SUSY and Exotica

by

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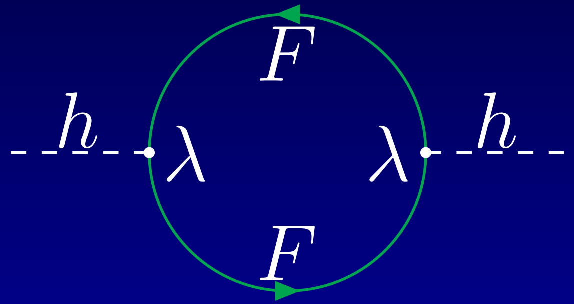
Talk outline

- SUSY Fits
- Impact of LHC data
- SUSY Tactics
- Exotica and $A_{FB}(t\bar{t})$

Please ask questions while I'm talking

Technical Hierarchy Problem

A problem with light, fundamental scalars. Their mass receives quantum corrections from heavy particles in the theory:



$$\sim -\frac{c\lambda^2}{16\pi^2}\int \frac{d^4k}{k^2 - m_F^2} + \ldots$$

Quantum correction to Higgs mass:

$$m_h^{phys} = m_h^{tree} + \mathcal{O}(\Lambda/100).$$

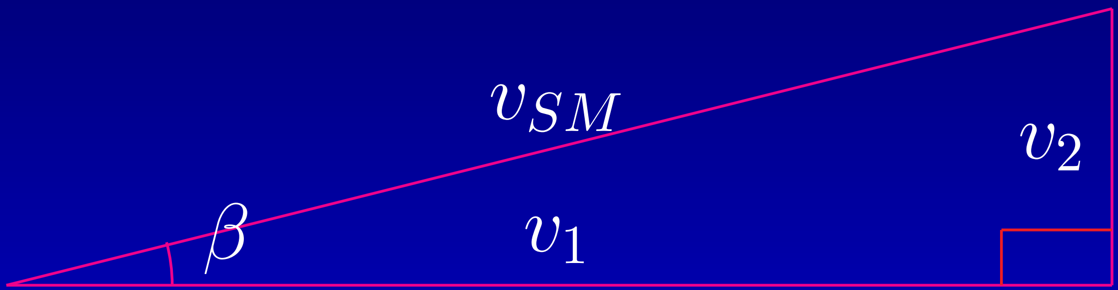
$\Lambda \sim 10^{19}$ GeV$/c^2$ is *heaviest mass scale* present. Higgs is eaten by W, Z to give $O(M_{W,Z}) \sim 90$ GeV/$c^2 \Rightarrow m_h^{tot} \lesssim 1$ TeV$/c^2$.

Electroweak Breaking

Both Higgs get vacuum expectation values:

$$\begin{pmatrix} H_1^0 \\ H_1^- \end{pmatrix} \to \begin{pmatrix} v_1 \\ 0 \end{pmatrix} \qquad \begin{pmatrix} H_2^+ \\ H_2^0 \end{pmatrix} \to \begin{pmatrix} 0 \\ v_2 \end{pmatrix}$$

and to get M_W correct, match with $v_{SM} = 246$ GeV:

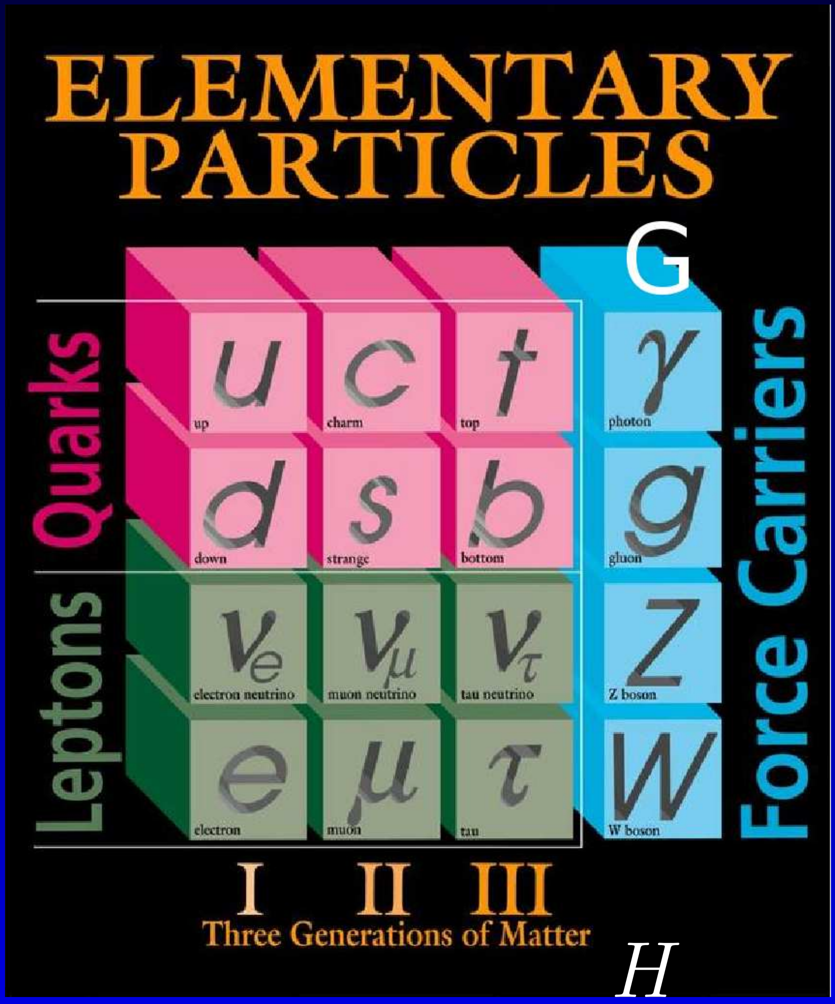


$\tan\beta = \frac{v_2}{v_1}$

$$\mathcal{L} = h_t \bar{t}_L H_2^0 t_R + h_b \bar{b}_L H_1^0 b_R + h_\tau \bar{\tau}_L H_1^0 \tau_R$$

$$\Rightarrow \frac{m_t}{\sin\beta} = \frac{h_t v_{SM}}{\sqrt{2}}, \qquad \frac{m_{b,\tau}}{\cos\beta} = \frac{h_{b,\tau} v_{SM}}{\sqrt{2}}.$$

Supersymmetric Copies



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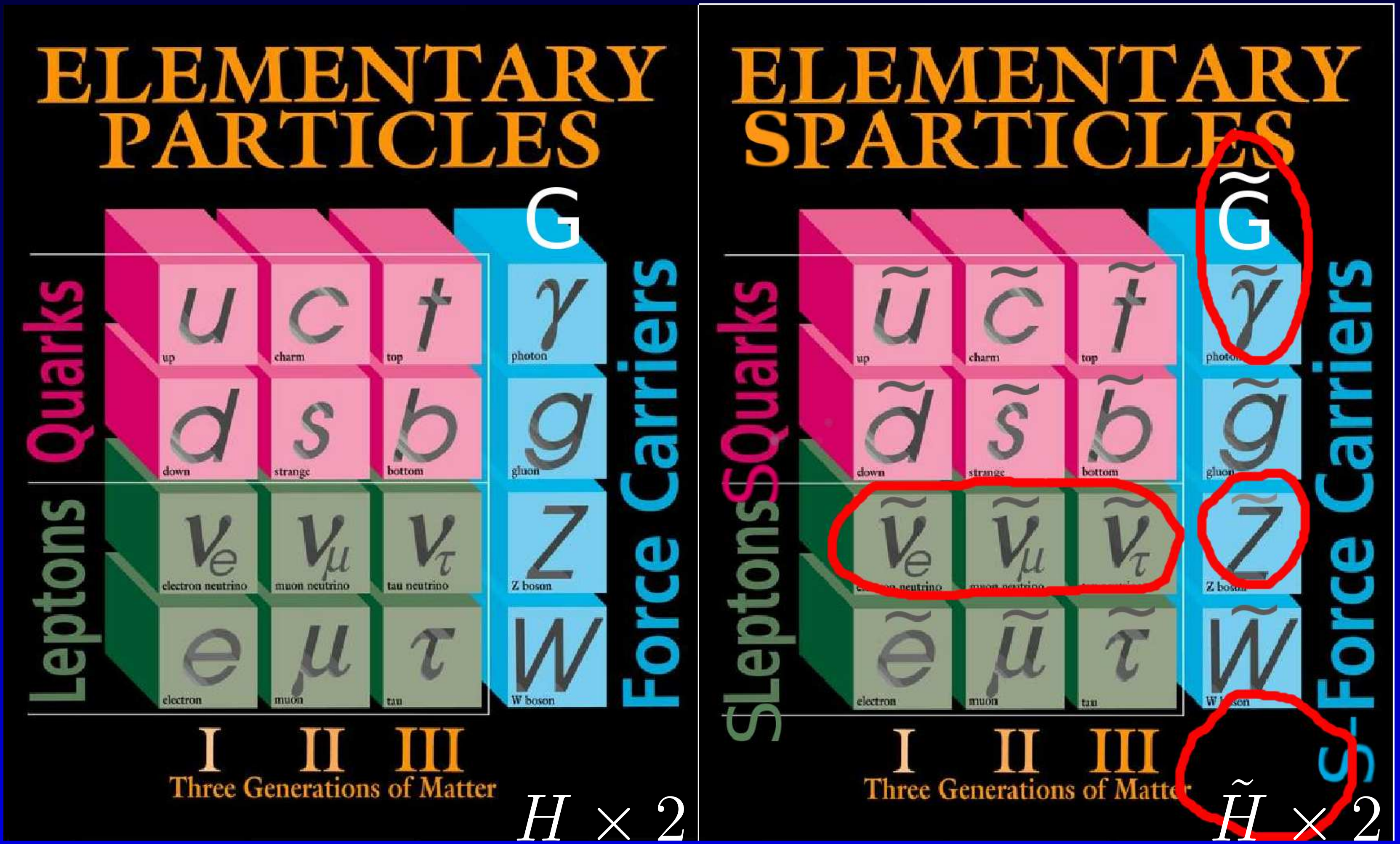


Implementation

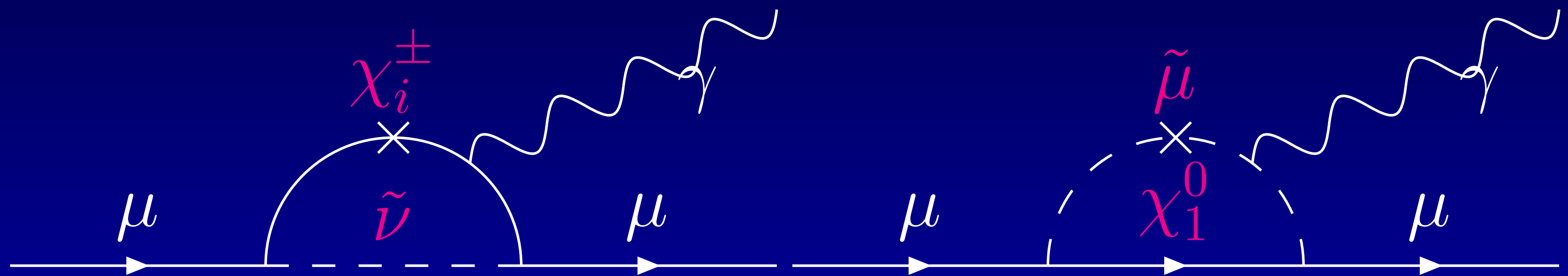
We use

- *95% C.L. direct search constraints*
- $\Omega_{DM} h^2 = 0.1143 \pm 0.02$ Boudjema *et al*
- $\delta(g-2)_\mu/2 = (29.5 \pm 8.8) \times 10^{-10}$ Stöckinger *et al*
- $B-$physics observables including $BR[b \to s\gamma]_{E_\gamma > 1.6 \text{ GeV}} = (3.52 \pm 0.38) \times 10^{-4}$
- Electroweak data W Hollik, A Weber *et al*

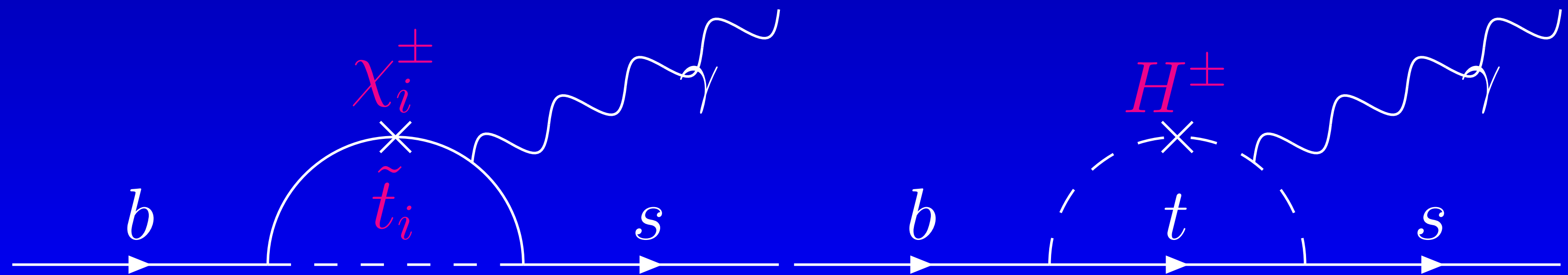
$$2 \ln \mathcal{L} = -\sum_i \chi_i^2 + c = \sum_i \frac{(p_i - e_i)^2}{\sigma_i^2} + c$$

Additional observables

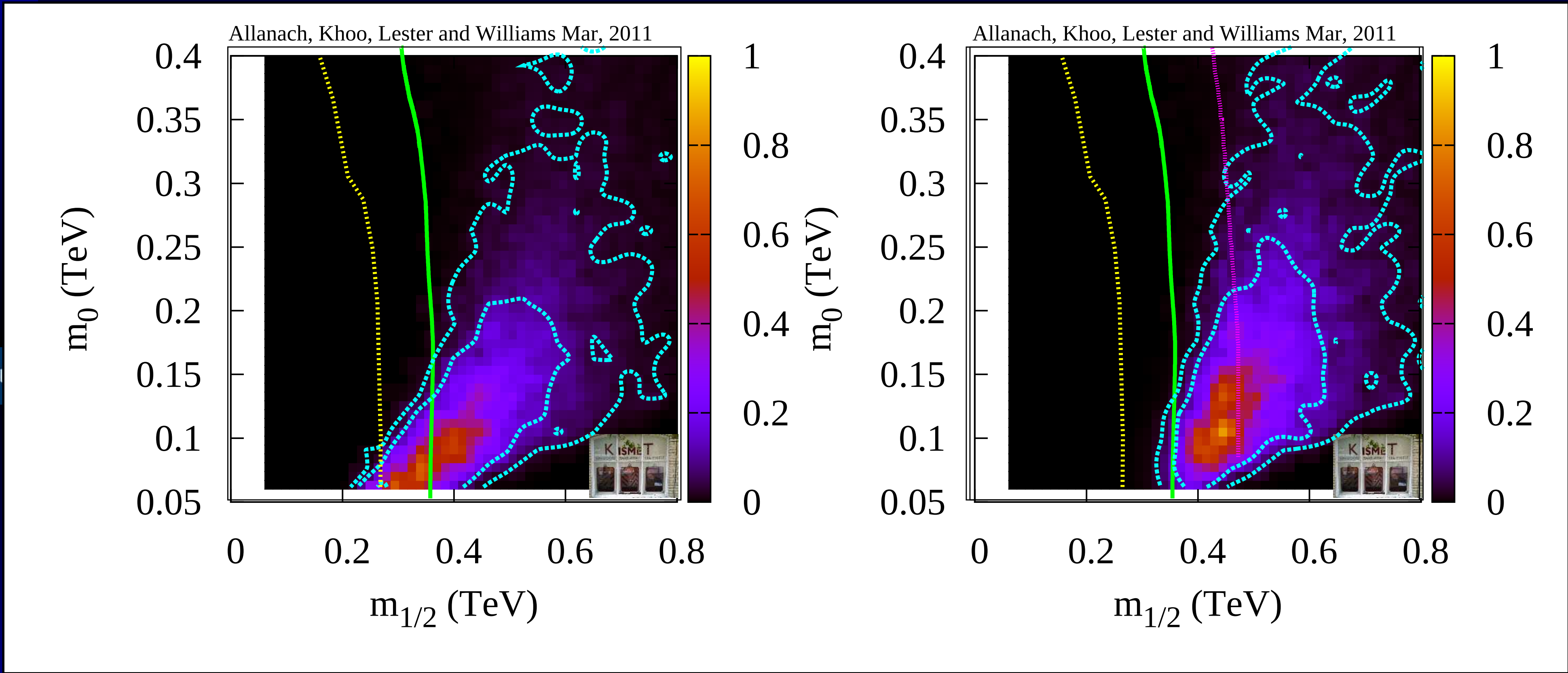
$$\delta\frac{(g-2)_\mu}{2} \sim 13 \times 10^{-10} \left(\frac{100\text{ GeV}}{M_{SUSY}}\right)^2 \tan\beta$$



$$BR[b \to s\gamma] \propto \tan\beta (M_W/M_{SUSY})^2$$

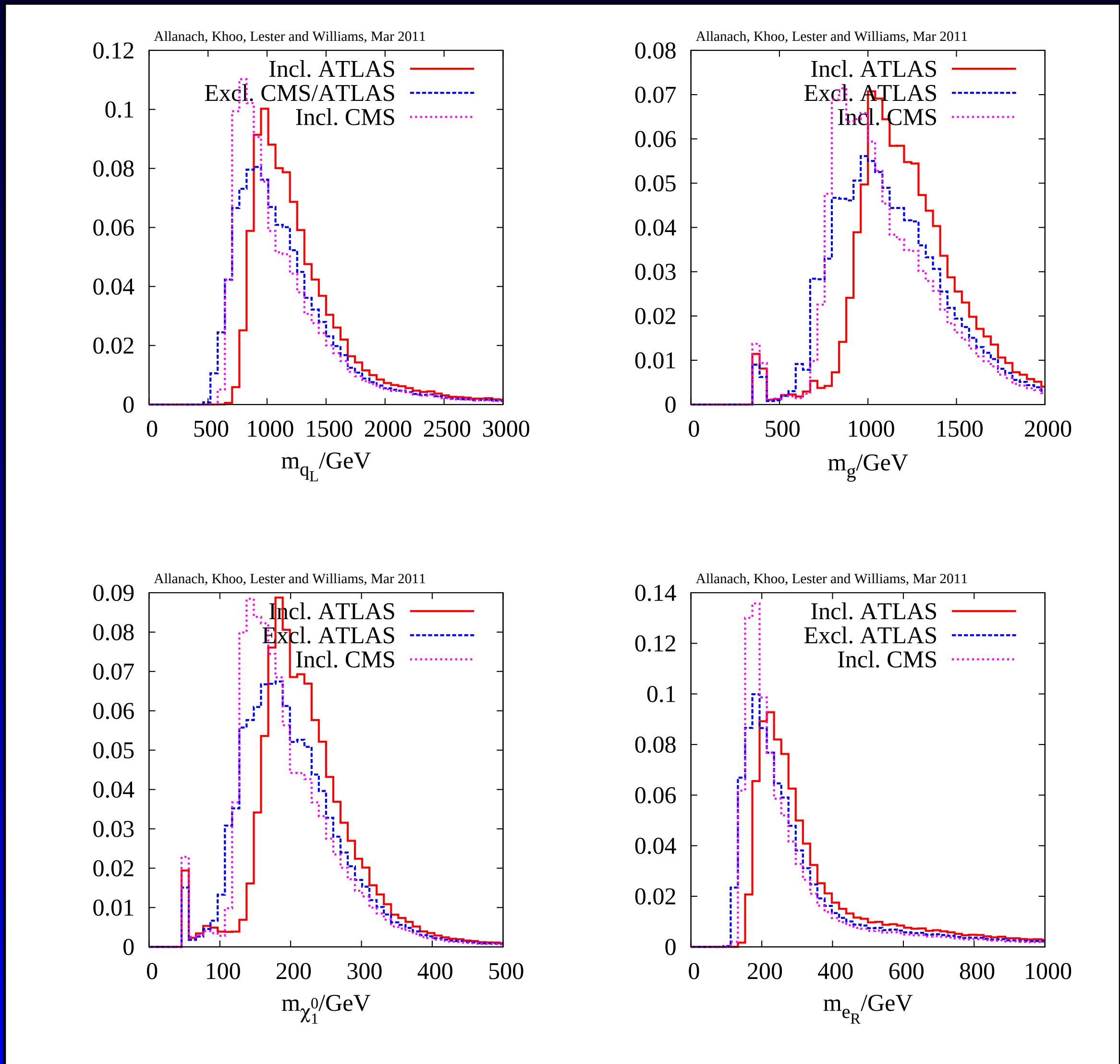


ATLAS Weighted Fits



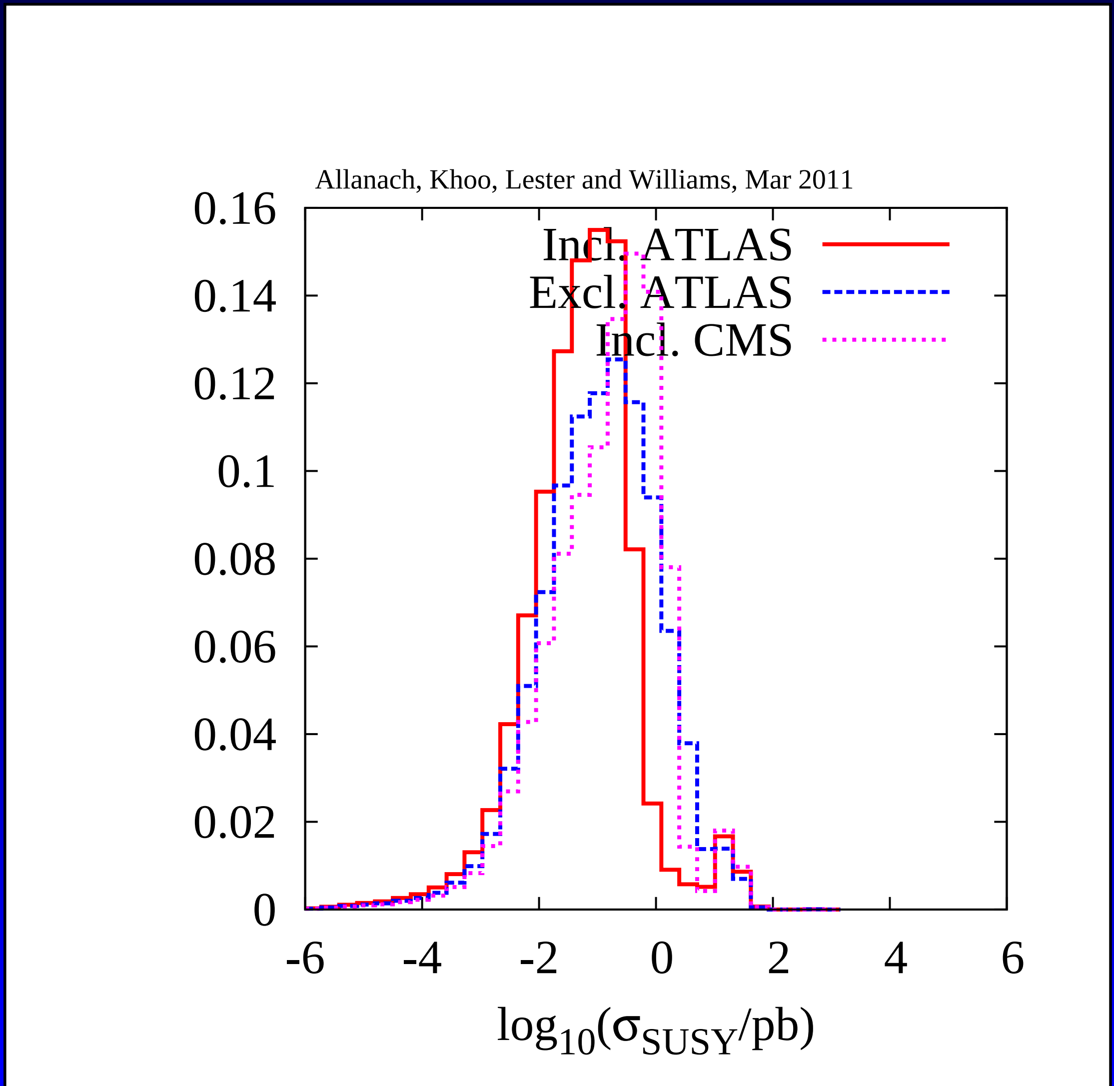
Again, we assume A_0-$\tan\beta$ independence and interpolate across m_0 and $m_{1/2}$. CMS 35 pb^{-1}, ATLAS 35 pb^{-1}, CMS 1 fb^{-1}

CMS/ATLAS Weighted Fits



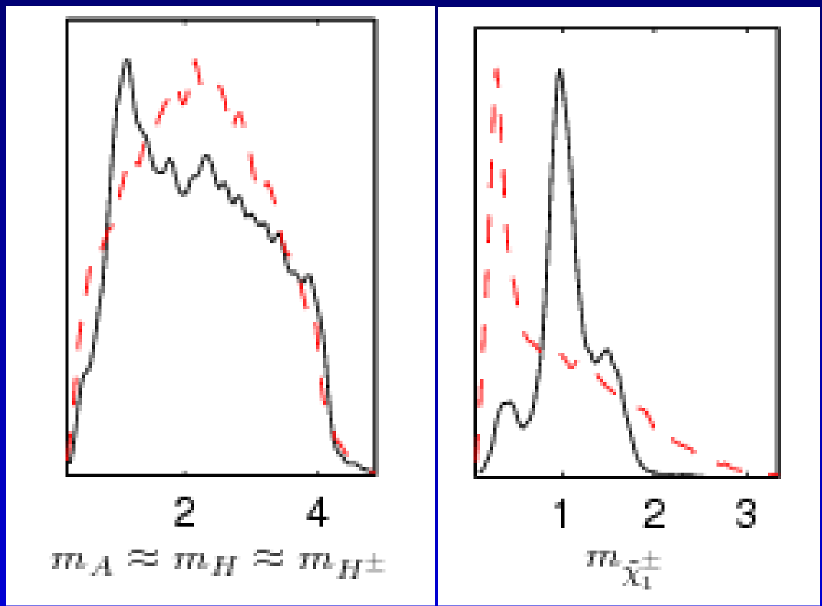
Prospects for SUSY

Look good! 5fb^{-1} expected over the next couple of years

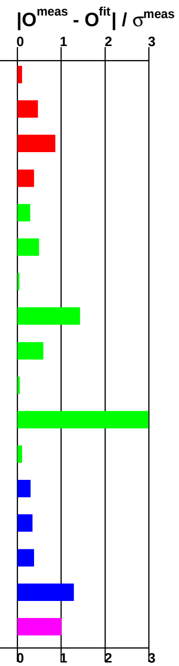


pMSSM Fits

25 pMSSM input parameters are: $M_{1,2,3}$, $A_{t,b,\tau,\mu}$, $m_{H_{1,2}}$, $\tan\beta$, $m_{\tilde{d}_{R,L}} = m_{\tilde{s}_{R,L}}$, $m_{\tilde{u}_{R,L}} = m_{\tilde{c}_{R,L}}$, $m_{\tilde{e}_{R,L}} = m_{\tilde{\mu}_{R,L}}$, $m_{\tilde{t},\tilde{b},\tilde{\tau}_{R,L}}$ m_t, $m_b(m_b)$ $\alpha_s(M_Z)^{\overline{MS}}$, $\alpha^{-1}(M_Z)^{\overline{MS}}$, M_Z. Combined Bayesian fit[a]:

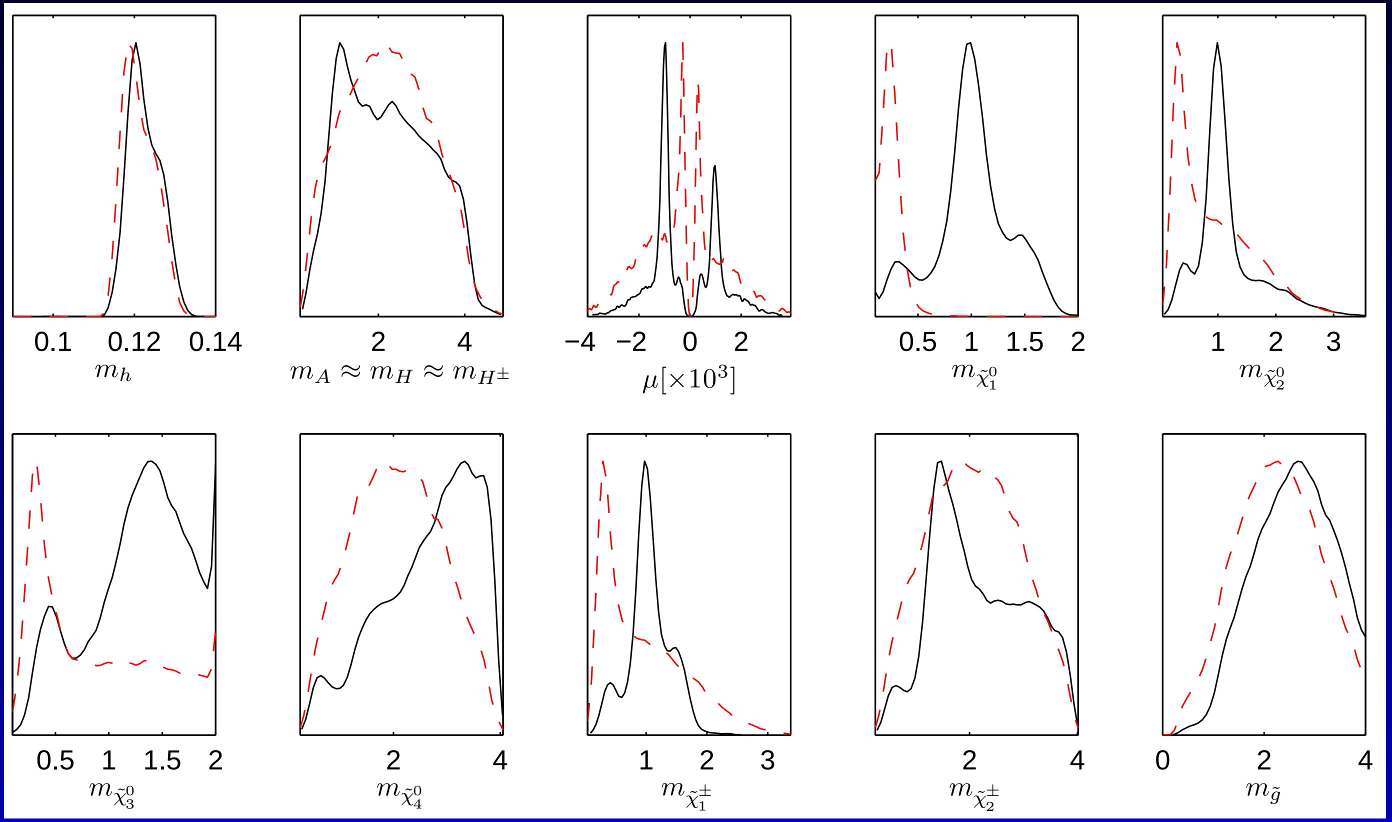


Observable	Measurement	Fit(Log)
m_W [GeV]	80.399 ± 0.025	80.402
Γ_Z [GeV]	2.4952 ± 0.0025	2.4964
$\sin^2\theta^{eff}_{lep}$	0.2324 ± 0.0012	0.2314
$\delta(g-2)_\mu \times 10^{10}$	30.20 ± 9.02	26.74
R^0_l	20.767 ± 0.025	20.760
R_b	0.21629 ± 0.00066	0.21962
R_c	0.1721 ± 0.0030	0.1723
A_e	0.1513 ± 0.0021	0.1483
A_b	0.923 ± 0.020	0.935
A_c	0.670 ± 0.027	0.685
A^b_{FB}	0.0992 ± 0.0016	0.1040
A^c_{FB}	0.071 ± 0.035	0.074
$BR(B \to X_s \gamma) \times 10^4$	3.55 ± 0.42	3.42
$R_{BR(B_u \to \tau\nu)}$	1.11 ± 0.32	1.00
$R_{\Delta M_{B_s}}$	1.15 ± 0.40	1.00
Δ_{0-}	0.0375 ± 0.0289	0.0748
$\Omega_{CDM}h^2$	0.11 ± 0.02	0.13



[a] S.S. AbdusSalam, BCA, F. Quevedo, F. Feroz, M. Hobson, PRD81 (2010) 985012, `arXiv:0904.2548`

Spectrum

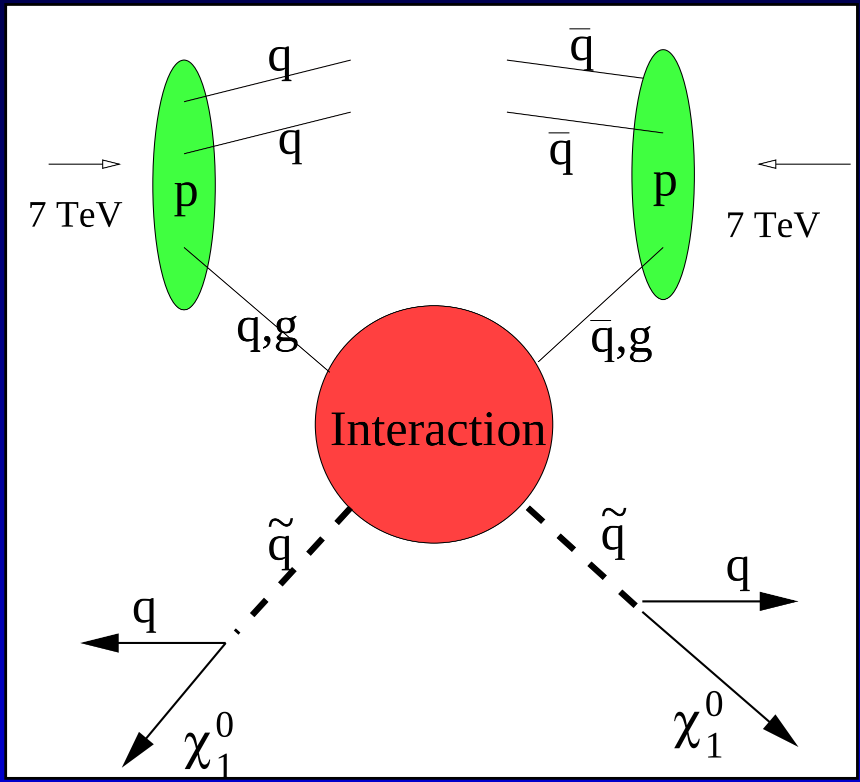


Obtained with `MultiNest`[a] algorithm in 16 CPU years. Prior dependence is *useful*: which predictions are robust?

[a] Feroz, Hobson `arxiv:0704.3704`

Collider SUSY Dark Matter Production

Strong sparticle production and decay to dark matter particles.



Any (light enough) dark matter candidate that couples to hadrons can be produced at the LHC

α_T, MET, M_{T_2} Searches

CMS: jets and missing energy arXiv:1101.1628

$\mathcal{L} = 35$ pb^{-1}. $H_T = \sum_{i=1}^{N_{jet}} |\mathbf{p}_T^{j_i}| > 350$ GeV.

$$\Delta H_T \equiv \sum_{j_i \in A} |\mathbf{p}_T^{j_i}| - \sum_{j_i \in B} |\mathbf{p}_T^{j_i}|. \tag{1}$$

One then calculates

$$\alpha_T = \frac{H_T - \Delta H_T}{2\sqrt{H_T^2 - \not{H}_T^2}} > 0.55 \tag{2}$$

where $\not{H}_T = \sqrt{(\sum_{i=1}^{N_{jet}} p_x^{j_i})^2 + (\sum_{i=1}^{N_{jet}} p_y^{j_i})^2}$.

M_{T_2} Search

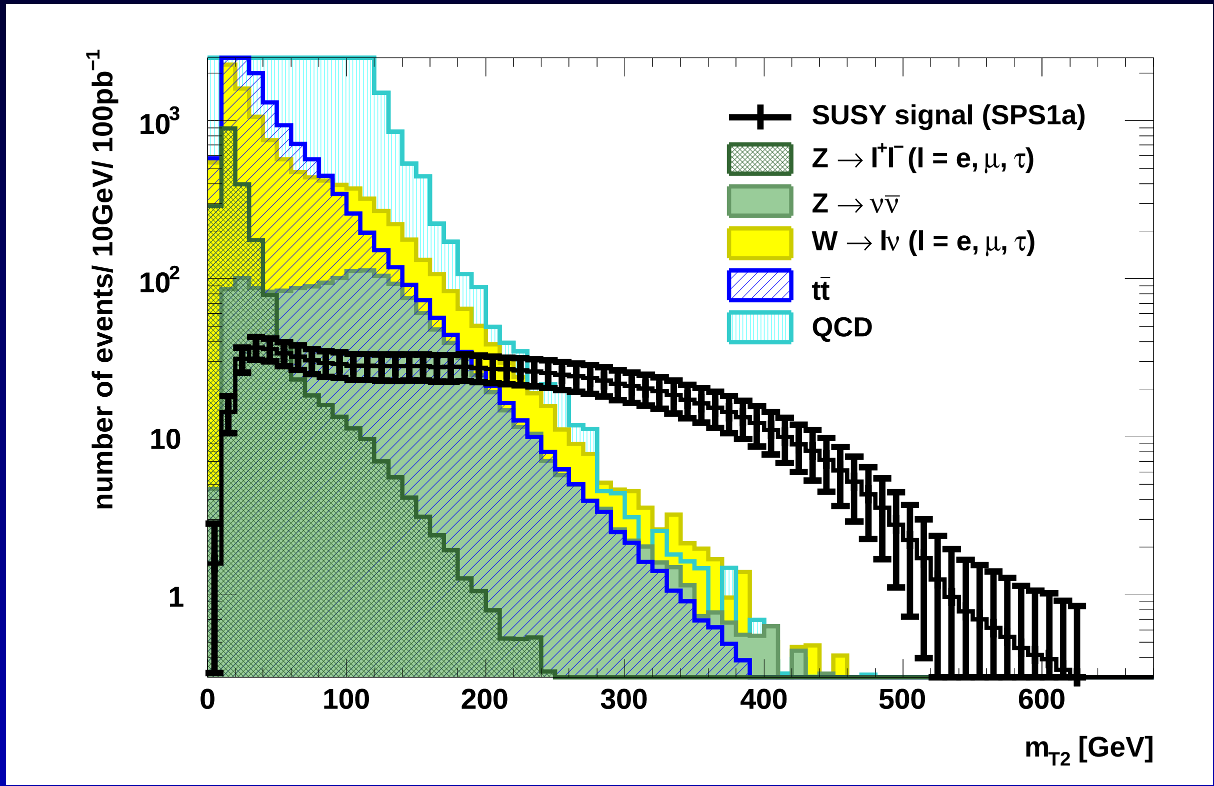


Figure 1: Only cuts: $N_j > 1$, $p_T > 50$ GeV, $\mathcal{L} = 100 pb^{-1}$ at $\sqrt{s} = 7$ TeV. Barr, Gwenlan PRD80 (2009) 074007.

Cue M_{T_2}

$$m_T^{(i)^2}(\mathbf{p}_T{}^{(i)}, \not{\mathbf{q}}_T{}^{(i)}) \equiv 2\left|\mathbf{p}_T{}^{(i)}\right|\left|\not{\mathbf{q}}_T{}^{(i)}\right| - 2\mathbf{p}_T{}^{(i)} \cdot \not{\mathbf{q}}_T{}^{(i)}$$

where $\not{\mathbf{q}}_T{}^{(i)}$ is a guess for the true, unknown missing transverse momentum $\not{\mathbf{p}}_T{}^{(i)}$. The variable M_{T2} is defined by:

$$M_{T2}(\mathbf{p}_T{}^{(1)}, \mathbf{p}_T{}^{(2)}, \not{\mathbf{p}}_T) \equiv \min_{\sum \not{\mathbf{q}}_T = \not{\mathbf{p}}_T} \left\{ \max\left(m_T^{(1)}, m_T^{(2)} \right) \right\}$$

The minimization is over all values of $\not{\mathbf{q}}_T{}^{(1,2)}$ consistent with $\sum \not{\mathbf{q}}_T = \not{\mathbf{p}}_T$. For the SUSY search, the unknown undetected particle masses are set to zero in M_{T2}.

M_{T2} v E_T^{miss}

BCA, Barr, Dafinca, Gwenlan, JHEP 1107 (2011) 104, `arXiv:1105.1024`

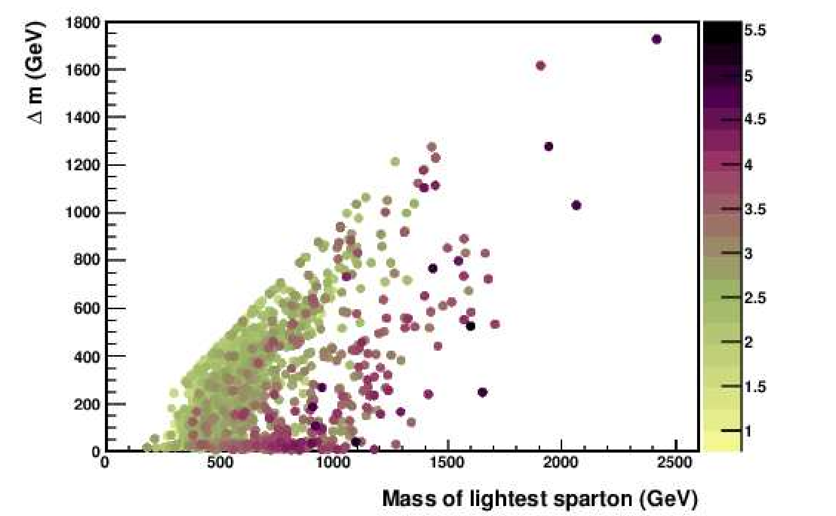
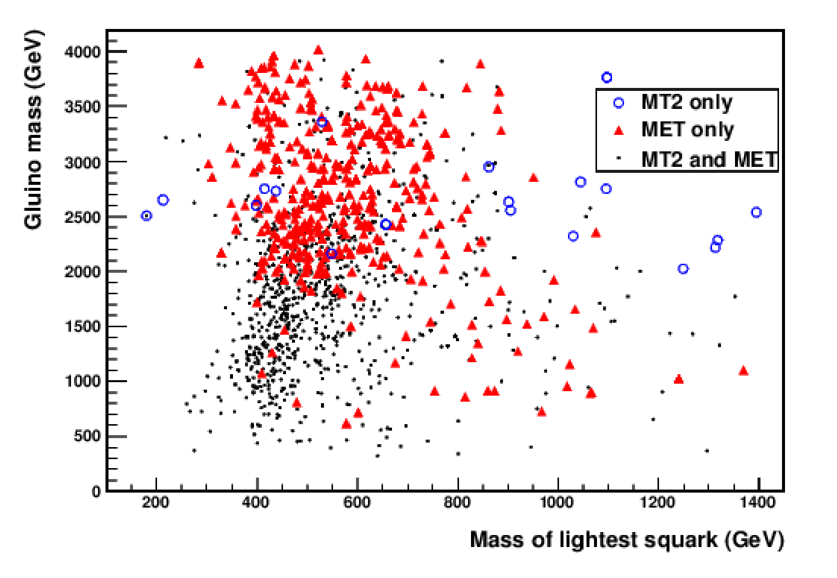


Figure 6: $\log_{10}$[luminosity (pb)$^{-1}$ needed for discovery] with the combined optimal M_{T2} and MET based strategy at $\sqrt{s}$ = 14 TeV in the ($\Delta m, m_{\text{lightest sparton}}$) plane. Δm is the mass difference between the lightest sparton and the LSP. The M_{T2} based strategy was optimized for an integrated luminosity of 1 fb^{-1}. Systematic uncertainties in the background have been neglected.



Compressed Spectra

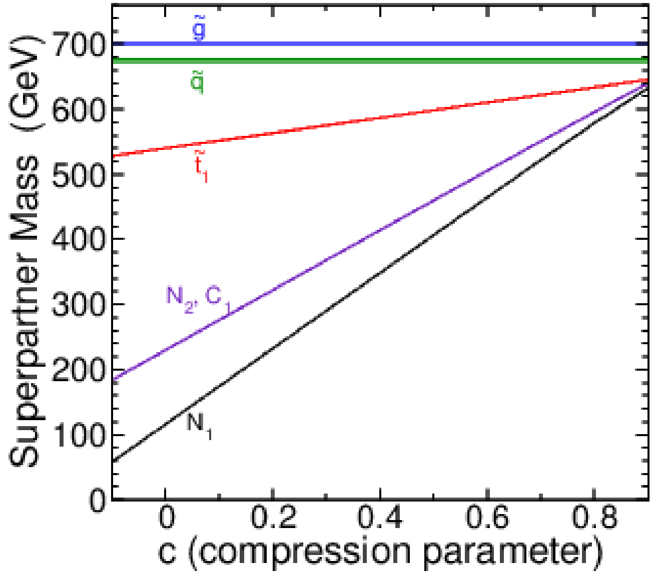


FIG. 1: The masses of the most relevant superpartners for the class of models defined in subsection III A, as a function of the compression parameter c, for fixed $M_{\tilde{g}} = 700$ GeV. The case $c = 0$ corresponds to an mSUGRA-like model.

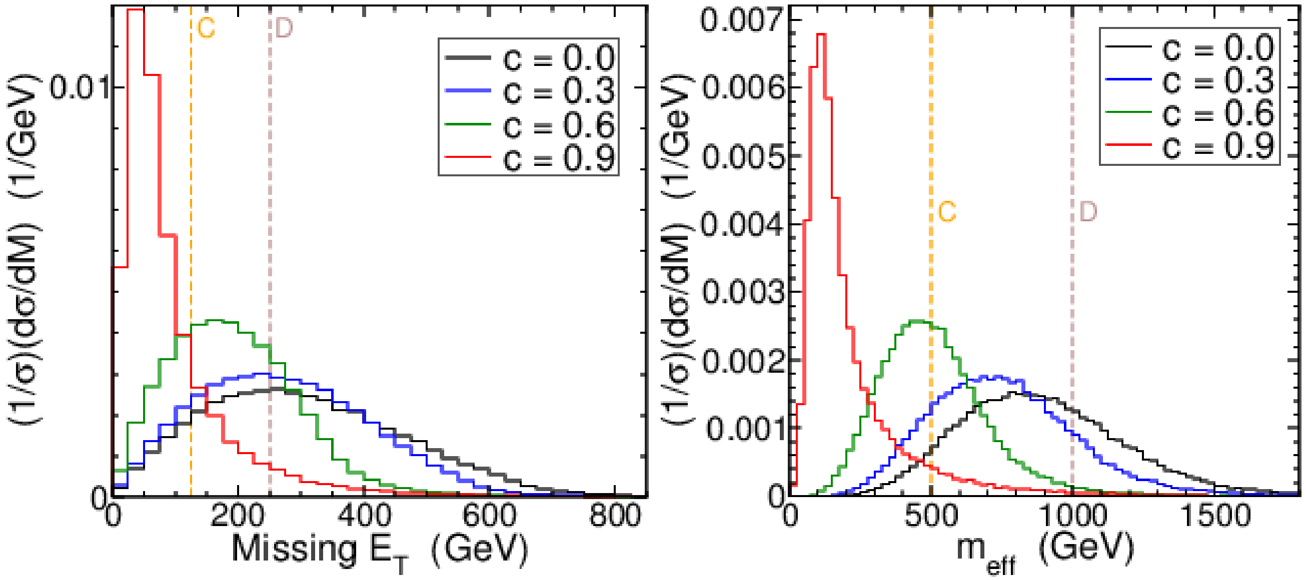
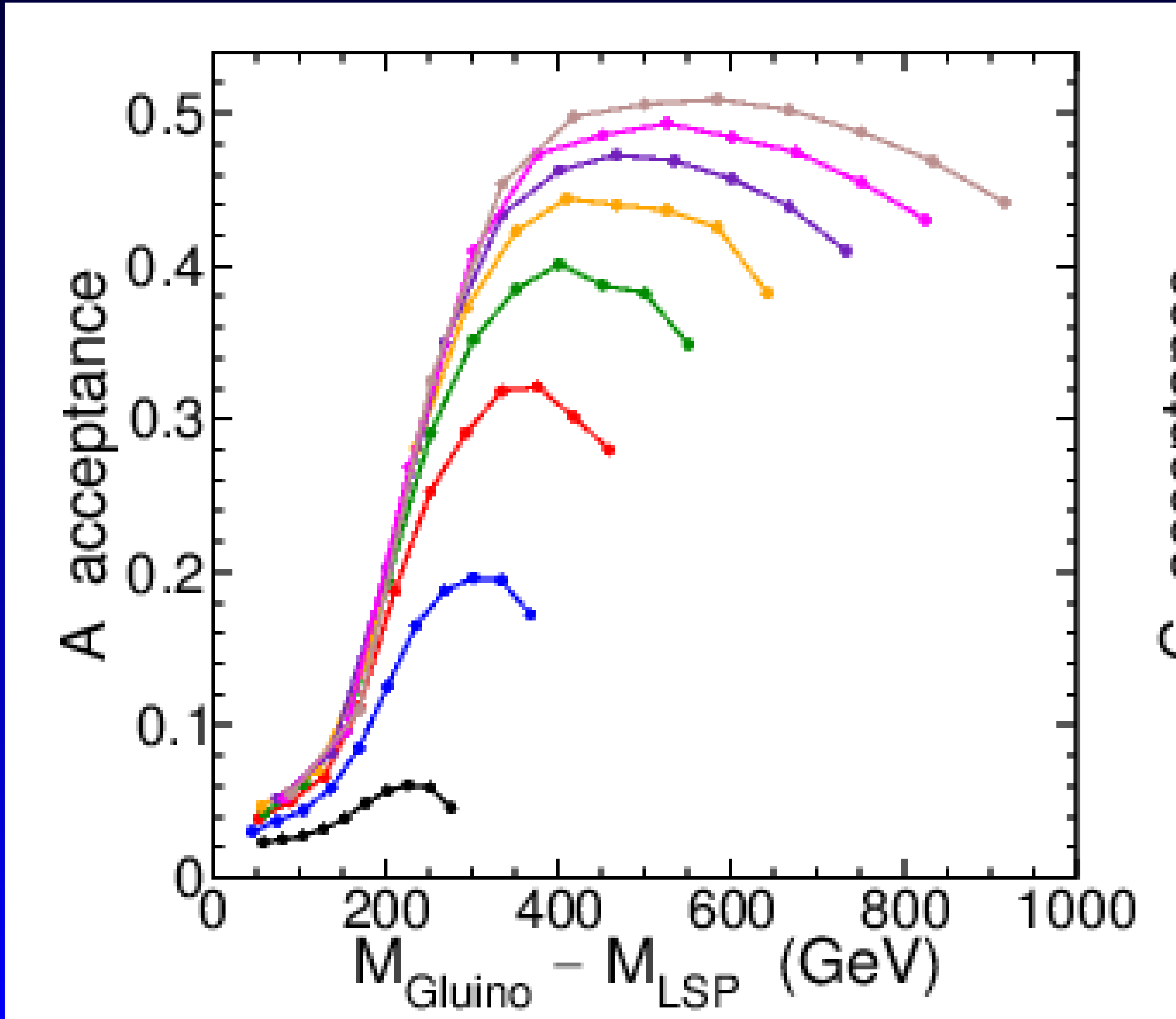


FIG. 2: The distributions before cuts of E_T^{miss} (left panel) and m_{eff} with 3 jets included (right panel) for models described in subsection III A with $M_{\tilde{g}} = 700$ GeV and $c = 0.0, 0.3, 0.6$, and 0.9, from right to left.

Compressed Spectra II

LeCompte, Martin, `arXiv:1105.4304`



Benchmarks

Currently we[a] are devising SUSY benchmark models. It's *imminent*.

- CMSSM, NUHM, mAMSB, mGMSB, RPV and some simplified models (via pMSSM) are defined.
- Defining interesting parameter planes: identifying important parameters which control the masses of sparticles in each case.
- Discrete set of points along monotonic lines: next point for the experiments to study is defined as the lightest one that is not ruled out to 95% CL.

[a] S.S. AbdusSalam, BCA H. Dreiner, J. Ellis, S. Heinemeyer, M. Krämer, M. Mangano, K.A. Olive, S. Rogerson, L. Roszkowski, M. Schlaffer, G. Weiglein

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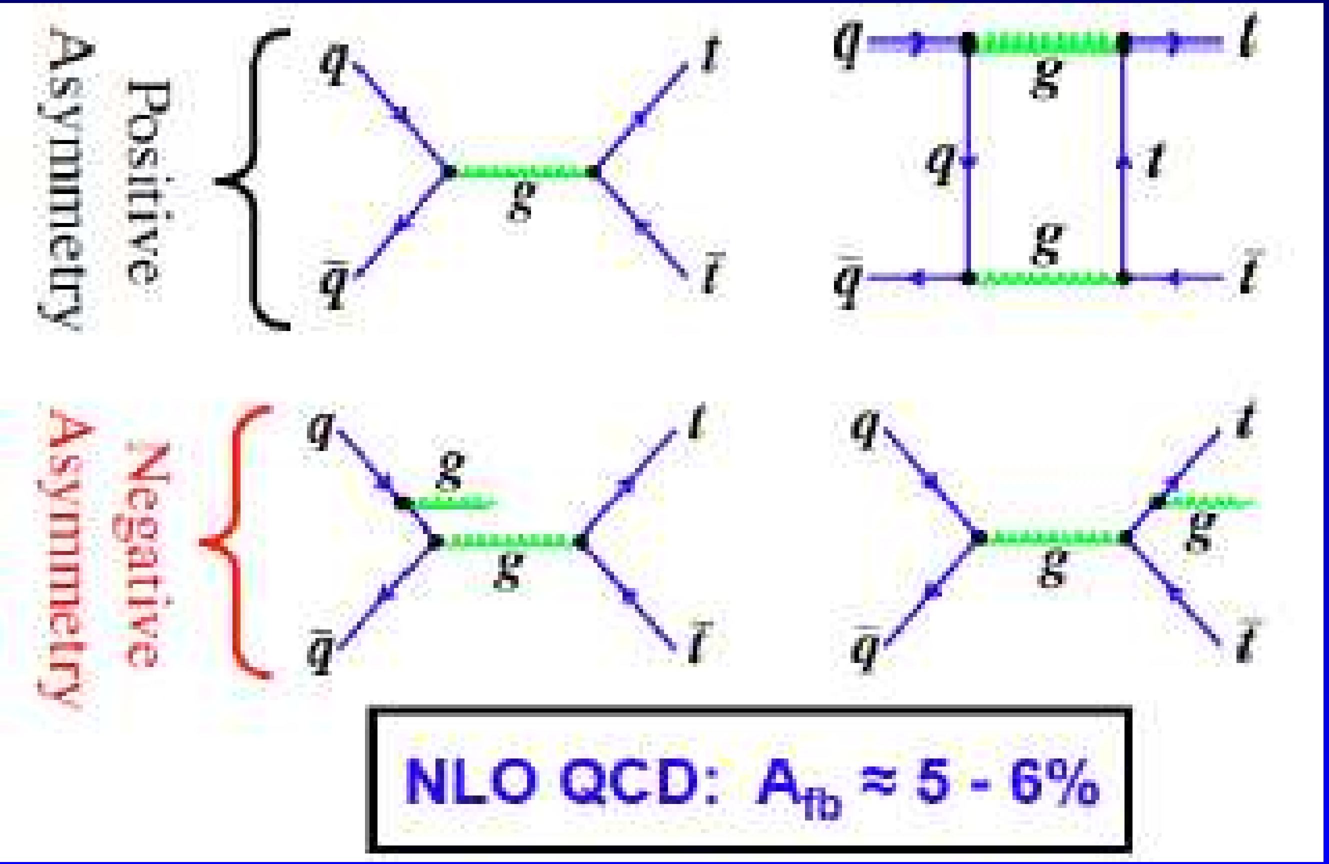
$A_{FB}(t\bar{t})$

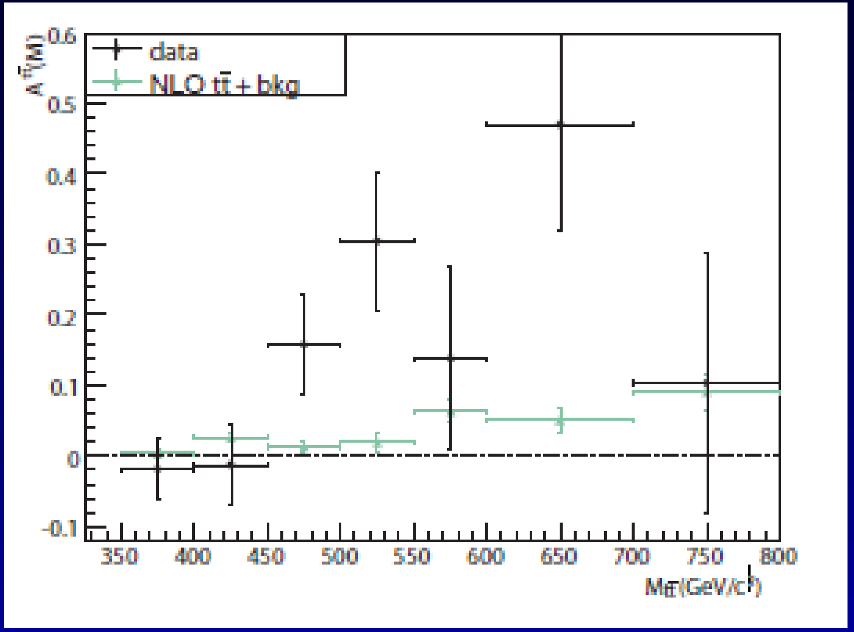
$$A_{FB} = \frac{N(y_t > y_{\bar{t}}) - N(y_{\bar{t}} > y_t)}{N(y_t > y_{\bar{t}}) + N(y_{\bar{t}} > y_t)}$$

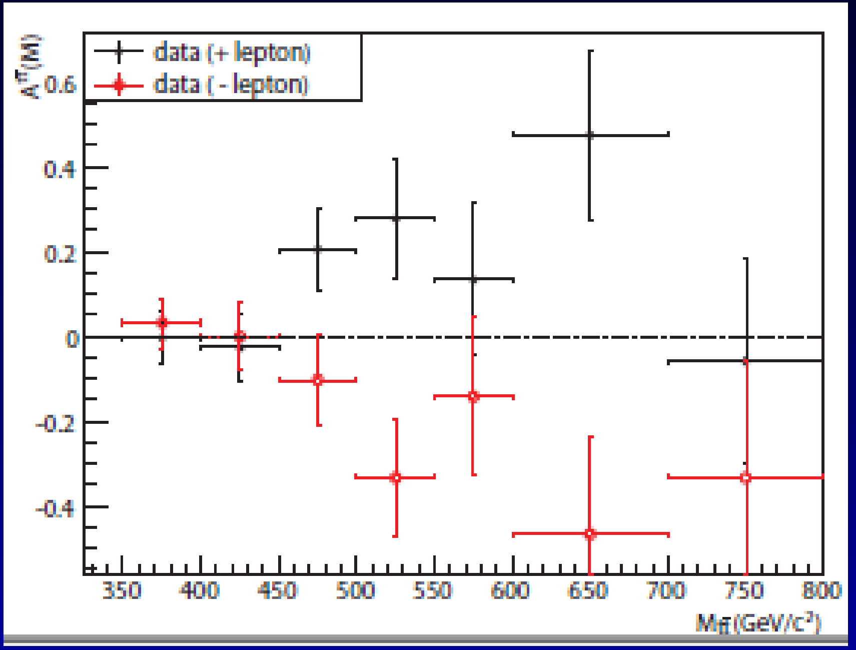
$$A_{FB}(CDF)_{lj+ll} = (20.9 \pm 6.6)\%,$$

$$A_{FB}(D0)_{lj} = (19.6 \pm 6.5)\%,$$



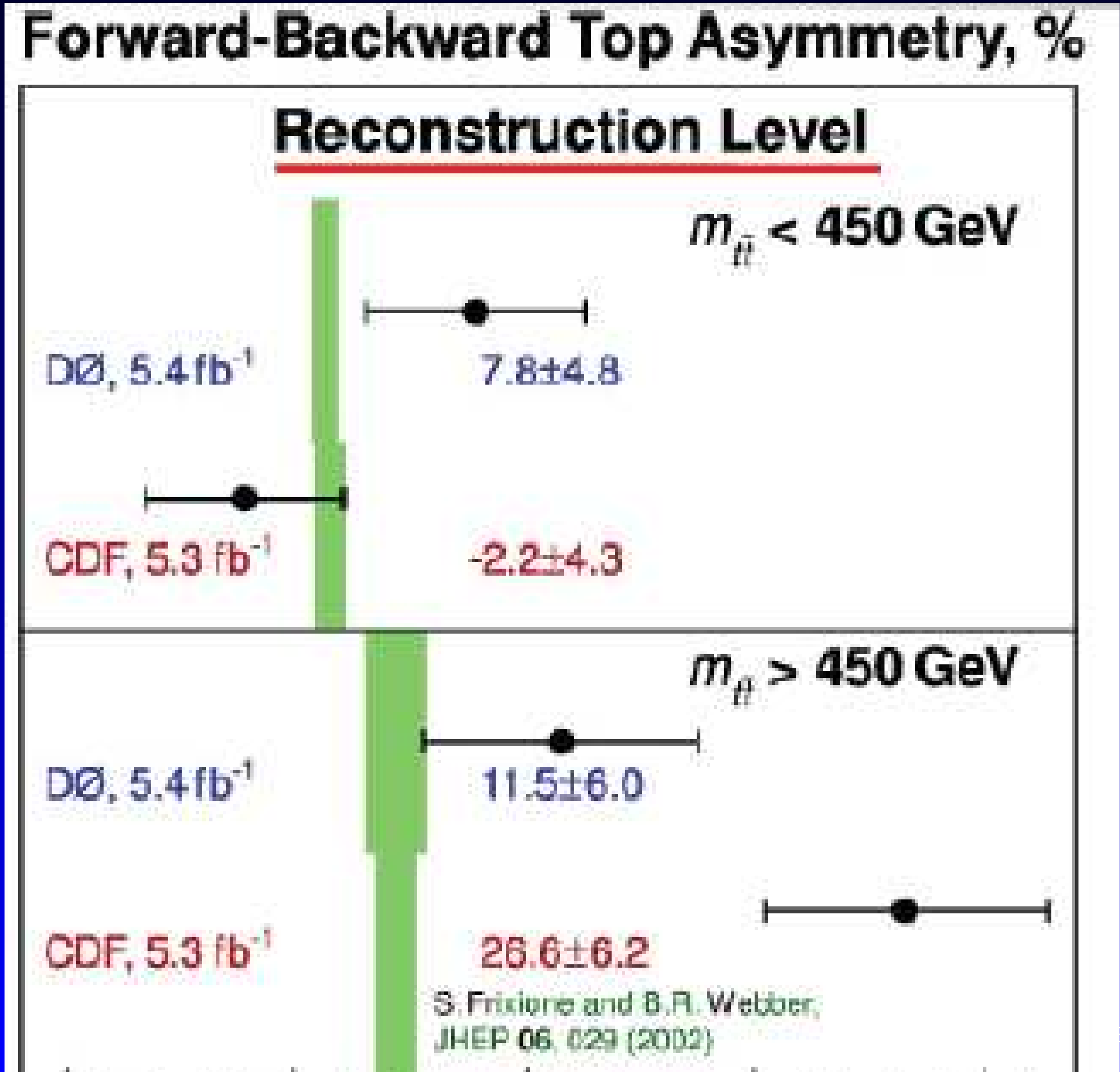
CDF





Seems to be increasing with mass. Lepton charge is nice verification.

$M_{t\bar{t}}$



Forward-Backward Top Asymmetry, %
Reconstruction Level
$m_{t\bar{t}}$ < 450 GeV
DØ, 5.4 fb^{-1}
7.8±4.8
CDF, 5.3 fb^{-1}
-2.2±4.3
$m_{t\bar{t}}$ > 450 GeV
DØ, 5.4 fb^{-1}
11.5±6.0
CDF, 5.3 fb^{-1}
26.6±6.2
S. Frixione and B.R. Webber,
JHEP 06, 029 (2002)

A_{FB} Exotica

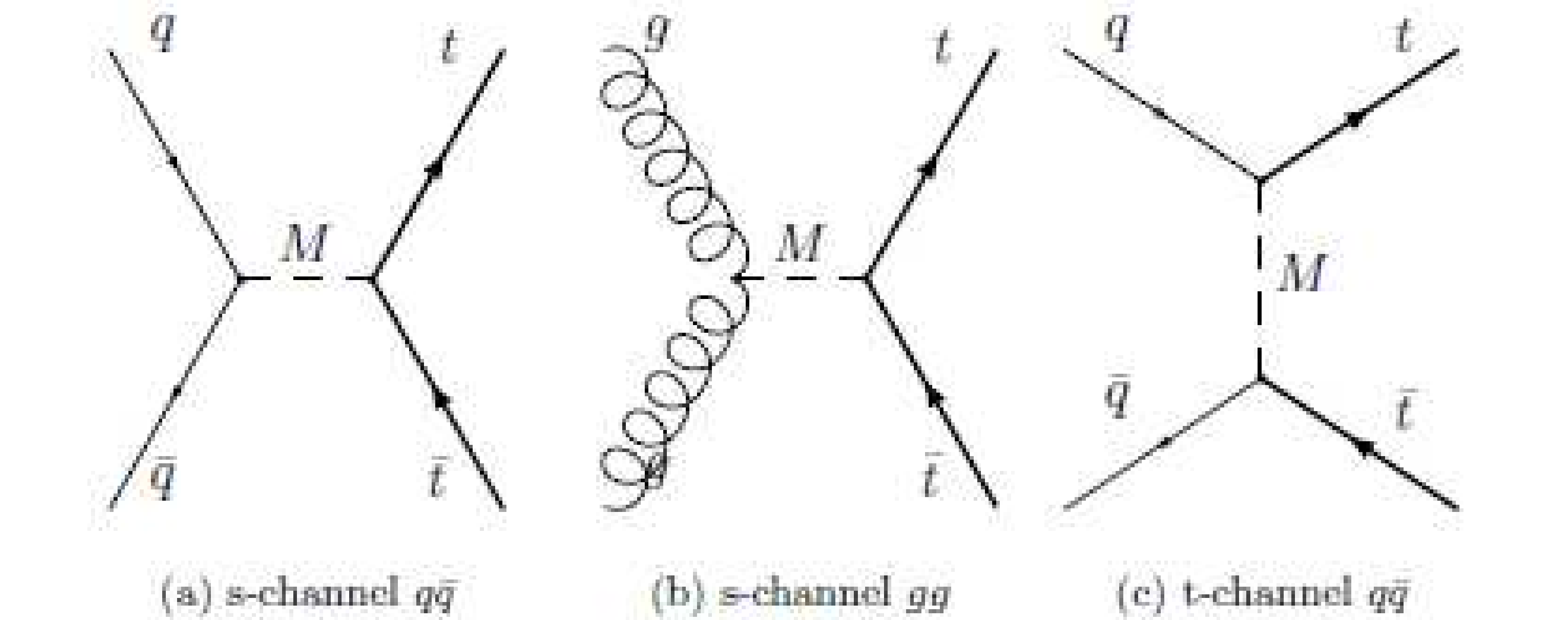


Figure 1: Tree level $t\bar{t}$ production diagram with mediator M exchange.

Must not disturb $\sigma_{t\bar{t}}$ or $d\sigma_{t\bar{t}}/dM_{t\bar{t}}$

- axigluons[a]
- Z'/W'[b]

LHC Asymmetry

Defined LHC charge asym

$$A_C = \frac{N(|y_t| > |y_{\bar{t}}|) - N(y_{\bar{t}} > |y_t|)}{N(|y_t| > |y_{\bar{t}}|) + N(y_{\bar{t}} > |y_t|)}$$

SM discovery would take 60 fb^{-1} at 5σ, but new physics quicker (Z' takes 2 fb^{-1})

$A_C^{CMS} = -1.6 \pm 3 \pm 1\%$ $A_C^{ATLAS} = -2.4 \pm 1.6 \pm 2.3\%$

Models

- Z' model is rather odd: only contains a vertex coupling utZ', eg $M_{Z'} = 800$ GeV, $g_Z = 3.4$: predicts significant *same sign tops*.
- W' models also covered by LHC experiments by now.
- Heavy axigluon models eg 2 TeV, g_q=-g_t=2.4 are ruled out by LHC m_{jj} searches
- Recent proposal[a]: axigluons $g =$0.4-0.8, $M = 50 - 90$ GeV. They evade jet data because the have masses *below* current limits. Non-resonant production suppresses new physics contribution to $\sigma_{t\bar{t}}$.

[a] Krnjaic, arXiv:1109.0648

Summary

- LHC analyses providing a nice amount of information for interpretation of data. There's always room for improvement...
- SUSY is late to the party, but not late enough to be reported missing
- CMSSM *could well be discovered this/next year*
- Current searches reach squark and gluino masses of 980 GeV. This will be extended to $\sim$ 1100 GeV next year, covering much of the good-fit region.
- $t\bar{t}$ asymmetry situation extremely murky. Many heavy axigluon models now ruled out.

Supplementary Material

CMS α_T Search

CMS: jets and missing energy `arXiv:1101.1628`

$\mathcal{L} = 35$ pb^{-1}. $H_T = \sum_{i=1}^{N_{jet}} |\mathbf{p}_T^{j_i}| > 350$ GeV.

$$\Delta H_T \equiv \sum_{j_i \in A} |\mathbf{p}_T^{j_i}| - \sum_{j_i \in B} |\mathbf{p}_T^{j_i}|. \quad (3)$$

One then calculates

$$\alpha_T = \frac{H_T - \Delta H_T}{2\sqrt{H_T^2 - \not{H}_T^2}} > 0.55 \quad (4)$$

where $\not{H}_T = \sqrt{(\sum_{i=1}^{N_{jet}} p_x^{j_i})^2 + (\sum_{i=1}^{N_{jet}} p_y^{j_i})^2}$.

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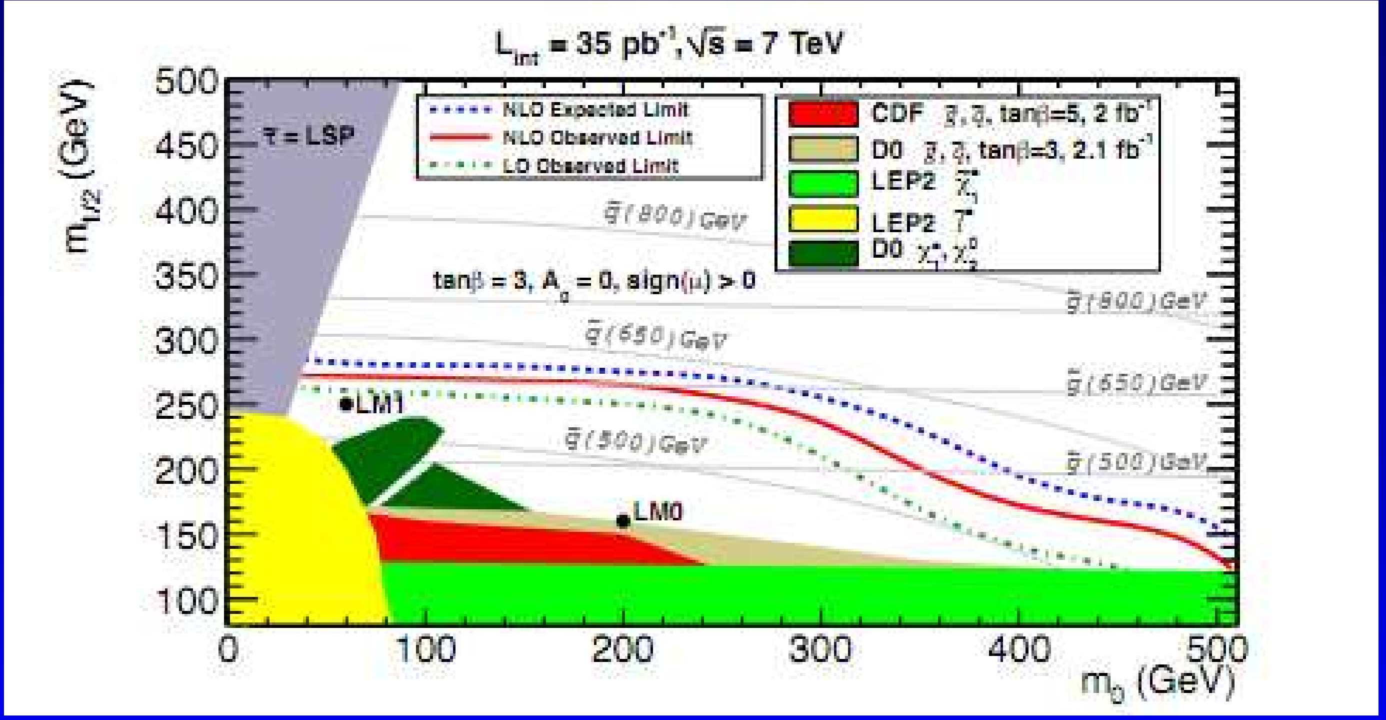
Results

Selection	Data	SM	QCD multijet	$Z \to \nu\bar{\nu}$	W + jets	$t\bar{t}$
$H_T > 250$ GeV	4.68M	5.81M	5.81M	290	2.0k	2.5k
$E_T^{j_2} > 100$ GeV	2.89M	3.40M	3.40M	160	610	830
$H_T > 350$ GeV	908k	1.11M	1.11M	80	280	650
$\alpha_T > 0.55$	37	30.5±4.7	19.5±4.6	4.2±0.6	3.9±0.7	2.8±0.1
$\Delta R_{ECAL} > 0.3 \vee \Delta\phi^* > 0.5$	32	24.5±4.2	14.3±4.1	4.2±0.6	3.6±0.6	2.4±0.1
$R_{miss} < 1.25$	13	9.3±0.9	0.03±0.02	4.1±0.6	3.3±0.6	1.8±0.1



ATLAS 0-lepton, jets and $\not p_T$

		A	B	C	D
Pre-selection	Number of required jets	≥ 2	≥ 2	≥ 3	≥ 3
	Leading jet $p_{\rm T}$ [GeV]	> 120	> 120	> 120	> 120
	Other jet(s) $p_{\rm T}$ [GeV]	> 40	> 40	> 40	> 40
	$E_{\rm T}^{\rm miss}$ [GeV]	> 100	> 100	> 100	> 100
Final selection	$\Delta\phi(\text{jet}, \vec{P}_{\rm T}^{\rm miss})_{\rm min}$	> 0.4	> 0.4	> 0.4	> 0.4
	$E_{\rm T}^{\rm miss}/m_{\rm eff}$	> 0.3	–	> 0.25	> 0.25
	$m_{\rm eff}$ [GeV]	> 500	–	> 500	> 1000
	$m_{\rm T2}$ [GeV]	–	> 300	–	–

Table 1: Criteria for admission to each of the four overlapping signal regions A to D. All variables are defined in §4.

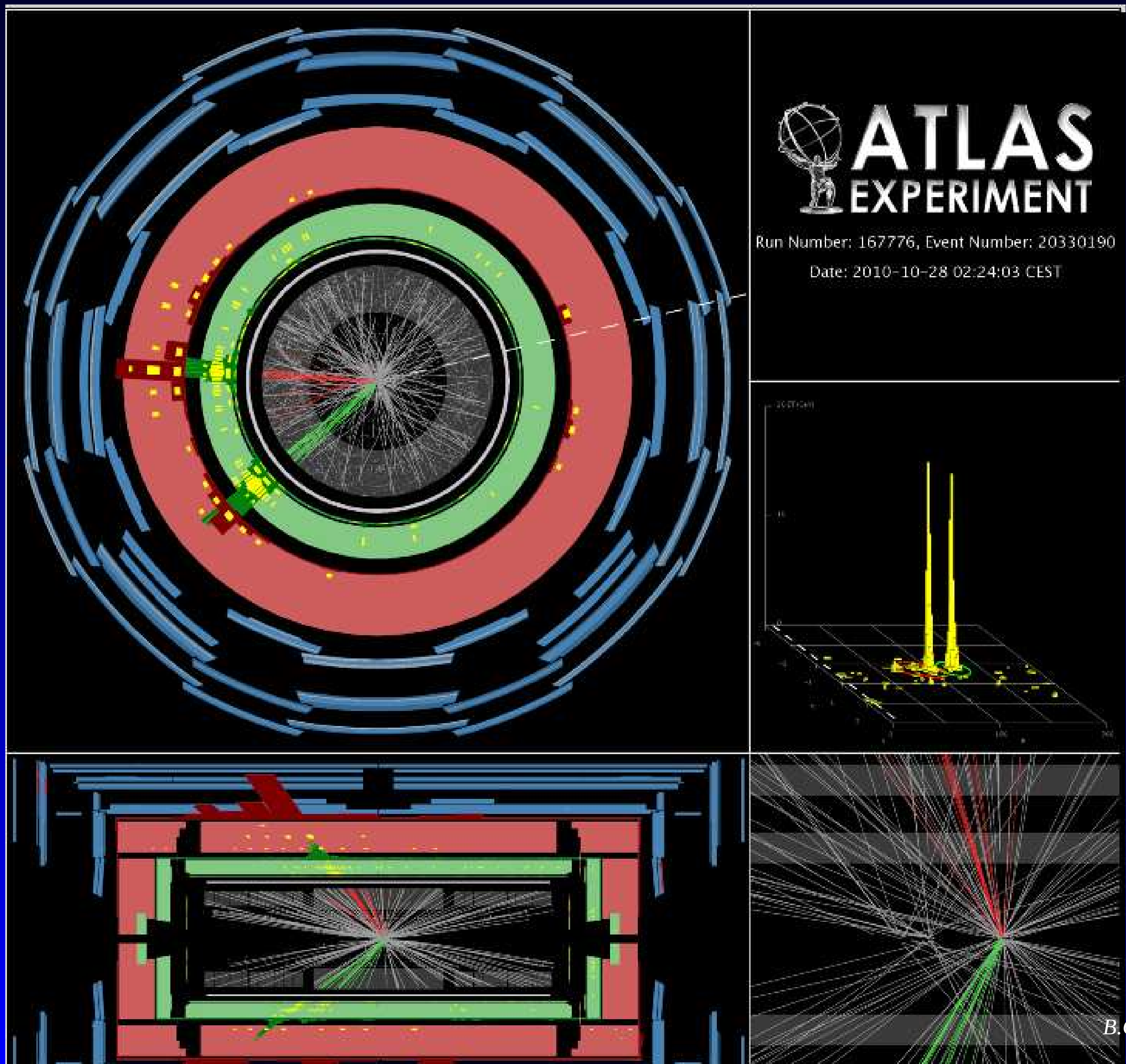
$$m_{eff} = \sum p_T^{(j)} + \not p_T,$$

$$m_T^{(i)^2}(\mathbf{p}_T{}^{(i)}, \not q_T{}^{(i)}) \equiv 2\left|\mathbf{p}_T{}^{(i)}\right| \left|\not q_T{}^{(i)}\right| - 2\mathbf{p}_T{}^{(i)} \cdot \not q_T{}^{(i)}$$

where $\not q_T{}^{(i)}$ is the transverse momentum of particle (i). For each event, it is a lower bound on $m(NLSP)$.

$$M_{T2}(\mathbf{p}_T{}^{(1)}, \mathbf{p}_T{}^{(2)}, \not p_T) \equiv \min_{\sum \not q_T = \not p_T} \left\{ \max\left(m_T^{(1)}, m_T^{(2)} \right) \right\}$$

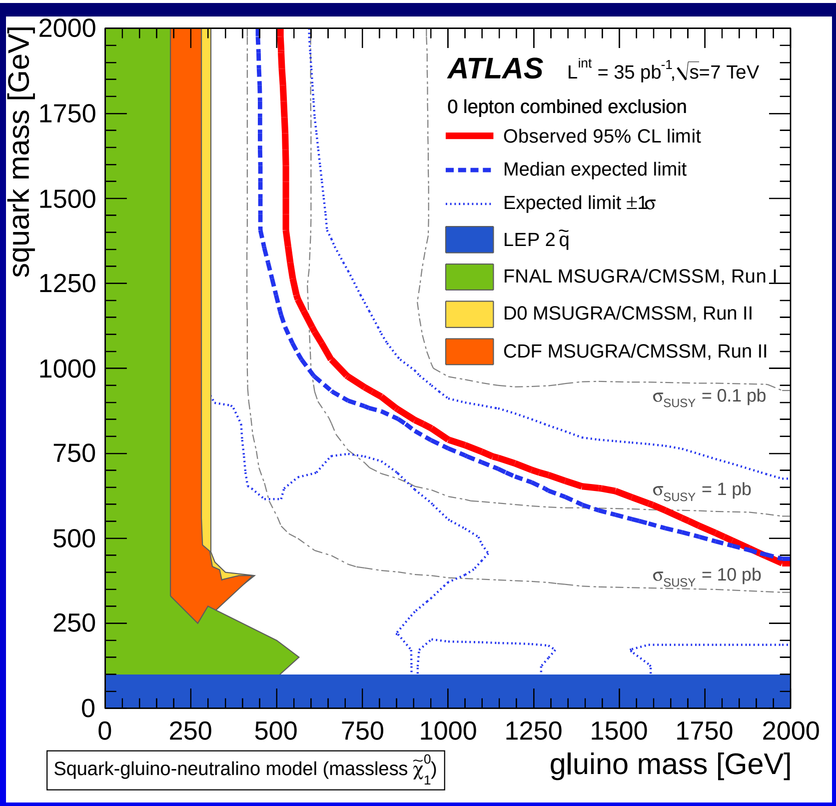
Candidate Event: High $E_T(j)$



MSSM Exclusion: Simplified Model

	Signal region A	Signal region B	Signal region C	Signal region D
QCD	$7\ ^{+8}_{-7}[\mathrm{u+j}]$	$0.6\ ^{+0.7}_{-0.6}[\mathrm{u+j}]$	$9\ ^{+10}_{-9}[\mathrm{u+j}]$	$0.2\ ^{+0.4}_{-0.2}[\mathrm{u+j}]$
W+jets	$50 \pm 11[\mathrm{u}]\ ^{+14}_{-10}[\mathrm{j}] \pm 5[\mathcal{L}]$	$4.4 \pm 3.2[\mathrm{u}]\ ^{+1.5}_{-0.8}[\mathrm{j}] \pm 0.5[\mathcal{L}]$	$35 \pm 9[\mathrm{u}]\ ^{+10}_{-8}[\mathrm{j}] \pm 4[\mathcal{L}]$	$1.1 \pm 0.7[\mathrm{u}]\ ^{+0.2}_{-0.3}[\mathrm{j}] \pm 0.1[\mathcal{L}]$
Z+jets	$52 \pm 21[\mathrm{u}]\ ^{+15}_{-11}[\mathrm{j}] \pm 6[\mathcal{L}]$	$4.1 \pm 2.9[\mathrm{u}]\ ^{+2.1}_{-0.8}[\mathrm{j}] \pm 0.5[\mathcal{L}]$	$27 \pm 12[\mathrm{u}]\ ^{+10}_{-6}[\mathrm{j}] \pm 3[\mathcal{L}]$	$0.8 \pm 0.7[\mathrm{u}]\ ^{+0.6}_{-0.0}[\mathrm{j}] \pm 0.1[\mathcal{L}]$
$t\bar{t}$ and t	$10 \pm 0[\mathrm{u}]\ ^{+3}_{-2}[\mathrm{j}] \pm 1[\mathcal{L}]$	$0.9 \pm 0.1[\mathrm{u}]\ ^{+0.4}_{-0.3}[\mathrm{j}] \pm 0.1[\mathcal{L}]$	$17 \pm 1[\mathrm{u}]\ ^{+6}_{-4}[\mathrm{j}] \pm 2[\mathcal{L}]$	$0.3 \pm 0.1[\mathrm{u}]\ ^{+0.2}_{-0.1}[\mathrm{j}] \pm 0.0[\mathcal{L}]$
Total SM	$118 \pm 25[\mathrm{u}]\ ^{+32}_{-23}[\mathrm{j}] \pm 12[\mathcal{L}]$	$10.0 \pm 4.3[\mathrm{u}]\ ^{+4.0}_{-1.9}[\mathrm{j}] \pm 1.0[\mathcal{L}]$	$88 \pm 18[\mathrm{u}]\ ^{+26}_{-18}[\mathrm{j}] \pm 9[\mathcal{L}]$	$2.5 \pm 1.0[\mathrm{u}]\ ^{+1.0}_{-0.4}[\mathrm{j}] \pm 0.2[\mathcal{L}]$
Data	87	11	66	2

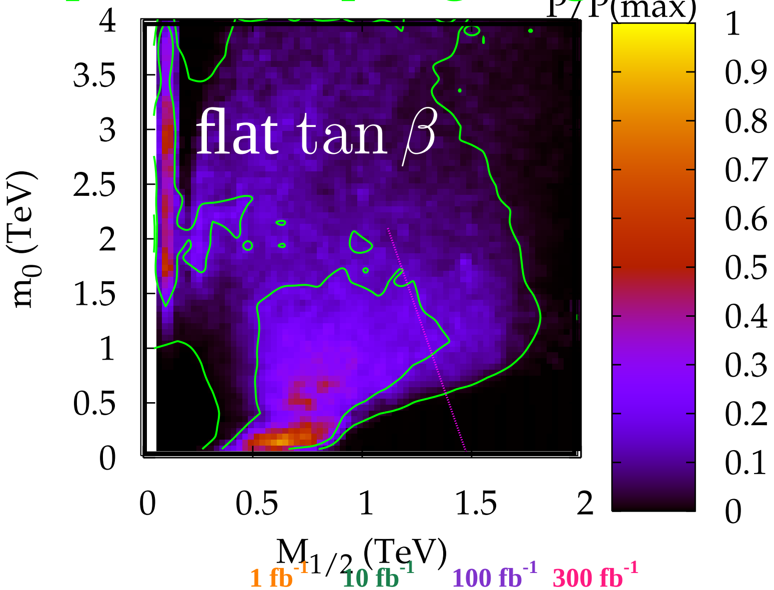
Table 2: Expected and observed numbers of events in the four signal regions. Uncertainties shown are due to "MC statistics, statistics in control regions, other sources of uncorrelated systematic uncertainty, and also the jet energy resolution and lepton efficiencies" [u], the jet energy scale [j], and the luminosity [$\mathcal{L}$].

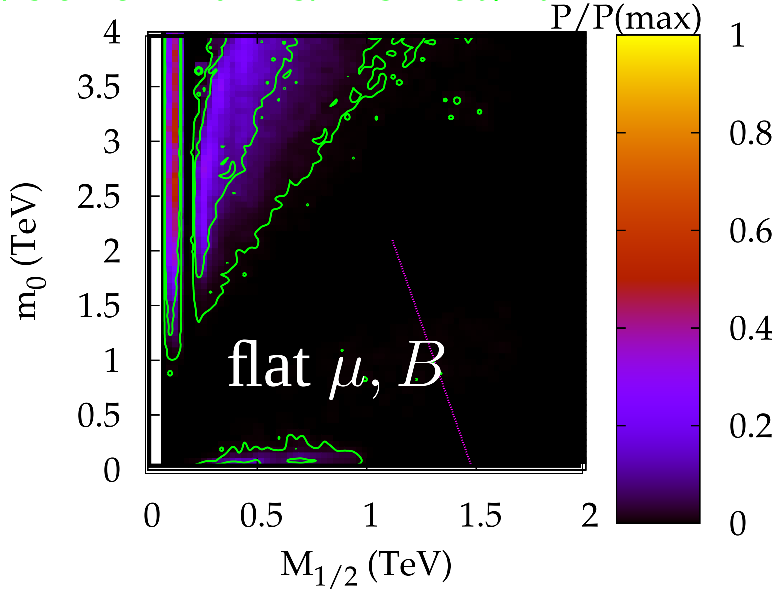


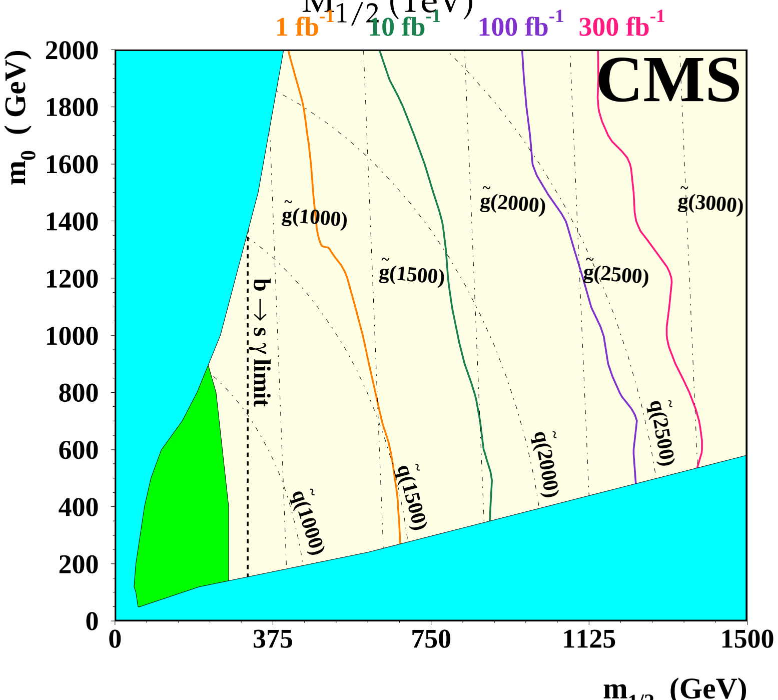
Killer Inference for Susy METeorology

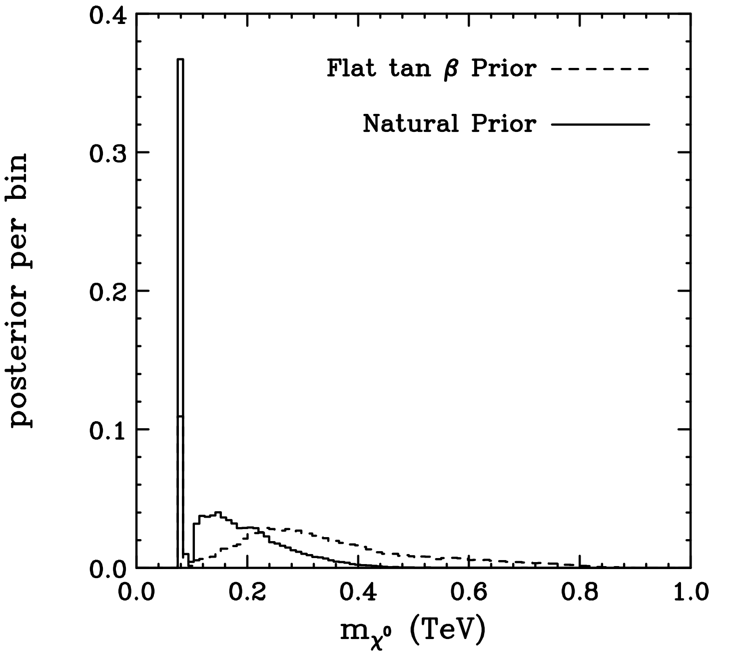
BCA, Cranmer, Weber, Lester, arXiv:0705.0487

http://users.hepforge.org/~allanach/benchmarks/kismet.html



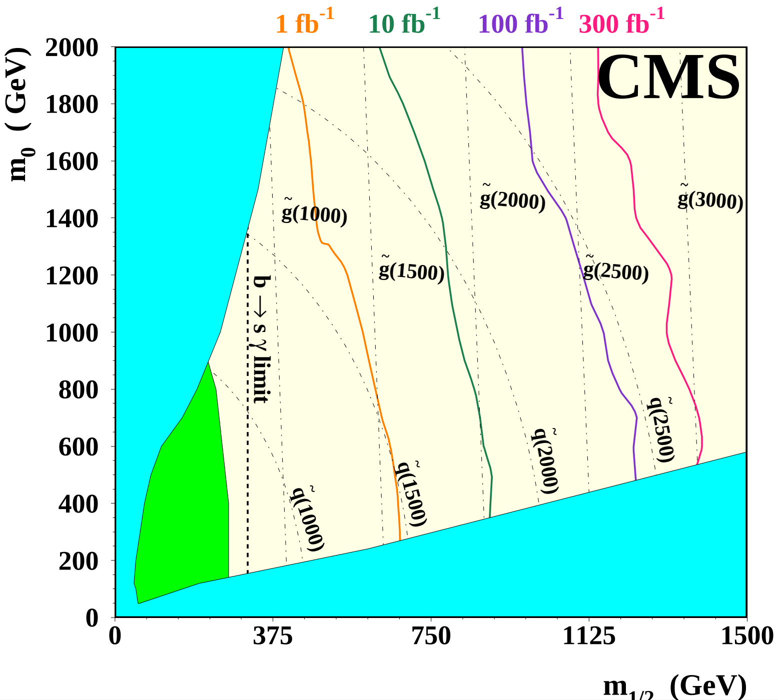


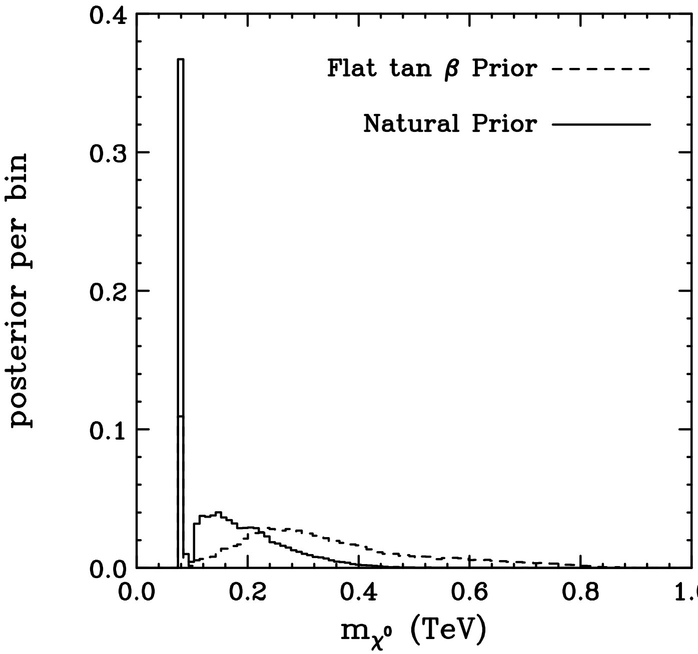




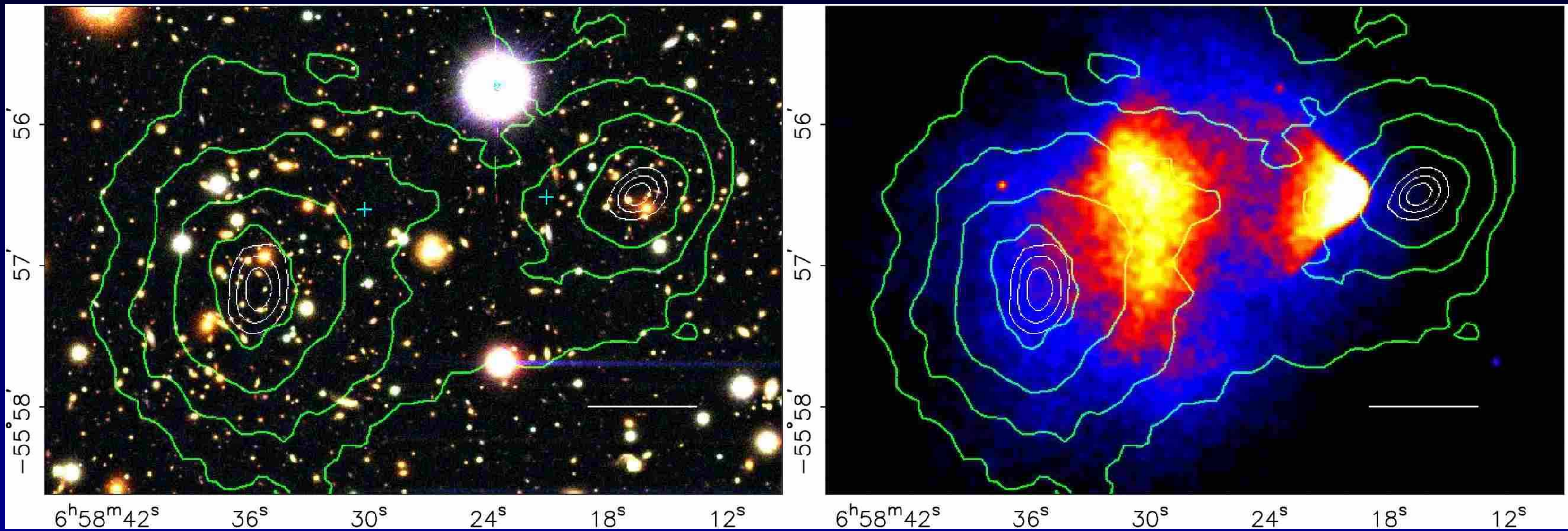
Killer Inference for Susy METeorology

BCA, Cranmer, Weber, Lester, arXiv:0705.0487

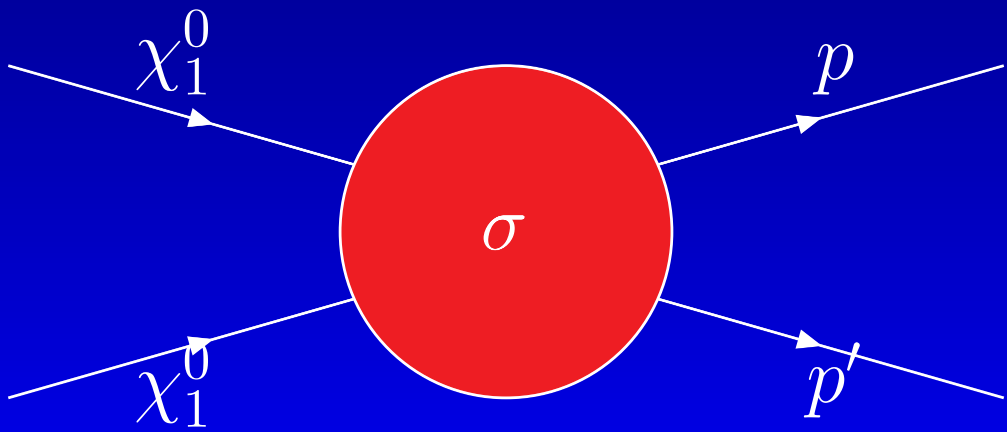




SUSY Dark Matter



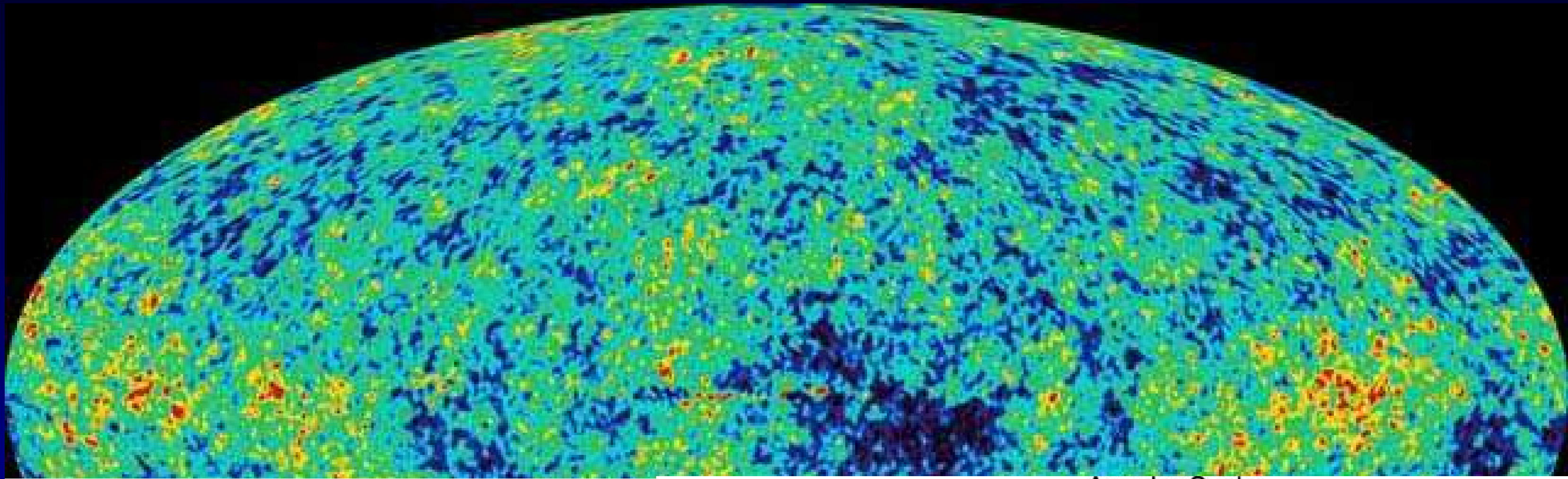
astro-ph/0608407

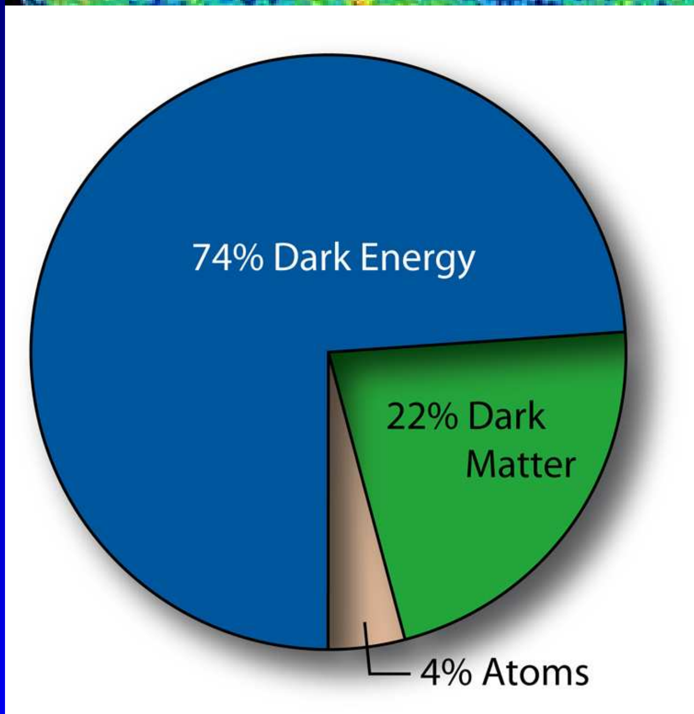


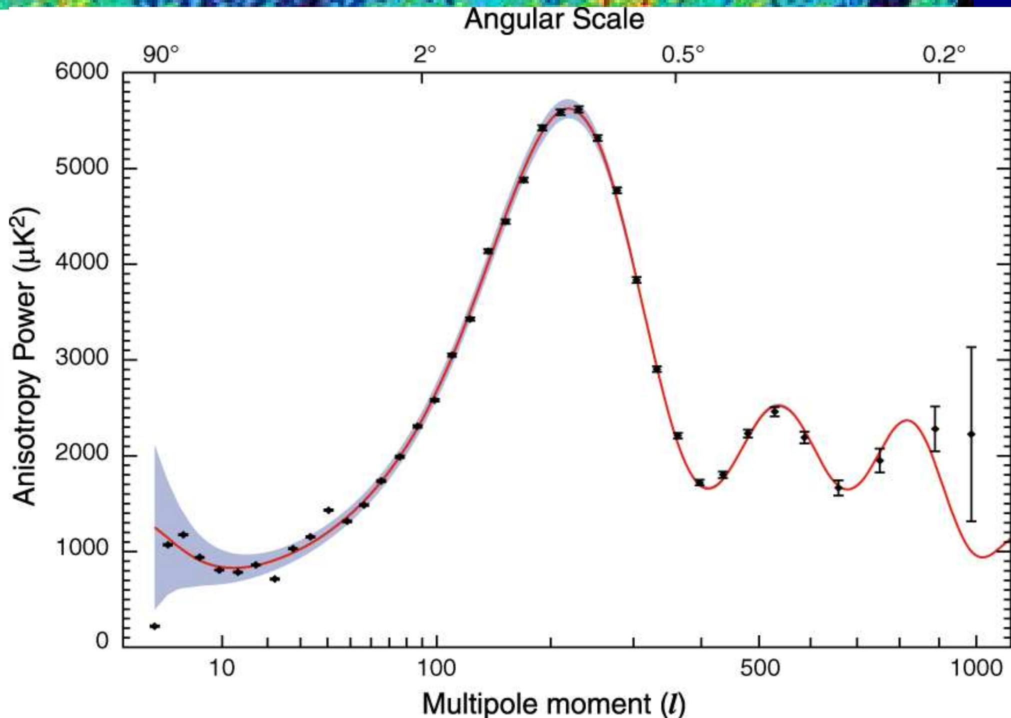
SUSY Prediction of Ωh^2

- Assume relic in thermal equilibrium with $n_{eq} \propto (MT)^{3/2} exp(-M/T)$.
- Freeze-out with $T_f \sim M_f/25$ once interaction rate $<$ expansion rate (t_{eq} critical)
- `microMEGAs` uses `calcHEP` to automatically calculate relevant Feynman diagrams for some given model Lagrangian: *flexible*.
- `darkSUSY`, `IsaRED` has MSSM annihilation channels hard-coded.
- Both `darkSUSY` and `micrOMEGAs` calculate (in-)direct predictions.

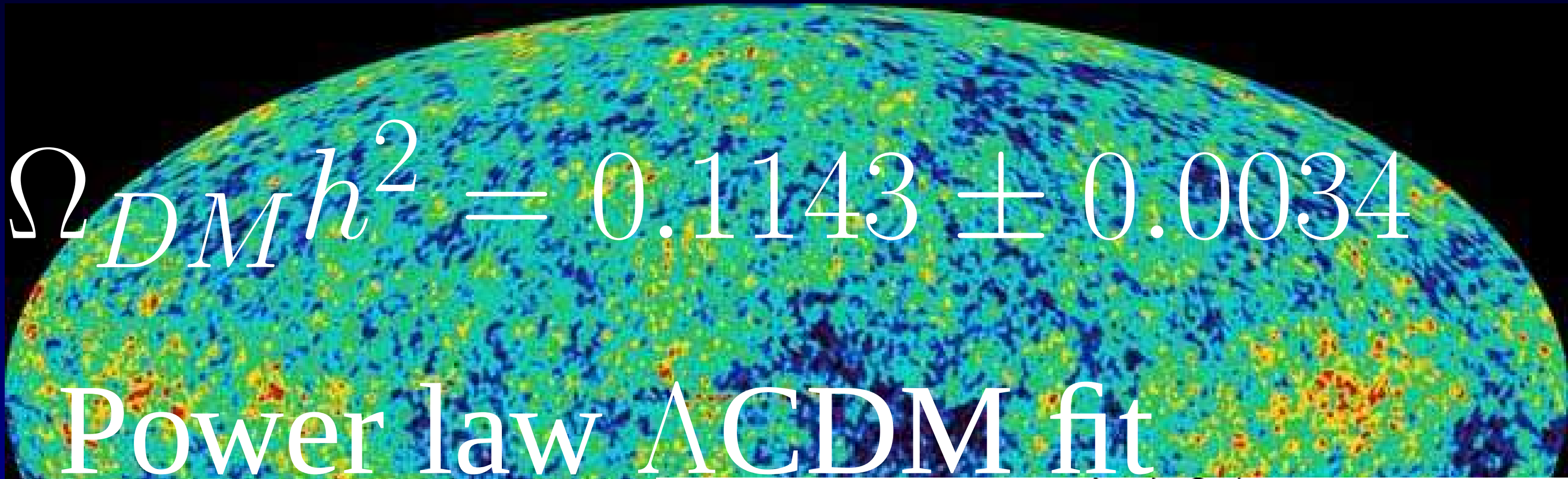
WMAP+BAO+Ia Fits





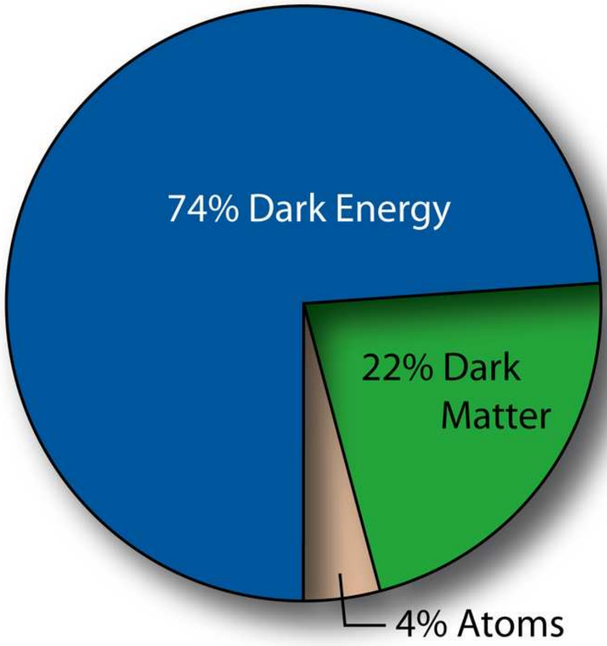


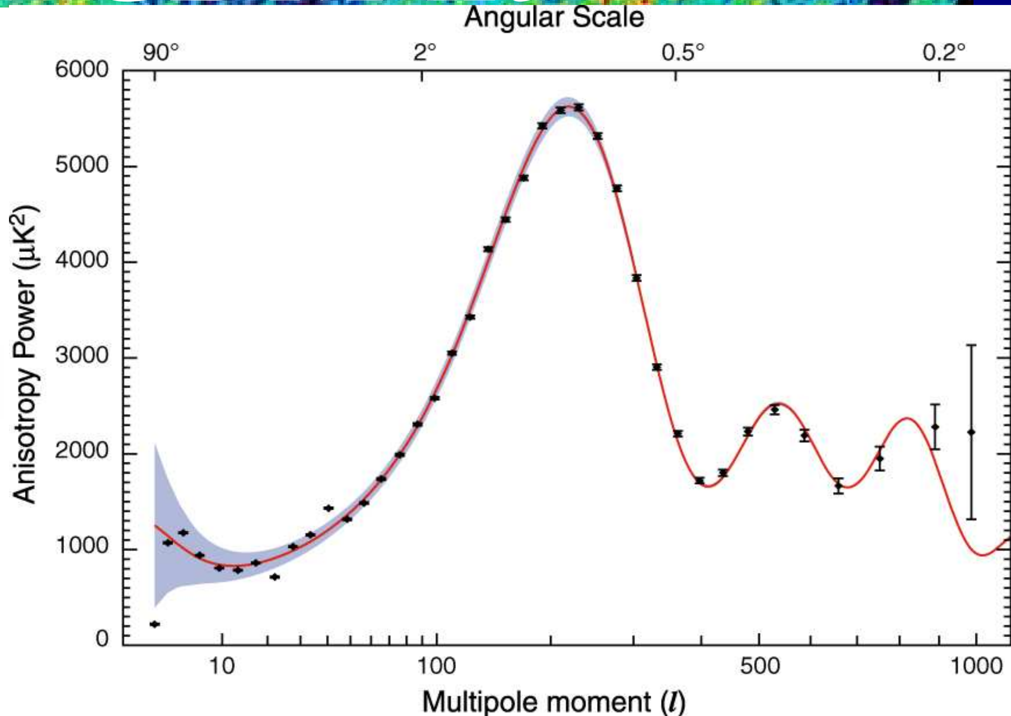
WMAP+BAO+Ia Fits



$$\Omega_{DM} h^2 = 0.1143 \pm 0.0034$$

Power law ΛCDM fit





mSUGRA Global Fits

There are 3 methodologies of doing these type of global fits:

- Markov Chain Monte Carlo: BCA *et al*; Ruiz de Austri *et al*: primary interpretation is *Bayesian*.
- MultiNest: Ruiz de Austri *et al*: Bayesian interpretation only.
- Minimising χ^2/Profile likelihood: Buchmueller *et al*. Impressive array of electroweak observables. Moving to a *hybrid* approach. Frequentist interpretation only.

Universality

Reduces number of SUSY breaking parameters from 100 to 3:

- $\tan\beta \equiv v_2/v_1$
- m_0, the common scalar mass (flavour).
- $M_{1/2}$, the common gaugino mass (GUT/string).
- A_0, the common trilinear coupling (flavour).

These conditions should be imposed at $M_X \sim O(10^{16-18})$ GeV and receive radiative corrections

$$\propto \ 1/(16\pi^2)\ln(M_X/M_Z).$$

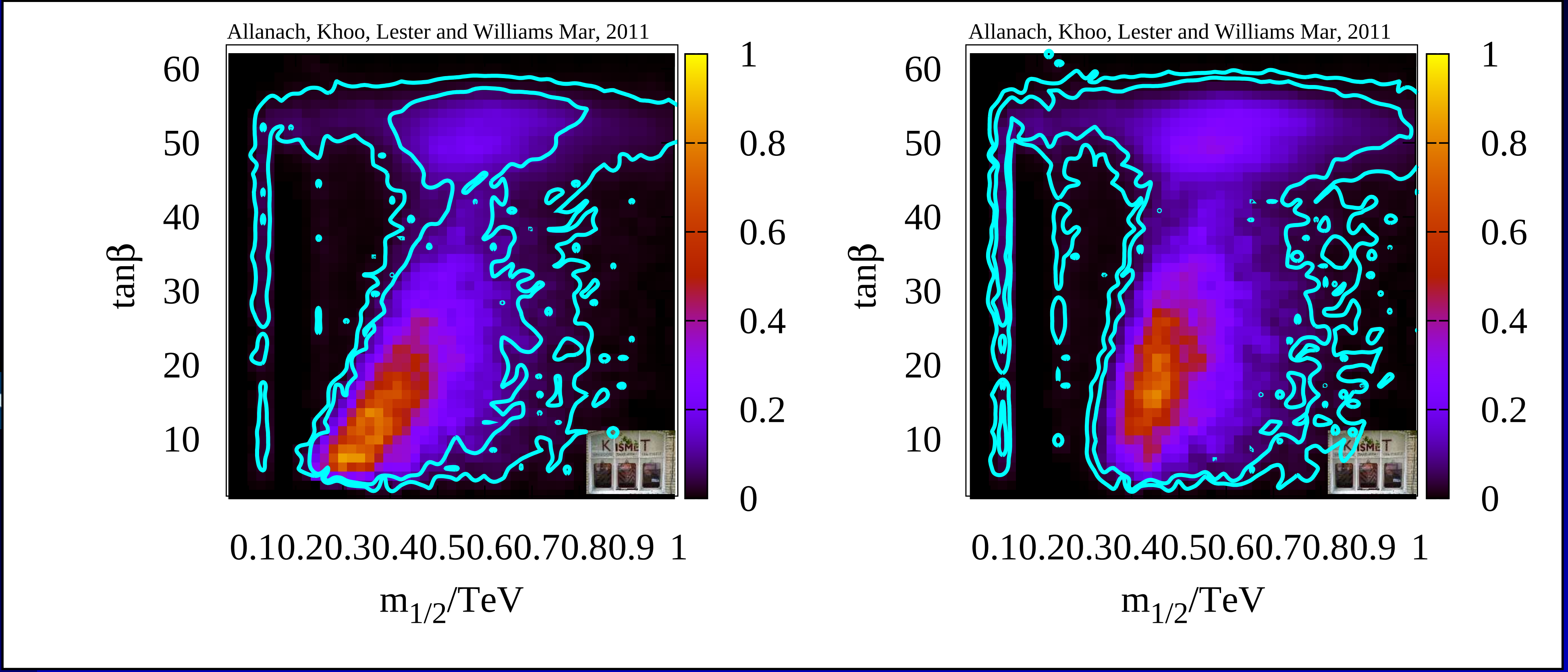
Also, Higgs potential parameter sgn(μ)=± 1.

Other work

- Shortly after my first, the `Mastercode` collaboration[a] performed fits of CMSSM, NUHM, VCMSSM, mSUGRA to CMS and ATLAS 1l data based on an informed guess of the likelihood function, fitted to the exclusion contours. Validated against one point. CMSSM results very similar to our analysis.
- More recently Akula et al[b] examined ATLAS 0 and 1-lepton analyses at varying A_0, $\tan\beta$ in a scan, showing where the indirect constraints apply.

[a] `arXiv://1102.4585`
[b] `arXiv://1103.1197`

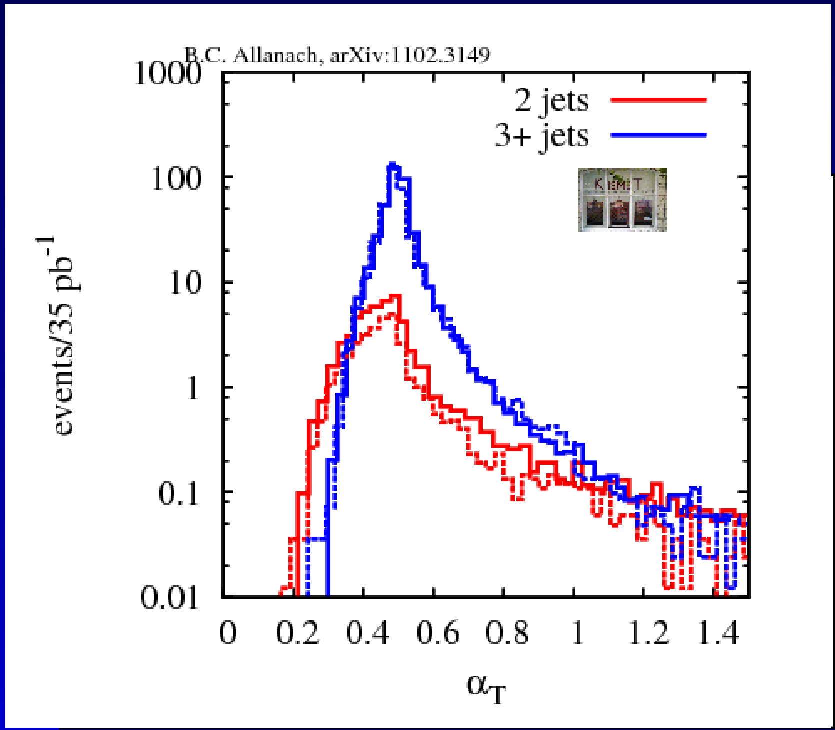
Log Fits



Before (left) and after (right) ATLAS 0-lepton exclusion limits.

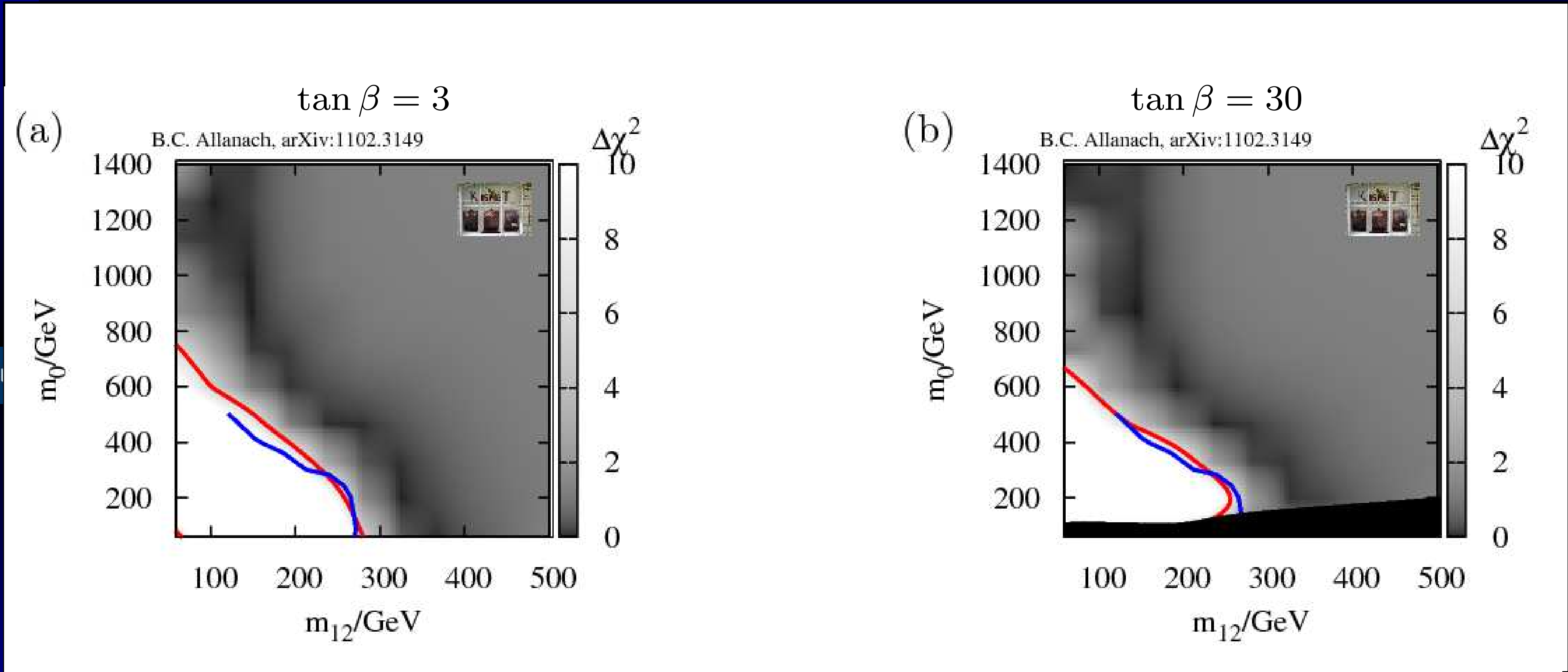
Validation of CMS Analysis

Used `SOFTSUSY3.1.7`, `Herwig++-2.4.2` and `fastjet-2.4.2` to simulate 10000 *signal* events α_T distributions with $H_T > 350$ GeV:



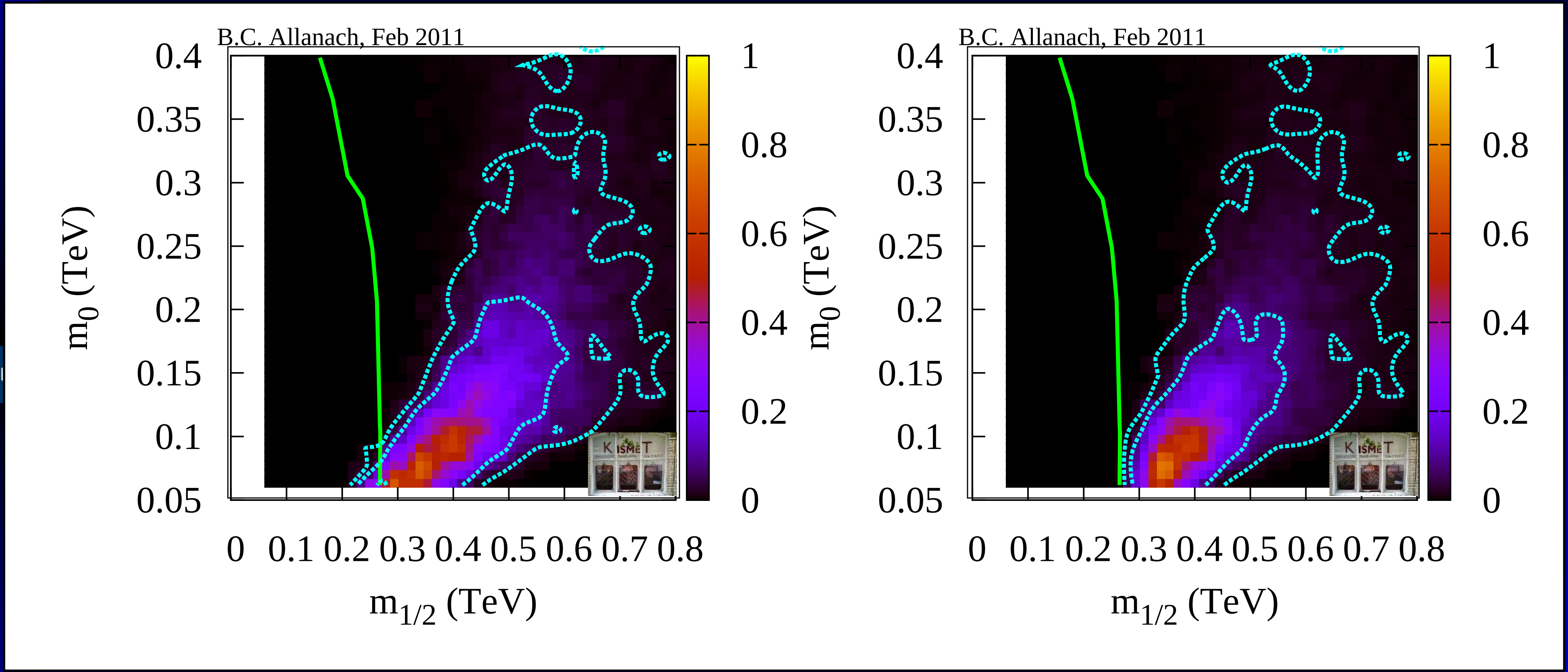
α_T distributions for SUSY point LM0 $m_0 = 200, m_{1/2} = 160, A_0 = -400, \tan\beta = 10$ by my simulation (solid) and CMS' (dashed).

CMS Validation II



$\Delta\chi^2$ approx $\tan\beta$, A_0 independent $\Rightarrow$ interpolate it across m_0 and $m_{1/2}$, then *re-weight fit with* $\Delta\chi^2$.

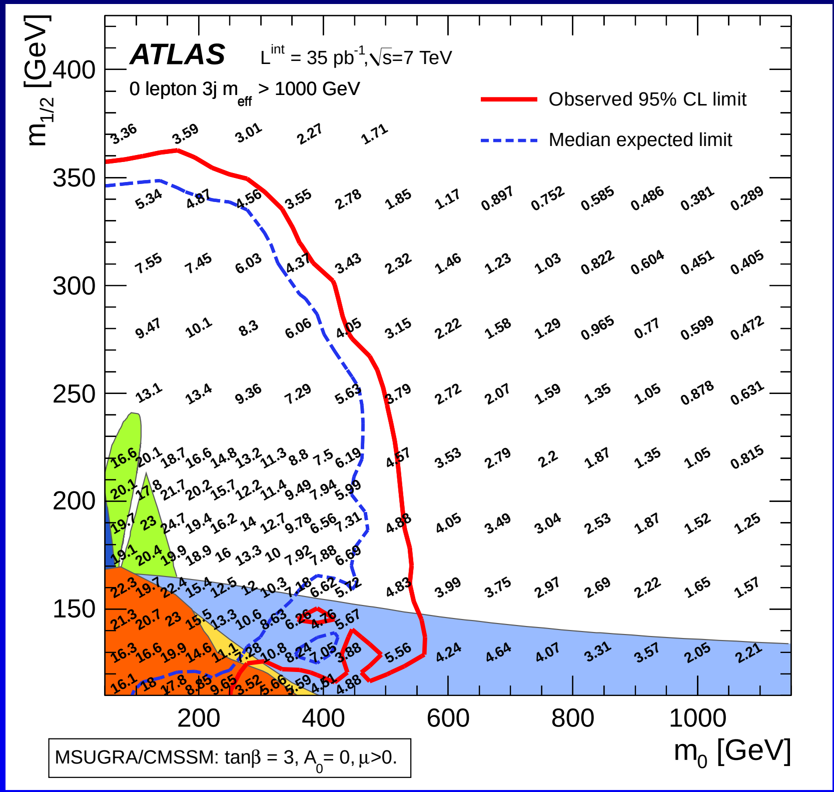
CMS Weighted Fits



Validation of ATLAS Analysis

Information $\vec{\Sigma}^{(i)} = (n_s^{(i)}, n_b^{(i)}, \sigma_s^{(i)}, \sigma_b^{(i)})$, expected number of events past cuts

$$\lambda(\vec{\Sigma}^{(i)}, \delta_s, \delta_b) = n_s^{(i)}(1 + \delta_s \cdot \sigma_s^{(i)}) + n_b^{(i)}(1 + \delta_b \cdot \sigma_b^{(i)}),$$

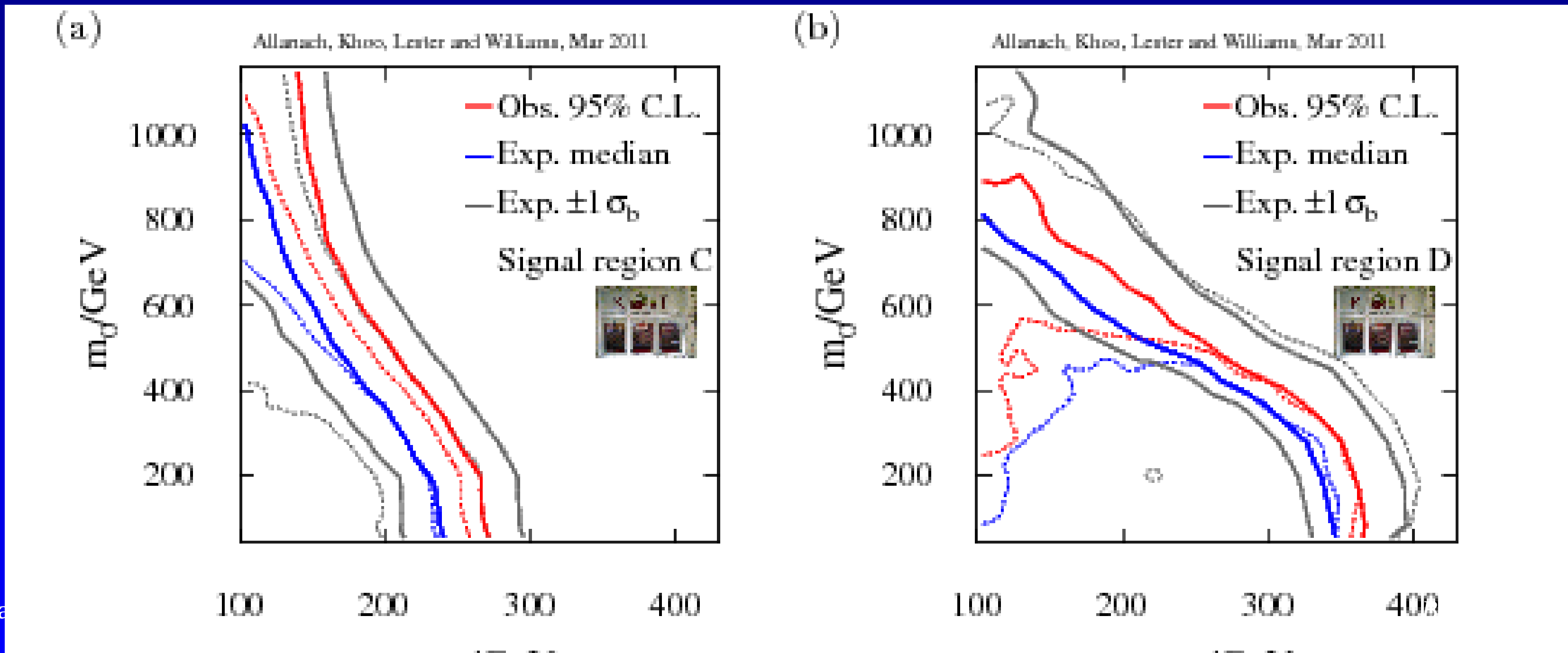


Validation of ATLAS Analysis II

For Poisson distribution p,

$$P_{sys}(n_o^{(i)}, \delta_s, \delta_b | \vec{\Sigma}^{(i)}) = \frac{1}{N^{(i)}} p\left(n_o^{(i)} | \lambda\right) e^{-\frac{1}{2}(\delta_b^2 + \delta_s^2)},$$

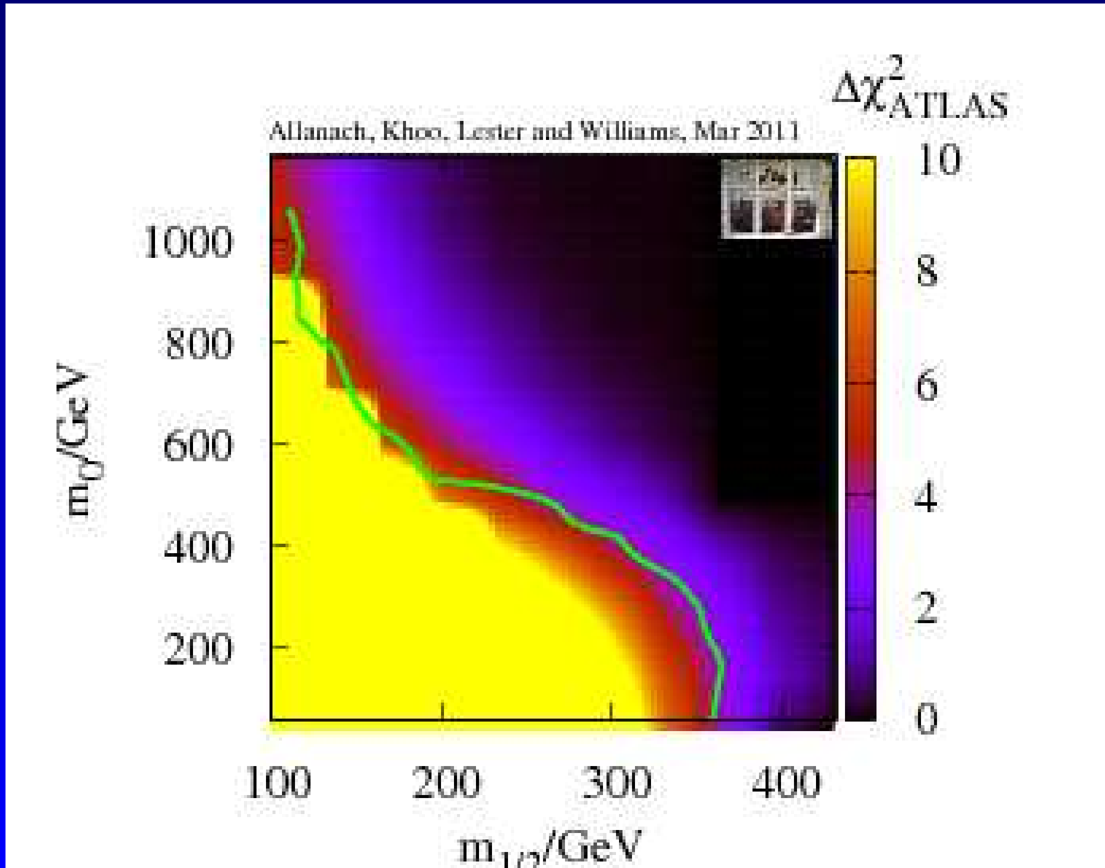
$$P_m(n_o^{(i)} | \vec{\Sigma}^{(i)}) = \int d\delta_s \int d\delta_b \, P_{sys}(n_o^{(i)}, \delta_s, \delta_b).$$



Validation of ATLAS Analysis III

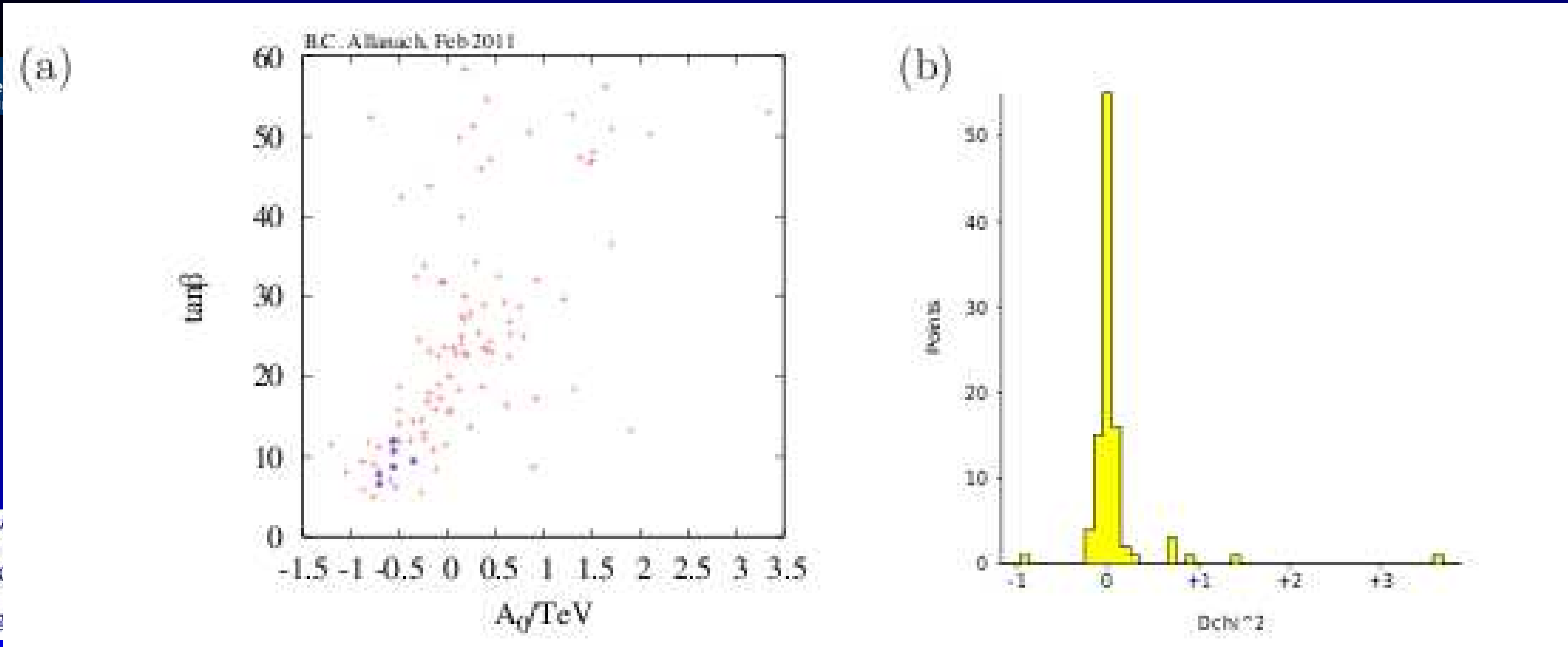
Found $\sigma_C = 0.6$, $\sigma_D = 0.3$ provide a reasonably good fit - by hand. $\vec{n} = (n_o^{(C)}, n_o^{(D)})$, $\vec{\lambda} = (\lambda_C, \lambda_D)$

$$P(\vec{n}|\vec{\lambda}) = p(n_o^{(D)}|\lambda_D)\, p(n_o^{(D)} - n_o^{(C)}|\lambda_C - \lambda_D). \quad (5)$$



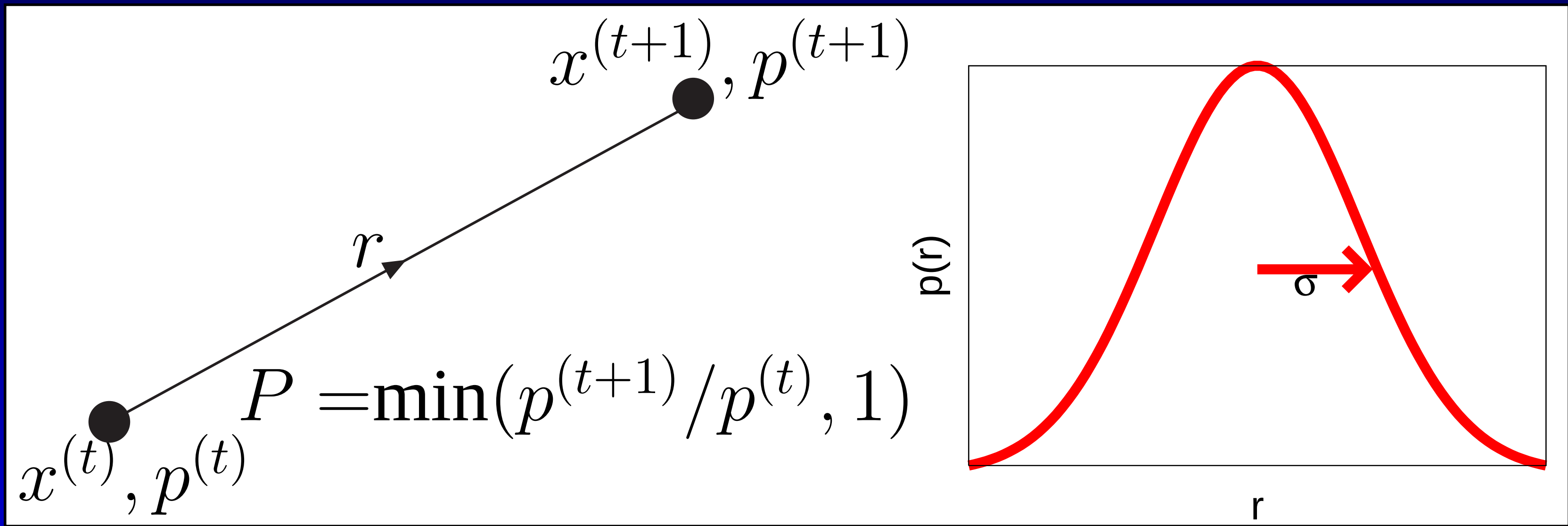
Checking A_0-$\tan\beta$ Independence

Choose samplings from the global fit at random and perform simulation on them. Then compare with $m_0 - m_{1/2}$ interpolation at $\tan\beta = 3$ and find how good the approximation is.



Markov-Chain Monte Carlo

Metropolis-Hastings Markov chain sampling consists of list of parameter points $x^{(t)}$ and associated posterior probabilities $p^{(t)}$.



Final density of x points $\propto p$. Required number of points goes *linearly* with number of dimensions.

Ice Cube

Neutralinos can become trapped in the sun $\tilde{h}^0 - Z$ coupling $\sigma_{\chi^0 \mathrm{p,SD}} \propto [|N_{1d}|^2 - |N_{1u}|^2]^2$ dominates. $A^{\odot} \equiv \sigma v/V$:

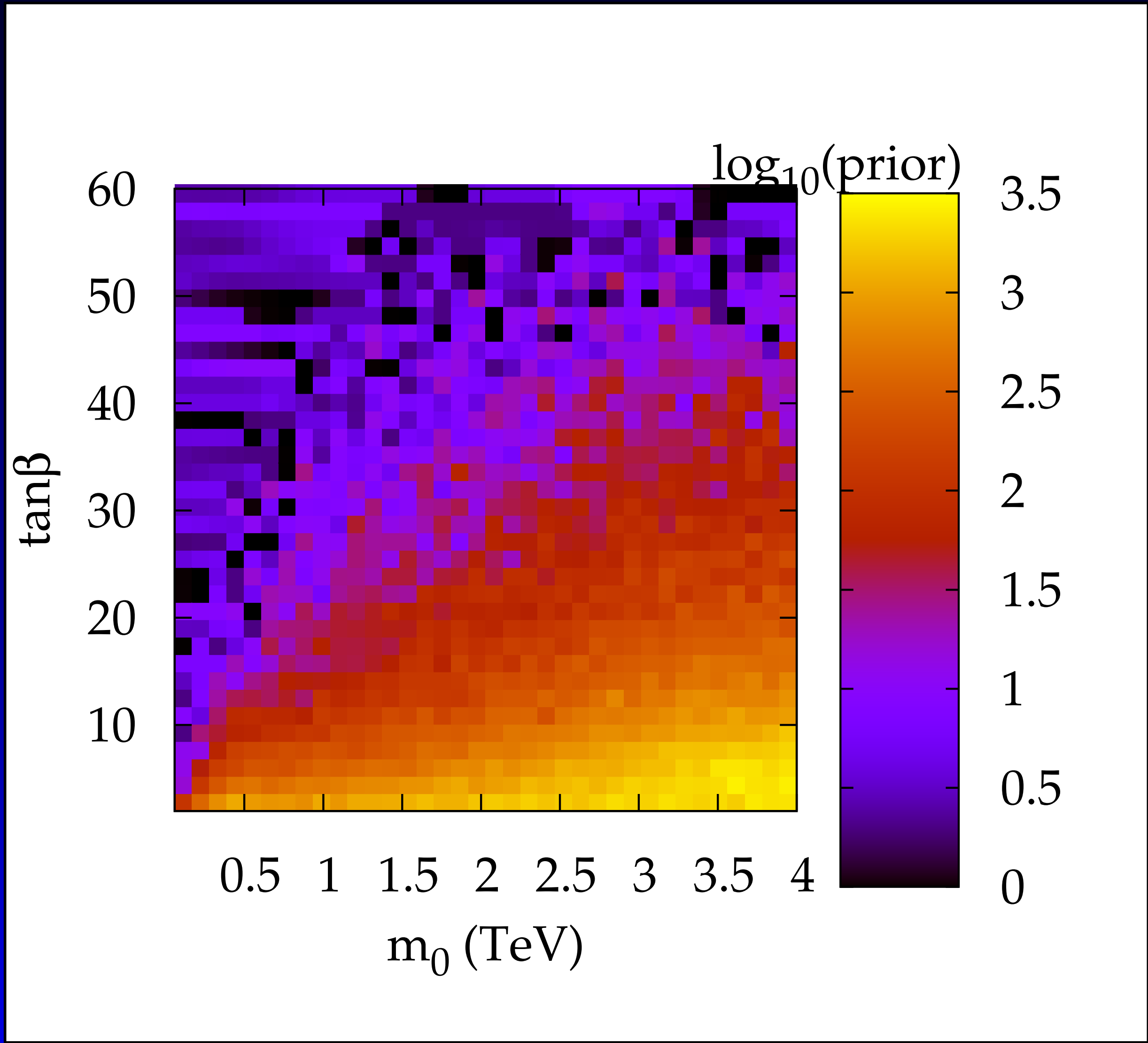
$$\dot{N} = C^{\odot} - A^{\odot} N^2,$$

$$\Gamma = \frac{1}{2} A^{\odot} N^2 = \frac{1}{2} C^{\odot} \tanh^2 \left(\sqrt{C^{\odot} A^{\odot}}\, t_{\odot} \right)$$

$$\frac{dN_{\nu_\mu}}{dE_{\nu_\mu}} = \frac{C_{\odot} F_{\mathrm{Eq}}}{4\pi D_{\mathrm{ES}}^2} \left(\frac{dN_\nu}{dE_\nu} \right)^{\mathrm{Inj}}$$

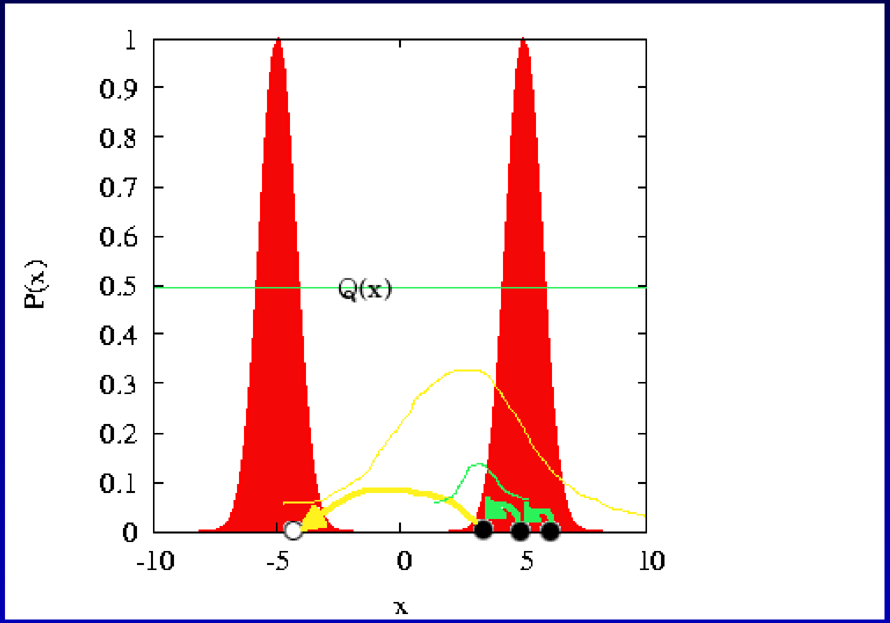
$$N_{\mathrm{ev}} \approx \int \int \frac{dN_{\nu_\mu}}{dE_{\nu_\mu}} \frac{d\sigma_\nu}{dy} R_\mu((1-y)\, E_\nu)\, A_{\mathrm{eff}}\, dE_{\nu_\mu}\, dy$$

Naturalness priors



Potential Problem

Often, people use a flat $Q(x)$. The trouble with this *"blind drunk"* sampling is the following situation:



Either large or small proposal widths σ lead to low efficiencies of sampling. Our proposal is to determine a $Q(x)$ closer to $P(x)$ *semi-automatically*.

Bank Sampling

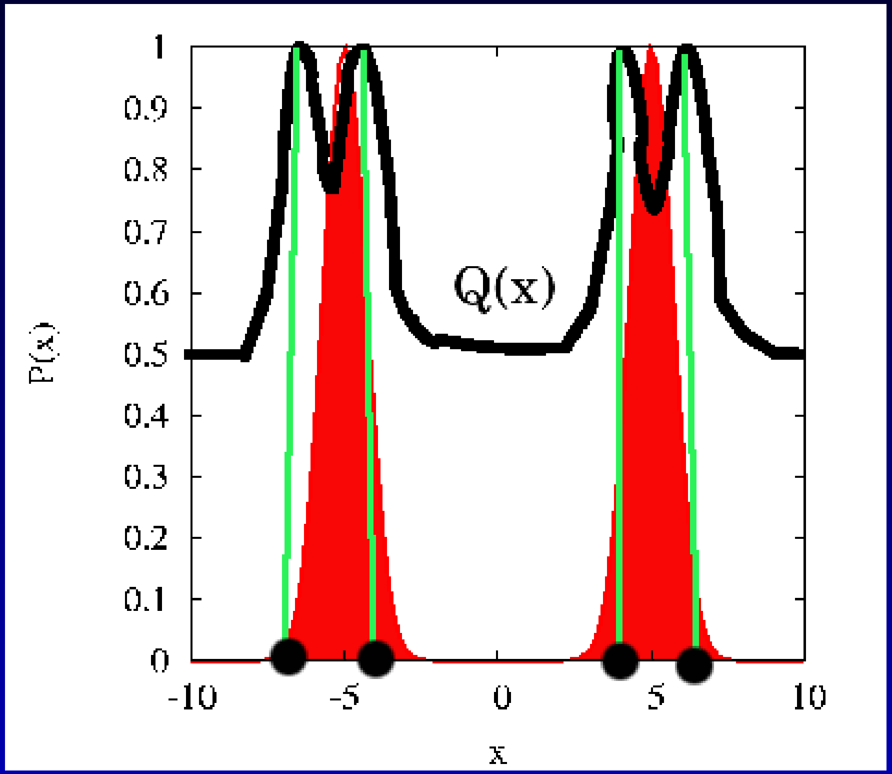


Figure 2: Bank points determined from previous runs: want to have at least one point in each maximum. *Knowledgeable drunk*

Proposal Distribution

$$Q_{bank}(\mathbf{x};\mathbf{x}^{(t)}) = (1-\lambda)K(\mathbf{x};\mathbf{x}^{(t)}) + \lambda \sum_{i=1}^{N} w_i K(\mathbf{x};\mathbf{y}^{(i)})$$

w_i are a set of N weights: $\sum_{i=1}^{N} w_i = 1$, $0 < \lambda < 1$, while K is the proposal distribution.

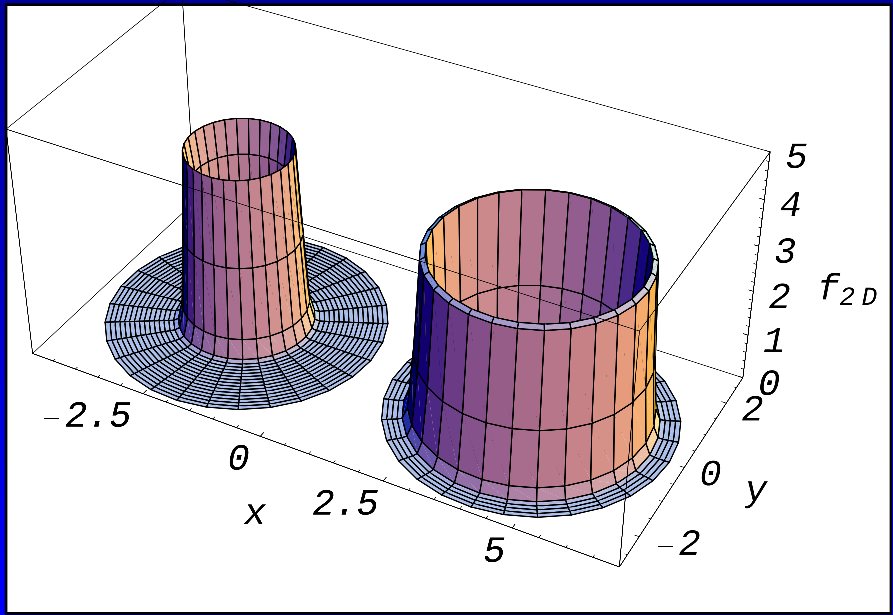
With probability $(1-\lambda)$ propose a local Metropolis step of the usual kind, i.e. "close" to the last point in the chain. With probability λ, teleport to the vicinity of one of the number of "banked" points, chosen with weight w_i from within the bank.

Example Distribution

$$f_{2D}(\mathbf{x}) = \mathrm{circ}(\mathbf{x}; c_1, r_1, w_1) + \mathrm{circ}(\mathbf{x}; c_2, r_2, w_2)$$

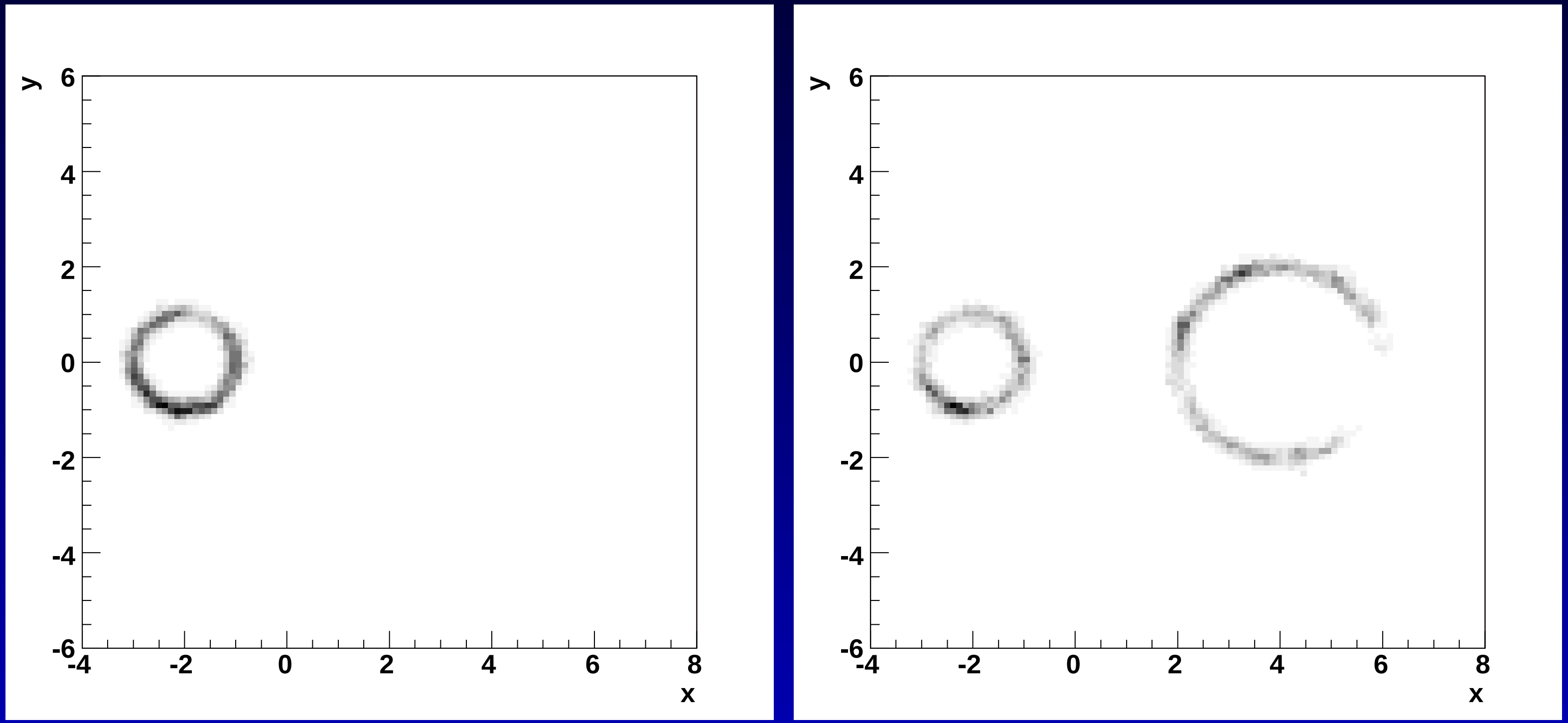
where $c_1 = (-2, 0)$, $r_1 = 1$, $w_1 = 0.1$, $c_2(+4, 0)$, $r_2 = 2$, $w_2 = 0.1$ and

$$\mathrm{circ}(\mathbf{x}; \mathbf{c}, r, w) = \frac{1}{\sqrt{2\pi w^2}} \exp\left[-\frac{(|\mathbf{x} - \mathbf{c}| - r)^2}{2w^2} \right].$$



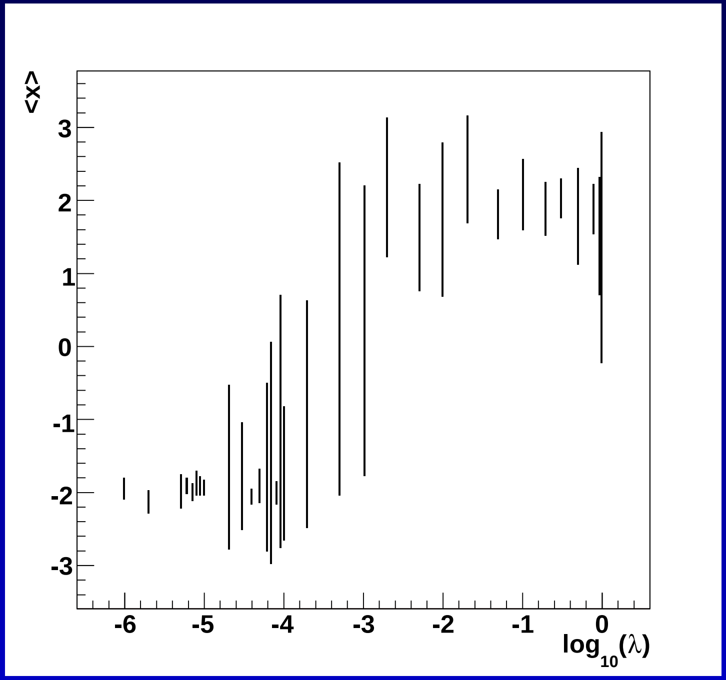
Bank vs Metropolis

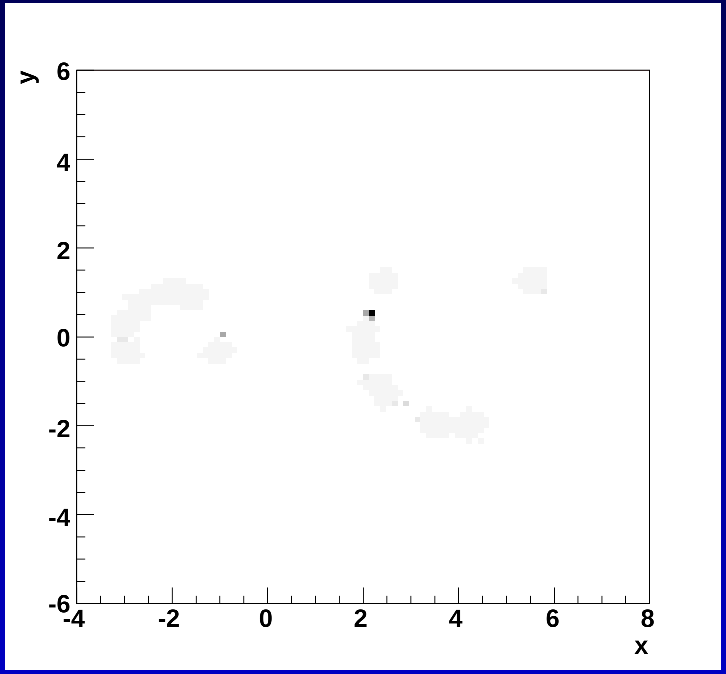
10 000 samples for MCMC and bank sampling:



Safety with respect to λ

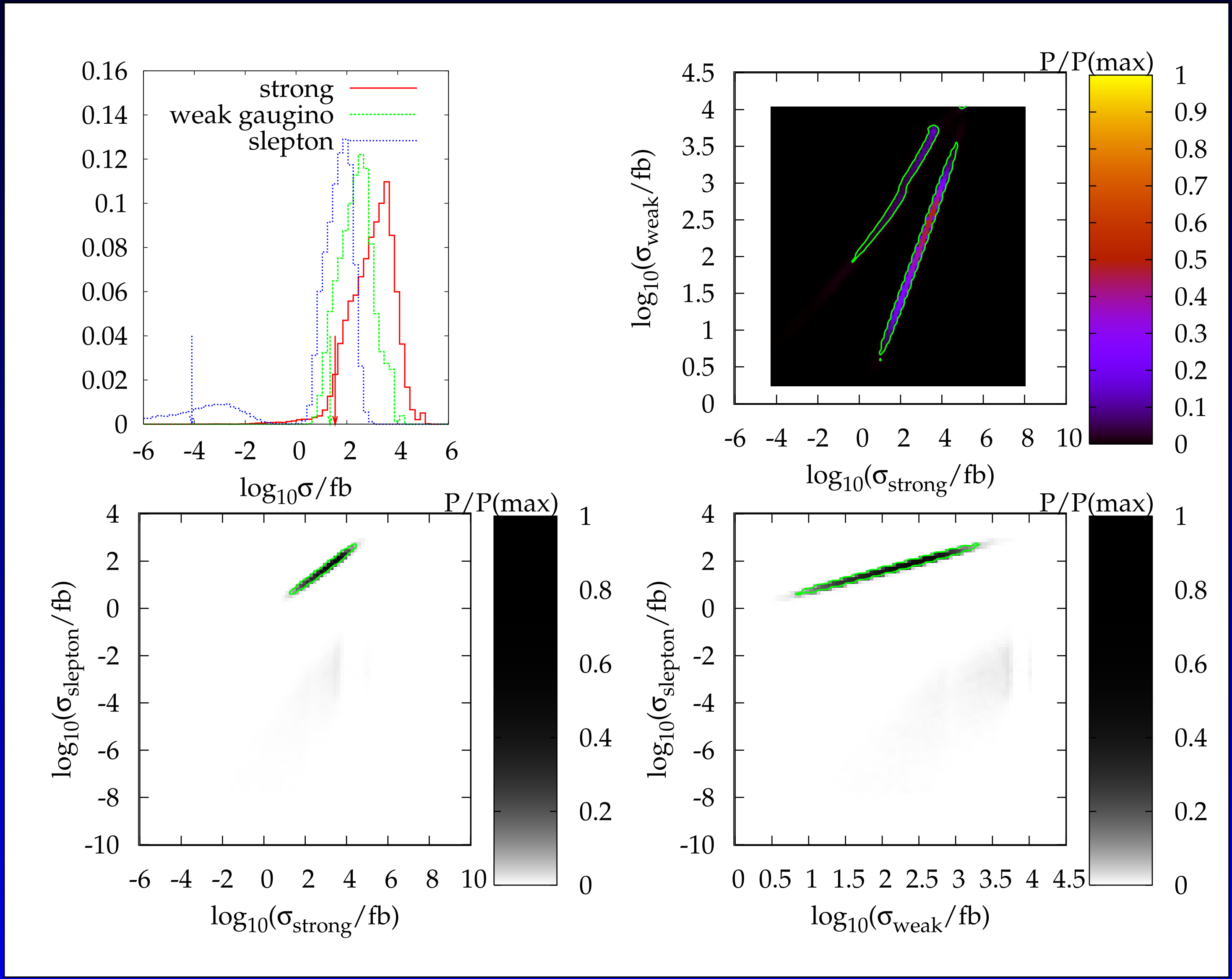
10 bank samplers, with 10 bank points generated in each circle: 10 000 samples. All started from $x = -2$. Correct $\langle x \rangle = 2$. $\lambda \approx 1$ is importance sampling limit.





$\mathcal{Q}$: What values of λ are "safe"?
$\mathcal{A}$: [0.001, 0.9]

LHC Cross-sections



Collider Check

Need corroboration with *direct detection*.

If we can pin particle physics down, a comparison between the predicted relic density and that observed is a test of the cosmological assumptions used in the prediction.[a]

Thus, if it doesn't fit, you change the cosmology until it does.

---

[a] BCA, G. Belanger, F. Boudjema, A. Pukhov, JHEP 0412 (2004) 020.; M. Nojiri, D. Tovey, JHEP 0603 (2006) 063

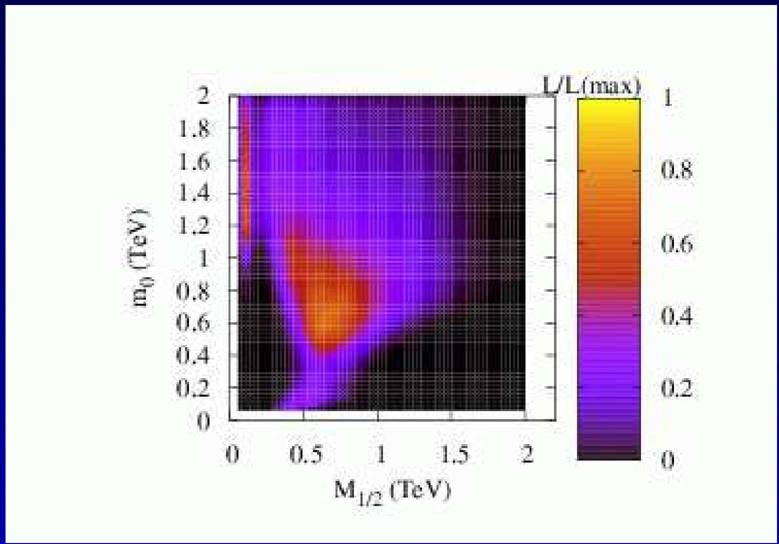
CMSSM Regions

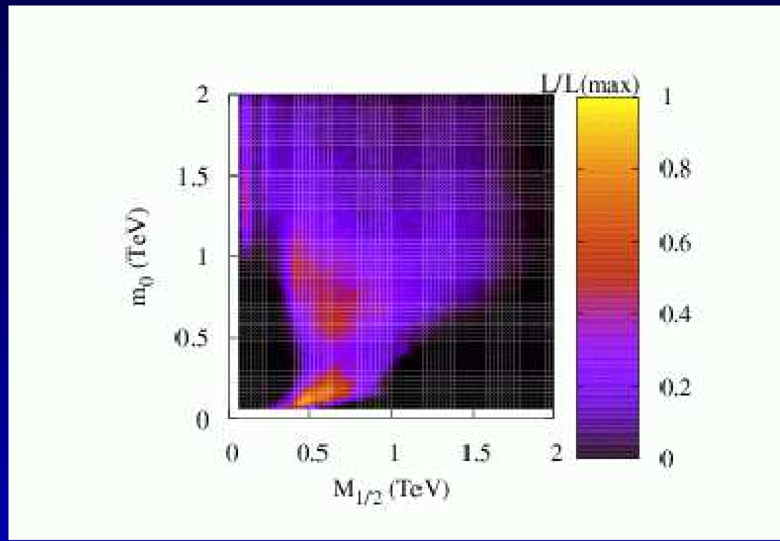
After WMAP+LEP2, bulk region diminished. Need specific mechanism to reduce overabundance:

- $\tilde{\tau}$ coannihilation: small m_0, $m_{\tilde{\tau}_1} \approx m_{\chi_1^0}$. Boltzmann factor $\exp(-\Delta M/T_f)$ controls ratio of species. $\tilde{\tau}_1 \chi_1^0 \to \tau\gamma$, $\tilde{\tau}_1 \tilde{\tau}_1 \to \tau\bar{\tau}$.
- Higgs Funnel: $\chi_1^0 \chi_1^0 \to A \to b\bar{b}/\tau\bar{\tau}$ at large $\tan\beta$. Also via[a] h at large m_0 small $M_{1/2}$.
- Focus region: Higgsino LSP at large m_0: $\chi_1^0 \chi_1^0 \to WW/ZZ/Zh/t\bar{t}$.
- $\tilde{t}$ coannihilation: high $-A_0$, $m_{\tilde{t}_1} \approx m_{\chi_1^0}$. $\tilde{t}_1 \chi_1^0 \to gt$, $\tilde{t}\tilde{t} \to tt$

[a] Datta, Djouadi, Drees, `hep-ph/0504090`

Comparison



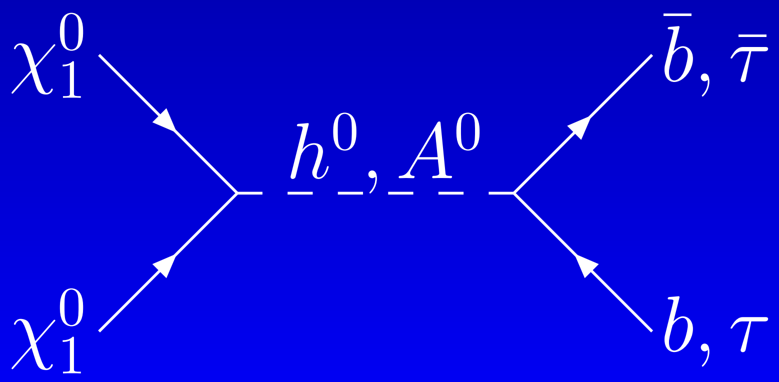


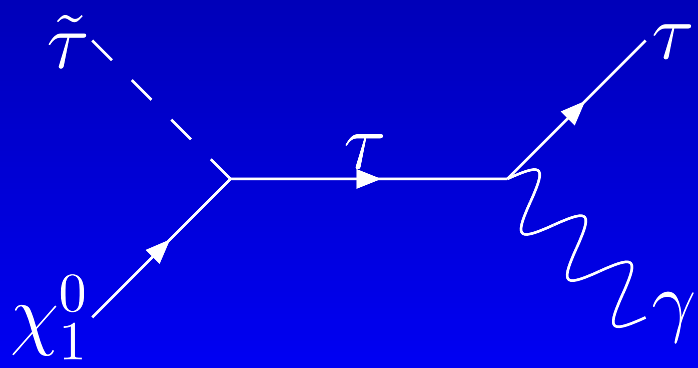
- LHS: allowing non thermal-χ_1^0 contribution
- RHS: only χ_1^0 dark matter
- *(flat priors)*

Annihilation Mechanism

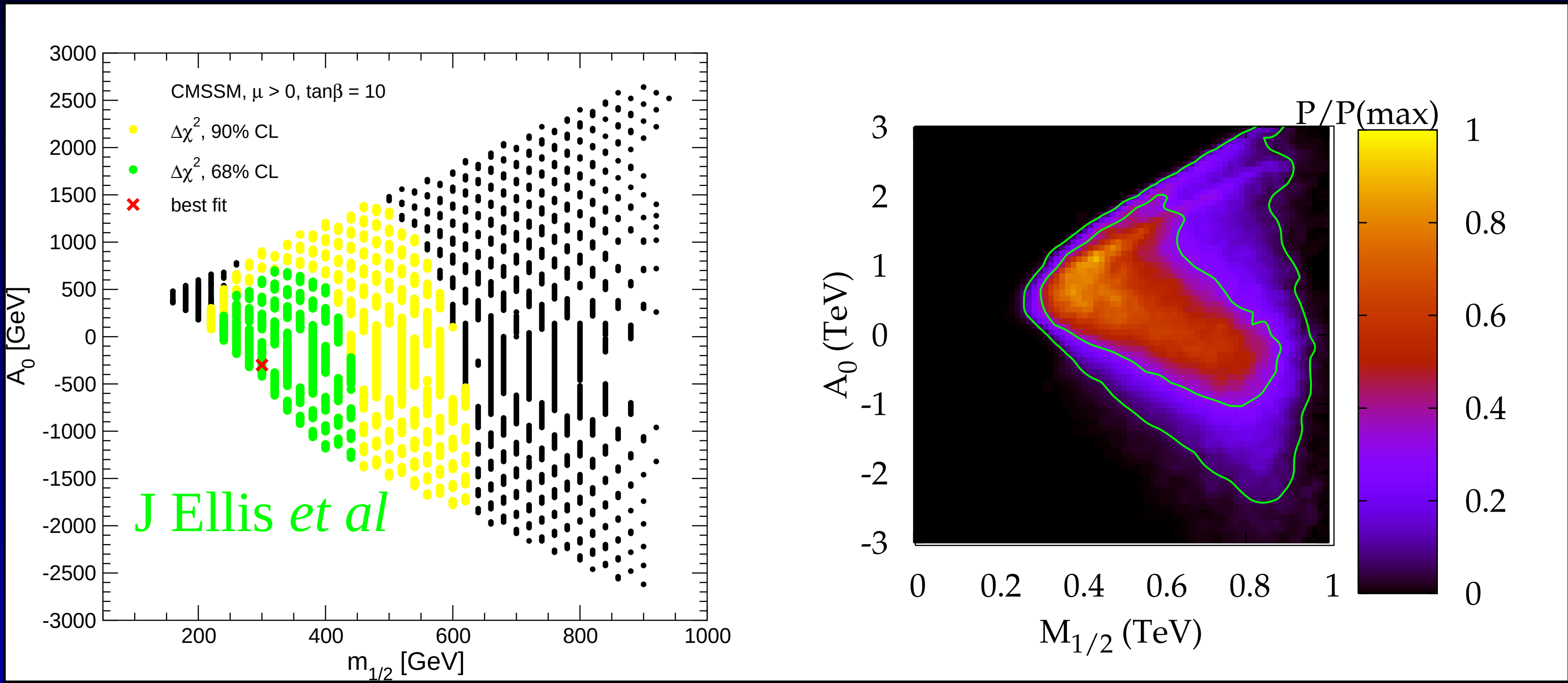
Define stau co-annihilation when $m_{\tilde{\tau}}$ is within 10% of $m_{\chi_1^0}$ and Higgs pole when $m_{h,A}$ is within 10% of $2m_{\chi_1^0}$.

mechanism	flat prior	natural prior
h^0-pole	0.025	0.07
A^0-pole	0.41	0.14
$\tilde{\tau}-$co-annihilation	0.26	0.18
rest	0.31	0.61



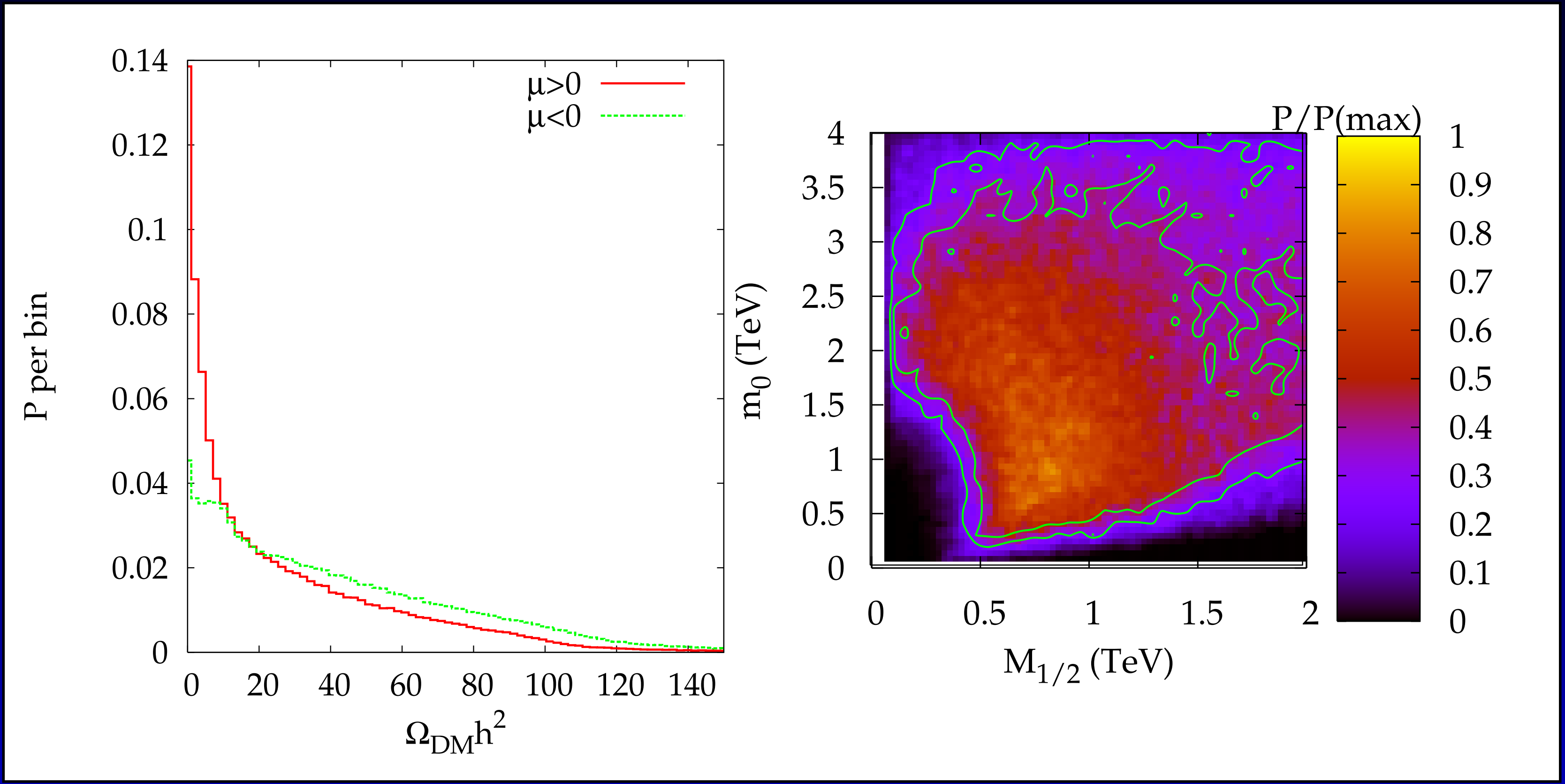


Comparison



- Fix $\tan\beta = 10$ and all SM inputs
- Restrict $m_0, M_{1/2} < 1$ TeV.
- *Same* fits!

No Dark Matter Fits



Huge χ^2 from the dark matter relic density.

Volume Effects

Can't rely on a good χ^2 in non-Gaussian situation

Likelihood and Posterior

Q: What's the chance of observing someone to be pregnant, given that they are female?

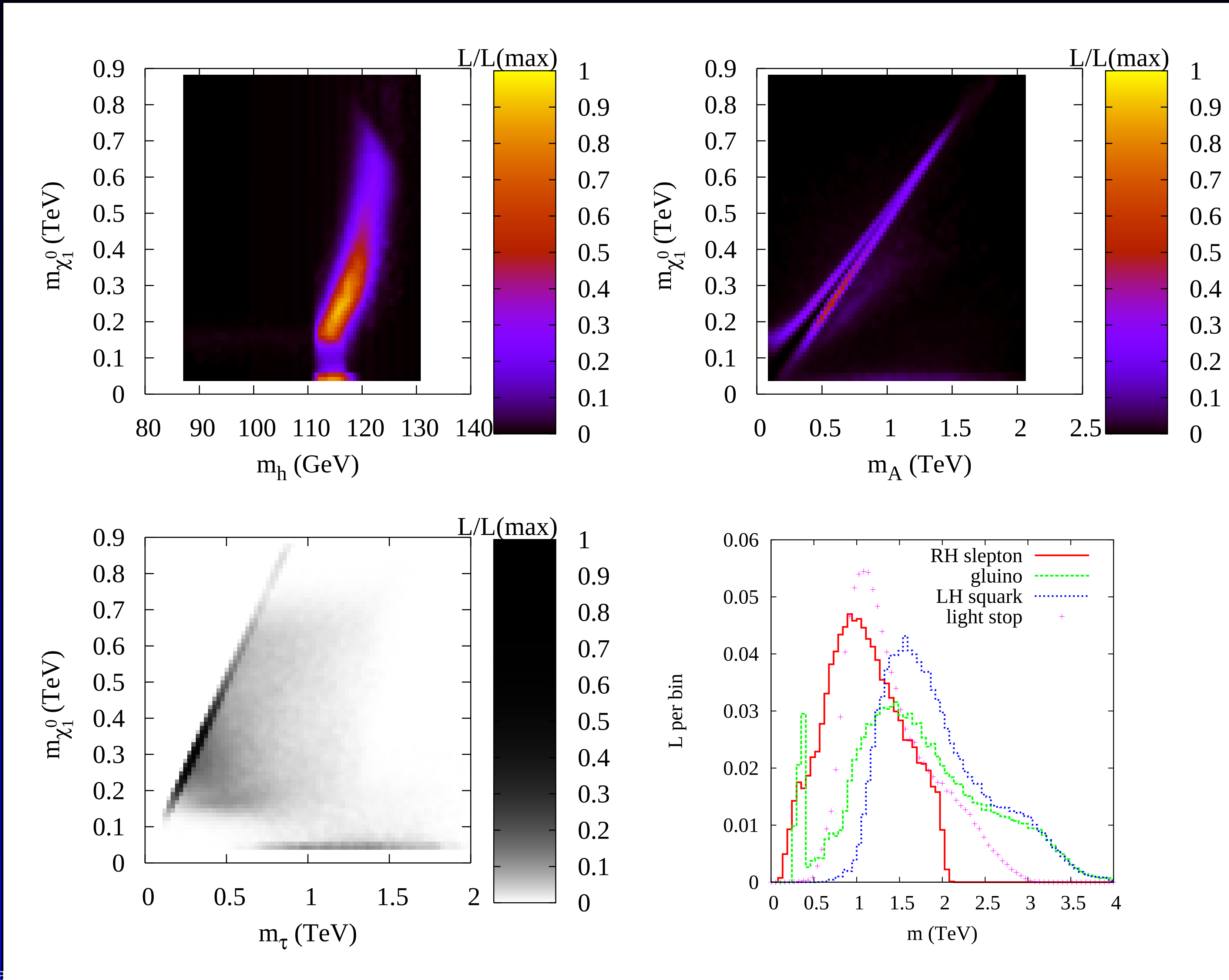
Likelihood

p(pregnant | female, human) = 0.01

Posterior

p(female | pregnant, human) = 1.00

Sanity Check



LHC vs LC in SUSY Measurement

- LHC (start date 2007) produces strongly interacting particles up to a few TeV. Precision measurements of mass *differences* possible if the decay chains exist: possibly per mille for leptons, several percent for jets.
- ILC has several energy options: 500-1000 GeV, CLIC up to 3 TeV. Linear colliders produce less strong particles but much easier to make precision measurements of masses/couplings.

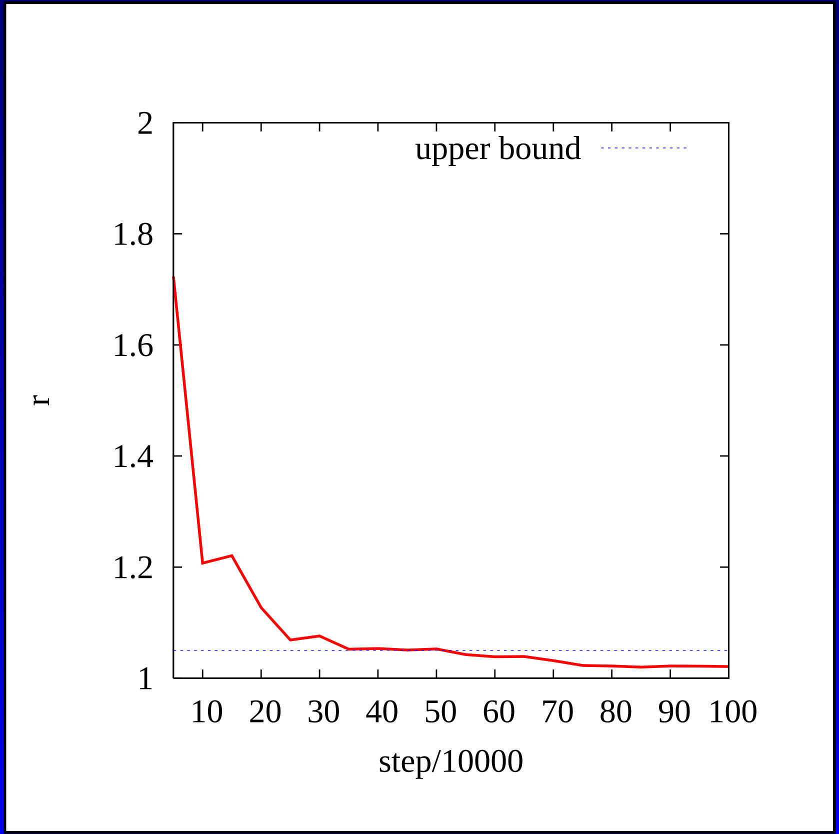
$\mathcal{Q}$: What energy for LC?

$\mathcal{Q}$: What do we get from LHC[a]?

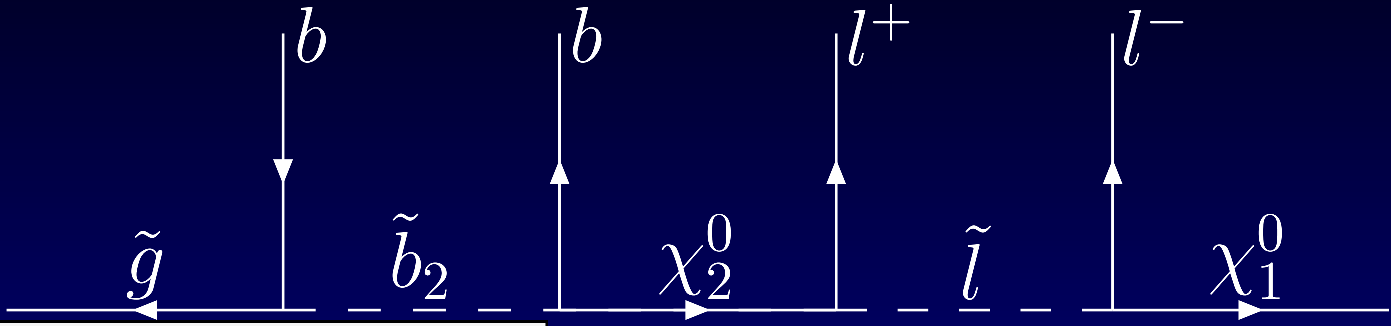
[a] LHC/ILC Working Group Report: `hep-ph/0410364`

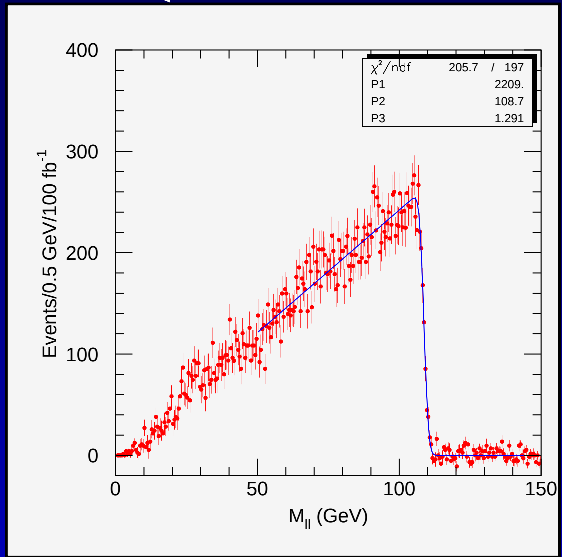
Convergence

We run $9 \times 1\,000\,000$ points. By comparing the 9 independent chains with random starting points, we can provide a statistical measure of convergence: an upper bound r on the excepted variance decrease for infinite statistics.



LHC SUSY Measurements





$$m_{ll}^2 = \frac{(m^2_{\chi^0_2} - m^2_{\tilde{l}})(m^2_{\tilde{l}} - m^2_{\chi^0_1})}{m^2_{\tilde{l}}}$$

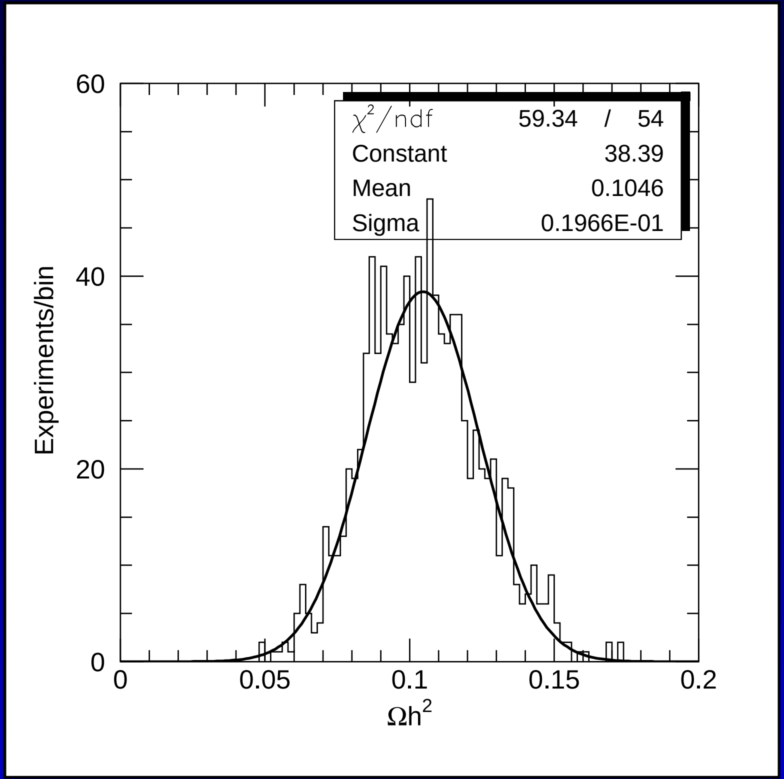
Q: Can we measure enough of these to pin SUSY[a] down?

---

[a]BCA, Lester, Parker, Webber, JHEP 0009 (2000) 004

Predicting Ωh^2

Not much left that's allowed but edge measurements allow reasonable Ωh^2 error[a] for 300 fb^{-1}.



$\mathcal{Q}$: What about other bits of parameter space?

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[a] M Nojiri, G Polesello, D Tovey, JHEP 0603 (2006) 063, `hep-ph/0512204`.

Bulk Region

M Nojiri, G Polesello, D Tovey, JHEP 0603 (2006) 063, `hep-ph/0512204`. for 300 fb^{-1}. SPA point $m_0 = 70$ GeV, $m_{1/2} = 250$ GeV, $A_0 = -300$ GeV, $\tan\beta = 10$, $\mu > 0$: $\Omega h^2 = 0.108$. Put in m_{ll}^{max}, m_{llq}^{max}, m_{lq}^{low}, m_{lq}^{high}, m_{llq}^{min}, $m_{l_L} - m_{\chi_1^0}$, $m_{ll}^{max}(\chi_4^0)$, $m_{\tau\tau}^{max}$, m_h.

$\tilde\chi_1^0\tilde\chi_1^0 \to \ell^+\ell^-$	40%
$\tilde\chi_1^0\tilde\chi_1^0 \to \tau^+\tau^-$	28%
$\tilde\chi_1^0\tilde\chi_1^0 \to \nu\bar\nu$	3%
$\tilde\chi_1^0\tilde\tau_1 \to Z\tau$	4%
$\tilde\chi_1^0\tilde\tau_1 \to A\tau$	18%
$\tilde\tau_1\tilde\tau_1 \to \tau\tau$	2%

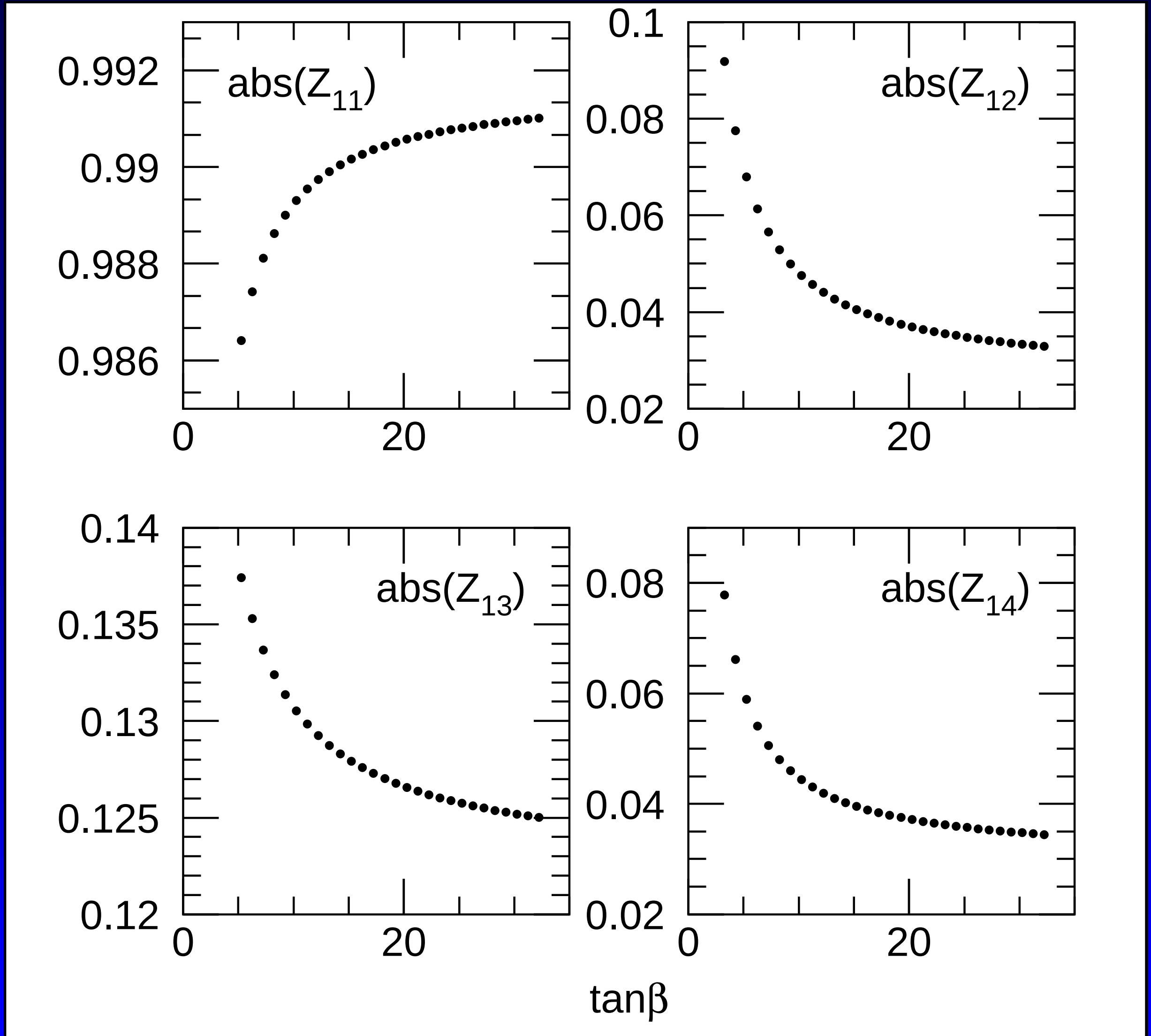
Neutralino mass matrix

Neutralino masses measured: $\chi^0_{1,2,4}$ but need mixing matrix to determine couplings. Left with $\tan\beta$.

$$\begin{bmatrix} M_1 & 0 & -m_Z c_\beta s_W & m_Z s_\beta s_W \\ 0 & M_2 & m_Z c_\beta c_W & -m_Z s_\beta c_W \\ -m_Z c_\beta s_W & m_Z c_\beta c_W & 0 & -\mu \\ m_Z s_\beta s_W & -m_Z s_\beta c_W & -\mu & 0 \end{bmatrix} \quad (6)$$

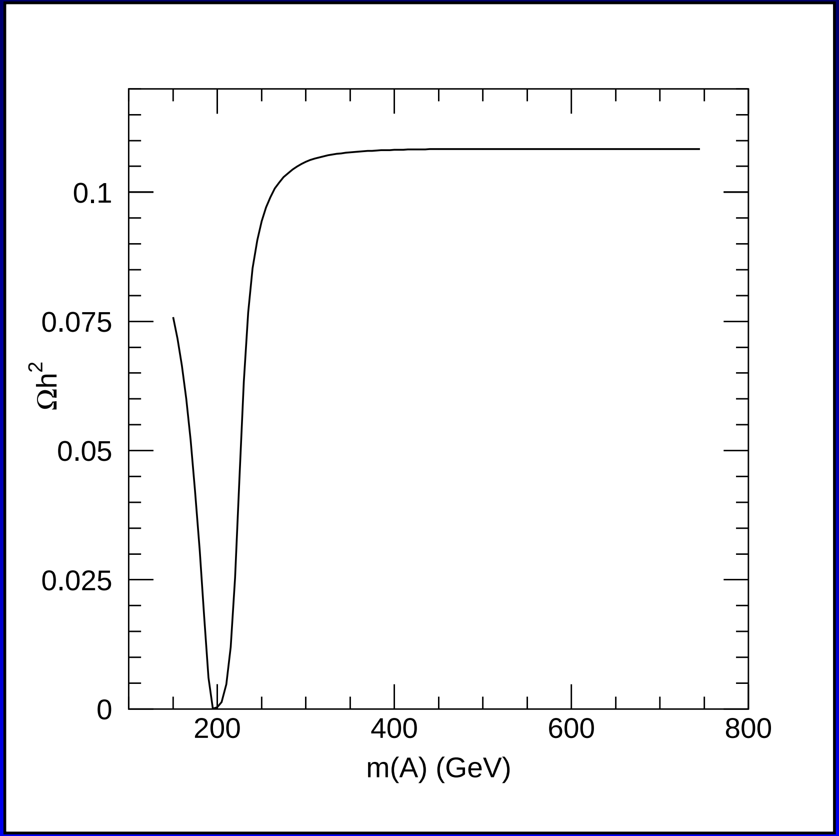
Neutralino mass matrix

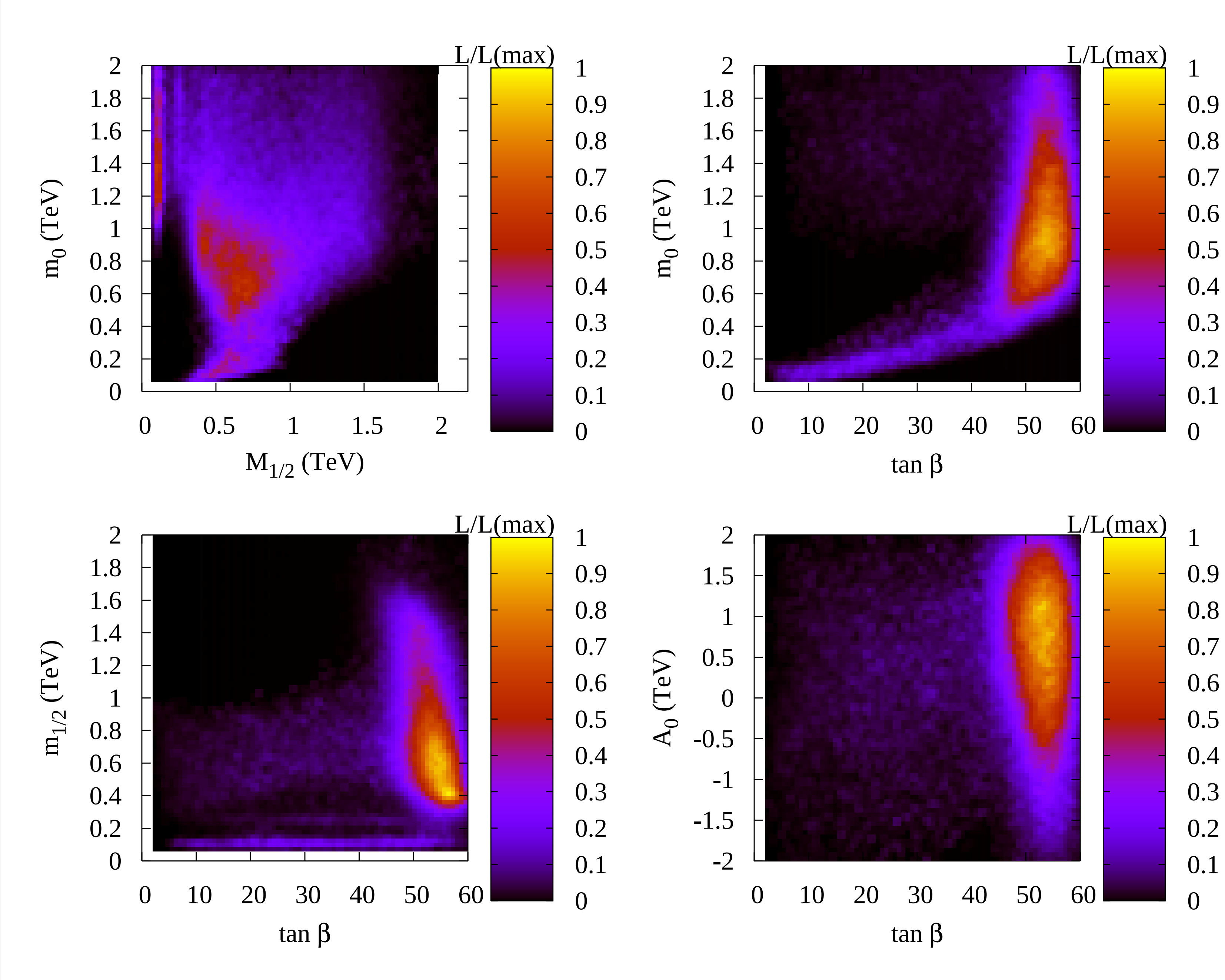
Neutralino masses measured: $\chi^0_{1,2,4}$ but need mixing matrix to determine couplings. Left with $\tan\beta$.



Slepton/A^0 Higgs

$\Gamma(\chi_2^0 \to \tilde{l}_R l)/\Gamma(\chi_2^0 \to \tilde{\tau}_1 \tau)$ then helps determine θ_τ for a given $\tan\beta$. Exclusion of A^0 helps you to exclude A^0 appearing in cascade decays. Meaurement of m_h provides constraints in $m_A - \tan\beta$ plane.




L/L(max)
m_0 (TeV)
M_1/2 (TeV)
tan β
m_1/2 (TeV)
A_0 (TeV)

v

Uncertainties in Relic Density

Bulk region: $\tilde{B}\tilde{B} \to Z, h \to l\bar{l}$. Coannihilation: $\tilde{\tau}\chi_1^0 \to \tau + X$

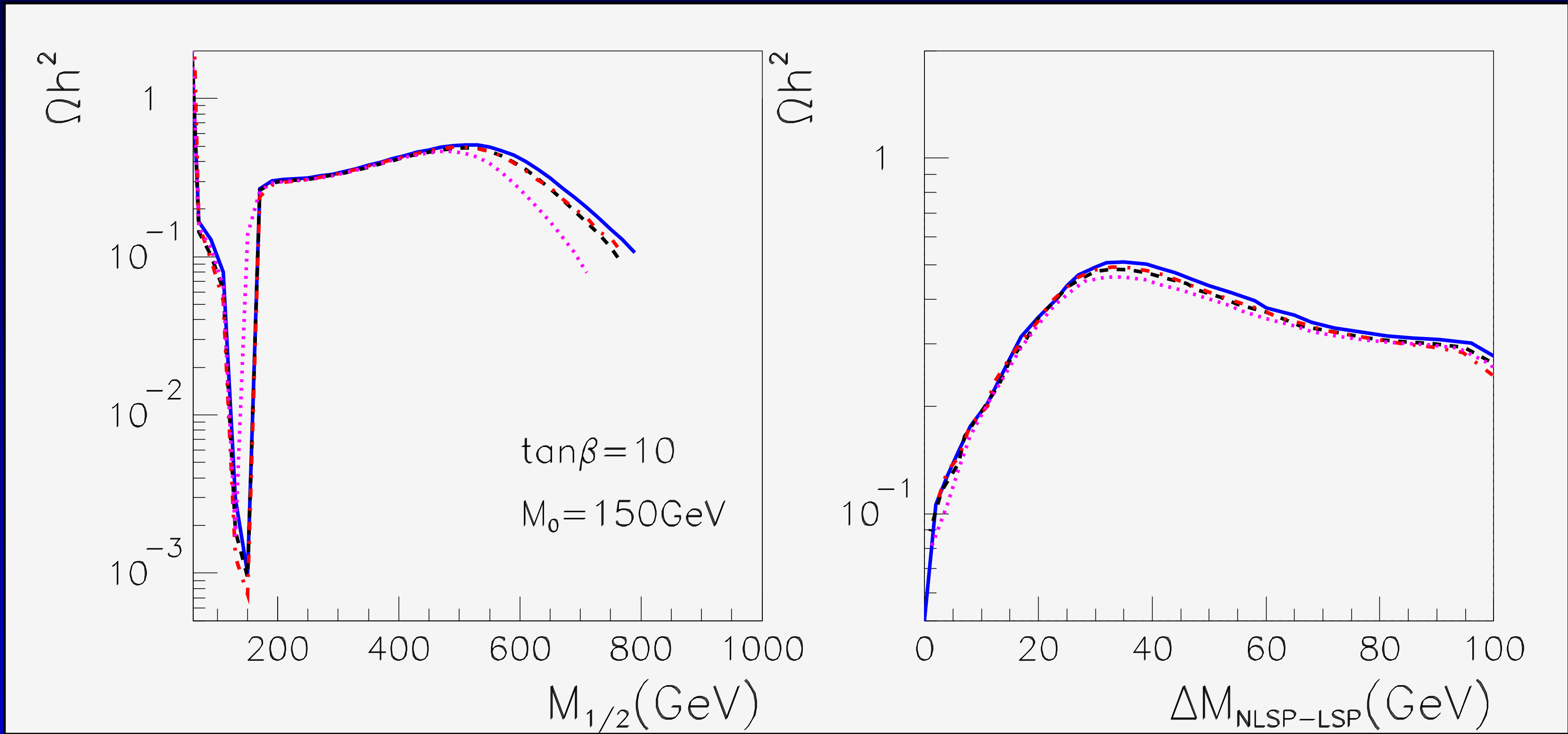


Figure 4: Bulk/coannihilation region. Full: `SoftSusy`, dotted: `SPheno`.

Focus Point

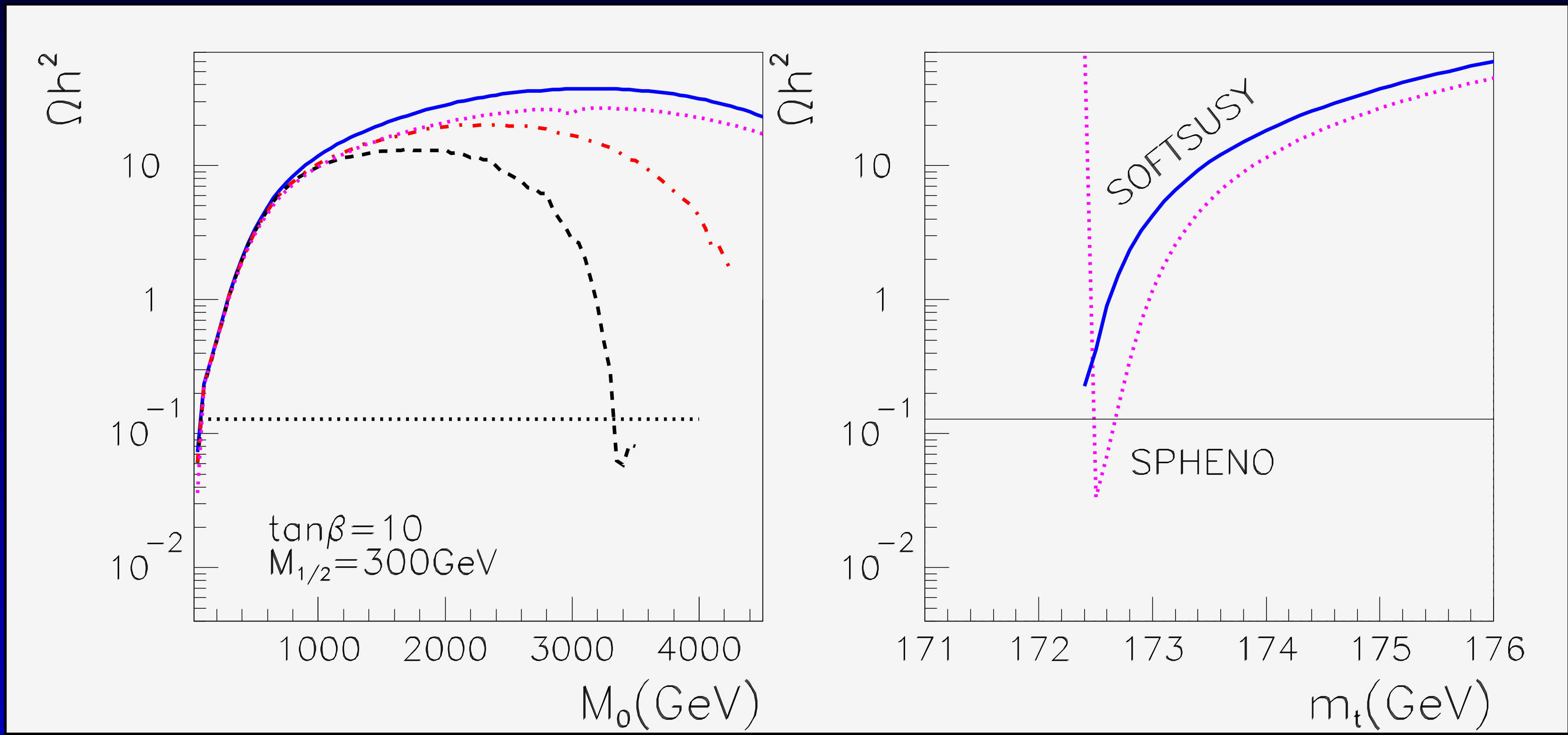


Figure 5: Focus point region. Full: `SoftSusy`, dotted: `SPheno`, dashed: `SuSpect`. Higgsino LSP annihilates into ZZ/WW

High $\tan\beta$

BCA, Belanger, Boudjema, Pukhov, Porod, hep-ph/0402161. Baer *et al*

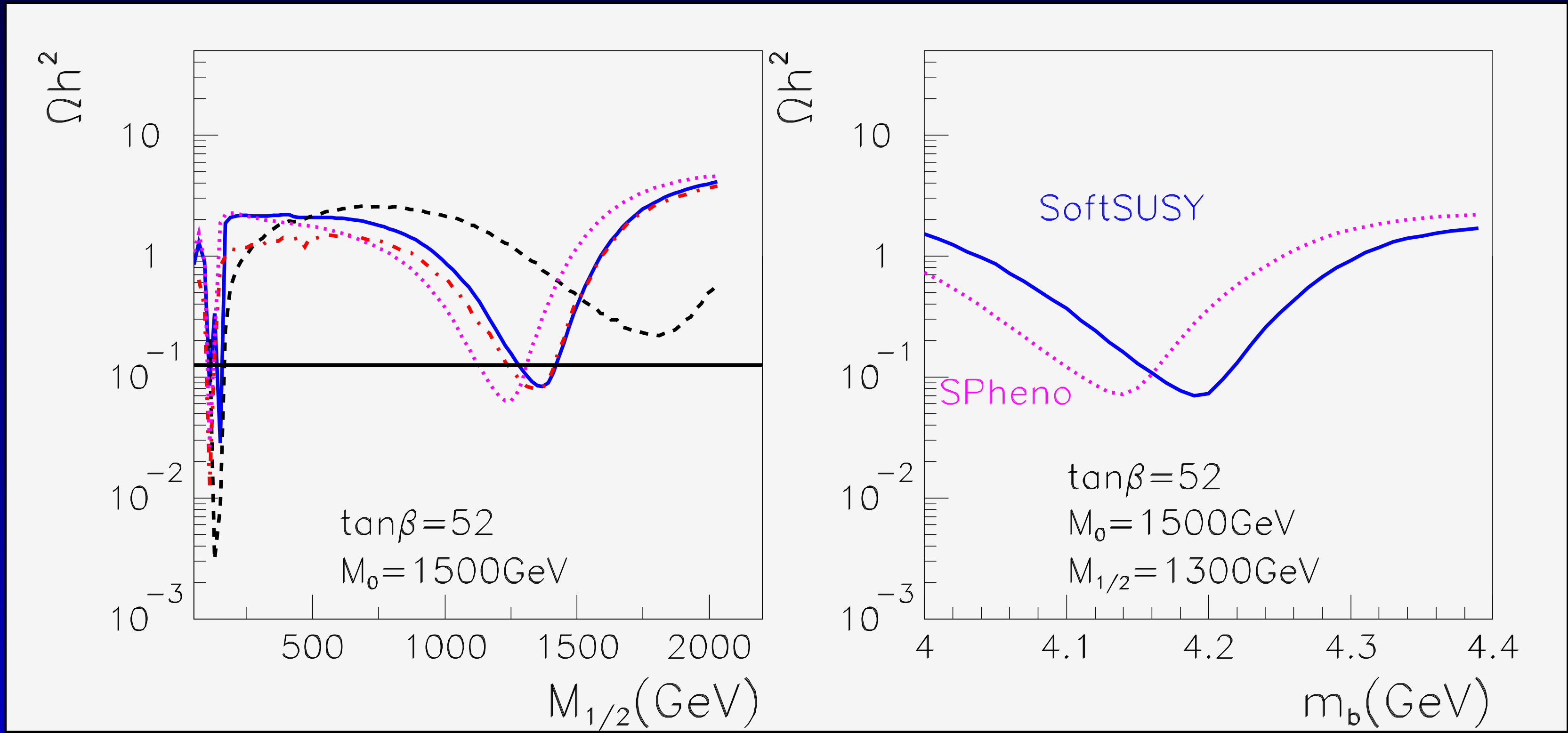
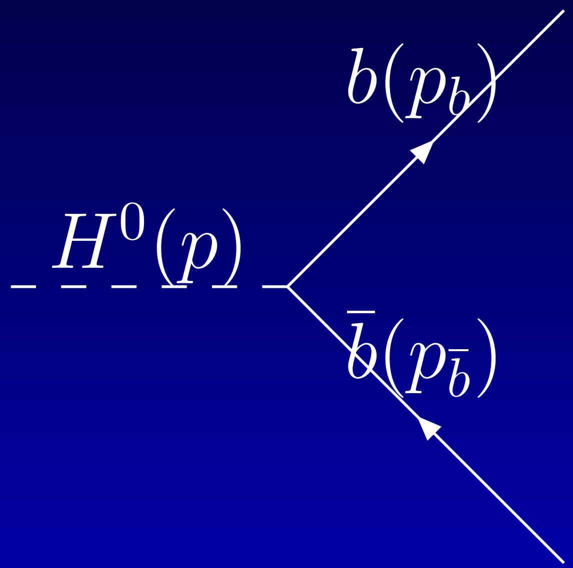


Figure 6: High $\tan\beta$ region. Full: `SoftSusy`, dotted: `SPheno`, dashed: `SuSpect`. Get annihilation into A.

SUSY Kinematics: a Reminder

Take a particle decaying into 2 particles, eg $H^0 \to b\bar{b}$.
We define the invariant mass of the $b\bar{b}$ pair such that:



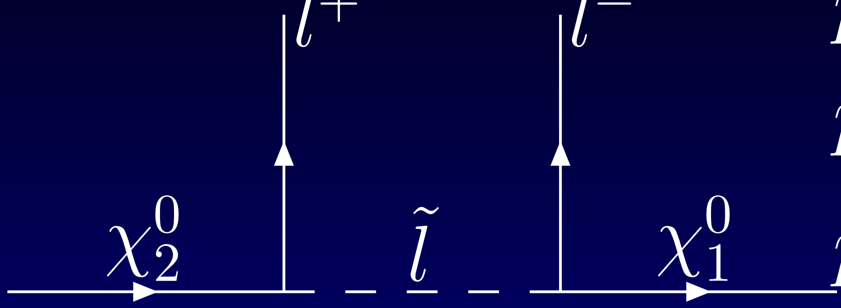
$$p^\mu = (\sqrt{m_H^2 + p^2}, \underline{p}) = p_b^\mu + p_{\bar{b}}^\mu$$

$$\Rightarrow p^2 = m_H^2 = (p_b + p_{\bar{b}})^2$$

Is *invariant* in boosted frames

Question: What happens to invariant mass in SUSY cascade decays, where we miss the final particle?

Cascade Decay



$$p^\mu_{\tilde{l}} = (m_{\tilde{l}}, \underline{0})$$
$$p^\mu_{l\pm} = (|\underline{p}_{l\pm}|, \underline{p}_{l\pm})$$
$$p^\mu_{\chi^0_{1,2}} = \left(\sqrt{{m_{\chi^0_{1,2}}}^2 + |\underline{p}_{\chi^0_{1,2}}|^2}, \underline{p}_{\chi^0_{1,2}}\right)$$

The invariant mass of the l^+l^- pair is

$$m^2_{ll} = (p_{l^+} + p_{l^-})^\mu (p_{l^+} + p_{l^-})_\mu = p^2_{l^+} + p^2_{l^-} + 2 p_{l^+} \cdot p_{l^-}$$
$$= 2|\underline{p}_{l^+}||\underline{p}_{l^-}|(1 - \cos\theta) \leq 4|\underline{p}_{l^+}||\underline{p}_{l^-}|.$$

Momentum conservation:

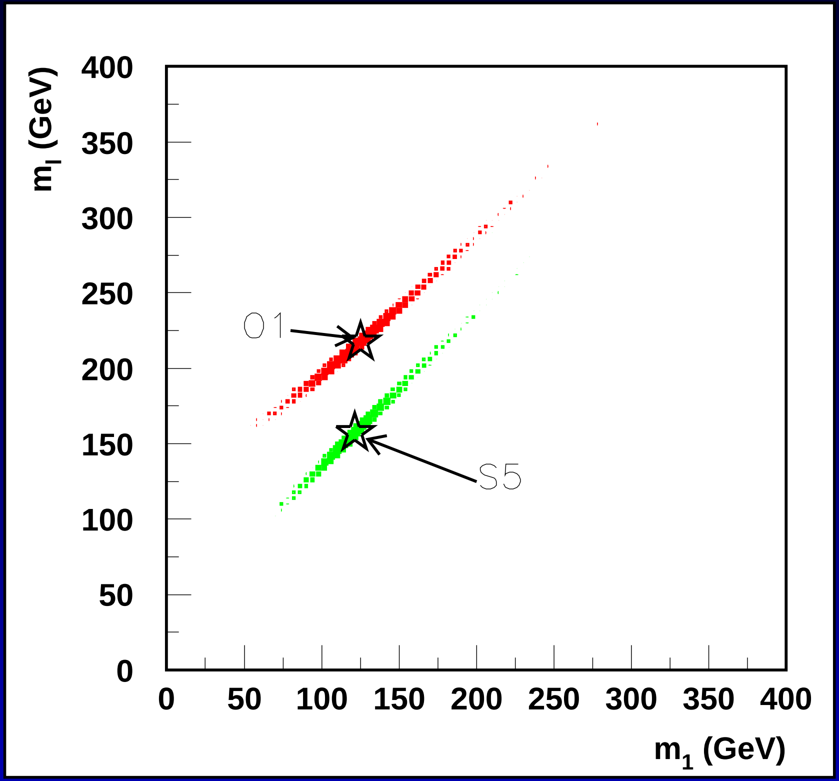
$$\Rightarrow \underline{p}_{\chi^0_2} + \underline{p}_{l^+} = \underline{0}, \qquad \underline{p}_{l^-} + \underline{p}_{\chi^0_1} = \underline{0}.$$

Energy conservation: $\sqrt{{m_{\chi^0_2}}^2 + |\underline{p}_{l^+}|^2} = m_{\tilde{l}} + |\underline{p}_{l^+}|$,

$$\Rightarrow |\underline{p}_{l^+}| = \frac{m^2_{\chi^0_2} - m^2_{\tilde{l}}}{2m_{\tilde{l}}}.$$ Similarly $$|\underline{p}_{l^-}| = \frac{m^2_{\tilde{l}} - m^2_{\chi^0_1}}{2m_{\tilde{l}}}.$$

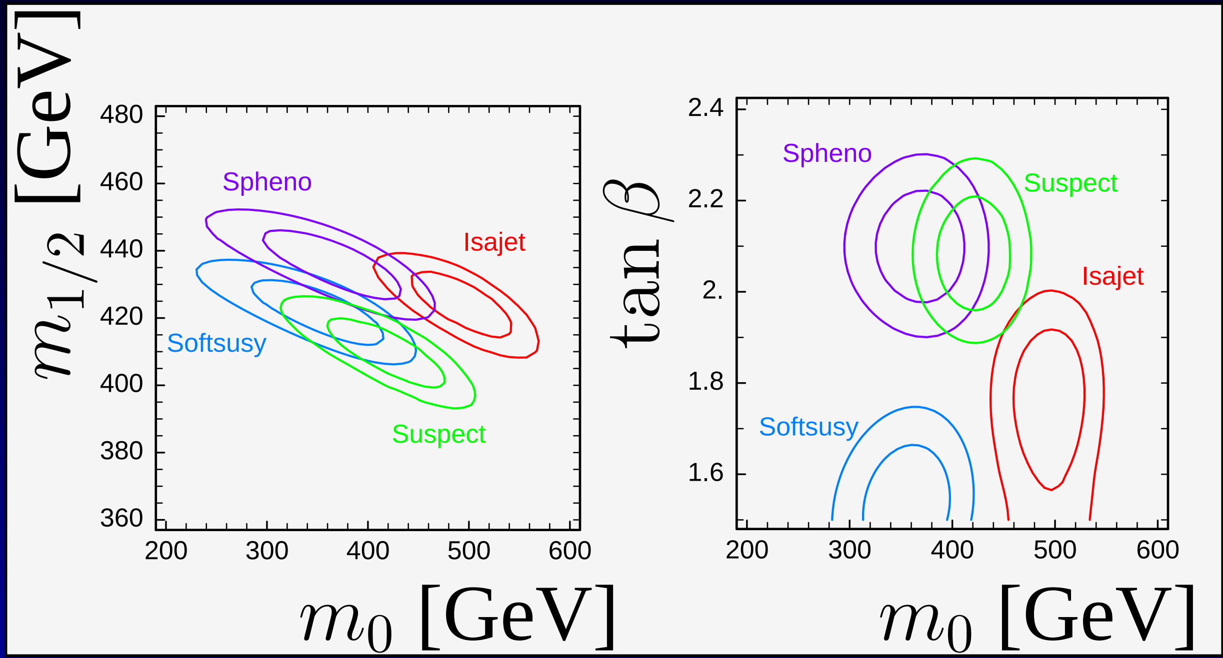
Edge to Mass Measurements

	width S5	width O1
χ_1^0	17	22
$\tilde{l}_R$	17	20
χ_2^0	17	20
$\tilde{q}$	22	20



Mass differences well constrained, but overall mass scale not so well constrained by LHC

Fitting to SUSY Breaking Model



- Experimenters pick a SUSY breaking point
- They derive observables and errors after detector simulation
- We fit[a] this "data" with our codes

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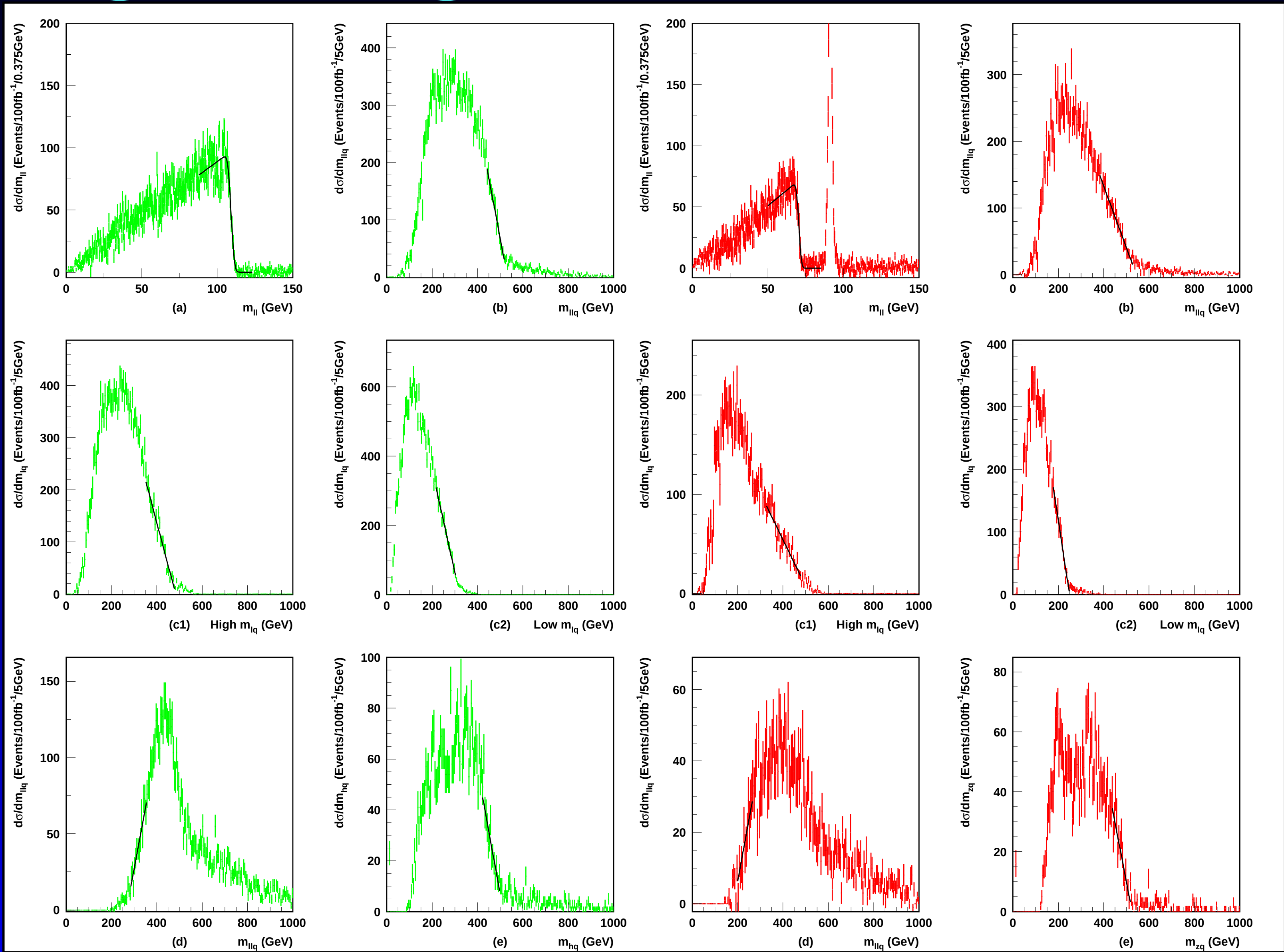
[a] BCA, S Kraml, W Porod, JHEP 0303 (2003) 016

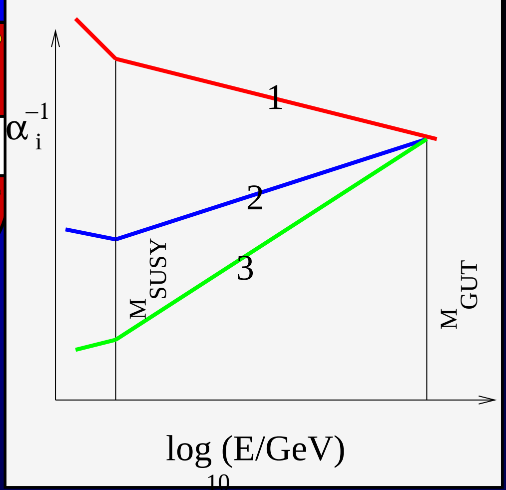
Edge Fitting at S5 and O1



Edge Positions

endpoint	S5 fit	O1 fit
m_{ll}	109.10±0.13	70.47±0.15
m_{llq} edge	532.1±3.2	544.1±4.0
lq high	483.5±1.8	515.8±7.0
lq low	321.5±2.3	249.8±1.5
llq thresh	266.0±6.4	182.2±13.5

Best case lepton mass measurements can be as accurate as 1 per mille, but jets are a few percent



SOFTSUSY

Get $g_i(M_Z)$, $h_{t,b,\tau}(M_Z)$.

↓

Run to M_S.

↓

REWSB, iterative solution of μ

↓

M_X. Soft SUSY breaking BC.

↓

Run to M_S. Calculate[a] sparticle pole masses.

↓

Run to M_Z (→ back to first step)

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[a] BCA, Comp. Phys. Comm. 143 (2002) 305.

Other Observables

Often more complicated, eg m_{llq} edge:

$$\max\left[\frac{(m^2_{\tilde q}-m^2_{\chi^0_2})(m^2_{\chi^0_2}-m^2_{\chi^0_1})}{m^2_{\chi^0_2}}, \frac{(m^2_{\tilde q}-m^2_{\tilde l})(m^2_{\tilde l}-m^2_{\chi^0_1})}{m^2_{\tilde l}},\right.$$

$$\left.\frac{(m_{\tilde q}m_{\tilde l}-m_{\chi^0_2}m_{\chi^0_1})(m^2_{\chi^0_2}-m^2_{\tilde l})}{m_{\chi^0_2}m_{\tilde l}}\right]$$

Also m_{lq}^{high}, m_{lq}^{low}, llq *threshold* [a], $M_{T_2}^2(m) =$

$$\min_{\not p_1+\not p_2=\not p_T}\left[\max\left\{m_T^2(p_T^{l_1},\not p_1,m), m_T^2(p_T^{l_2},\not p_2,m)\right\}\right],$$

$\max[M_{T_2}(m_{\chi^0_1})] = m_{\tilde l}]$ for dislepton production.

Same order prior

We wish to encode the idea that "SUSY breaking terms should be of the same order of magnitude"

$$p(m_0|M_S) = \frac{1}{\sqrt{2\pi w^2} m_0} \exp\left(-\frac{1}{2w^2} \log^2 (\frac{m_0}{M_S}) \right),$$

$$p(A_0|M_S) = \frac{1}{\sqrt{2\pi e^{2w}} M_S} \exp\left(-\frac{1}{2e^{2w}} \frac{A_0^2}{M_S^2} \right),$$

We don't know SUSY breaking scale M_S:

$$p(m_0, M_{1/2}, A_0, \mu, B) = \int_0^\infty dM_S \, p(m_0, M_{1/2}, A_0, \mu, B|M_S) \, p(M_S)$$

Naturalness

$$M_Z^2 = \tan 2\beta \left[m_{H_2}^2 \tan\beta - m_{H_1}^2 \cot\beta \right] - 2\mu^2$$

Cancellation implied by sparticle mass bounds. Quantify by

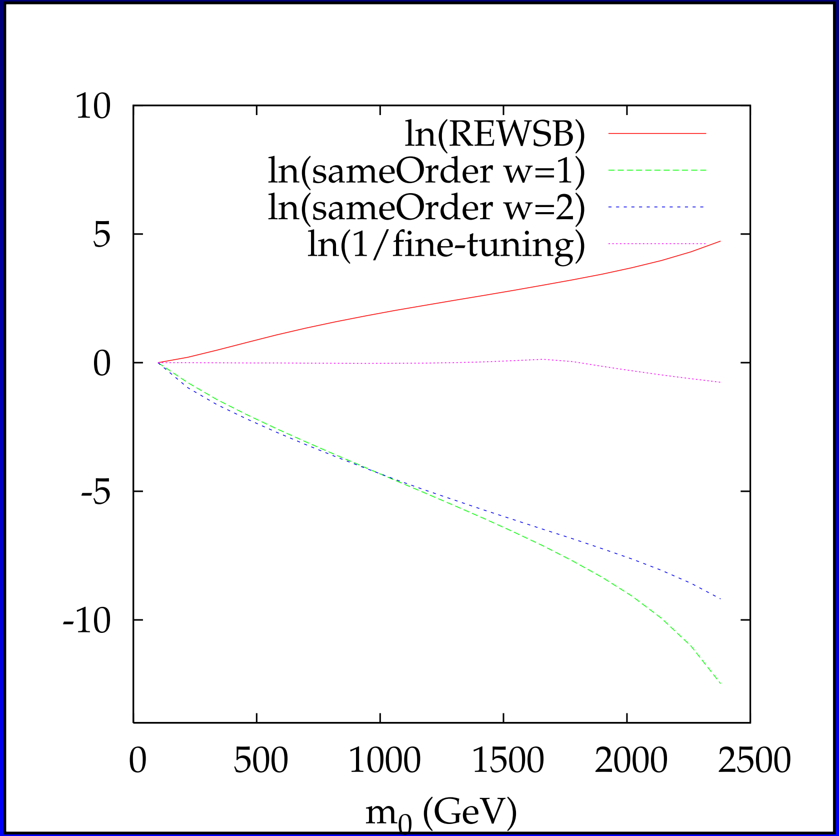
$$f = \max_x \{ \| \frac{d \ln M_Z^2}{d \ln x} \| \}$$

where $x \in \{M_{1/2}, m_0, A_0, \mu, B\}$. We will choose the prior to be $1/f$.

Fine Tuning

Compare with usual definition of *fine-tuning*:

$$f = \max_p \frac{d \ln M_Z}{d \ln p}$$



SPS1a Point

$M_{1/2} = 250$ GeV

$\tan\beta = 10$ GeV

$A_0 = -100$ GeV