

Running-mass definition in the heavy-quark electroproduction

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(in collaboration with J.Blümlein, and S.Moch)

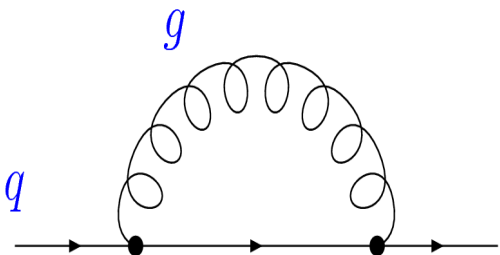
- Quark mass definitions
- Improved heavy-quark threshold corrections
- Running masses in the FFN and VFN schemes
- Updated ABM PDF fit
- Summary

Pole mass definition

The pole mass is defined as a the QCD Lagrangian parameter and is commonly used in the QCD calculations

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \sum_{\text{flavors}} \bar{q} (i\not{D} - m_q) q$$

Pole mass is defined for the free (unobserved) quarks

$$\not{p} - m_q - \Sigma(p, m_q) \Big|_{p^2 = m_q^2}$$


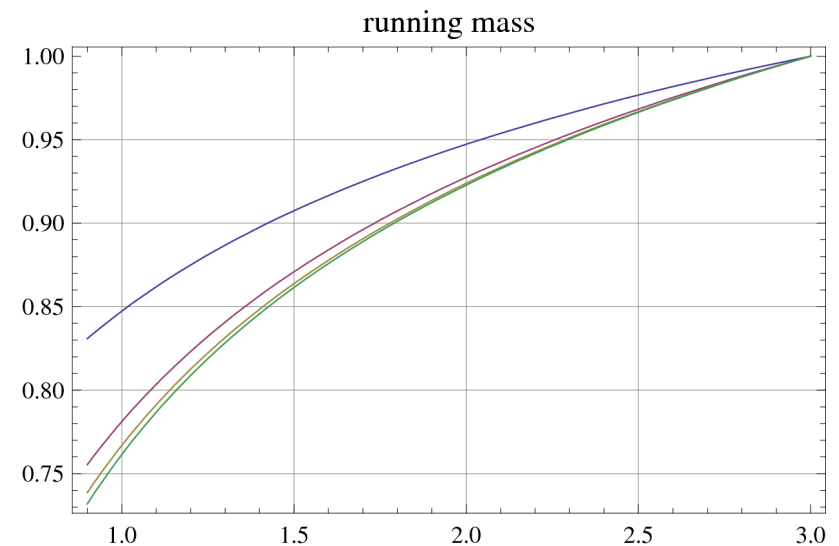
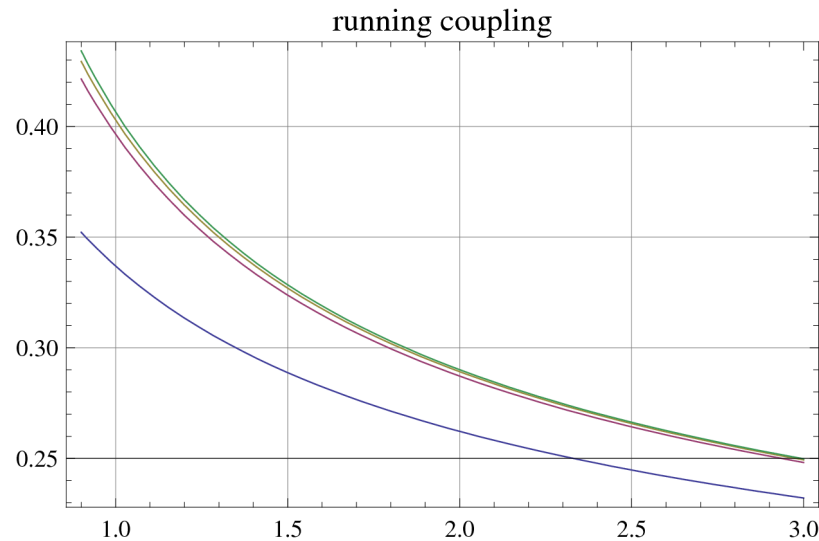
The quantum corrections due to the self-energy loop integrals receive contribution down to scale of $O(\Lambda_{\text{QCD}})$ → **sensitivity to the high order corrections, particularly at the production threshold**

Running quark mass

The renormgroup equation for mass is similar to one for the coupling constant

$$\mu^2 \frac{d}{d\mu^2} \alpha_s(\mu) = \beta(\alpha_s)$$

$$\mu^2 \frac{d}{d\mu^2} m(\mu) = \gamma(\alpha_s)m(\mu)$$



The corrections up to 4-loops are known

van Ritbergen, Vermaseren, Larin PLB 400, 379 (1997)

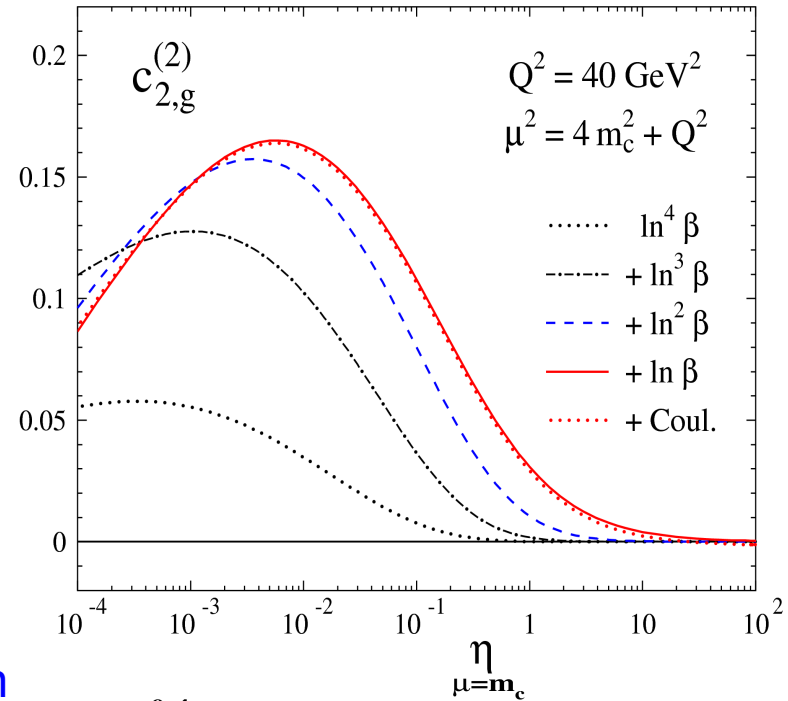
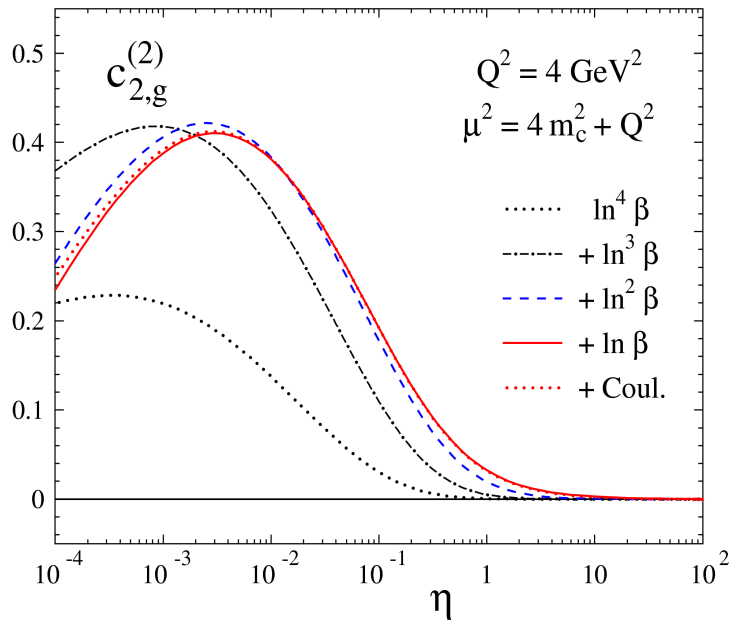
Chetyrkin PLB 404, 161 (1997)

Vermaseren, Larin, van Ritbergen PLB 405, 327 (1997)

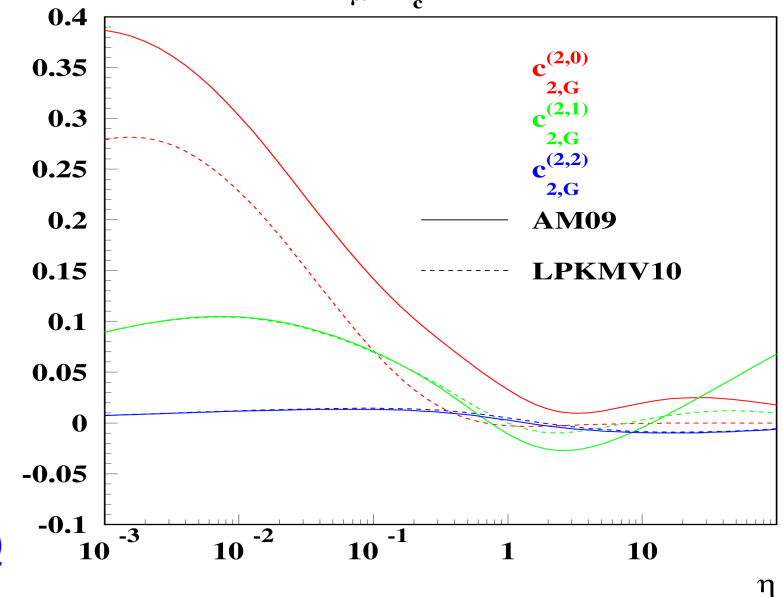
The choice of $\mu_R = m_c$ is close to the hard scattering data kinematic \rightarrow better perturbative convergence and reduced scale dependence

- The $t\bar{t}$ production in hadronic collisions Laengefeld, Moch, Uwer PRD 80, 054009 (2009)
- The heavy-quark electroproduction in the approximate NNLO (full NLO + NNLO threshold resummation) sa, Moch [hep-ph 1011.5790]

Approximate NNLO heavy-quark coefficients

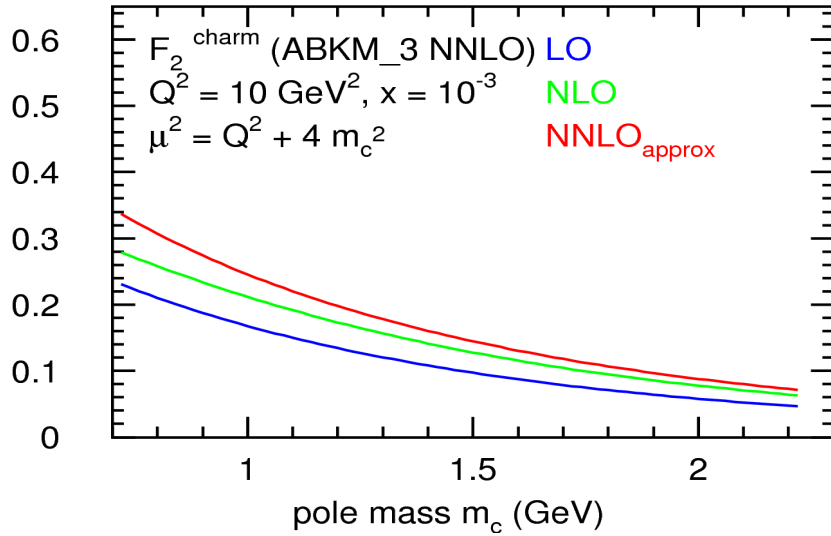


- At small x and small Q the main contribution comes from $\eta < 1$ due to the gluon distribution shape (threshold production)
- The large logs $\sim \ln^{2n}(\beta)$ can be resummed in all orders, this gives a good approximation to the exact NNLO expression at small β with the tower of large logs
- The first log and Coulomb terms have been recently added $\rightarrow F_2^c$ gets somewhat smaller at small Q and somewhat bigger at large Q

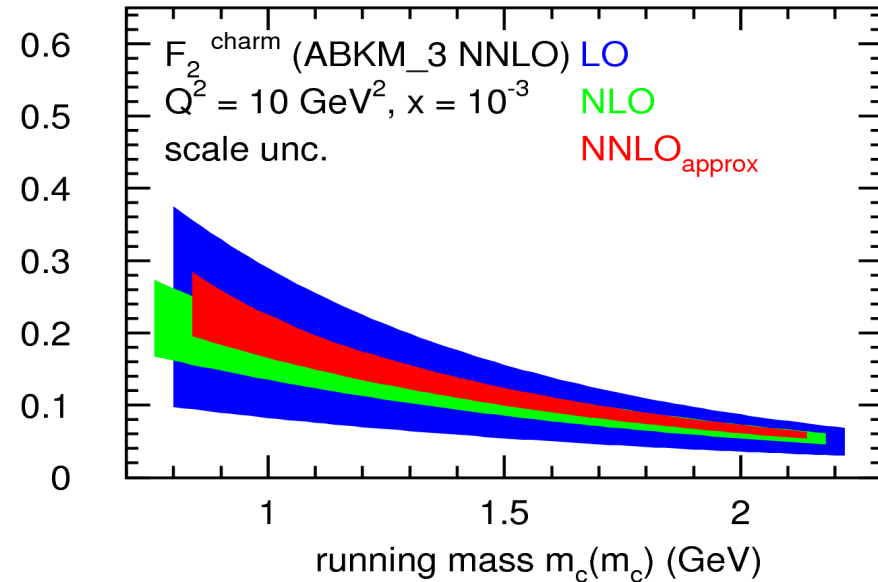
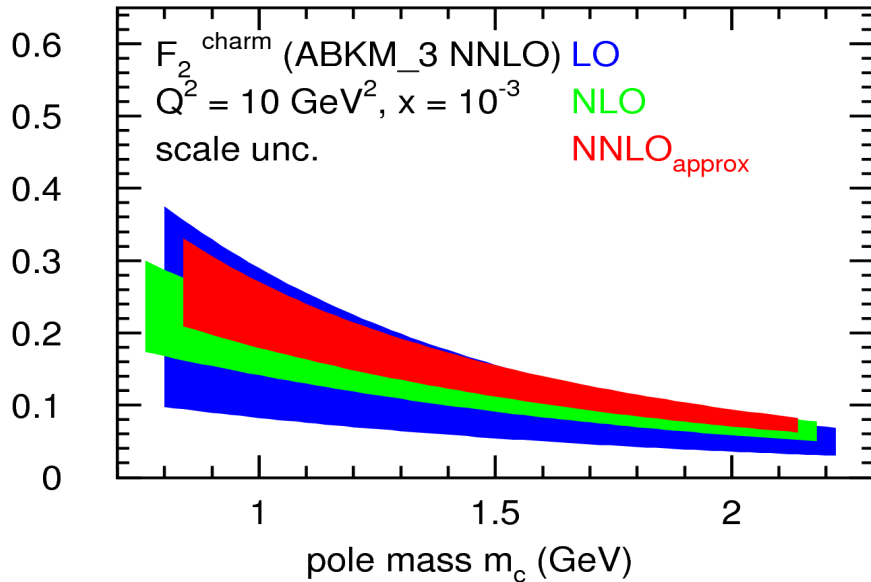
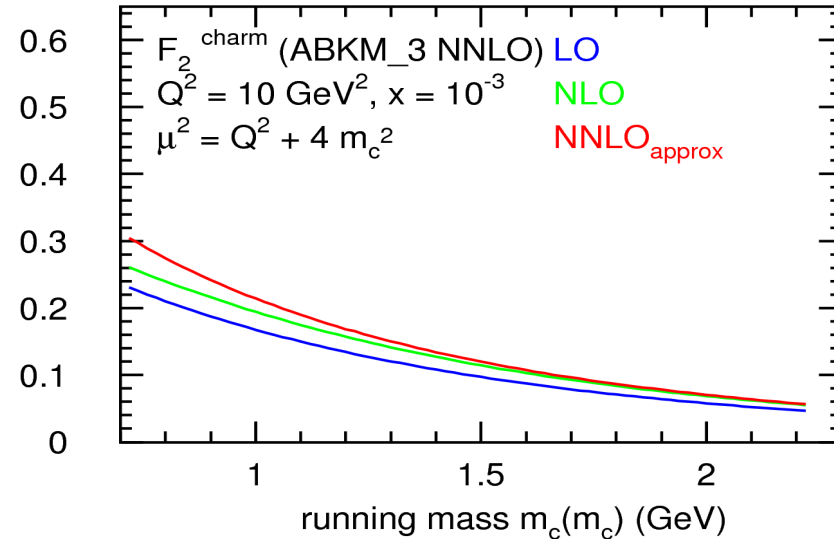


Running mass definition for the DIS SFs

Pole mass



Running mass



c-quark DIS production

The NNLO(approx.) FFNS ABM *predictions* based on the running mass definition are in nice agreement with the new HERA data

N³LO corrections?

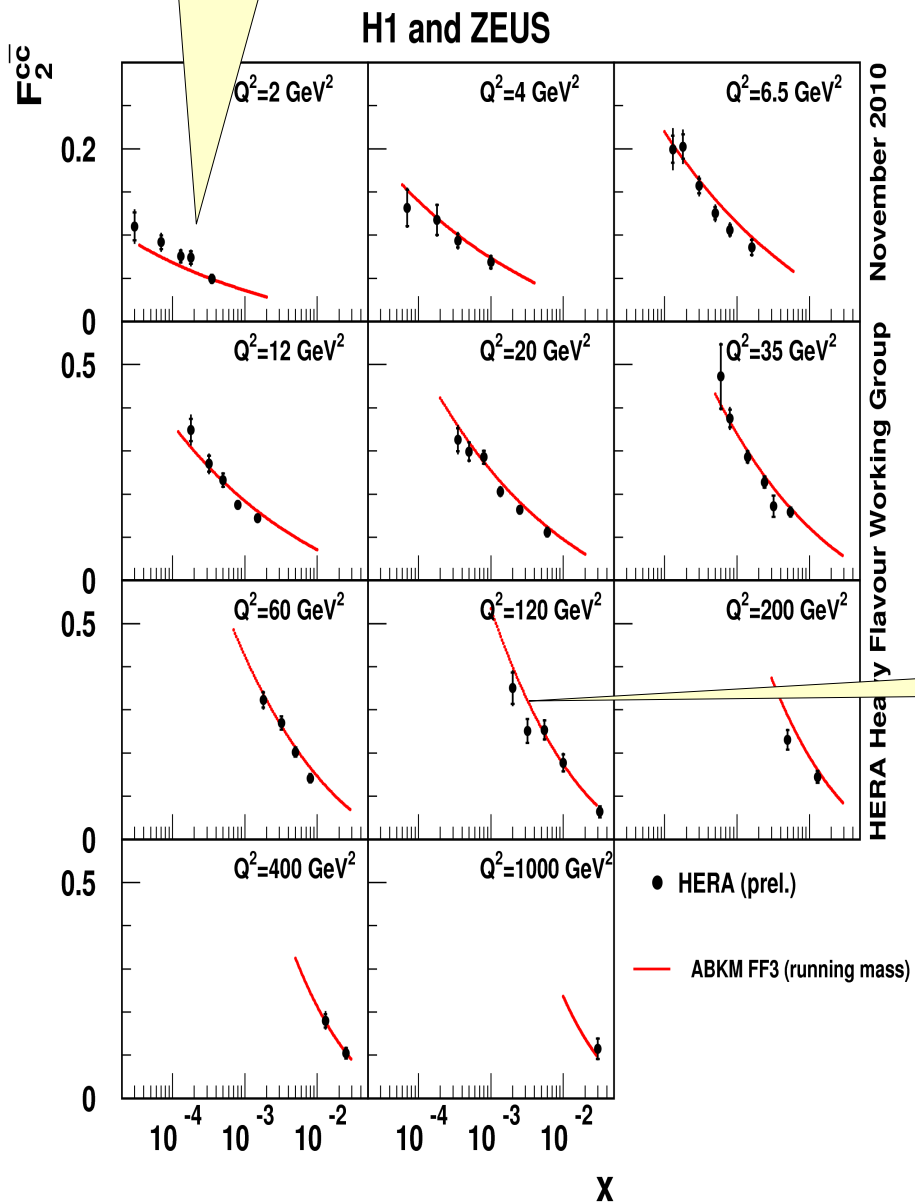
$$m_c(m_c) = 1.27 \pm 0.08 \text{ GeV} \quad (\text{PDG '10})$$

ABKM09 fit with the running-mass definition

$$m_c(m_c) = 1.18 \pm 0.06 \text{ GeV} \quad (\text{incl. } F_2 + \text{PDG})$$

HERA data prefer value of m_c close to the PDG one

No need of the resummation



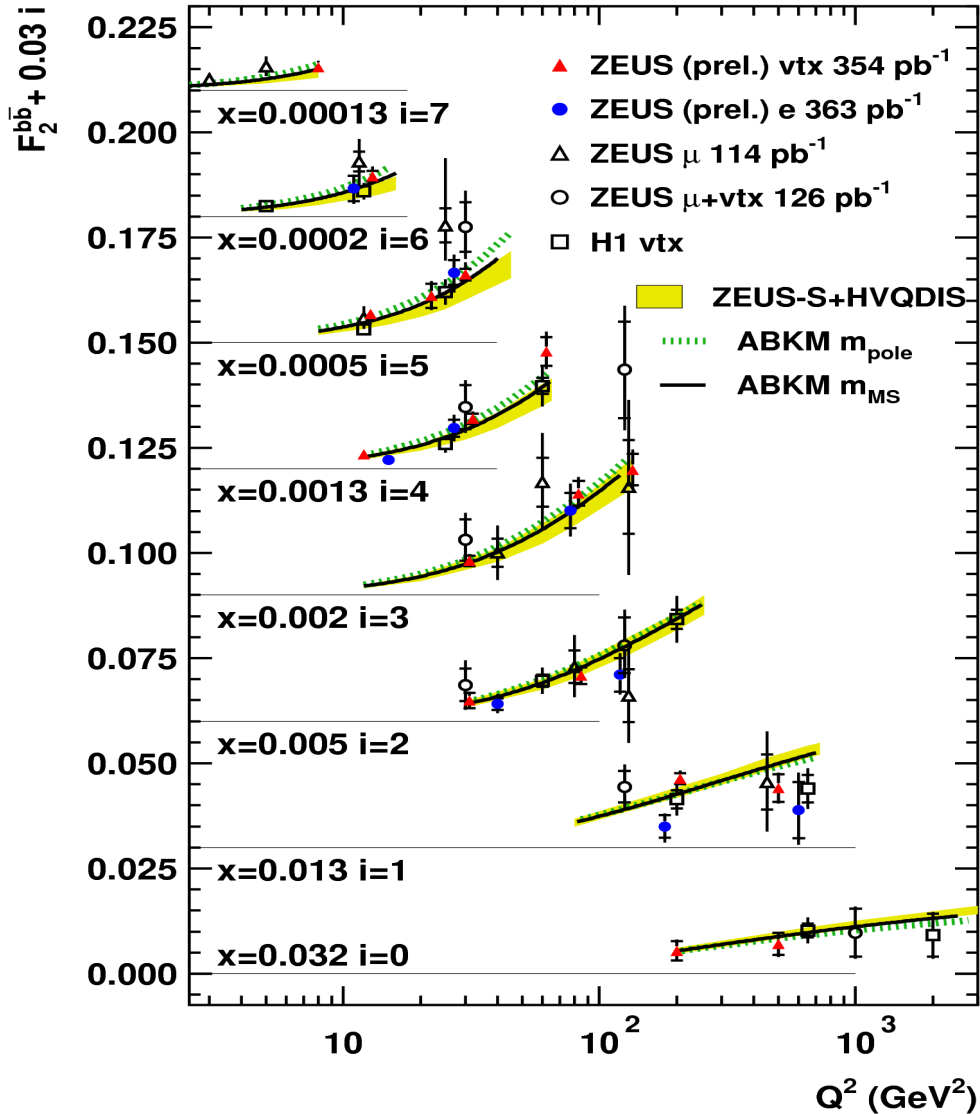
At $Q \gg m_c$ first Mellin NNLO moments are known

Ablinger et al. NPB 844, 26 (2011)

Bierenbaum, Blümlein, Klein NPB 829, 417 (2009)

b-quark production

ZEUS



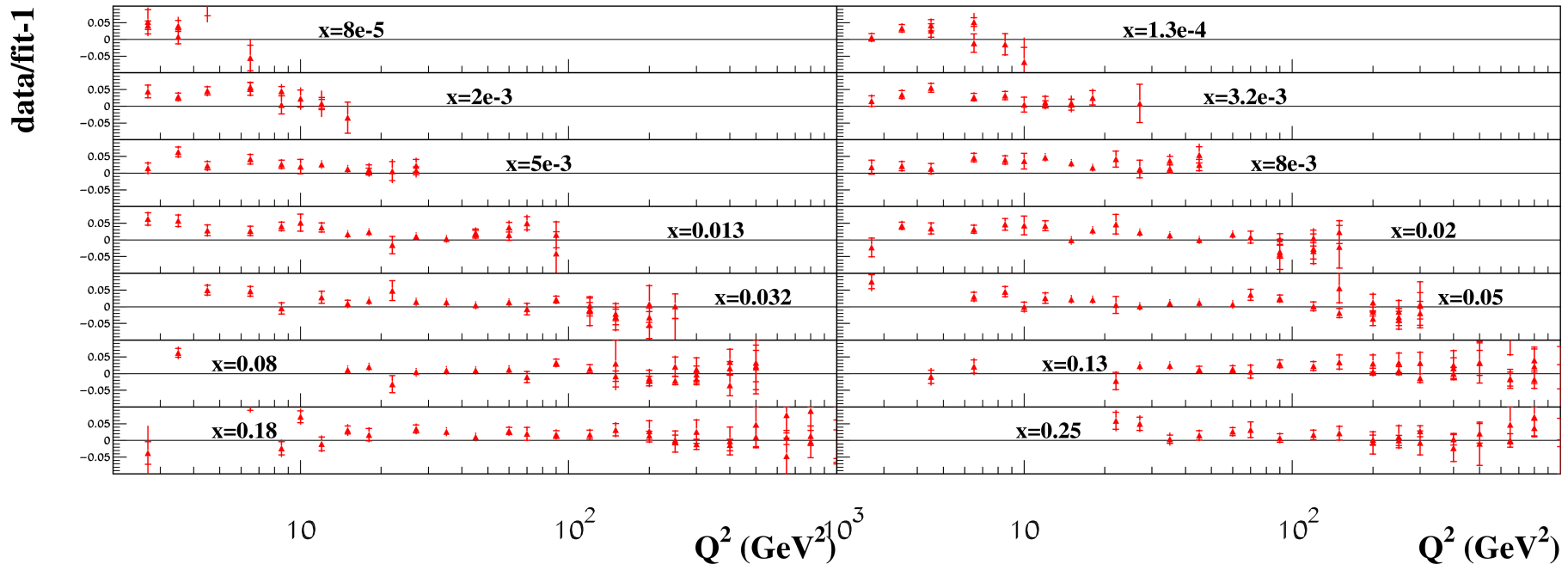
(courtesy of A.Geiser and P.Roloff)

For the b-quark production NNLO_{approx} predictions work well → the threshold approximation is better justified

No sensitivity to m_b → fixed at the PDG value $m_b(m_b) = 4.19 \pm 0.12$ GeV

High-Q inclusive DIS data

H1 and ZEUS Collaborations JHEP 1001, 109 (2010)



- The PDF shape was modified to accommodate new data

$$xS(x) = \exp [a \ln x (1 + \beta \ln x) (1 + \gamma_1 x)] (1 - x)^b$$

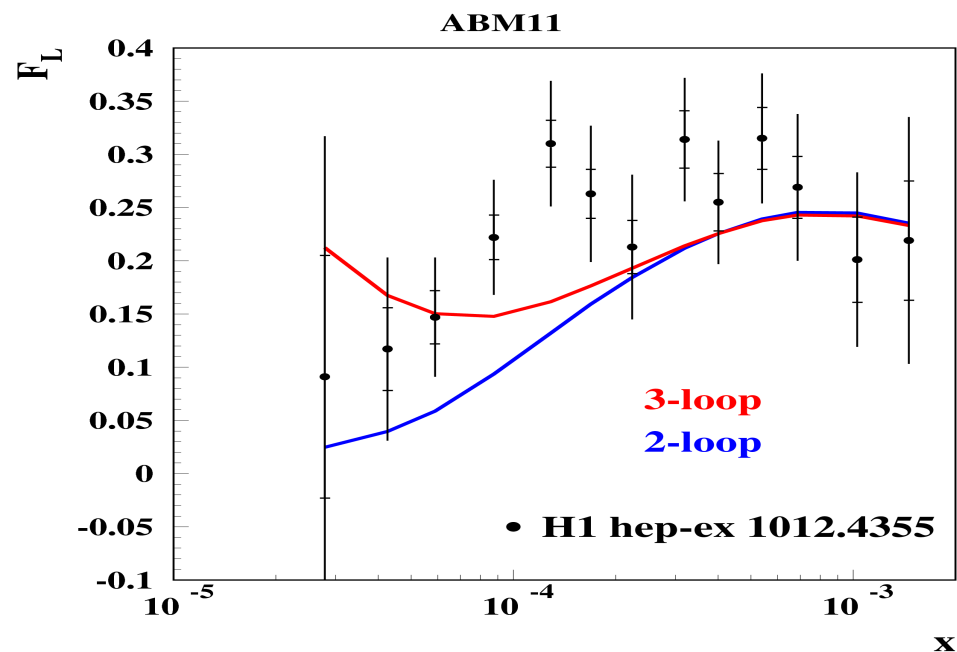
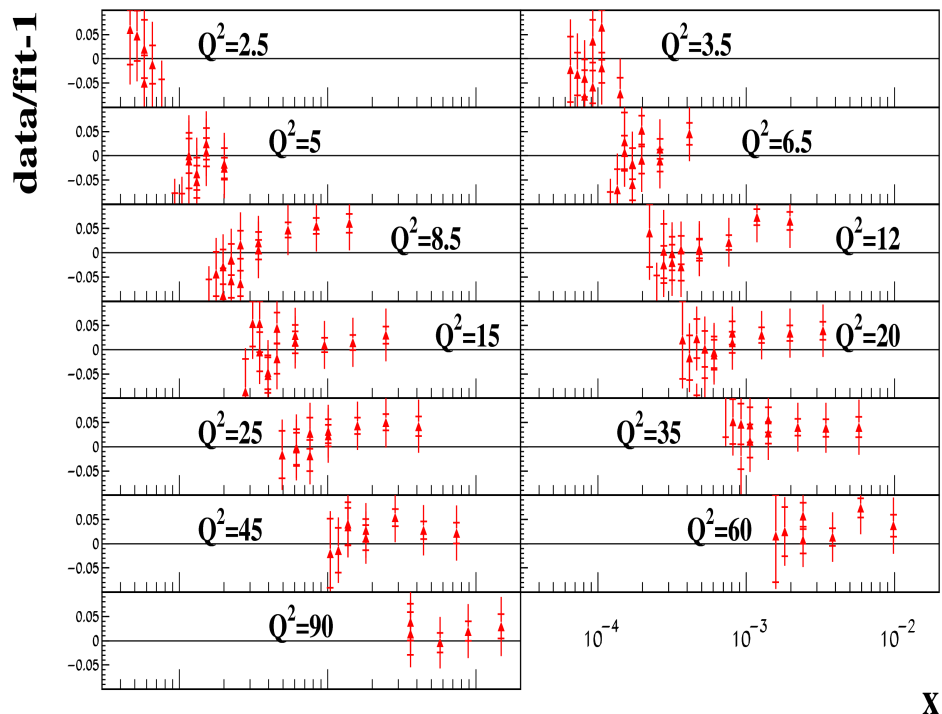
$$xu_V(x) = \exp [a \ln x (1 + \gamma_1 x + \gamma_2 x^2 + \gamma_3 x^3)] (1 - x)^b$$

- $\chi^2/\text{NDP}=1.1$, with account of the systematic error correlations (114 sources). Slightly worse for the small-Q part, the same observed in the model-independent fit

sa, Blümlein, Moch [hep-ph 1007.3657]

$$m_c(m_c)=1.27\pm 0.08 \text{ GeV} \quad m_b(m_b)=4.19\pm 0.13 \text{ GeV} \quad (\text{PDG '10})$$

Low-Q inclusive DIS data



The data prefer quite big 3-loop corrections to F_L at small x

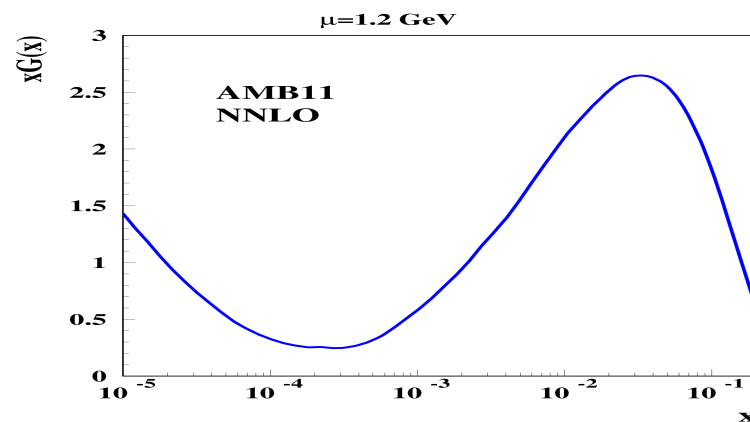
Moch, Vermaseren, Vogt PLB 606, 123 (2005)

- The low-energy H1 data are quite sensitive to F_L at small x and Q

H1 Collaboration [hep-ex 1012.4355]

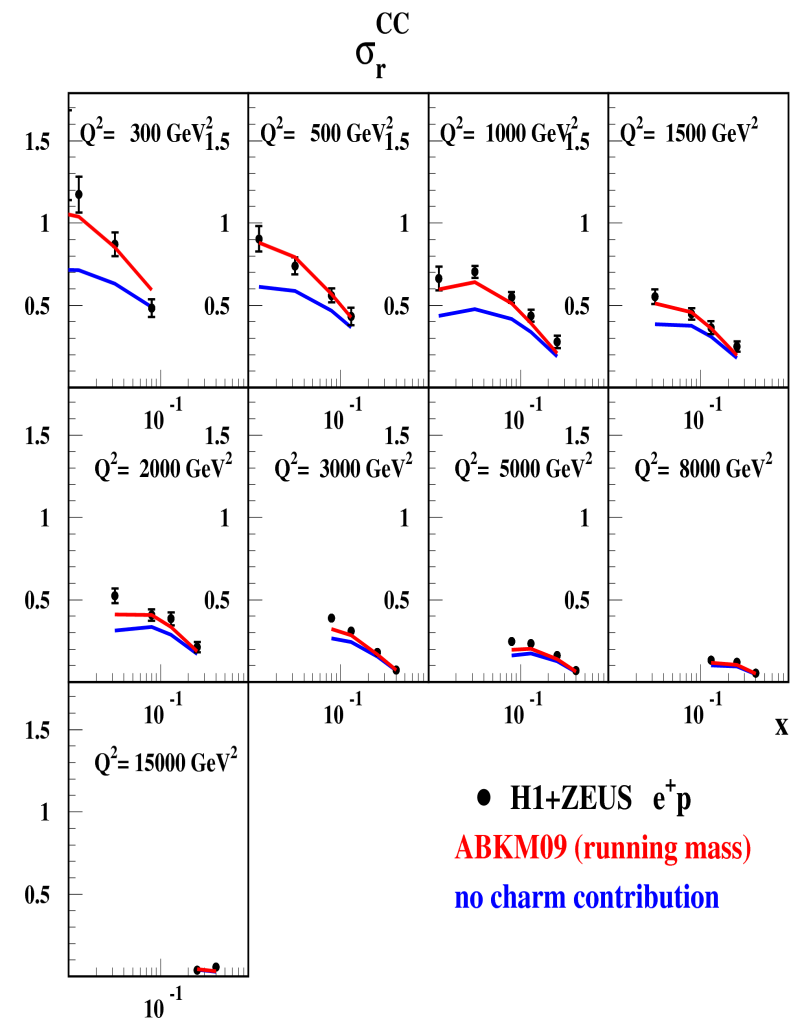
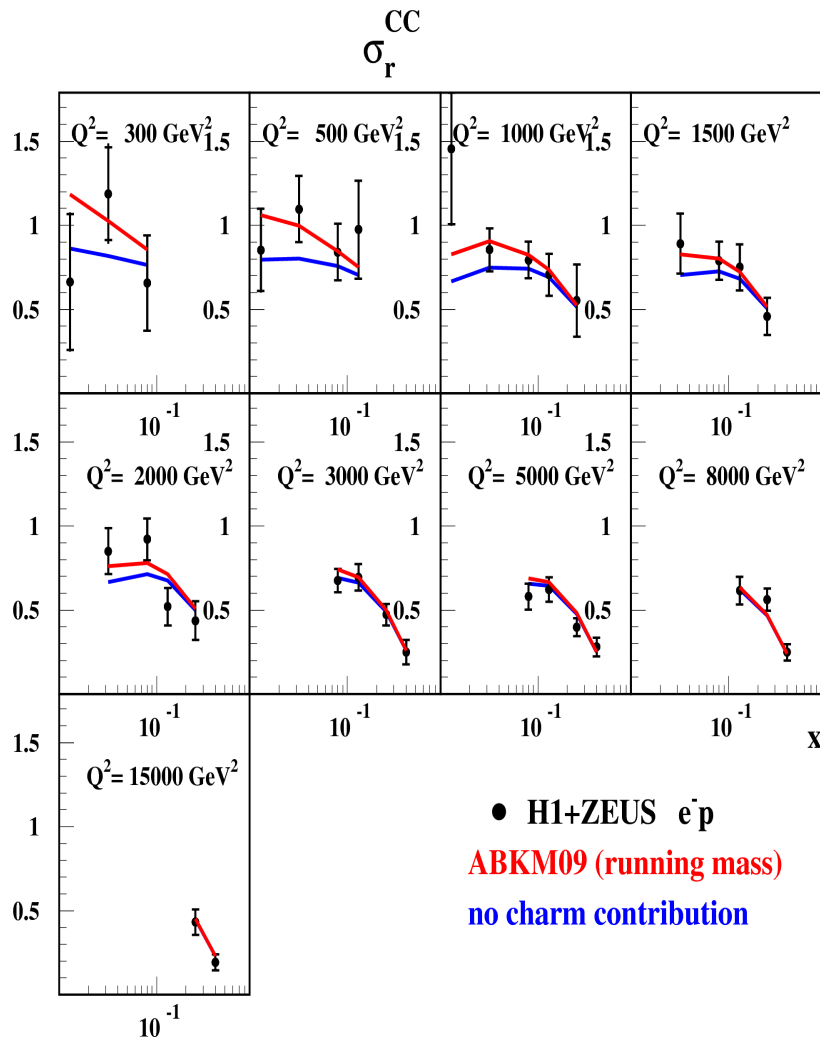
- The data can be easily accommodated in the fit: the value of $\chi^2/NDP=1.05$; no clear sign of the collinear evolution violation

- Positive small- x gluons are preferred by the data at low scale



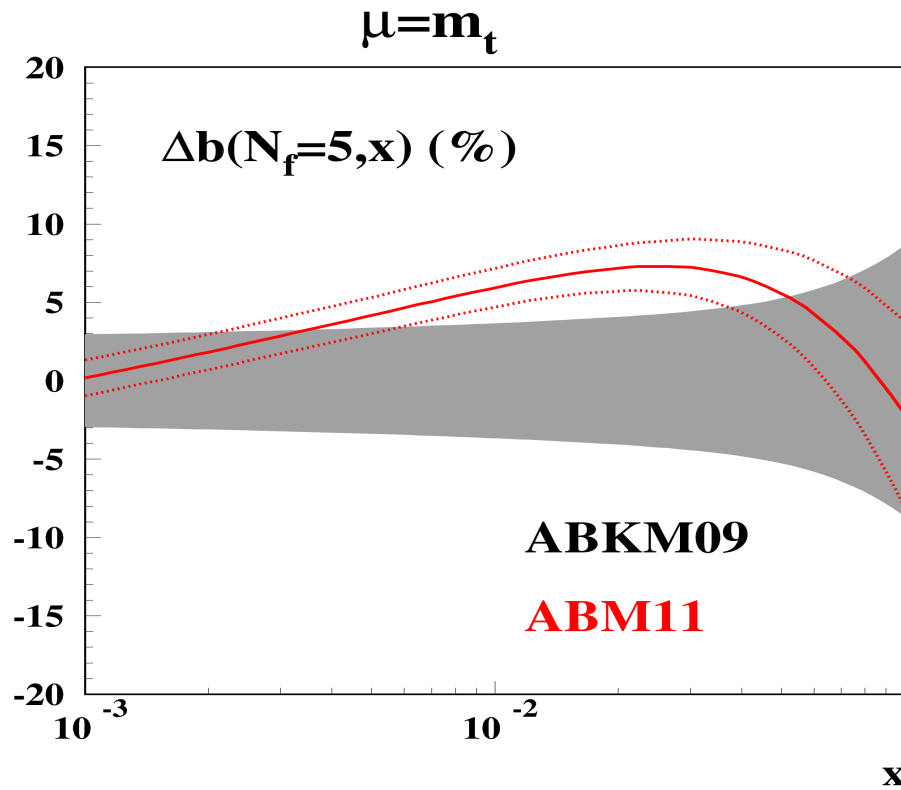
CC inclusive data

H1 and ZEUS Collaborations JHEP 1001, 109 (2010)



- Nice agreement with ABKM09 predictions
- Impact of the data on ABKM09 fit is marginal
- With the improved accuracy at future facilities, (at EIC?), the strange distribution can be better constrained.

Running mass and the VFN scheme



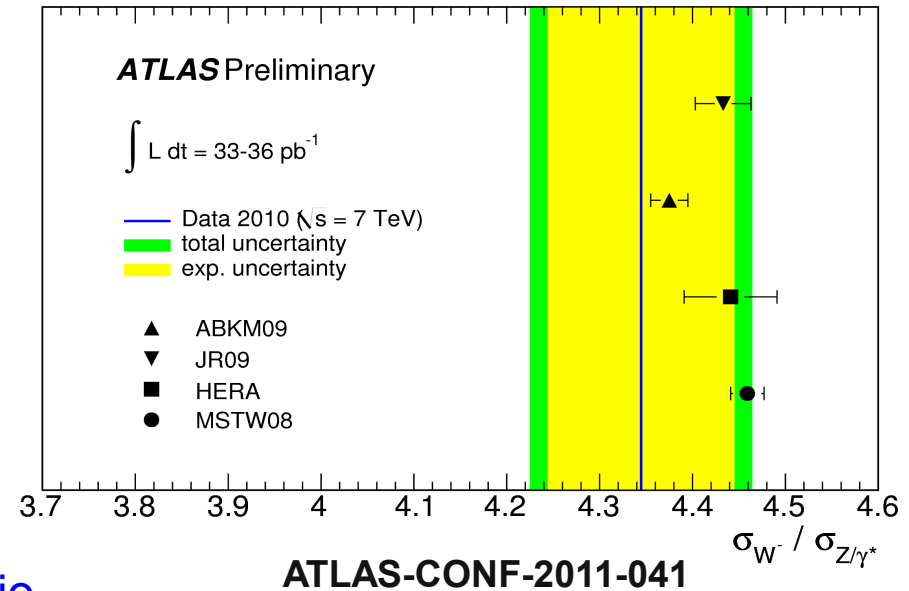
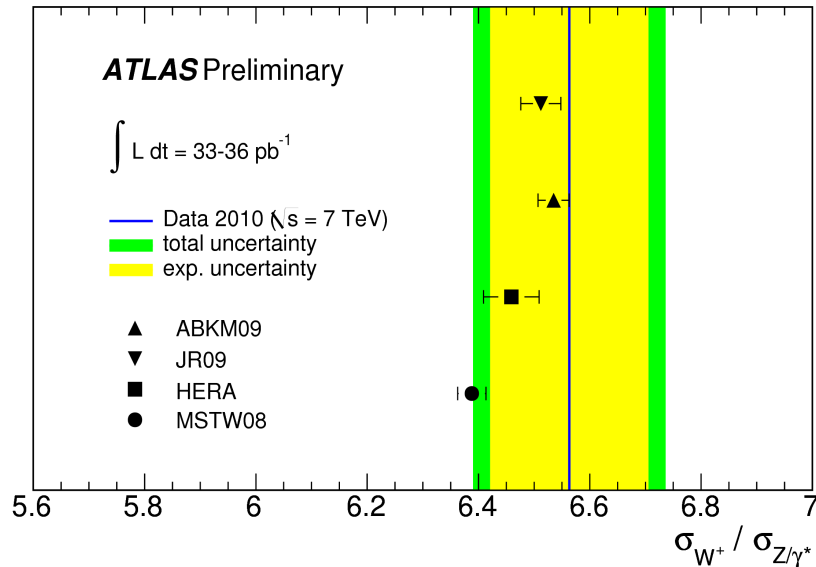
The 4- and 5-flavour PDFs are generated from the ABM11 fit performed with the running-mass definition; the massive OMEs with the running-mass definition are used

The change in the heavy-quark distribution is due to:

- change in the 3-flavor distributions from ABKM09 to ABM11
- change in the masses: $m_b = 4.5 \rightarrow 4.19 \pm 0.12$ GeV (PDG '10)
- modification of the massive OMEs

The b-quark distribution uncertainty is reduced \rightarrow impact on the single-top production

NNLO benchmarks



The luminosity uncertainty cancels in the ratio

ABKM09

$$\alpha_s(M_Z) = 0.1135(14)$$

| | $\sigma(W^+)$ (nb) | $\sigma(W^-)$ (nb) | $\sigma(Z)$ (nb) | $\sigma(M_H = 165 \text{ GeV})$ (pb) |
|----------|--------------------|--------------------|------------------|--------------------------------------|
| Tevatron | 26.1(3) | | 7.69(8) | 0.25(2) |

LHC7

| | | | | |
|--|---------|---------|---------|----------|
| | 58.9(9) | 39.4(6) | 28.4(5) | 7.05(23) |
|--|---------|---------|---------|----------|

ABM11 = ABKM09 + running mass definition + new HERA data

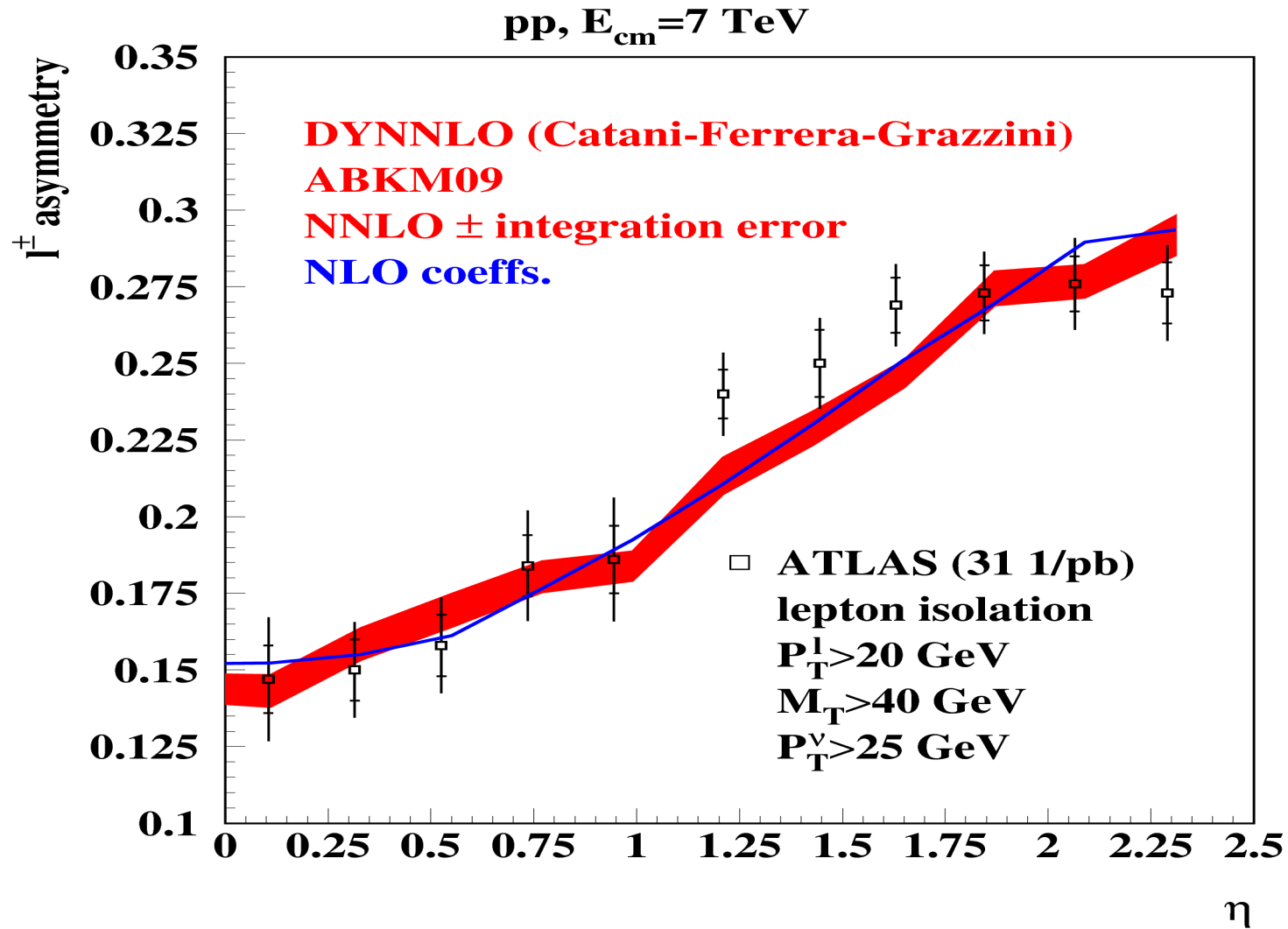
$$\alpha_s(M_Z) = 0.1134(11)$$

Tevatron

| | | | | |
|--|-----------|--|---------|-----------|
| | 26.41(23) | | 7.76(7) | 0.241(16) |
|--|-----------|--|---------|-----------|

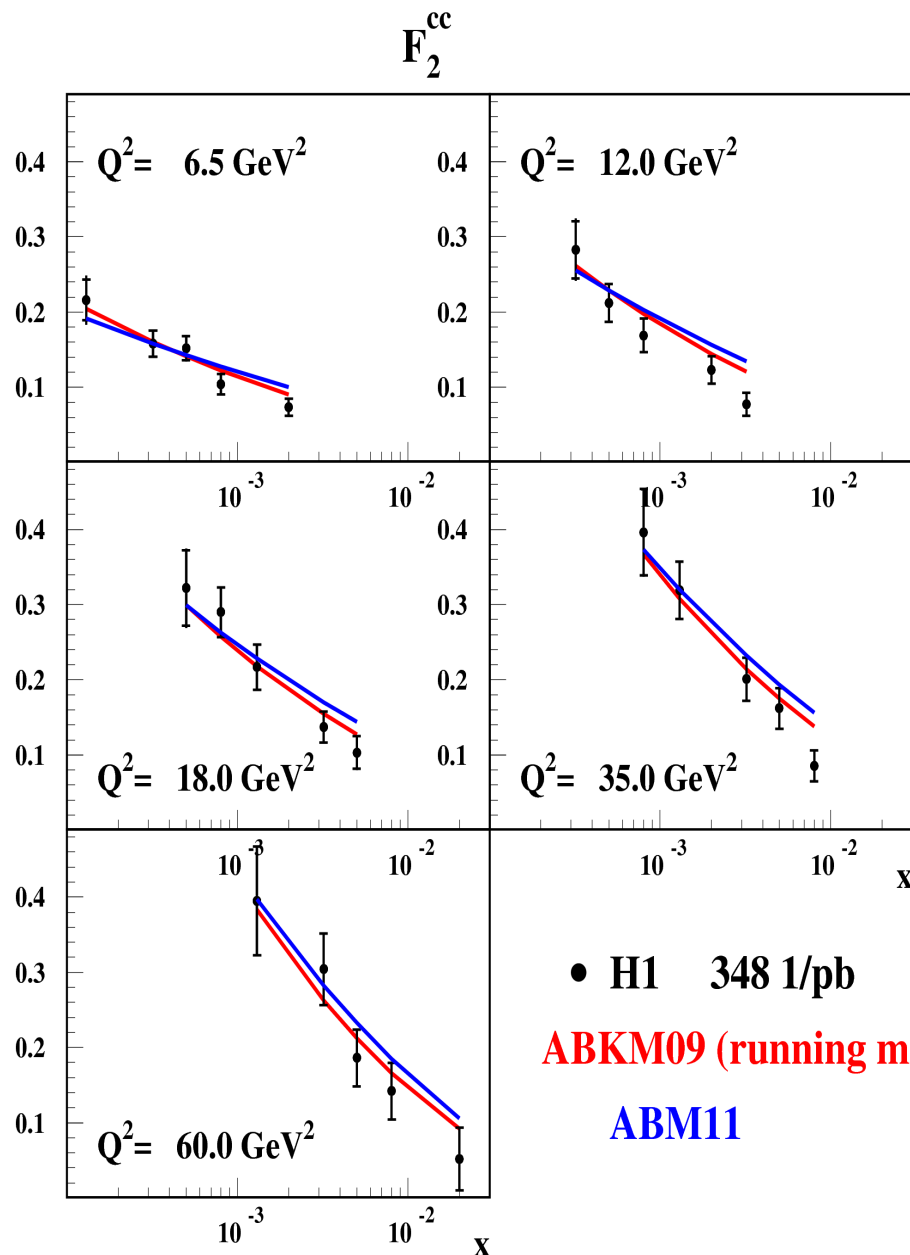
LHC7

| | | | | |
|--|---------|---------|---------|----------|
| | 58.9(7) | 39.6(5) | 28.5(3) | 7.21(18) |
|--|---------|---------|---------|----------|



- The ABKM09 predictions are in nice agreement with the ATLAS data
- The updated ABM11 fit gives similar results

Toward determination of m_c (m_c) from DIS



Improved accuracy can be obtained with the semi-inclusive data included due to correlation between quark and gluon PDFs is reduced

ABKM09 (running mass):

$$m_c(m_c) = 1.18 \pm 0.06 \text{ GeV (incl. } F_2 \text{ + PDG)}$$

With semi-inclusive data added

$$m_c(m_c) = 1.10 \pm 0.04 \text{ GeV (incl. } F_2 + F_2^{cc} \text{)} \\ \text{(prel.)}$$

• H1 348 1/pb

ABKM09 (running mass)

ABM11

Work in progress

$$m_c(m_c) = 1.27 \pm 0.08 \text{ GeV (PDG '10)}$$

Summary and outlook

- The running mass definition is implemented for the DIS semi-inclusive structure functions
 - Improved perturbative stability and the scale variation uncertainty
 - Consistent treatment of the mass in DIS and other processes, like e^+e^- initiated
 - First determination of running mass from the DIS data
- Better determination on the heavy-quark PDFs
- Improved uncertainty foreseen with inclusion of the HERA combined charm data
 - Resolving correlation between gluon and sea distribution

The heavy-quark electro-production

The dominant mechanism is photon-gluon fusion, contributes up to 30% to the inclusive structure functions. The massive coefficient functions are known up to the NLO.

$$C_{2,g}^{LO} = c^{(0,0)} \quad \text{Witten NPB 104, 445 (1976)}$$

$$C_{2,g}^{NLO} = c^{(1,0)} + c^{(1,1)} \ln(\mu_F^2/m_c^2)$$

Laenen, Riemersma, Smith, van Neerven NPB 392, 162 (1993)

FFNS

- Only 3 light flavors in the initial state are considered.
- Accurate at $Q \sim m_c$
- At large Q the fixed-order results may be insufficient due to big logs $\sim \ln^n(Q/m_c)$ must be resummed
- Involved high-order calculations: The full NNLO corrections are missed, however numerically important threshold resummation results are available

Laenen, Moch PRD 59, 034027 (1999)

Lo Presti, Kawamura, Moch, Vogt [hep-ph 1008.0951]

ZMVFNS

Collins, Tung NPB 278, 934 (1986)

- At $Q \gg m_c$ the heavy quarks are considered as massless \rightarrow the NNLO evolution and the coefficient functions up to N³LO are ready
- The big logs $\sim \ln^n(Q/m_c)$ are in a natural way resummed in the QCD evolution
- Matching conditions for the 3(4)-flavor and the 4(5)-flavor massless theories
- A smooth matching with the FFNS in the limit of $Q \rightarrow m_c$ must be provided

