Simultaneously Extracting G_F and Constraining the PMNS Matrix in SM4

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Flavour and the Fourth Family
Durham, September 15, 2011

Outline

Based on H. Lacker & A. Menzel, JHEP07(2010)006

Properties of the PMNS matrix

- Unitary: $U^\dagger \cdot U = U \cdot U^\dagger = \mathbb{1}$
- $\Rightarrow$ 9 parameters: $\exists$ 6 rotation angles $\Rightarrow$ 3 complex phases remain
- Parametrization used: Botella/Chau,Phys. Lett. B 168, 1986

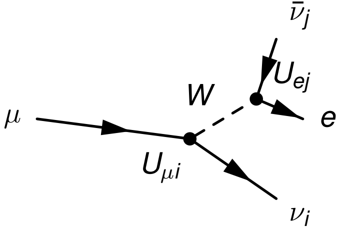
$$\begin{pmatrix} U^*_{e1} & U^*_{e2} & U^*_{e3} & U^*_{e4} \\ U^*_{\mu 1} & U^*_{\mu 2} & U^*_{\mu 3} & U^*_{\mu 4} \\ U^*_{\tau 1} & U^*_{\tau 2} & U^*_{\tau 3} & U^*_{\tau 4} \\ U^*_{E1} & U^*_{E2} & U^*_{E3} & U^*_{E4} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & c_u & s_u \\ 0 & 0 & -s_u & c_u \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & c_v & 0 & s_v e^{-i\phi_3} \\ 0 & 0 & 1 & 0 \\ 0 & -s_v e^{i\phi_3} & 0 & c_v \end{pmatrix} \cdot$$

$$\cdot \begin{pmatrix} c_w & 0 & 0 & s_w e^{-i\phi_2} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -s_w e^{i\phi_2} & 0 & 0 & c_w \end{pmatrix} \cdot \begin{pmatrix} & V_{PMNS3} & & 0 \\ & & & 0 \\ & & & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

The unitarity of the SM3 submatrix is now lost!

Determination of G_F in SM3

$$1/\tau_\mu = \Gamma\left(\mu^- \to e^- \bar{\nu}_e \nu_\mu(\gamma)\right) = \frac{G_F^2 m_\mu^5}{192\pi^3} R_\mu F_{\mu,e}$$



Neutrinos are not detected!
$\Rightarrow$ Each of the two neutrinos contains components of
ν_1, ν_3, ν_3 as described by the PMNS matrix!

R_μ: Electroweak radiative corrections,
$F_{\mu,e}$: Phase space factor

Determination of G_F in SM3

Strictly speaking, the theoretical expression is therefore

$$1/\tau_\mu = \Gamma\left(\mu^- \to e^- \bar{\nu}_e \nu_\mu(\gamma)\right) = \frac{G_F^2 m_\mu^5}{192\pi^3} R_\mu F_{\mu,e} \cdot \sqrt{\sum_{i,j=1}^{3} \left|U_{\mu i}\right|^2 \left|U_{ej}\right|^2}$$

but due to the 3×3 unitarity of the PMNS matrix:

$$\sum_{i=1}^{3} \left|U_{\ell i}\right|^2 = 1$$

SM4 considerations

- $\Gamma(Z^0) \quad \Rightarrow \quad$ In case of Dirac-neutrinos: $m_{\nu_4} > M_{Z^0}/2 \gg m_\mu$. Therefore, ν_4 is not produced.

  $\Rightarrow$ As before, the expression reads

$$1/\tau_\mu = \frac{G_F^2 m_\mu^5}{192\pi^3} R_\mu F_{\mu,e} \cdot \sqrt{\sum_{i,j=1}^{3} \left|U_{\mu i}\right|^2 \left|U_{ej}\right|^2}$$

- However, now we have 4×4 unitarity (instead of 3×3)

Consequences for G_F

A priori, all we can state about the sums $\sum_{i,j=1}^{3} \left|U_{\mu i}\right|^2$ is:

- $0 \leq \sum_i^3 |U_{\ell i}|^2 \leq 1$
- $0 \leq |U_{\ell i}| \leq 1$
- $\sum_i^3 |U_{\ell i}|^2 = (1 - |U_{\ell 4}|^2)$

What we actually extract in this experiment is therefore

$$G_F \sqrt{(1 - |U_{e4}|^2)(1 - \left|U_{\mu 4}\right|^2)}$$

The number $G_{F,SM3} = 1.1663788(7) \cdot 10^{-5}\ GeV^{-2}$ (MuLan 2011) is only a lower limit on the actual SM4 value!

Step 1: Leptonic τ-decays

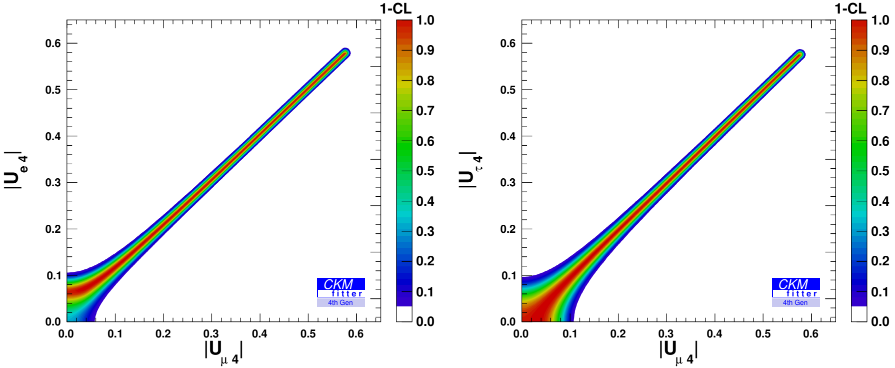
- $\Gamma\left(\tau^- \to e^-, \mu^- \bar{\nu}_{e,\mu} \nu_\tau(\gamma)\right)$ is predicted by the same expression as the decay of the $\mu (\mathcal{B}_x = \Gamma_x \cdot \tau)$.

τ_μ	$2.1969803 \pm 0.0000022\ \mu$s	MuLan
τ_τ	$(290.6 \pm 1.0) \cdot 10^{-15}$s	LEP, CLEO
$\mathcal{B}(\tau \to e\nu\bar{\nu}(\gamma))$	$(17.85 \pm 0.05)\%$	PDG 2010
$\mathcal{B}(\tau \to \mu\nu\bar{\nu}(\gamma))$	$(17.36 \pm 0.05)\%$	
$\frac{\mathcal{B}(\tau^- \to \mu^- \nu_\tau \bar{\nu}_\mu(\gamma))}{\mathcal{B}(\tau^- \to e^- \nu_\tau \bar{\nu}_e(\gamma))}$	$0.9796 \pm 0.0016 \pm 0.0036$[1]	BaBar[2]

[1] 1.8 σ deviation from SM3 prediction of perfect lepton universality

[2] arXiv:0912.0242

A first upper limit...



- $|U_{e4}| \approx |U_{\mu 4}| \approx |U_{\tau 4}|$: Lepton universality
- Lower correlation at smaller $|U_{\ell i}|$: Uncertainties, 1.8σ deviation
- Upper limit at 0.577: Consequence of unitarity

A first upper limit...

No input used for PMNS matrix elements

$$\begin{pmatrix} U^*_{e1} & U^*_{e2} & U^*_{e3} & U^*_{e4} \\ U^*_{\mu 1} & U^*_{\mu 2} & U^*_{\mu 3} & U^*_{\mu 4} \\ U^*_{\tau 1} & U^*_{\tau 2} & U^*_{\tau 3} & U^*_{\tau 4} \\ U^*_{E1} & U^*_{E2} & U^*_{E3} & U^*_{E4} \end{pmatrix}$$

$$|U_{e4}|^2 + |U_{\mu 4}|^2 + |U_{\tau 4}|^2 \leq 1$$

$$\Rightarrow |U_{\ell 4}|^2 \leq 1/3$$

$$|U_{ell4}| \leq \sqrt{1/3} \approx 0.577$$

Therefore, in case of maximum $|U_{\ell 4}|$:

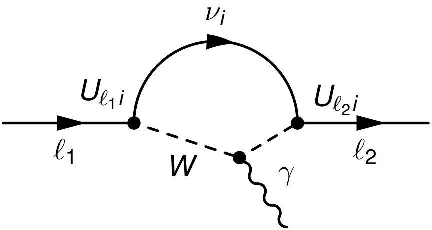
$$G_F \sqrt{(1 - |U_{e4}|^2)(1 - |U_{\mu 4}|^2)}$$

$$= G_F \sqrt{(1 - \frac{1}{3})(1 - \frac{1}{3})}$$

$$= 2/3$$

$$\Rightarrow G_F = 3/2 G_{F,SM3}$$

Marked enhancement: Radiative decays of μ and τ



90 % CL Limits		
$\mathcal{B}(\mu \to e\gamma)$	$< 2.4 \cdot 10^{-12}$	MEG collaboration[3]
$\mathcal{B}(\tau \to e\gamma)$	$< 3.3 \cdot 10^{-8}$	PDG 2010
$\mathcal{B}(\tau \to \mu\gamma)$	$< 4.4 \cdot 10^{-8}$	

[3] arXiv:1107:5547v3

Predicted branching ratio [4]

$$\mathcal{B} = \frac{3\alpha_{EM}(m_{\ell_1}^2)}{32\pi} x^2 \left[6(I(x)^{(2)} - I(x)^{(3)})\right]^2 \frac{\left|U_{\ell_1 4} U_{\ell_2 4}^*\right|^2}{(1 - \left|U_{\ell_2 4}\right|^2)(1 - \left|U_{\ell_2 4}\right|^2)}$$

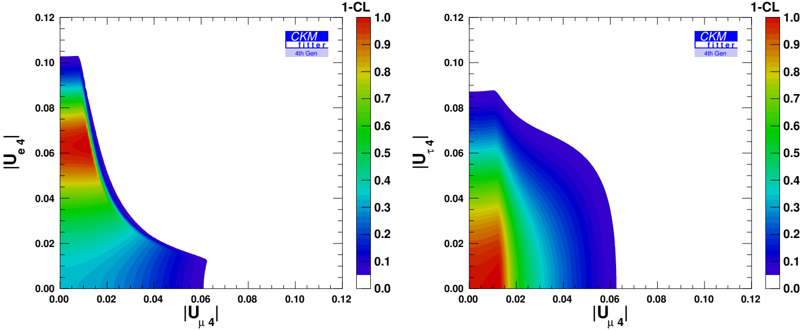
where

$$I(x) = I^{(n)}(x) = \int_0^1 \frac{z^n}{z + x(1-z)} \, \mathrm{d}z \qquad \text{and} \qquad x = \left(\frac{m_{\nu_4}}{m_W}\right)^2$$

ν_i=ν_4 dominates due to high mass

[4] Altarelli, Nucl. Phys. B 125(1977)

Radiative decays of μ and τ



- Contour $\frac{\text{const}}{|U_{e4}|}$ due to $\mathcal{B} \propto \left| U_{\ell_1 4} U^*_{\ell_2 4} \right|^2$
- In other graphs: Less precise upper limits of τ decays (as compared to μ decay) obscure this feature
- $|U_{e4}| \neq 0$ preferred (not significant)

Further improvement: Semileptonic Decays of Mesons

Each $\ell\nu$-Vertex receives an additional factor of

$$\sum_{i=1}^{3} |U_{\ell_i}|^2 \cdot \frac{G_F}{G_{F,SM3}}$$

Process	$\|V_{us}\| \cdot f_+(0)$	Reference
$K_L \to \pi e \nu$	0.2163(6)	FLAVIAnet 2010
$K_L \to \pi \mu \nu$	0.2166(6)	
$K_S \to \pi e \nu$	0.2155(13)	
$K^\pm \to \pi e \nu$	0.2160(11)	
$K^\pm \to \pi \mu \nu$	0.2158(14)	

Leptonic Decays

Process	Value	SM3-prediction	Reference
$\frac{\pi\to e\nu}{\pi\to\mu\nu}$	$1.230 \pm 0.004 \cdot 10^{-4}$	1.2354 ± 0.0002[5]	PDG 2010
$\frac{K\to e\nu}{K\to\mu\nu}$	$(2.488 \pm 0.01) \cdot 10^{-5}$	2.477 ± 0.001[6]	NA62 2011

In the SM4, the deviation between theory and experiment constrains the following factor common to both formulas (in SM4):

$$\sqrt{\frac{1-|U_{e4}|^2}{1-|U_{\mu 4}|^2}}$$

[5] Finkemeier, Phys.Lett.B 387(1996)

[6] Talk at EPS conference 2011

Update on JHEP paper

Update on JHEP-Paper! Due to

- Update $K_{\ell 3}$ (FLAVIAnet 2008 $\longrightarrow$ FLAVIANET 2010)
- Including recent NA62 determination of R_K
- New measurement of muon lifetime τ_μ by MuLan
- New upper limit on $\mathcal{B}(\mu \to e\gamma)$ (MEG, July 2011)

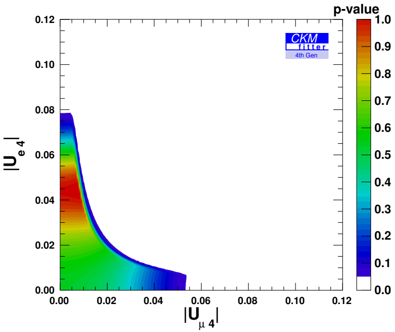
the p-value of $|U_{e4} = 0|$ changed from 2.6 % to $\approx$ 25 %!

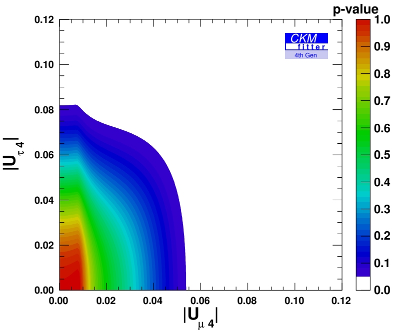
Results - PMNS matrix

$$U = \begin{pmatrix} * & * & * & < 0.073 \\ * & * & * & < 0.045 \\ * & * & * & < 0.072 \\ < 0.092 & < 0.092 & < 0.092 & > 0.9958 \end{pmatrix}$$

- Strong mixing between SM3 neutrinos and the fourth neutrino is excluded
- PMNS matrix is really close to 3×3 unitarity

Results - PMNS matrix





Results - G_F

G_F [10^{-5} GeV^{-2}]		$G_{F,SM3}$
$\tau, \mu \to \ell\nu\bar{\nu}(\gamma)$	1.46 ± 0.29	
$+\ \tau, \mu \to \ell\gamma$	$1.1689^{+0.0015}_{-0.0017}$	1.16637887(7)[7]
+ meson decays	$1.16756^{+0.00098}_{-0.00104}$	

- Relative Uncertainty of G_F increases by a factor of ≈ 14000 compared to textbook value $G_{F,SM3}$
- Very slight increase of central value

[7] MuLan 2011

Consequences

- Moduli of CKM matrix elements are measured in semileptonic processes $\rightarrow$ How are these values understood in an SM4 context?
- G_F cannot be used as a fixed value any more due to the increased uncertainty (Electroweak Precision fit!)
- Extraction of G_F means simultaneous extraction of PMNS elements

CKM matrix elements $|V_{ij}|$

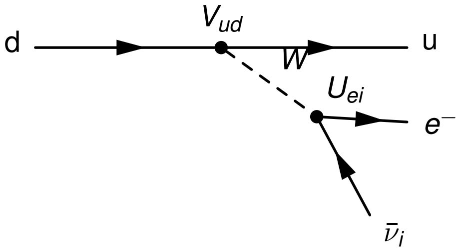
The published $|V_{ij}|$ can be used in SM4 if they either

- are measured in such way that they do not depend on G_F and / or PMNS matrix elements:
  - $|V_{cd}|$ when measured in $\bar{\nu}$N-scattering (CHDS, CHARM II)
  - $R = \frac{|V_{tb}|^2}{|V_{td}|^2+|V_{ts}|^2+|V_{tb}|^2}$
- their uncertainty is so large that the different value and increased error on G_F in SM4 have no effect: $|V_{ub}|$, $|V_{cs}|$, $|V_{cb}|$, $|V_{ts}|$, $|V_{td}|$ or
- if they are available as separate results for different leptons in the final state - until now, only $|V_{us}|$.

$|V_{ud}|$ satisfies none of these conditions.

$|V_{ud}|$

Measured in superallowed β decays



$$\frac{G_F}{G_{F,SM3}} |V_{ud}| \sqrt{1 - |U_{e4}|^2}$$

SM3:	SM4:
$\lvert V_{ud}\rvert = 0.97425 \pm 0.00022$	$\lvert V_{ud}\rvert = 0.97421^{+0.00024}_{-0.00023}$

At a glance:

- Extracting G_F by only measuring τ_μ is not possible in SM4
- G_F can be constrained by τ_μ with data from leptonic decays of τ, radiative decays of μ and τ and certain (semi)leptonic meson decays.
- In that process, the PMNS matrix elements $U_{\ell 4}$ and $U_{E1...4}$ are constrained simultaneously.
- The value of G_F obtained that way has an uncertainty 100 times the size of error on the PDG value of $G_{F,SM3}$ (and 14000 the size of the error on the recent MuLan value)
- Until now, the only CKM element whose value is affected by the changes in G_F is $|V_{ud}|$

Outlook:

$\lvert\epsilon_K\rvert$	●
Δm_d, Δm_s	●
x_D	●
A_{SL}, $A_{SL}(B_s)$, $A_{SL}(B_d)$	●
$R_{B\to s\gamma}$ aus $B \to X_s\gamma$	●
$\mathcal{B}(B_q \to \mu^+\mu^-)$	●
Single-top-Analysen, Interpretation of direct searches (Tevatron, LHC)	●

Revision of the Electroweak Precision Fit, taking into account the larger uncertainty on G_F and the PMNS/CKM dependency.

But...

While the GF/PMNS part is reliably working, the inclusion of other observables such as

- ϵ_K from K^0 mixing
- Δ_{m_d} and Δ_{m_s} from $B_{d,s}$ mixing
- Dimuon asymmetry A_{SL} and its constituents $A_{SL,B_{d,s}}$
- $R_{b\to s\gamma} = \frac{\Gamma(B\to X_s\gamma)}{\Gamma(B\to X_c\ell\nu)}$

in one fit is yet to be finished.

On the other hand...

nearly-satisfied 3×3 unitarity of the PMNS matrix means it is not necessary to include full PMNS dependency in our fits at the present level of precision.

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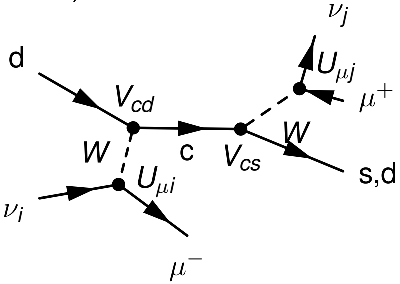
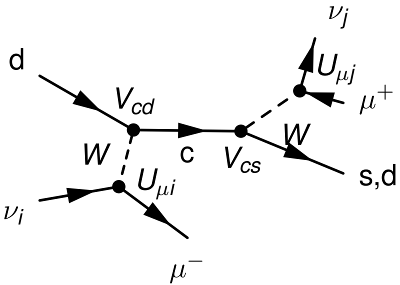
$|V_{cd}|$

Does not change:

Extracted from $\frac{\sigma(dimuon)}{\sigma(singlemuon)}$ measured in neutrino beam experiments (CDHS, CHARM II, CCFR):





- PMNS sum from vertex $\nu_i \mu^- W$ cancels in ratio of σs
- PMNS sum from vertex $\nu_j \mu^+ W$ implicit in measured $\mathcal{B}(X_c \to \mu X)$

Other BSM models

Unification of spin and charge to explain family replication (Breskvar et al., arXiv 0606159)

The symmetry breaking discussed there requires the PMNS matrix

$$U = \begin{pmatrix} 0.697 & 0.486 & 0.177 & 0.497 \\ 0.486 & 0.697 & 0.497 & 0.177 \\ 0.177 & 0.497 & 0.817 & 0.234 \\ 0.497 & 0.177 & 0.234 & 0.817 \end{pmatrix}$$

which is excluded by our results.

Other BSM models

See-saw models (e.g. Frampton, Hung & Sher, Phys. Rept. 330)

Mixing angle between 3. und 4 family θ_{34} satisfies

$$\sin^2\theta_{34} \approx \frac{m_\tau}{m_E}.$$

In the Botella-Chau parametrization we used, $\sin\theta_{34} \approx |U_{\tau 4}|$.
$\Rightarrow$ Agrees with our results if $m_E \gtrsim 343$ GeV

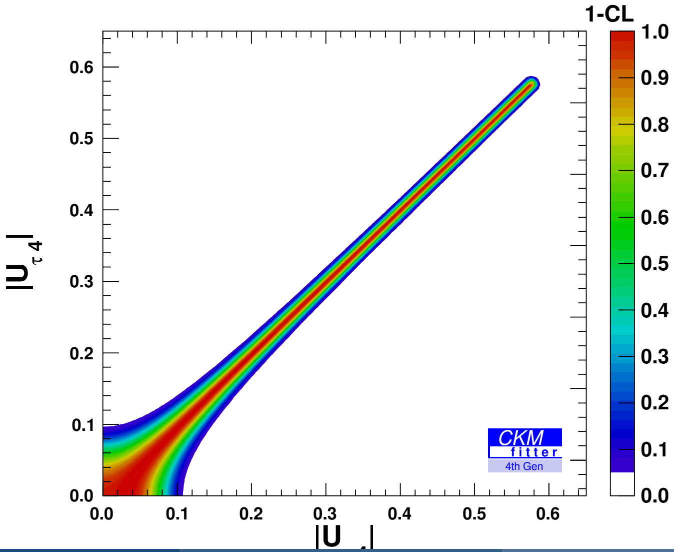
(See also Otto Eberhardt's talk

Other BSM models

Explaining the anomalous magnetic moment of the muon by the presence of ν_4 (Hou, Lee, Ma, Phys. Rev D 79, 2009)

Requires $|U_{e4}| = O(0.7)$. $\Rightarrow$ Does not agree with our results.

Plots not shown yet



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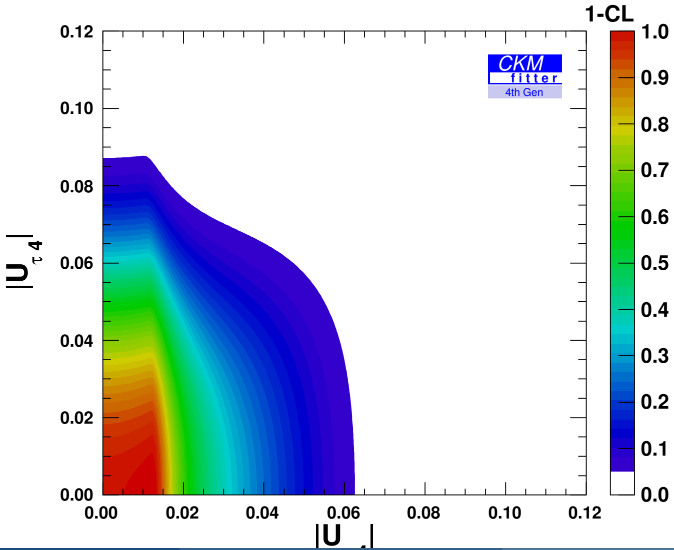

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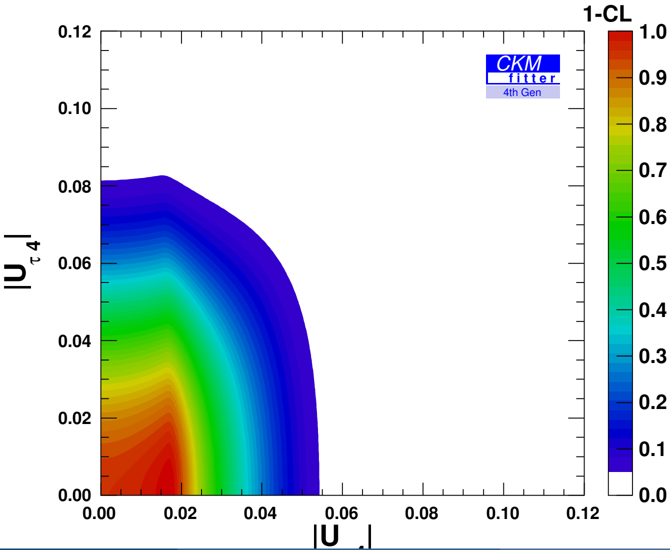

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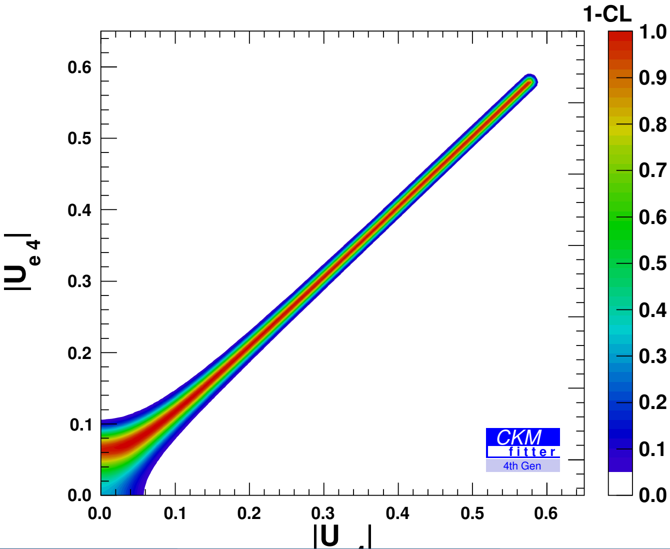

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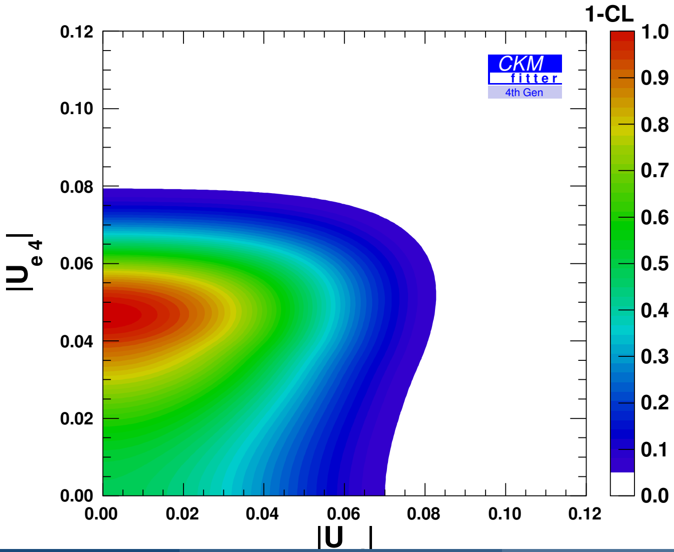

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