Key Observables in the Flavour Sector

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Plan of talk

- Flavour Physics (B and K) and SM4
- Relevant Parameters: allowed ranges or fit values
- Key Observables:
  - SM4 allowed ranges
  - SM4 predictions
- Conclusions

Flavour physics: Mixing and decay

- Low energy observables in flavor physics play an important role for an indirect search of NP
- NP in B: look in CP asymmetries, branching fractions, mass and lifetime differences
- FCNC processes play an important role for the detection of NP effects
- Data from K, D and B_d mesons have been consistent with SM picture $\Rightarrow$ Most of the anomalous results have been found in $b \to s$ transitions $\Rightarrow$ B_s meson, interesting and important portal for indirect detection of NP

Why extra fermion generation?

- In the SM, the number of fermion generations is not fixed by any symmetry principle
- Simple extension of SM $\Rightarrow$ LHC has the potential to discover or fully exclude it
- Not ruled out by data:
  - EW precision data does not exclude the presence of the 4th generation
  - Neutrino oscillation data still allow the presence of an extra heavy neutrino
- It can address some of the currently open questions $\Rightarrow$

  large baryon asymmetry, fermion mass hierarchy problem, Dark Matter problem...

Observables as Input !

References : Soni et.al. PRD82 (2010),
Buras et.al. JHEP1009 (2010),
Nandi & Soni, PRD83 (2011)

CP violation and K Physics

Indirect CP violation in $K_L \to \pi\pi$,

$$|\epsilon_K| \propto f_K^2 \hat{B}_K k_\epsilon \mathrm{Im}\Big[\eta_c (\lambda_{ds}^{c*})^2 S(x_c) + 2\eta_{ct}(\lambda_{ds}^{c*}\lambda_{ds}^{t*}) S(x_c, x_t) + \eta_t (\lambda_{ds}^{t*})^2 S(x_t)$$

$$+ 2\eta_{ct'}(\lambda_{ds}^{c*}\lambda_{ds}^{t'*}) S(x_c, x_{t'}) + 2\eta_{tt'}(\lambda_{ds}^{t*}\lambda_{ds}^{t'*}) S(x_t, x_{t'}) + \eta_{t'}(\lambda_{ds}^{t'*})^2 S(x_{t'})\Big]$$

$\lambda_{ds}^{i*} = V_{id}V_{is}^*$, $S(x_c, x_t) \Rightarrow$ Inami-Lim functions

Theory parameters: $B_K = 0.725 \pm 0.026$, $f_K = 0.1558 \pm 0.0017$, $\kappa_\epsilon = 0.94 \pm 0.02$

Measured value: $|\epsilon_k|_{exp} = (2.32 \pm 0.007) \times 10^{-3}$ $\Rightarrow$ Known with 0.3% accuracy

$$Br(K^+ \to \pi^+ \nu\bar{\nu}) = \kappa_+ \Bigg[\left(\frac{\mathrm{Im}(\lambda_{ds}^t)}{\lambda^5} X(x_t) + \frac{\mathrm{Im}(\lambda_{ds}^{t'})}{\lambda^5} X(x_{t'}) \right)^2$$

$$+ \left(\frac{\mathrm{Re}(\lambda_{ds}^c)}{\lambda} P_0(X) + \frac{\mathrm{Re}(\lambda_{ds}^t)}{\lambda^5} X(x_t) + \frac{\mathrm{Re}(\lambda_{ds}^{t'})}{\lambda^5} X(x_{t'}) \right)^2 \Bigg]$$

$\kappa_+ = (5.36 \pm 0.026) \times 10^{-11}$, $P_0(X) = 0.42 \pm 0.03 \Rightarrow$ NNLO QCD

Measured Value: $BR(K^+ \to \pi^+ \nu\nu) = (1.7 \pm 1.1) \times 10^{-10}$

Measured Direct CP violation in $K \to \pi\pi \Rightarrow \epsilon'/\epsilon$ = $(1.65 \pm 0.26) \times 10^{-3}$

Theory (ϵ'/ϵ): Large hadronic uncertainties ...

B_q-$\bar{B}_q$ $(q = d, s)$ mixing

The mass difference: $\Delta M_q = M_H - M_L \simeq 2|M_{12}^q|$

$$M_{12}^q \propto \hat{B}_{bq} f_{B_q}^2 \left[\eta_t (\lambda_{qb}^{t*})^2 S(x_t) + \eta_{t'} (\lambda_{qb}^{t'*})^2 S(x_{t'}) + \; 2\eta_{tt'} (\lambda_{qb}^{t*}) (\lambda_{qb}^{t'*}) S(x_t, x_{t'})\right]$$

Theory parameters: $f_{bd}\sqrt{B_{bd}} = 0.224 \pm 0.015$ GeV, $\xi = \frac{f_{bs}\sqrt{B_{bs}}}{f_{bd}\sqrt{B_{bd}}} = 1.232 \pm 0.042$

Measured, $\Delta M_d = (0.507 \pm 0.005)\, ps^{-1}$ & $\Delta M_s = (17.77 \pm 0.12)\, ps^{-1}$ (pdg)

Two Important Measurements:

Time dependent CP asymmetry from all charmonium B_d decay modes:
$\sin 2\beta_d^{\rm eff} = 0.676 \pm 0.020$ (HFAG) $\Rightarrow$ Indications that the predicted $\sin 2\beta_d^{\rm eff}$ is larger than the measured value. (see talk by Soni)

Time dependent CP asymmetry in $B_s \to J/\psi\phi$ (at 68% C.L) :

$$\begin{aligned} 2\beta_s^{eff} &\in [0.04, 1.04] \cup [2.16, 3.10] \quad CDF(5.2\, fb^{-1}) \\ &\in 0.76^{+0.38}_{-0.36}\, (stat) \pm 0.02 (syst) \quad DO(6.1\, fb^{-1}) \\ &\in 0.13 \pm 0.18\, (stat) \pm 0.07 (syst) \quad LHCb(337\, pb^{-1}) \end{aligned}$$

Here, $2\beta_q^{eff} = Arg\left(-M_{12}^q / \Gamma_{12}^q\right)$

FCNC: $b \to s$ transitions

- In the leading log approximation the $B \to X_s\gamma$ branching fraction (perturbative) :

$$R = \frac{Br(B\to X_s\gamma)}{Br(B\to X_c e\bar{\nu}_e)} = \frac{|\lambda^t_{sb}|^2}{|V_{cb}|^2}\,\frac{6\alpha|C_7^{\rm tot}(m_b)|^2}{\pi f(\hat{m}_c)\kappa(\hat{m}_c)}$$

$$C_7^{\rm tot}(\mu) = C_7(\mu) + \frac{\lambda^{t'}_{sb}}{\lambda^t_{sb}} C_7^{t'}(\mu), \quad V_{cb} = (41.2 \pm 1.1)\times 10^{-3}$$

Theory inputs: $\hat{m}_c = 0.29 \pm 0.02, \quad f(\hat{m}_c) = 0.542 \pm 0.045, \quad \kappa(\hat{m}_c) = 0.88 \pm 0.003$

Measurements: $BR(B \to X_s\gamma) = (3.55 \pm 0.25)\times 10^{-4}$ (PDG)

$BR(B \to X_c\ell\nu) = (10.61 \pm 0.17)\times 10^{-2}$ (PDG)

- The branching fraction for the decay $B \to X_s\ell^+\ell^-$ $z \equiv q^2/m_b^2$:

$$Br(B \to X_s\,\ell^+\,\ell^-) \quad \propto \quad |\lambda^t_{sb}|^2 \int (1-z)^2 \left[(1+2z)\left(|C_9^{\rm tot}|^2 + |C_{10}^{\rm tot}|^2\right) + 4\left(1+\frac{2}{z}\right)|C_7^{\rm tot}|^2 + 12{\rm Re}(C_7^{\rm tot}C_9^{\rm tot*})\right] dz$$

$$\mathcal{BR}(B \to X_s\ell^+\ell^-)_{\rm low(1\,GeV^2 \le q^2 \le 6\,GeV^2)} = (1.60 \pm 0.50)\times 10^{-6}, \quad \text{PDG}$$

$$\mathcal{BR}(B \to X_s\ell^+\ell^-)_{\rm high(14.4\,GeV^2 \le q^2 \le m_b^2)} = (0.44 \pm 0.12)\times 10^{-6}, \quad \text{PDG}$$

Other important inputs

- Non-decoupling oblique correction:

  $$T_4 \propto \left(|V_{t'b'}|^2\delta m_{t'b'} + |V_{t'b}|^2\delta m_{t'b} + |V_{tb'}|^2\delta m_{tb'} - |V_{t'b}|^2\delta m_{tb} + |V_{t's}|^2\delta m_{t's}\right)$$

  Terms sensitive to $m_{t'}$ or $m_{b'}$: $\delta m_{12} \propto \left(m_1^2 + m_2^2 - \frac{2m_1^2 m_2^2}{m_1^2 - m_2^2}\ln(m_1^2/m_2^2)\right)$

  SM4 elements that can be constrained: $|V_{t'b}|^2$ and $|V_{t's}|^2$ ($V_{t'b'} \approx V_{tb}$ & $V_{t'b} \approx V_{tb'}$)

  Measured Value: $T_4 = 0.11 \pm 0.14$, PDG

- Vertex corrections to $Z \to b\bar{b}$ (4G sensitive term) : $\Gamma(Z \to b\bar{b}) \propto \left(1 + \delta_b\right)$

  The loop corrections to the vertex mediated by W boson:

  $$\delta_b \approx 10^{-2}\left(\left(-\frac{m_t^2}{2m_Z^2} + 0.5\right)|V_{tb}|^2 + \left(-\frac{m_{t'}^2}{2m_Z^2} + 0.5\right)|V_{t'b}|^2\right)$$

  Vertex correction sensitive to $|V_{t'b}|^2$

  Measured Value: $R_{bb} = \frac{\Gamma(Z\to b\bar{b})}{\Gamma(Z\to hadrons)} = 0.216 \pm 0.001$

Parameters : Scan or χ^2 Fitting

Parameters need to Scan or Fit

- Unitarity: $\lambda^t_{qb} = 1 - \lambda^u_{qb} - \lambda^c_{qb} - \lambda^{t'}_{qb}$ and $\lambda^t_{ds} = 1 - \lambda^u_{ds} - \lambda^c_{ds} - \lambda^{t'}_{ds}$
- Unknowns : $|\lambda^{t'}_{sb}|e^{i\delta^s_{t'}}$, $|\lambda^{t'}_{db}|e^{i\delta^d_{t'}}$, $|\lambda^{t'}_{ds}|e^{i(\delta^s_{t'} - \delta^d_{t'})}$, $|V_{t'b}|^2$, $|V_{t's}|^2$..
  - Further Replacements: $|V_{t'b}|^2 = \frac{\lambda^{t'}_{sb}\lambda^{t'*}_{db}}{\lambda^{t'}_{ds}}$, $|V_{t's}|^2 = \frac{\lambda^{t'}_{sb}\lambda^{t'*}_{ds}}{\lambda^{t'}_{db}}$
- Other inputs: $\gamma = (75.0 \pm 22.0)^\circ$ and $|V_{ub}| = (32.8 \pm 3.94) \times 10^{-4}$

Methods :

- Scan Method (RAN1): [Nandi & Soni, PRD83 (2011)]
  Constrain used $\Rightarrow$ T_4 + R_{bb} + $|\epsilon_k|$ + ΔM_d + ΔM_s+ $Br(B \to X_s\gamma)$ + $Br(B \to X_s\ell^+\ell^-)$ +$\mathcal{BR}(K^+ \to \pi^+\nu\nu)$
- χ^2 Fitting using MINUIT [$\chi^2_A = ((A - A^c)/A^{err})^2$] :
  Total χ^2 = $\chi^2_{|\epsilon_K|}$ + $\chi^2_{K^+\to\pi^+\nu\bar{\nu}}$ + $\chi^2_{\Delta M_s}$ + $\chi^2_{\Delta M_d}$ + $\chi^2_{B\to X_s\, l^+\, l^-:\mathrm{low}}$ + $\chi^2_{B\to X_s\, l^+\, l^-:\mathrm{high}}$ + $\chi^2_{B\to X_s\gamma}$ + $\chi^2_{R_{bb}}$ + $\chi^2_{A_b}$ + $\chi^2_{T_4}$ + χ^2_γ+$\chi^2_{V_{ub}}$

...Work in Progress (Alok, Dighe, London, Nandi)

Results: Scan and Fit

Elements	Allowed ranges		
	Scan	Fitting (at 1σ)	
	$m_{t'}$ (GeV) = [375,575]	$m_{t'}$ = 400 GeV	$m_{t'}$ = 575 GeV
$\lvert\lambda^{t'}_{db}\rvert\times 10^4$	< 21.0	0.26 ± 0.50	0.13 ± 0.24
$\lvert\lambda^{t'}_{sb}\rvert\times 10^2$	< 1.5	0.85 ± 0.64	0.37 ± 0.47
$\lvert\lambda^{t'}_{ds}\rvert\times 10^4$	< 37.4	0.48 ± 0.70	0.27 ± 0.39
$\delta^s_{t'}$ (rad)	Fig.	1.39 ± 0.30	1.41 ± 0.48
		4.93 ± 0.30	4.91 ± 0.48
$\delta^d_{t'}$ (rad)	[0,2π]	4.53 ± 0.46	4.42 ± 0.78
		1.79 ± 0.44	1.65 ± 0.75
$\chi^2/d.o.f$	N.A.	3.57/5	3.81/5

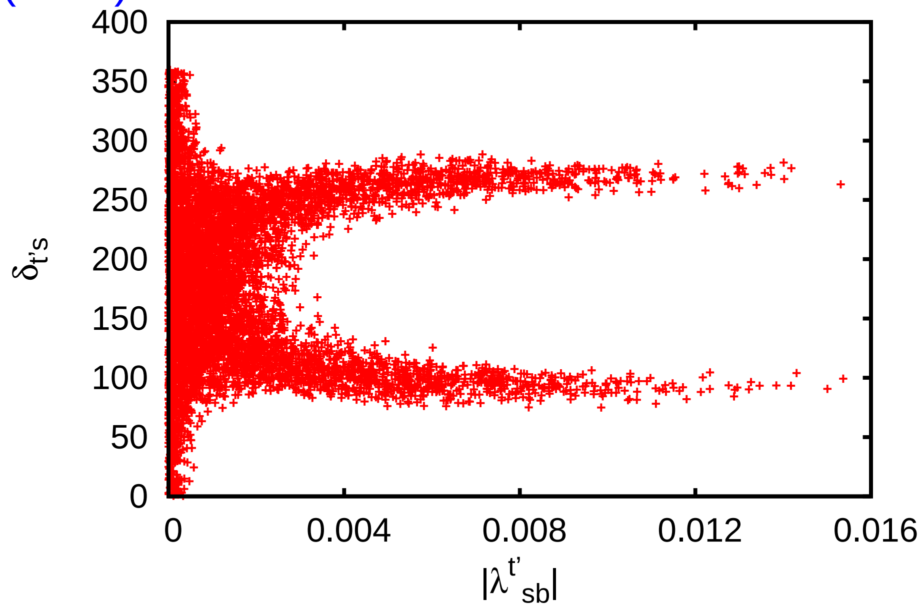
Indirect Constraint:

$|V_{t'b}| = 0.07 \pm 0.09$ for $m_{t'} = 400$ GeV, 0.04 ± 0.06 for $m_{t'} = 575$ GeV

$|V_{t's}| = 0.13 \pm 0.16$ for $m_{t'} = 400$ GeV, 0.09 ± 0.12 for $m_{t'} = 575$ GeV

$\lambda_{sb}^{t'}$: Magnitude vs Phase

Nandi & Soni, PRD83 (2011) ..

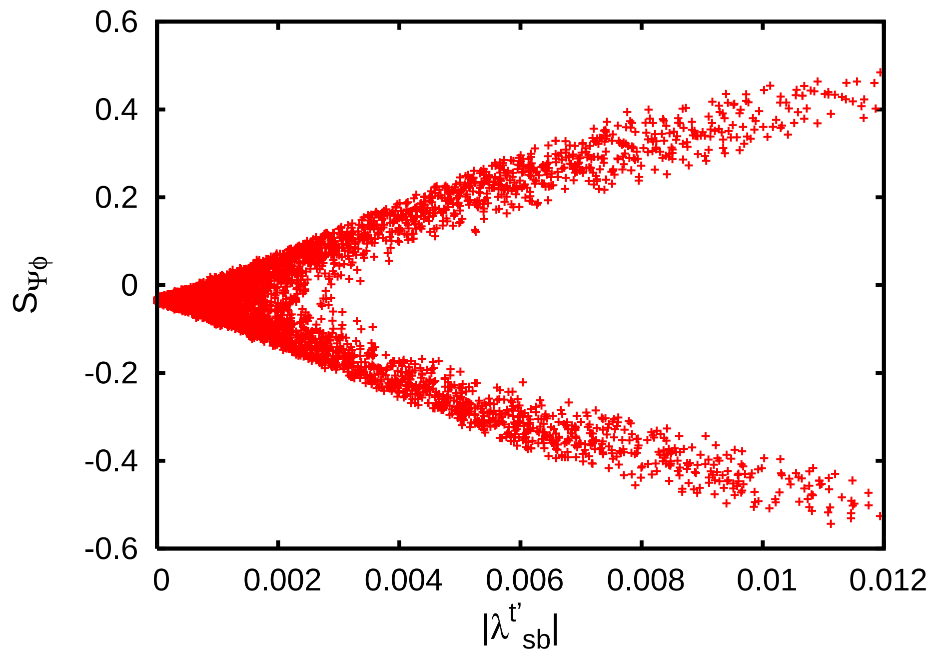


Here $m_{t'} = [375\text{GeV}, 575\text{GeV}]$..

- The magnitude of different SM4 product couplings reduce with the increase of $m_{t'}$
- For slightly higher values of $|\lambda_{sb}^{t'}|$, $\delta_{t'}^{s}$ has two distinct solutions...
- Due to the constrain from $|\epsilon_K|$, along with $\delta_{t'}^{s}$, $\delta_{t'}^{d}$ has two distinct solutions..

Correlation: $S_{\psi\phi}$ with $|\lambda_{sb}^{t'}|$

For $m_{t'} = [375\text{GeV}, 575\text{GeV}]$...Nandi & Soni, PRD83 (2011)



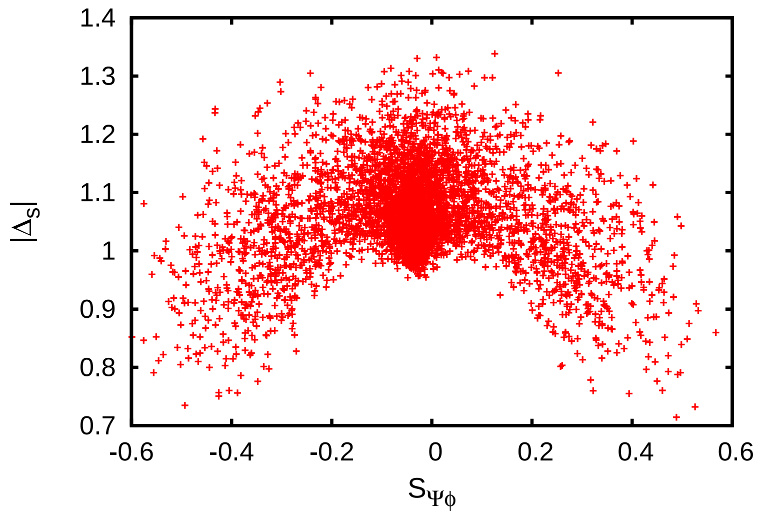
Similar effects on $S_{\psi\phi}$:
Soni et.al. PRD82 (2010),
Buras et.al. JHEP1009 (2010)

- Depending upon the constrain on $|\lambda_{sb}^{t'}|$, $S_{\psi\phi} = \sin\phi_s$ can reach up to ± 0.6..
- SM4 prediction: $\phi_s = \pm(0.29 \pm 0.23)$ (rad)
  $= \pm(0.18 \pm 0.23)$ (rad) ....work in progress

$\Rightarrow$ Allowed by the present data within the error bar..

.....Work in Progress (Alok, Dighe, London, Nandi)

B_s-$\bar{B}_s$ mixing: Size in SM4



......Nandi & Soni, PRD83 (2011)

$\Rightarrow M_{12}^s = M_{12}^{s,SM} |\Delta_s| e^{i\phi_s}$

$\Rightarrow$ In SM, $|\Delta_s| \approx 1$

$\Rightarrow m_{t'} = [375\text{GeV}, 575\text{GeV}]$...

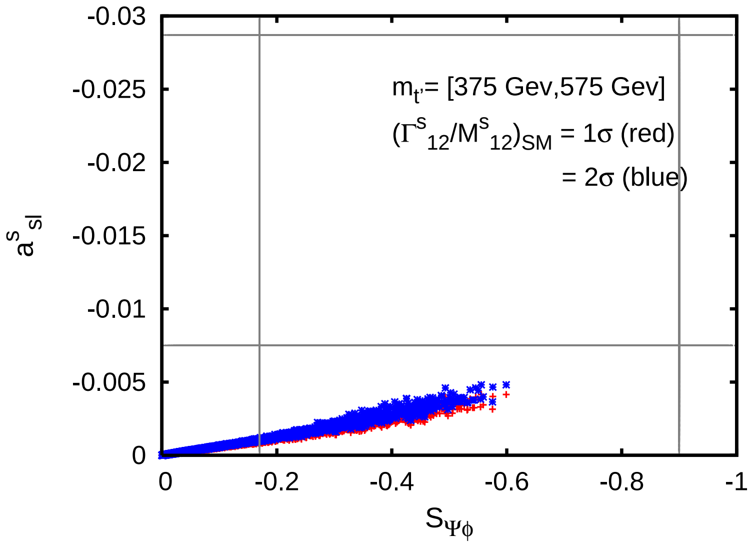
- Uncertainty in $|\Delta_s|$ is limited by the lattice calculations...
- $|\Delta_s|$ decrease with the increase of $|S_{\psi\phi}|$..
- SM4 prediction: $|\Delta_s| = 0.92 \pm 0.11$...$m_{t'} = 400$ GeV

  $= 0.95 \pm 0.12$ ...$m_{t'} = 575$ GeV

.....Work in Progress (Alok, Dighe, London, Nandi)

Correlation: a^s_{sl} vs $S_{\psi\phi}$

Soni et.al. PRD82 (2010), Buras et.al. JHEP1009 (2010), Nandi & Soni, PRD83 (2011)



.....Nandi & Soni, PRD83 (2011)

$\Rightarrow a^s_{sl} = \frac{\Gamma^{s,sm}_{12}}{M^{s,sm}_{12}} \frac{\sin\phi_s}{|\Delta_s|}$

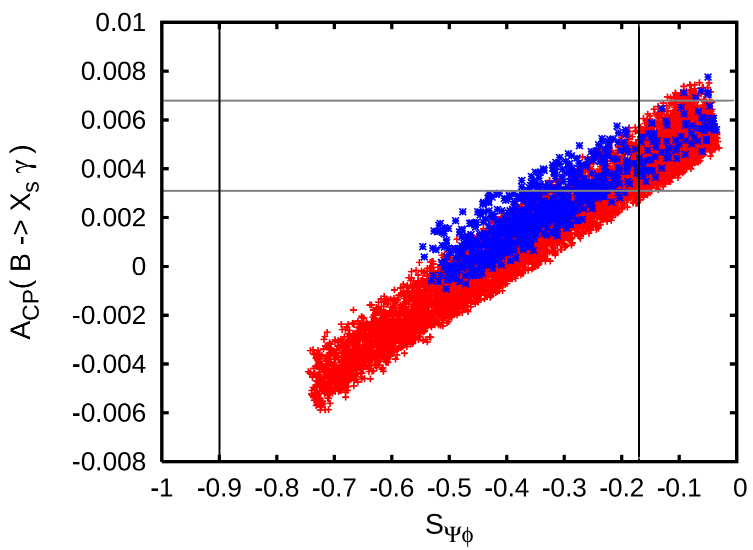
$\Rightarrow$ In SM, $\frac{\Gamma^{s,sm}_{12}}{M^{s,sm}_{12}} = (4.97 \pm 0.94) \times 10^{-3}$..

$\Rightarrow a^{s,sm}_{sl} = (2.1 \pm 0.6) \times 10^{-5}$ JHEP 0706, 072 (2007)

- $\Rightarrow A^b_{sl} = (-0.787 \pm 0.172 \pm 0.093)\%$ by DØ Col. 1101.6308 [hep-ex]
  $\Rightarrow$ 3.9σ deviation from SM prediction
- Extracted $a^s_{sl} = (-1.81 \pm 1.06)\%$
- SM4 prediction , $a^s_{sl} = \pm(0.002 \pm 0.001)$...$m_{t'} = 400$ GeV
  $= \pm(0.001 \pm 0.001)$ ...$m_{t'} = 575$ GeV
  .....Work in Progress (Alok, Dighe, London, Nandi)

Correlation: $S_{\psi\phi}$ and $A_{CP}(B \to X_s\gamma)$

Soni et.al, PRD82 (2010), Buras et.al, JHEP1009 (2010)



....Soni et.al, PRD (2010)

$$A_{CP}^{B\to X_s\gamma} = \frac{\Gamma(\bar{B} \to X_s\gamma) - \Gamma(B \to X_{\bar{s}}\gamma)}{\Gamma(\bar{B} \to X_s\gamma) + \Gamma(B \to X_{\bar{s}}\gamma)}$$

$$= \left(0.44^{+0.24}_{-0.13}\right)\%,\ SM(E_\gamma > 1.6\text{GeV})$$

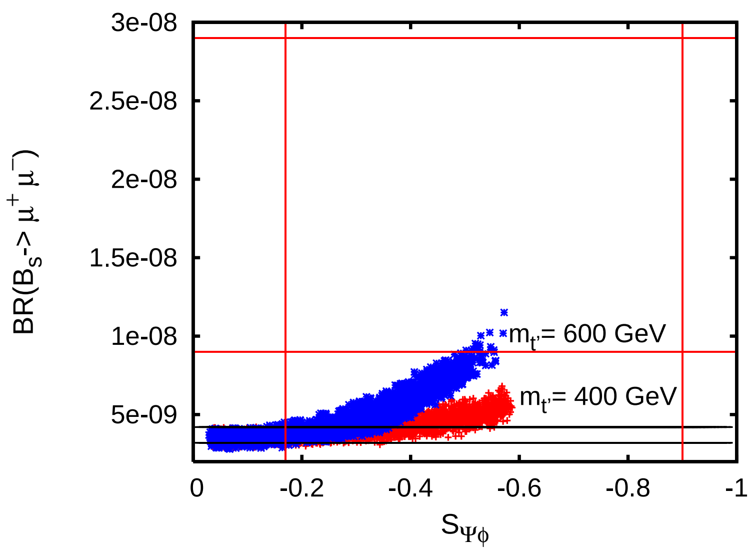
$...Hurth\ et.al\ (2003)$

- The current world average: $(-1.2 \pm 2.8)\%$ .....Barberio et.al (2008)
- Future Experiment: $0.4\% - 0.5\%$ Super-B....Browder et.al. (2008)

Obs	$m_{t'}$ =400 GeV		$m_{t'}$ =575 GeV	
	$\phi_s = 0.29 \pm 0.23$	-0.29 ± 0.23	0.18 ± 0.23	-0.18 ± 0.23
$A_{CP}^{B\to X_s\gamma}$ (%)	1.23 ± 0.51	-0.05 ± 0.50	0.89 ± 0.41	0.27 ± 0.40

Correlation: $S_{\psi\phi}$ Vs $Br(B_s \to \mu^+\mu^-)$

Soni et.al., PRD82 (2010), Buras et.al, JHEP1009 (2010)



.....Soni et.al, PRD (2010)

- $Br(B_s \to l^+l^-) \propto B_{bs} \frac{|C_{10}^{\rm tot}|^2 |\Delta'|}{\Delta M_s}$
- SM prediction: $(3.35 \pm 0.32) \times 10^{-9}$

  ..Buras, PLB566 (2003)

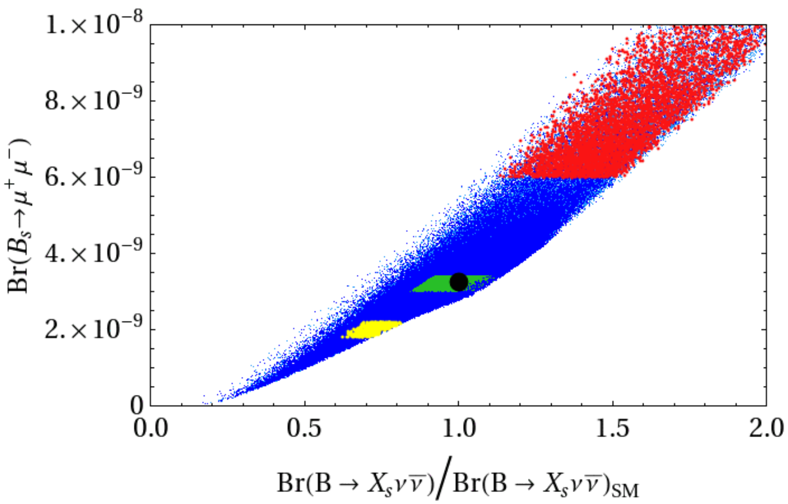
- LHCb bound: $Br(B_s \to \mu^+\mu^-) < 1.1 \times 10^{-8}$ 95% CL,...Talk by Raven at LPI (2011)
- In SM4, $Br(B_s \to \mu^+\mu^-)$ increases with the increase of $S_{\psi\phi}$
- SM4 predictions : $(3.47 \pm 1.92) \times 10^{-9}$ for $m_{t'} = 400\mathrm{GeV}$

  $(3.32 \pm 2.76) \times 10^{-9}$ for $m_{t'} = 575\mathrm{GeV}$

  .....Work in Progress (Alok, Dighe, London, Nandi)

Correlation: $Br(B_s \to \mu^+\mu^-)$ and $Br(B \to X_s \nu\bar{\nu})$

Soni et.al., PRD82 (2010), Buras et.al, JHEP1009 (2010)



.....Buras et.al, JHEP (2010)

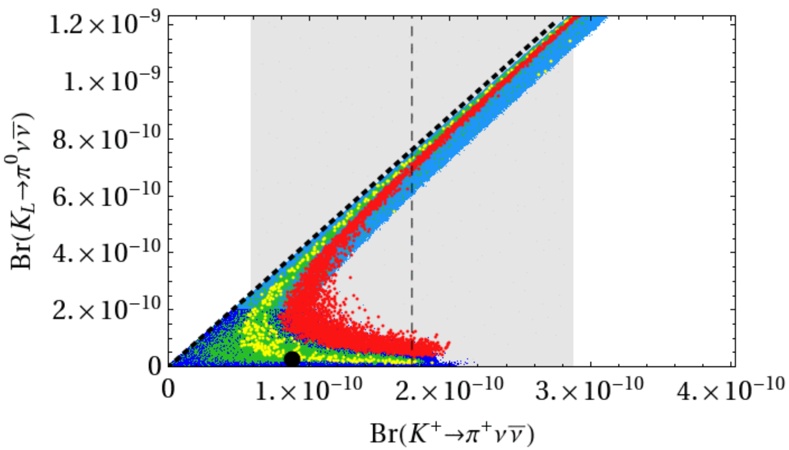
- $\frac{Br(B\to X_s\nu\bar{\nu})}{|\lambda^t_{sb}X_0(x_t)|^2} \propto \left|1 + \frac{\lambda^{t'}_{sb}}{\lambda^t_{sb}}\frac{X_0(x_{t'})}{X_0(x_t)}\right|^2$
- SM prediction: $(2.7 \pm 0.2) \times 10^{-5}$ JHEP04 (2009) 022

- For larger values of $S_{\psi\phi}$, an enhancement over the SM prediction is possible...
- Current experimental bound: $Br\,(B \to X_s \nu\bar{\nu}) < 6.4 \times 10^{-4}$
- SM4 predictions : $(2.04 \pm 0.66) \times 10^{-5}$ for $m_{t'} = 400\text{GeV}$
  $(2.04 \pm 0.95) \times 10^{-5}$ for $m_{t'} = 575\text{GeV}$

.....Work in Progress (Alok, Dighe, London, Nandi)

Correlation: $\mathbf{Br(K_L \to \pi^0 \nu\bar{\nu})}$ **Vs** $K^+\pi^+\nu\bar{\nu}$

Soni et.al., PRD82 (2010), Buras et.al, JHEP1009 (2010)



- $Br(K_L \to \pi^0 \nu\bar{\nu}) \propto$
  $\left[\left(\frac{Im\lambda_t}{\lambda^5} X(x_t) + \frac{Im\lambda_{t'}}{\lambda^5} X(x_{t'})\right)^2\right]$
- SM predictions:
  $Br(K_L \to \pi^0 \nu\bar{\nu}) = (2.8 \pm 0.6) \times 10^{-11}$
  $Br(K^+ \to \pi^+ \nu\bar{\nu}) = (8.5 \pm 0.7) \times 10^{-11}$
  ...Buras et.al, PRL95 (2005)

- $Br(K_L \to \pi^0 \nu\bar{\nu})$ increases with the increase of $Br(K^+ \to \pi^+ \nu\bar{\nu})$
  $\Rightarrow$ current SM4 allowed range $< 1.2 \times 10^{-8}$
- SM4 predictions : $(2.44 \pm 3.21) \times 10^{-11}$ for $m_{t'} = 400\mathrm{GeV}$
  $(3.22 \pm 6.04) \times 10^{-11}$ for $m_{t'} = 575\mathrm{GeV}$
  .....Work in Progress (Alok, Dighe, London, Nandi)

A_{CP} and A_{FB} in $b \to s\ell^+\ell^-$

Soni et.al., PRD82 (2010)

- Direct CP asymmetry in $B \to X_s\ell^+\ell^-$:
  - In the SM, $A_{CP}(B \to X_s\ell^+\ell^- \approx 0$ in the high-q^2 region
  - In SM4, $A_{CP}(B \to X_s\ell^+\ell^-)$ can be enhanced up to 1% and it is highly correlated with $S_{\psi\phi}$
  - Super-B can measure $A_{CP}(B \to X_s\ell^+\ell^-$ with an $\approx (1 \to 2)$% accuracy
  - No significant deviations from SM in the low-q^2 region..
- The Forward-Backward asymmetry $A_{FB}(q^2)$ in inclusive or exclusive $b \to s\ell^+\ell^-$ decay:
  - No significant deviations from SM....
  - Zero crossing of $A_{FB}(q^2)$ is also consistent with SM....

Conclusions

- Shortly after the discovery of third generation, a fourth generation was an obvious extension $\Rightarrow$ Consistent with Electroweak precision test
- Heavier quarks with masses $(\mathbf{400}-\mathbf{600})\ GeV$ can play a crucial role in dynamical electroweak-symmetry breaking...
- SM4 offers a simple explanation to many anomalies seen in B and B_s system..
- Several processes wherein SM4 causes large deviations from the expectations of SM3; for example $\mathbf{K_L} \to \pi^{\mathbf{0}}\nu\bar{\nu}$, $S_{\psi\phi}$, $A_{CP}(B \to X_s\gamma$... *e.t.c* $\Rightarrow$ may provide further indirect evidence for an additional family of quark
- Precise measurement of $S_{\psi\phi}$ or $Br(B_s \to \mu^+\mu^-)$ will improve our understanding of 4G effects in $b \to s$ transitions....

Thank you !