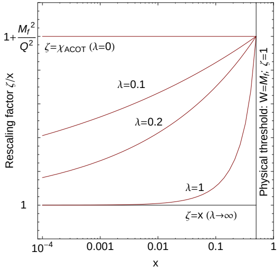
S-ACOT input parameters

At $Q \approx m_c$, F_2^c depends significantly on

1. **Charm mass:** $m_c = 1.3$ GeV in CT10
2. **Factorization scale:** $\mu = \sqrt{Q^2 + \kappa m_c^2}$; $\kappa = 1$ in CT10
3. **Rescaling variable** $\zeta(\lambda)$ for matching in $\gamma^* c$ channels
   (*Tung et al., hep-ph/0110247; Nadolsky, Tung, PRD79, 113014 (2009)*)

$$F_i(x, Q^2) = \sum_{a,b} \int_\zeta^1 \frac{d\xi}{\xi} \, f_a(\xi, \mu) \, C^a_{b,\lambda}\left(\frac{\zeta}{\xi}, \frac{Q}{\mu}, \frac{m_i}{\mu}\right)$$

$$x = \zeta \Big/ \left(1 + \zeta^\lambda \cdot (4m_c^2)/Q^2\right) \,, \text{ with } 0 \leq \lambda \lesssim 1$$

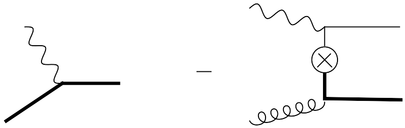


CT10 uses

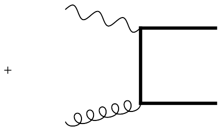
$$\zeta(0) \equiv \chi \equiv x\left(1 + 4m_c^2/Q^2\right),$$

motivated by momentum conservation

Classes of Feynman diagrams I

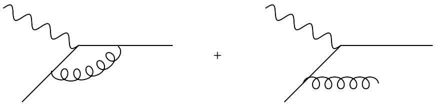


NLO Subtraction

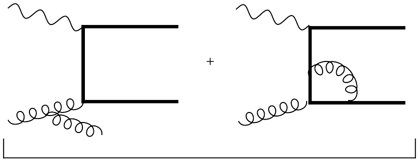


NLO γ^* g

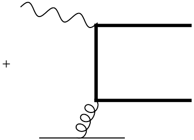
ACOT I: Phys.Rev.D50:3085−3101,1994
ACOT II: Phys.Rev.D50:3102−3118,1994



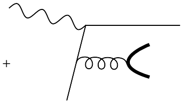
NLO γ^* c



NNLO: γ^* g



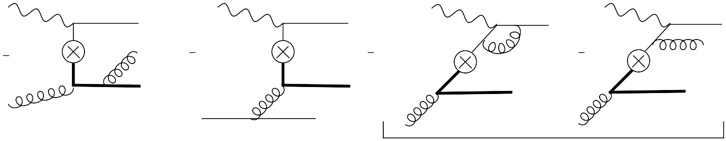
NNLO: γ^* Σ



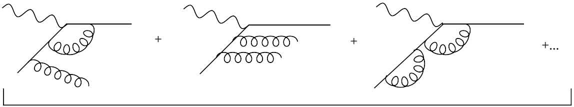
NNLO: γ^* q NS

Riemersma et. al. Phys.Lett. B347 (1995)

Classes of Feynman Diagrams II



NNLO Subtractions   Buza, Matiounine, Smith, Van Neerven, Eur. Phys. J. C 1998



NNLO γ^* c   Moch, Vermaseren and Vogt, Nucl.Phys.B724, 2005

Cancellations between Feynman diagrams

Validity of the S-ACOT calculation was verified by checking for certain cancellations at $Q \approx m_c$ and $Q \gg m_c$

- $Q \approx m_c$:

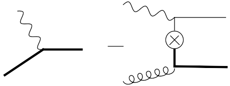
$$D^{(2)}_{C1} \ll D^{(2)}_{C0} \ll D^{(1)}_{C0} \leq F_2^c(x, Q)$$

- $Q \gg m_c$:

$$D^{(2)}_{g} \ll D^{(1)}_{g} < F_2^c(x, Q)$$

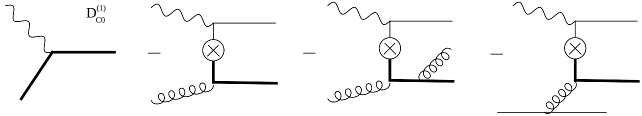
These cancellations are indeed observed in our results

NNLO: Cancellations at $Q^2 \approx m_c^2$



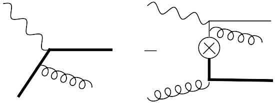
$$D_{C0}^{(1)} = C_Q^{(0)} \otimes c - a_s C_Q^{(0)} \otimes A_{Qg}^{(1)} \otimes g; \qquad a_s = \frac{\alpha_s}{(4\pi)} \tag{1}$$

$D_{C0}^{(1)}$



$D_{C0}^{(2)}$

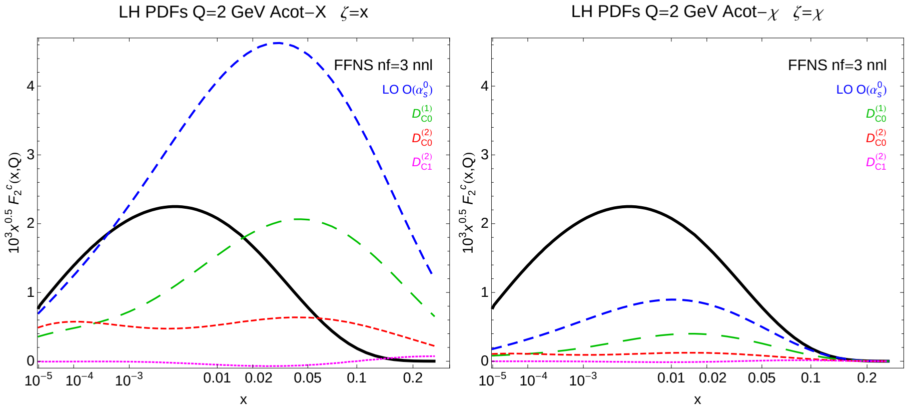
$$D_{C0}^{(2)} = D_{C0}^{(1)} - a_s^2 C_Q^{(0)} \otimes A_{Qg}^{(2),S} \otimes g - a_s^2 C_Q^{(0)} \otimes A_{Q\Sigma}^{(2),PS} \otimes \Sigma \tag{2}$$



$$D_{C1}^{(2)} = C_Q^{(1)} \otimes c - a_s^2\, C_Q^{(1)} \otimes A_{Qg}^{(1)} \otimes g \tag{3}$$

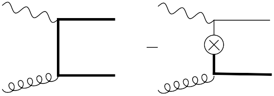
$D_{C1}^{(2)}$

NNLO: Cancellations at $Q^2 \approx m_c^2$



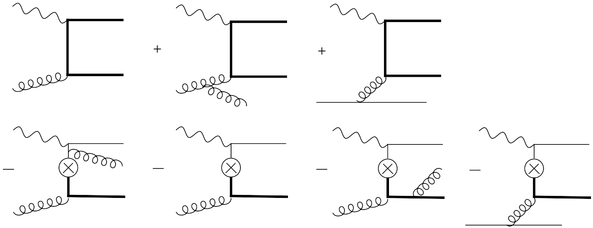
$D^{(2)}_{C1} \ll D^{(2)}_{C0} \ll D^{(1)}_{C0} \leq$ FFN at NNLO both for $\zeta = x$ and $\zeta = \chi$.

NNLO: Cancellations at $Q \gg m_c$



$D_g^{(1)}$

$$D_g^{(1)} \equiv C_g^{(1)} = a_s \left(F_g^{(1)} \otimes g - C_Q^{(0)} \otimes A_{Qg}^{(1),S} \otimes g \right) \qquad (4)$$

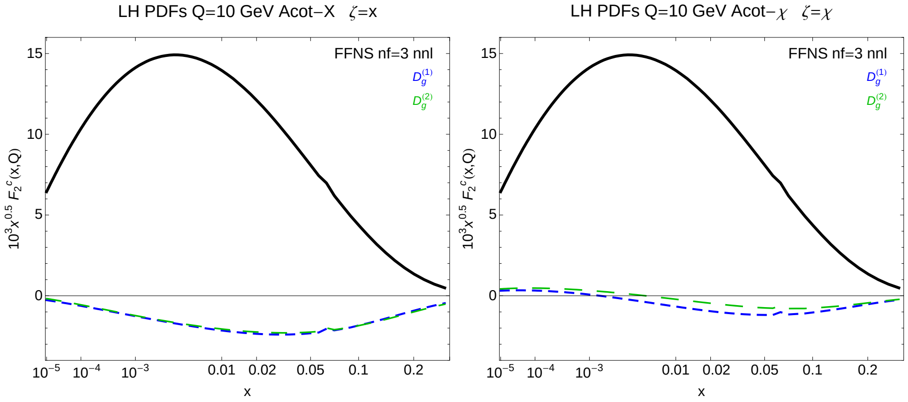


$D_g^{(2)}$

$$D_g^{(2)} = D_g^{(1)} + a_s^2 \Big[\tilde{F}_g^{(2)} \otimes g + \tilde{F}_\Sigma^{(2)} \otimes \Sigma - C_Q^{(1)} \otimes A_{Qg}^{(1),S} \otimes g$$
$$- C_Q^{(0)} \otimes A_{Qg}^{(2),S} \otimes g - C_Q^{(0)} \otimes A_{Q\Sigma}^{(2),PS} \otimes \Sigma \Big] \qquad (5)$$

$D_g^{(1)}$ is of order of α_s^2 while $D_g^{(2)}$ is of order of α_s^3.

F_2^c at NNLO: Cancellations at $Q = 10$ GeV



$D_g^{(2)} \ll D_g^{(1)} <$ FFN at NNLO $<$ ACOT

$\log \frac{Q^2}{m_c^2}$ terms in FFN are cancelled well by subtractions.