Heavy-quark contributions: flavor-number schemes

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Heavy-quark contributions: flavor-number schemes

Heavy-quark contributions to QCD predictions for hadron colliders:

$$\sigma(p_1,p_2) = \sum_{ij} \int dx_1\, dx_2\, \hat{\sigma}_{ij}\left(x_1, x_2, \tfrac{m^2}{\mu^2}, \alpha_s(\mu^2)\right) f_i(x_1,\mu^2)\, f_j(x_2,\mu^2)$$

Besides explicit dependences, PDFs are determined by global fits

The primary input for these analyses are DIS inclusive data

Heavy quarks contribute considerably, e.g., for the dominant $F_2(x,Q^2)$:

$$\mathbf{F_2^c \lesssim 30\%}, \qquad \mathbf{F_2^b \lesssim 3\%}$$

Thus their appropriate treatment is *vital* for a correct interpretation of the upcoming results at LHC

There are several formalisms (“schemes”): FFNS, VFNS, GM-VFNS’s

When is which scheme adequate? Is this clear or are there uncertainties?

Heavy-quark contributions: flavor-number schemes

Exact fully-massive calculations: FFNS

Theoretical status of heavy-quark contributions to DIS

A glance through the neutral-current contributions

Perturbative stability of the FFNS

Effective heavy-quark distributions: VFNS

Relevance of the VFNS

General-mass VFNS

GM-VFNS implementations

Plausibility of the GM-VFNS

Heavy-quark treatments in global fits

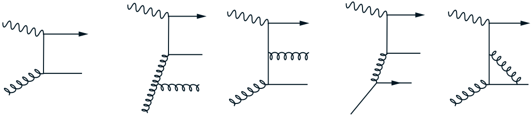
Exact fully-massive calculations: FFNS

HQ generated in hard collisions, *not collinearlly*, short “lifetime” ($\neq$ parton)

Experimentally: **no** $(< 1\%)$ **intrinsic** heavy-quark content in the nucleon

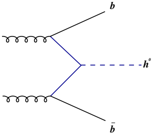
Final-state $\equiv$ **extrinsic** heavy-quark content $\Rightarrow$ **fully-massive calculations** initiated by gluons and light (u, d, s) quarks $\equiv$ **FFNS** (in this context)

For electroproduction up to NLO:

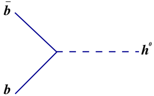


The only *drawback* of the FFNS is the calculational difficulty, for instance:

FFNS: $g\,g \to b\,\bar{b}\,H$



VFNS: $b\,\bar{b} \to H$



Thus *effective* HQ distributions (VFNS) are often used to ease the calculations

Theoretical status of heavy-quark contributions to DIS

The inclusive coefficient functions for neutral-current DIS are known at LO [Witten 75, Glück and Reya 79] and NLO [Laenen, Riemersma, Smith, van Neerven 93]

There is a fully exclusive NLO calculation [Harris and Smith 95]: **HVQDIS**, **in which all the experimental analysis at HERA is based**

For charged-current DIS the inclusive Wilson coefficients are also known up to NLO [Gottschalk 81],[Glück, Godbole, Reya 88], [Glück, Kretzer, Reya 96] and there is an NLO exclusive calculation [Glück, Kretzer, Reya 97]

At NNLO only the asymptotic ($Q^2 \gg m^2$) coefficient functions for charged current [Buza, van Neerven 97] and neutral current [Bierenbaun, Blümlein, Klein 09] are known

No complete NNLO calculation of heavy-quark contributions in DIS exist!

Only one group [Blümlein et al.] trying to improve in this (talk by J. Blümlein)

Some *approximations* can be made using small-x [Catani, Cialfoni, Hautmann 91] and threshold [Laenen and Moch 99] resummations

A glance through the neutral-current contributions

Gluon dominated (starts at LO), therefore “small-x”-dominated:

$$F^h_{k=2,L}(x,Q^2,m^2) = \frac{\alpha_s(\mu^2)Q^2}{4\pi^2 m^2} \sum_{f=g,q,\bar{q}} \int_{z_{th}}^{1} dz\, f(z,\mu^2)\, \hat{F}^h_{k=2,L}(\tfrac{x}{z},\mu^2,Q^2,m^2), \quad z_{th} = x(1+\tfrac{4m^2}{Q^2})$$

about 80% originates in the region $z_{th} \leq z \leq 3z_{th}$ [Vogt 96]

⇒ **threshold region is always important** (irrespective of Q^2)

$$\hat{F}^h_k = e_h^2 \delta_{fg}\, c^{(0)}_{k,g}(\eta,\xi) + 4\pi\alpha_s(\mu^2)\left[e_h^2\left(c^{(1)}_{k,f}(\eta,\xi) + \bar{c}^{(1)}_{k,f}(\eta,\xi)\ln\frac{\mu^2}{m^2}\right)\right.$$
$$\left. + e_f^2\left(d^{(1)}_{k,f}(\eta,\xi) + \bar{d}^{(1)}_{k,f}(\eta,\xi)\ln\frac{\mu^2}{m^2}\right)\right], \ \eta = \frac{\hat{s}}{4m^2} - 1,\ \xi = \frac{Q^2}{m^2}$$

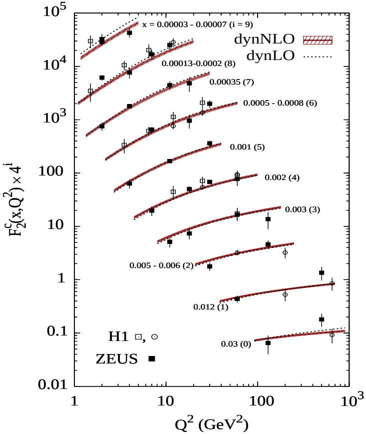
The coefficients functions contain $\ln\frac{Q^2}{m^2}$’s (**not mass divergences**)

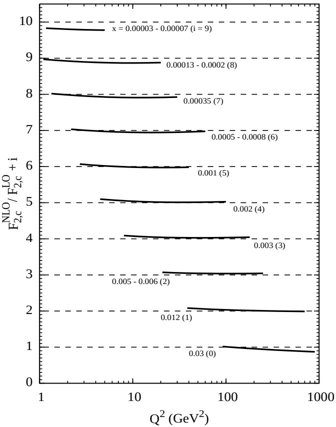
Are these terms dangerous or is the FFNS stable for DIS phenomenology?

The fact that all data are well described by HVQDIS implies that these terms are not destroying the convergence of the QCD perturbative expansion

Perturbative stability of the FFNS

Nevertheless we can look at the results for the highest available Q^2:





It is also very *stable* with respect to variations in μ_r and μ_f [Glück, Reya, Stratmann 94]

FFNS gets trough *all* "stability tests"!!

⇒ there is no need to resum *supposedly* "large logarithms"

Recent studies point to stability also at hadron colliders [Campbell, Frederix, Maltoni, Tramontano 09]

Effective heavy-quark distributions: VFNS

Starting from the FFNS: $F_i^{FF}(x,Q^2) = \sum_k C_{ik}^{FF,3}(\frac{Q^2}{m^2}) \otimes f_k^3(Q^2), \quad k = 0,1,2,3$

and noting that the mass depence factorizes in the **asymptotic limit**:

$$\lim_{Q^2 \gg m^2} C_{ik}^{FF,3}(\frac{Q^2}{m^2}) \equiv \sum_j C_{ij}^{VF,4} \otimes A_{jk}(\frac{Q^2}{m^2}), \quad \begin{cases} A_{jk} = \text{massive OME's } (5 \times 4) \\ C_{ij} = \text{light-parton coefficient functions} \end{cases}$$

one can construct *effective* heavy-quark PDFs (VFNS) [Buza, Matiouine, Smith, van Neerven 98]:

$$f_j^4(Q^2) = \sum_k A_{jk}(\frac{Q^2}{m^2}) \otimes f_k^3(Q^2) \Rightarrow F_i^{VF}(x,Q^2) = \sum_k C_{ik}^{VF,4} \otimes f_k^4(Q^2), k = 0,1,2,3,4$$

A's are process independent ⇒ **preserves universality!!**

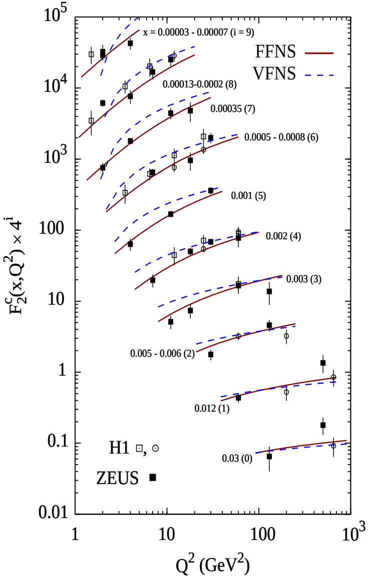
In practice **massless evolution** increasing n_f at unphysical "thresholds" $Q^2 \simeq m^2$

This resums (RGE) the $\ln \frac{Q^2}{m^2}$'s of the final-state contributions $\neq$ intrinsic HQs

The effective VFNS HQ-PDFs are *assumed* to be correct asymptotically but:

Is this scheme relevant for phenomenology?

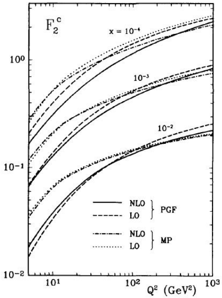
Relevance of the VFNS



Even though $W^2 \gg 4m_c^2$ for *all* data:

At small-x (small Q^2): **gross overestimate**

It does better at larger x (larger Q^2)



[Glück,Reya, Stratmann 94]

It never reduces to the exact (FFNS) result

(not even at very large Q^2)

$\Rightarrow$ *dropped terms* ($\propto \frac{m^2}{Q^2}$) *are always relevant*

The VFNS should not be used for global analyses!! (this is well-known since a long time)

Relevance of the VFNS

VFNS reliable for large invariant mass of the produced system: $W_{th}^2 \gg m^2$

$\Rightarrow$ non-relativistic threshold effects supressed [Glück, Reya, PJD 08]

For charm production in neutral-current DIS: $\frac{W_{\text{th}}}{m_c} = 2 \Rightarrow$ VFNS fails

For the previous example, Higgs-boson production in $b\bar{b}$ fusion:

$$\frac{W_{\text{th}}}{m_b} = \frac{2m_b + m_H}{m_b} \simeq \frac{m_H}{m_b} \gg 1 \Rightarrow \text{VFNS should work}$$

Note that VFNS PDFs can be generated from FFNS PDFs (3-flavor)

Input determined using the FFNS for DIS data!!

This combines the virtues of both **FFNS + VFNS** schemes

Typical scheme-choice uncertainties? Example, W production at LHC:

$$\sigma^{\text{NLO}}(pp \to W^+ + W^- + X) = \begin{cases} 186.5 \pm 4.9_{\text{pdf}} \,{}^{+4.8}_{-5.5}\,\big|_{\text{scale}}\ \text{nb} & \text{(VFNS)} \\ 192.7 \pm 4.7_{\text{pdf}} \,{}^{+3.8}_{-4.8}\,\big|_{\text{scale}}\ \text{nb} & \text{(FFNS)} \end{cases}$$

Sufficiently accurate ($\approx 10\%$) **for LHC and Tevatron** energies

General-mass VFNS

Some authors prefer VFNS's with built-in heavy-quark-masses effects

These should be close to the FFNS result *near the transition points*:

$$F_i^{FF}(x,Q^2) \simeq F_i^{GM}(x,Q^2) = \sum_k C_{ik}^{GM,4} \otimes f_k^4(Q^2), \quad k = 0,1,2,3,4$$

$$\Rightarrow C_{ik}^{FF,3}(\tfrac{Q^2}{m^2}) \simeq \sum_j C_{ij}^{GM,4}(\tfrac{Q^2}{m^2}) \otimes A_{jk}(\tfrac{Q^2}{m^2})$$

and tend to the (zero-mass) VFNS asymptotically:

$$\lim_{Q^2 \gg m^2} C_{ik}^{GM,4}(\tfrac{Q^2}{m^2}) = C_{ij}^{VF,4}$$

Note however that the C_{ij}'s cannot be obtained from the equations above:

The theory does not completely define a GM-VFNS for finite $\frac{Q^2}{m^2}$

Additional *guesswork* needed to fix the "scheme" in the intermediate region

Moreover, this is only feasible for totally *inclusive* structure functions

$\Rightarrow$ **the experimental results (and thus the PDFs) still depend on the FFNS**

(Does it makes sense to change scheme at this point??)

GM-VFNS implementations

Often go like $F^{GM} \equiv F^{FF} + F^{ZM} - F^{\text{model}}$, with $\begin{cases} F^{\text{mod}} \to F^{\text{FF}} \text{ for } Q^2 \gg m^2 \\ F^{\text{mod}} \to F^{\text{ZM}} \text{ for } Q^2 \simeq m^2 \end{cases}$

(Besides proposals for including masses in the RGE [Martin, Roberts, Ryskin, Stirling 98], later abandoned)

There are many implementations ...

ACOT: Original idea [Aivazis, Collins, Olness, Tung 94] with a simple LO "subtraction term" for F_2; later modifications [Kretzer, Tung *et al.*] extend it to NLO and included simplifications and rescaling variables → S-ACOT(χ) (talk by P. Nadolsky)

BMSN [Buza, Matiounine, Smith, van Neerven 98]: Generalization of original ACOT, $F^{\text{mod}} = F^{\text{FF,asymp}}$ (see also CSN [Chuvakin, Smith, van Neerven 00])

RT: originally [Roberts, Thorne 98] emphasis on matching F_2 and its first derivative at transition points; later simplified and extended to NNLO [Thorne 06]; uses a "frozen" term beyond $Q^2 = m^2$ (talk by R. Thorne)

FONNL [Forte, Laenen, Nason, Rojo 10]: Essentially similar to BMSN

(Although some of them are known "not to work" properly)

Plausibility of the GM-VFNS

GM-VFNS's are *unnecesary* for HERA (FFNS) and for Tevatron or LHC (VFNS):

What is the advantage of GM-VFNS's?

The PDFs acommodate themselves to the GM-VFNS coefficients for DIS:
Process/model-dependent distributions? **Is this dangerous for universality?**

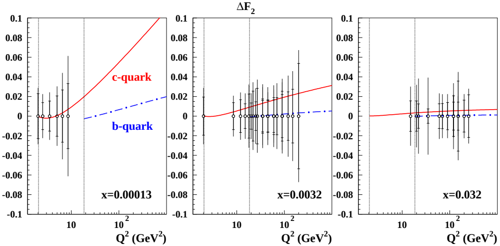
My opinion:

We should not try to model heavy-quark contributions, but stick to schemes unequivocally defined on solid theoretical grounds

How do the differences compare with the experimental errors?

We can compare $F^{h,FFNS} - F^{h,BMSN}$ with ΔF_2^{tot} [Alekhin, Blümlein, Klein, Moch 10]

⇒ generally effects are smaller (but in some regions in the limit)



Heavy-quark treatments in global fits

Traditionally the most “popular choice” for global fits was the **VFNS**, with the exception of the GRV group, which used the FFNS *already* for GRV94

This changed after the release of CTEQ6.5 (2007), where a GM-VFNS was used and the effects of heavy-quark masses in the predictions at hadron colliders were “re-discovered”: **today their importance is generally recognized**

Current choices of the (main) PDF groups are:

CTEQ: ACOT-like

MSTW and HERAPDF: TR-like

AB(K)M: FFNS (and BMSN for comparison)

NNPDF: FONLL

(G)JR: FFNS and VFNS (generated from the FFNS)

The experimental analyses use the FFNS (exclusive calculations needed)

Summary and conclusions

An appropriate treatment of heavy quarks in global PDF analyses is **vital** for present and future results in high-energy colliders

The **FFNS is a stable and reliable framework** for the treatment of heavy-quark contributions (there is no need to resum supposedly "large logarithms")

The current theoretical status is, and will be for a while, NLO
(Unfortunately there is only one group working towards improving the situation)

The **VFNS is** not valid for global analyses but **sufficiently accurate for LHC and Tevatron** energies (the input distributions must be generated in the FFNS)

Other "popular" choices (which we do *not recommend*) are the so-called GM-VFNS's (model/process dependent)

Schemes choices are a source of theoretical uncertainties in the predictions for high-energy colliders which cannot (and should not) be hidden "by convention"