AMEND

Michael A. Schmidt

Institute for Particle Physics Phenomenology
Durham

16 Sep 2011

Flavour and the Fourth Family

based on:
Y. Farzan, S. Pascoli, MS [*JHEP* **10** (2010) 111] and work in progress

AMEND

A Model Explaining Neutrino masses and Dark matter

Michael A. Schmidt

Institute for Particle Physics Phenomenology
Durham

16 Sep 2011

Flavour and the Fourth Family

based on:
Y. Farzan, S. Pascoli, MS [*JHEP* **10** (2010) 111] and work in progress

Radiative Neutrino Masses and Dark Matter

Michael A. Schmidt

Institute for Particle Physics Phenomenology
Durham

16 Sep 2011

Flavour and the Fourth Family

based on:
Y. Farzan, S. Pascoli, MS [*JHEP* **10** (2010) 111] and work in progress

Radiative Neutrino Masses and Dark Matter

Michael A. Schmidt

Institute for Particle Physics Phenomenology
Durham

16 Sep 2011

Flavour and New Electroweak-Scale Particles

based on:
Y. Farzan, S. Pascoli, MS [*JHEP* **10** (2010) 111] and work in progress

Outline

Outline

Neutrino Masses and Leptonic Mixing

Global Fit to Neutrino Oscillations [Schwetz,Tortola,Valle (2011)]

	Best-fit	Allowed range (3σ)
$\sin^2\theta_{12}$	0.312	$0.27\dots0.36$
$\sin^2\theta_{23}$	0.52	$0.39\dots0.64$
$\sin^2\theta_{13}$	0.013	$0.001\dots0.035$ (n.h.)
	0.016	$0.001\dots0.039$ (i.h.)
Δm^2_{21} $[10^{-5}\,\text{eV}^2]$	7.59	$7.09\dots8.19$
Δm^2_{31} $[10^{-3}\,\text{eV}^2]$	2.50	$2.14\dots2.76$ (n.h.)
	-2.40	$-2.13\cdots-2.67$ (i.h.)

Neutrino Masses and Leptonic Mixing

Global Fit to Neutrino Oscillations [Schwetz,Tortola,Valle (2011)]

	Best-fit	Allowed range (3σ)
$\sin^2\theta_{12}$	0.312	$0.27\ldots0.36$
$\sin^2\theta_{23}$	0.52	$0.39\ldots0.64$
$\sin^2\theta_{13}$	0.013	$0.001\ldots0.035$ (n.h.)
	0.016	$0.001\ldots0.039$ (i.h.)
Δm^2_{21} $[10^{-5}\,\text{eV}^2]$	7.59	$7.09\ldots8.19$
Δm^2_{31} $[10^{-3}\,\text{eV}^2]$	2.50	$2.14\ldots2.76$ (n.h.)
	-2.40	$-2.13\cdots-2.67$ (i.h.)

Mass measurements

- Tritium end-point measurement $\sum_i |U_{ei}|^2 m_i^2 \leq (2.3\,\text{eV})^2$ (95% CL)

  [MAINZ experiment]

- Neutrinoless double beta decay $\sum_i U_{ei}^2 m_i < (0.35-1.05)\,\text{eV}$

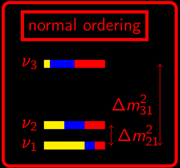
  [Heidelberg-Moscow, NEMO3, CUORICINO experiment]

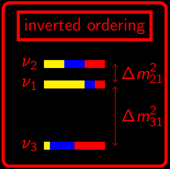
- Cosmology $\sum m_i \leq (0.44-1.5)\,\text{eV}$ [González-García, Maltoni, Salvado (2010)]

Neutrino Mass Generation

Open Questions

- Nature of neutrinos: Dirac vs. Majorana
- Absolute neutrino mass scale
- Mass ordering
- Only hint for third mixing angle θ_{13}
- CP Phases δ, φ_1, φ_2

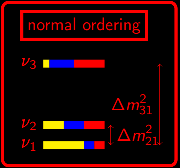


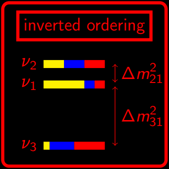


Neutrino Mass Generation

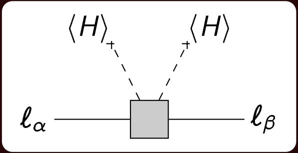
Open Questions

- Nature of neutrinos: Dirac vs. Majorana
- Absolute neutrino mass scale
- Mass ordering
- Only hint for third mixing angle θ_{13}
- CP Phases δ, φ_1, φ_2





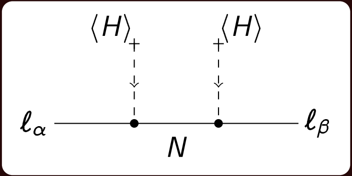
Weinberg operator



Assumption: Some underlying physics generates this effective operator and therefore leads to non-vanishing neutrino masses.

At Tree-Level: Seesaw Mechanism

Standard Seesaw [Minkowski;Yanagida;Glashow;Gell-Mann,Ramond,Slansky;Mohapatra,Senjanovic]



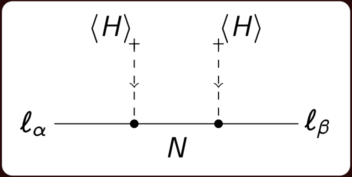
$$\mathcal{M} = \begin{pmatrix} 0 & m_D \\ \cdot & M_{NN} \end{pmatrix} \Rightarrow m_\nu \approx -m_D M_{NN}^{-1} m_D^T$$

$$\left(m_D \sim \mathcal{O}\left(\Lambda_{ew}\right)\ , \quad M_{NN} \sim \mathcal{O}\left(\Lambda_{B-L}\right)\right)$$

leads to effective mass of light neutrinos $m_\nu \lesssim \mathcal{O}\left(1\,\mathrm{eV}\right)$

At Tree-Level: Seesaw Mechanism

Standard Seesaw [Minkowski;Yanagida;Glashow;Gell-Mann,Ramond,Slansky;Mohapatra,Senjanovic]



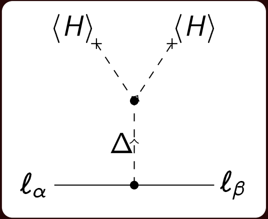
$$\mathcal{M} = \begin{pmatrix} 0 & m_D \\ \cdot & M_{NN} \end{pmatrix} \Rightarrow m_\nu \approx -m_D M_{NN}^{-1} m_D^T$$

$$(m_D \sim \mathcal{O}(\Lambda_{ew}) \ , \quad M_{NN} \sim \mathcal{O}(\Lambda_{B-L}))$$

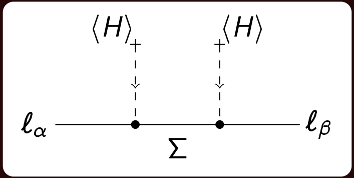
leads to effective mass of light neutrinos $m_\nu \lesssim \mathcal{O}(1\,\text{eV})$

Variants

Type II (scalar triplet) seesaw

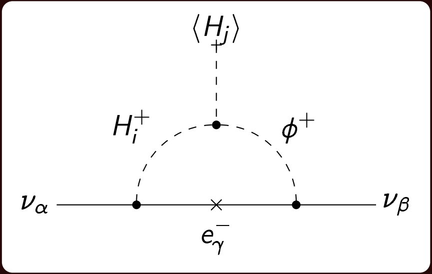


Type III (fermionic triplet) seesaw



Radiative Neutrino Mass Generation

Zee Model[Zee (1980)]

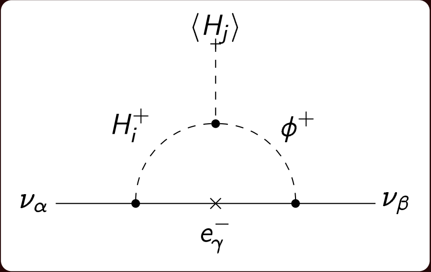


- 2 Higgs doublets H_j
- charged scalar ϕ^+

$$(m_\nu)_{\alpha\beta} \sim \frac{(Y_e^i m_e Y_\phi)_{\alpha\beta}}{16\pi^2} \frac{\langle H_j \rangle M_{jk}}{m_\phi^2} \left[\ln \frac{m_\phi^2}{m_H^2} \right]_{ki}$$

Radiative Neutrino Mass Generation

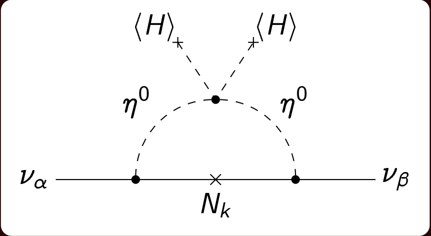
Zee Model[Zee (1980)]



- 2 Higgs doublets H_j
- charged scalar ϕ^+

$$(m_\nu)_{\alpha\beta} \sim \frac{(Y_e^i m_e Y_\phi)_{\alpha\beta}}{16\pi^2} \frac{\langle H_j \rangle M_{jk}}{m_\phi^2} \left[\ln \frac{m_\phi^2}{m_H^2} \right]_{ki}$$

Ma Model[Ma (2006)]



$$(m_\nu)_{\alpha\beta} = \sum_k \frac{Y_{\alpha k} Y_{\beta k} M_k}{16\pi^2} \left[\frac{m_R^2}{m_R^2 - M_k^2} \ln \frac{m_R^2}{M_k^2} - \frac{m_I^2}{m_I^2 - M_k^2} \ln \frac{m_I^2}{M_k^2} \right]$$

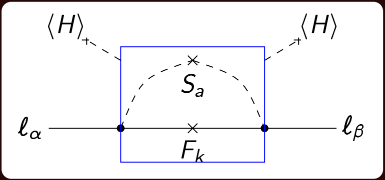
Non-exhaustive List of References

- A. Zee, *Phys. Lett.* **B93** (1980) 389.
- K. S. Babu, *Phys. Lett.* **B203** (1988) 132.
- A. Pilaftsis,*Z. Phys. C* **55** (1992) 275
- E. Ma, *Phys. Rev. Lett.* **81** (1998) 1171–1174.
- L. M. Krauss, S. Nasri, and M. Trodden, *Phys. Rev.* **D67** (2003) 085002.
- K. Cheung and O. Seto, *Phys. Rev.* **D69** (2004) 113009.
- T. Asaka, S. Blanchet, and M. Shaposhnikov, *Phys. Lett.* **B631** (2005) 151–156.
- E. Ma, *Phys. Rev.* **D73** (2006) 077301.
- J. Kubo, E. Ma, and D. Suematsu, *Phys. Lett.* **B642** (2006) 18–23.
- E. J. Chun and H. B. Kim, *JHEP* **10** (2006) 082.
- T. Hambye, K. Kannike, E. Ma, and M. Raidal, *Phys. Rev.* **D75** (2007) 095003.
- J. Kubo and D. Suematsu, *Phys. Lett.* **B643** (2006) 336–341.
- C. Boehm et al., *Phys. Rev.* **D77** (2008) 043516.
- M. Aoki, S. Kanemura, and O. Seto, *Phys. Rev. Lett.* **102** (2009) 051805.
- M. Aoki, S. Kanemura, and O. Seto, *Phys. Rev.* **D80** (2009) 033007.
- D. Aristizabal Sierra et al., *Phys. Rev.* **D79** (2009) 013011.
- D. Suematsu, T. Toma, and T. Yoshida, *Phys. Rev.* **D79** (2009) 093004.
- Y. Farzan, *Phys. Rev.* **D80** (2009) 073009.
- B. Bajc et al., *JHEP* **05** (2010) 048.
- Y. Farzan, S. Pascoli, and M. Schmidt, *JHEP* **10** (2010) 111.
- D. Suematsu, T. Toma, and T. Yoshida, *Phys. Rev.* **D82** (2010) 013012.
- D. Suematsu, and T. Toma, 1011.2839
- H. Higashi, T. Ishima, and D. Suematsu, 1101.2704
- . . .

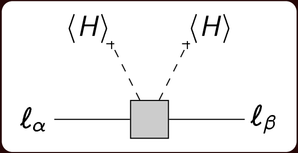
Outline

1. Introduction
2. Radiative Neutrino Mass Generation and Dark Matter
3. AMEND: A Model Explaining Neutrino masses and Dark matter
4. Conclusions and Outlook

A Closer Look at Ma-type Models



$\Rightarrow$

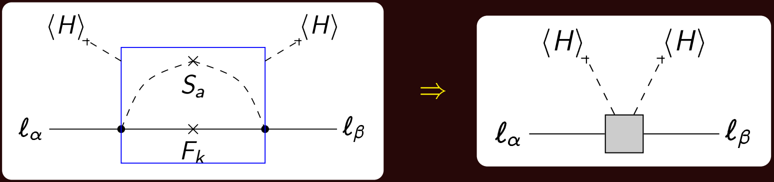


$$(m_\nu)_{\alpha\beta} \sim \sum_a \sum_k \frac{Y^a_{\alpha k} Y^a_{\beta k} M_k}{16\pi^2} \left[\frac{m_R^{a2}}{m_R^{a2} - M_k^2} \ln \frac{m_R^{a2}}{M_k^2} - \frac{m_I^{a2}}{m_I^{a2} - M_k^2} \ln \frac{m_I^{a2}}{M_k^2} \right]$$

Conditions for Majorana Neutrino Mass Term

- Coupling Lepton doublet ℓ, scalar S_a and fermion F_k is allowed

A Closer Look at Ma-type Models

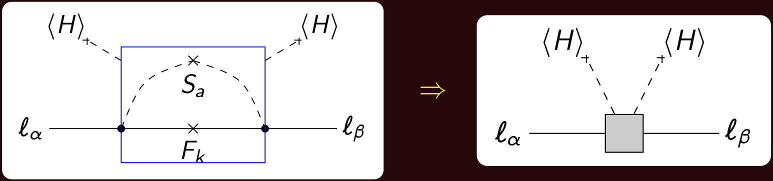


$$(m_\nu)_{\alpha\beta} \sim \sum_a \sum_k \frac{Y^a_{\alpha k} Y^a_{\beta k} M_k}{16\pi^2} \left[\frac{m_R^{a2}}{m_R^{a2} - M_k^2} \ln \frac{m_R^{a2}}{M_k^2} - \frac{m_I^{a2}}{m_I^{a2} - M_k^2} \ln \frac{m_I^{a2}}{M_k^2} \right]$$

Conditions for Majorana Neutrino Mass Term

- Coupling Lepton doublet ℓ, scalar S_a and fermion F_k is allowed
- Massive scalar S_a and fermion F_k in loop

A Closer Look at Ma-type Models

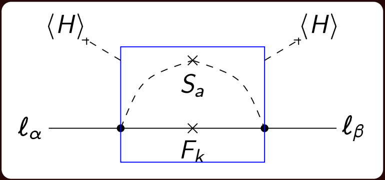


$$(m_\nu)_{\alpha\beta} \sim \sum_a \sum_k \frac{Y^a_{\alpha k} Y^a_{\beta k} M_k}{16\pi^2} \left[\frac{m_R^{a2}}{m_R^{a2} - M_k^2} \ln \frac{m_R^{a2}}{M_k^2} - \frac{m_I^{a2}}{m_I^{a2} - M_k^2} \ln \frac{m_I^{a2}}{M_k^2} \right]$$

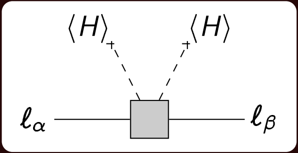
Conditions for Majorana Neutrino Mass Term

- Coupling Lepton doublet ℓ, scalar S_a and fermion F_k is allowed
- Massive scalar S_a and fermion F_k in loop
- Mass splitting between scalar and pseudoscalar in loop

A Closer Look at Ma-type Models



$\Rightarrow$

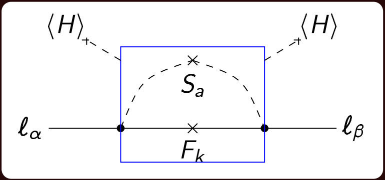


$$(m_\nu)_{\alpha\beta} \sim \sum_a \sum_k \frac{Y^a_{\alpha k} Y^a_{\beta k} M_k}{16\pi^2} \left[\frac{m_R^{a2}}{m_R^{a2} - M_k^2} \ln \frac{m_R^{a2}}{M_k^2} - \frac{m_I^{a2}}{m_I^{a2} - M_k^2} \ln \frac{m_I^{a2}}{M_k^2} \right]$$

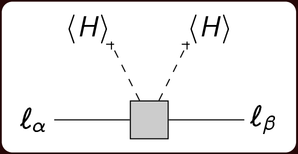
Conditions for Majorana Neutrino Mass Term

- Coupling Lepton doublet ℓ, scalar S_a and fermion F_k is allowed
- Massive scalar S_a and fermion F_k in loop
- Mass splitting between scalar and pseudoscalar in loop
- $\Delta L = 2$ lepton number violation: e.g. Majorana mass term for F_k

A Closer Look at Ma-type Models



$\Rightarrow$

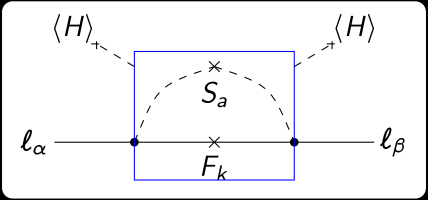


$$(m_\nu)_{\alpha\beta} \sim \sum_a \sum_k \frac{Y^a_{\alpha k} Y^a_{\beta k} M_k}{16\pi^2} \left[\frac{m_R^{a2}}{m_R^{a2} - M_k^2} \ln \frac{m_R^{a2}}{M_k^2} - \frac{m_I^{a2}}{m_I^{a2} - M_k^2} \ln \frac{m_I^{a2}}{M_k^2} \right]$$

Conditions for Majorana Neutrino Mass Term

- Coupling Lepton doublet ℓ, scalar S_a and fermion F_k is allowed
- Massive scalar S_a and fermion F_k in loop
- Mass splitting between scalar and pseudoscalar in loop
- $\Delta L = 2$ lepton number violation: e.g. Majorana mass term for F_k
- Generally discrete symmetry needed to forbid tree-level mass and/or avoid FCNCs $\Rightarrow$ lightest component of F_k, S_a is stable $\Rightarrow$ DM candidate

Framework

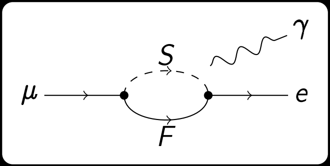


- Lepton number violation either in propagator or in vertex
- Models with discrete symmetry $\mathbb{Z}_2$ and coupling to lepton doublet ℓ.
- New particles F_k, S_a are odd under $\mathbb{Z}_2$ (SM particles even).
- Embed discrete symmetry into a continuous symmetry $\mathrm{U}(1)_X$ with $\mathrm{U}(1)_X \to \mathbb{Z}_2$.

	$\mathrm{U}(1)_X$	$\mathbb{Z}_2$
ℓ_α	0	+
F_k	1	-
S_a	-1	-

- Here, we only consider scalar DM. The scalar mass eigenstates are denoted by δ_i ordered by its mass with δ_1 being the lightest.
- It works analogously for fermionic DM.

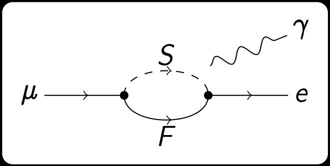
Lepton Flavour Violation



$$\Gamma(\mu \to e\gamma) = \frac{\alpha_{\rm em} m_\mu^5 |g_\mu g_e|^2}{(384\pi^2)^2 M_+^4} \left| I\left(\frac{M_0^2}{M_+^2}\right)\right|^2$$

- M_0 – mass of neutral part.
- M_+ – mass of charged part.
- $I(0) = 1$ and $I(t) \xrightarrow{t\to\infty} 0$

Lepton Flavour Violation



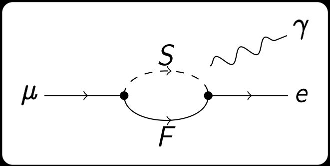
$$\Gamma(\mu \to e\gamma) = \frac{\alpha_{\rm em} m_\mu^5 |g_\mu g_e|^2}{(384\pi^2)^2 M_+^4} \left| I\left(\frac{M_0^2}{M_+^2}\right)\right|^2$$

Experimental Limits [PDG 2011, MEG 2011]

$$\mathrm{Br}(\mu \to e\gamma) < 2.4 \cdot 10^{-12}$$
$$\mathrm{Br}(\tau \to e\gamma) < 1.1 \cdot 10^{-7}$$
$$\mathrm{Br}(\tau \to \mu\gamma) < 4.5 \cdot 10^{-8}$$

- M_0 – mass of neutral part.
- M_+ – mass of charged part.
- $I(0) = 1$ and $I(t) \xrightarrow{t\to\infty} 0$

Lepton Flavour Violation



$$\Gamma(\mu \to e\gamma) = \frac{\alpha_{\rm em} m_\mu^5 |g_\mu g_e|^2}{(384\pi^2)^2 M_+^4} \left| I\left(\frac{M_0^2}{M_+^2}\right)\right|^2$$

Experimental Limits [PDG 2011, MEG 2011]

$$\mathrm{Br}(\mu \to e\gamma) < 2.4 \cdot 10^{-12}$$
$$\mathrm{Br}(\tau \to e\gamma) < 1.1 \cdot 10^{-7}$$
$$\mathrm{Br}(\tau \to \mu\gamma) < 4.5 \cdot 10^{-8}$$

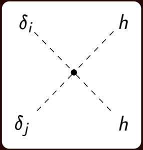
- M_0 – mass of neutral part.
- M_+ – mass of charged part.
- $I(0) = 1$ and $I(t) \xrightarrow{t\to\infty} 0$

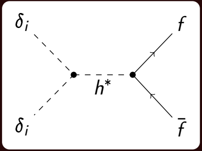
Bounds

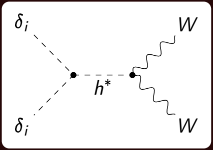
- $M_+/g \gtrsim 10\,\mathrm{TeV}$
- unless special flavour structure: $g_e \ll g_\mu$ (or $g_\mu \ll g_e$)

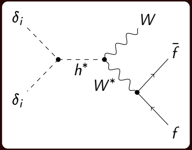
Dark Matter Annihilation Channels

Scalar Interactions



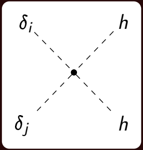


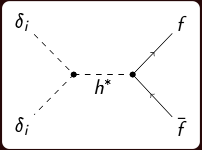


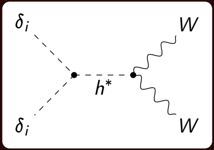


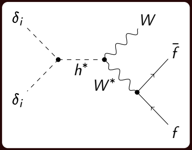
Dark Matter Annihilation Channels

Scalar Interactions

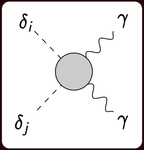


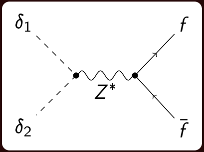


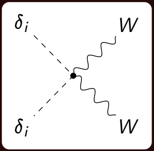


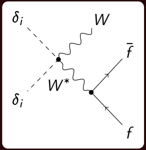


Gauge Interactions



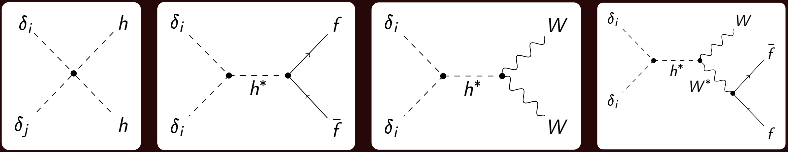




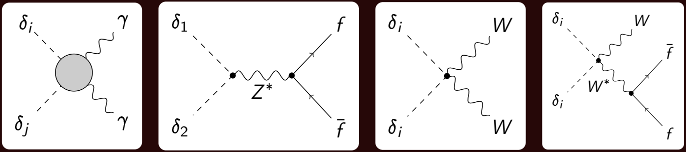


Dark Matter Annihilation Channels

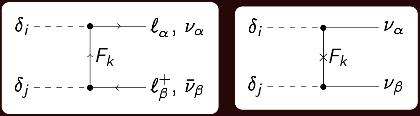
Scalar Interactions



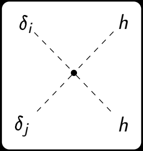
Gauge Interactions

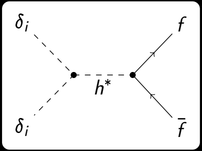


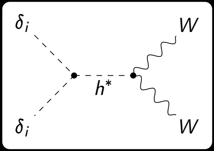
Fermion Exchange

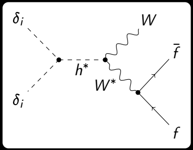


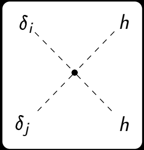
Dark Matter Annihilation (Scalar Interactions)



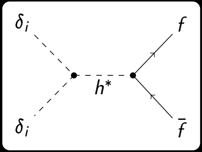






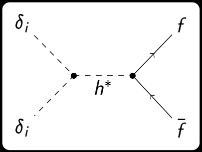


$$\Rightarrow \langle \sigma_{hh} v \rangle \simeq \frac{|\lambda_L|^2 (M_1^2 - m_h^2)^{1/2}}{16\pi M_1^3}$$



$$\Rightarrow \left\langle \sigma^{h}_{f\bar{f}} v \right\rangle \simeq N_c \frac{|\lambda_L|^2}{\pi} \frac{m_f^2}{(4\, M_1^2 - m_h^2)^2} \frac{(M_1^2 - m_f^2)^{3/2}}{M_1^3}$$

Dark Matter Annihilation (Scalar Interactions)



$$\Rightarrow \left\langle \sigma^h_{f\bar{f}} v \right\rangle \simeq N_c \frac{|\lambda_L|^2}{\pi} \frac{m_f^2}{(4\,M_1^2 - m_h^2)^2} \frac{(M_1^2 - m_f^2)^{3/2}}{M_1^3}$$

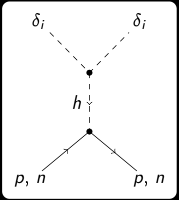
In general for Higgs mediated annihilation (for $m_h = 140\,\text{GeV}$, $M_1 = 75\,\text{GeV}$):

$$\langle \sigma(\delta_1 \delta_1 \to h^* \to \ldots) v \rangle = (2 m_h \Gamma(h \to \ldots))|_{m_h \to 2M_1} \frac{1}{4M_1^2} \frac{4|\lambda_L|^2 v_h^2}{(4M_1^2 - m_h^2)^2}$$

Using HiggsBounds: $\Gamma|_{2\times 75\,\text{GeV}} = 17.4\ \text{MeV} \Rightarrow \lambda_L \approx 0.037$

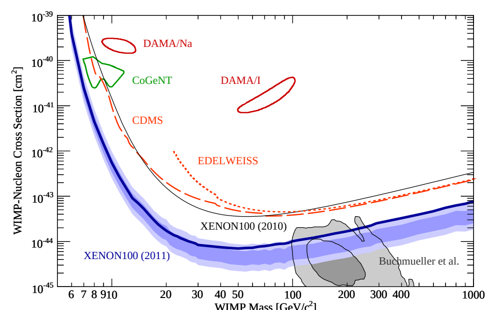
Direct DM Detection (eSI)

$$\frac{dR}{dE_R}(E_R,t) = \frac{\rho_\chi}{2M_1 m_r^2} A^2 \sigma_p F^2(E_R) \int_{v_{min}}^{v_{esc}} d^3 v \frac{f_{local}(\vec{v},t)}{v}$$



$$\sigma_p = \frac{|\lambda_L|^2}{\pi} \frac{\mu^2_{\delta_1 n} m_p^2}{M_1^2 m_h^4} f^2$$

$$\approx 7.0\times10^{-45} \left(\frac{\lambda_L}{0.037}\right)^2 \left(\frac{75\text{ GeV}}{M_1}\right)^2 \left(\frac{140\text{ GeV}}{m_h}\right)^4 \left(\frac{f}{0.3}\right)^2 \text{cm}^2$$



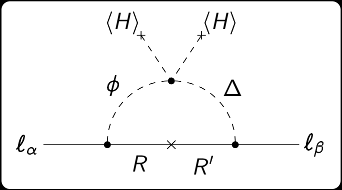
[XENON100 (2011)]

Outline

Model

Particle Content

	SU(2)	U(1)	U(1)$_X$	$\mathbb{Z}_2$
$\ell_L^{(i)}$	2	-1/2	0	+
R_R	2	-1/2	1	-
R'_R	2	1/2	-1	-
Δ	3	1	1	-
ϕ	1	0	-1	-



- Symmetry explanation for smallness of couplings U(1)$_X \to \mathbb{Z}_2$
- $\Rightarrow$ here explicit, spontaneous symmetry breaking also possible
- Symmetry protects smallness from large quantum corrections
- Lepton number violation in vertex

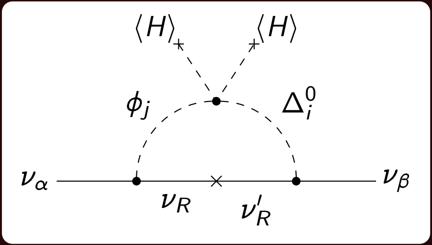
Fermion Sector

$$-m_{RR}(R'^C)^\dagger \cdot R - g_\alpha \phi^\dagger R^\dagger \ell_{L\alpha} - \tilde{g}_\alpha \phi R^\dagger \ell_{L\alpha} - (\tilde{g}_\Delta)_\alpha R'^\dagger \cdot \Delta \cdot \ell_{L\alpha} + \text{h. c.}$$

technical details

Neutrino Masses

One Loop Diagram Generating Neutrino Masses



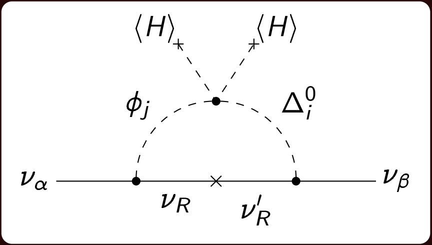
- neutral scalar mass eigenstates δ_i
- with scalar masses M_i
- α_1 mixing between $\delta_{1,3}$
- α_2 mixing between $\delta_{2,4}$

$$(m_\nu)_{\alpha\beta} = g_\alpha (\tilde{g}_\Delta)_\beta \tilde{\eta} + \tilde{g}_\alpha (\tilde{g}_\Delta)_\beta \eta + (\alpha \leftrightarrow \beta)$$

$$\eta = \eta(m_{RR}, M_i, \alpha_i) \qquad \tilde{\eta} = \tilde{\eta}(m_{RR}, M_i, \alpha_i)$$

Neutrino Masses

One Loop Diagram Generating Neutrino Masses



- neutral scalar mass eigenstates δ_i
- with scalar masses M_i
- α_1 mixing between $\delta_{1,3}$
- α_2 mixing between $\delta_{2,4}$

$$(m_\nu)_{\alpha\beta} = g_\alpha (\tilde{g}_\Delta)_\beta \tilde{\eta} + \tilde{g}_\alpha (\tilde{g}_\Delta)_\beta \eta + (\alpha \leftrightarrow \beta)$$

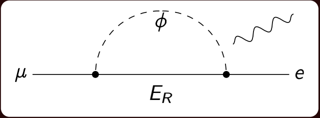
$$g\tilde{g}_\Delta \simeq 4.0 \times 10^{-6} \frac{m_\nu}{0.05 \text{ eV}} \frac{70 \text{ GeV}}{M_1} \frac{50 \text{ MeV}}{\delta} \frac{m_{RR}}{300 \text{ GeV}} \frac{0.1}{|\sin\alpha_1|} \left(\frac{m_{RR}^2}{m_{RR}^2 - m_\Delta^2} \ldots \right)^{-1}$$

$$g\tilde{g}_\Delta \simeq 4.5 \times 10^{-6} \frac{m_\nu}{0.05 \text{ eV}} \frac{300 \text{ GeV}}{m_{RR}} \frac{1 \text{ GeV}^2}{\tilde{m}_{\phi\Delta}^2} \left(\frac{m_\Delta}{500 \text{ GeV}} \right)^2 \frac{m_{RR}^2 - m_\Delta^2}{m_\Delta^2} \left(\log \frac{m_{RR}^2}{m_\Delta^2} \right)^{-1}$$

$$\tilde{g}\tilde{g}_\Delta \simeq 1.8 \times 10^{-10} \frac{m_\nu}{0.05 \text{ eV}} \frac{300 \text{ GeV}}{m_{RR}} \frac{0.1}{\sin\alpha_1} \frac{m_{RR}^2 - m_\Delta^2}{m_\Delta^2} \left(\log \frac{m_{RR}^2}{m_\Delta^2} \right)^{-1}$$

Lepton Flavour Violation and $(g\text{-}2)_\mu$

Lepton Flavour Violation



$$\mathrm{Br}(\mu \to e\gamma) = 2.5 \cdot 10^{-9} \left(\frac{300\ \mathrm{GeV}}{m_{RR}}\right)^4 \left|\frac{g_\mu^*}{0.1}\frac{g_e}{0.1}\right|^2$$

$$\mathrm{Br}(\tau \to \alpha\gamma) = 4.5 \cdot 10^{-10} \left(\frac{300\ \mathrm{GeV}}{m_{RR}}\right)^4 \left|\frac{g_\tau^*}{0.1}\frac{g_\alpha}{0.1}\right|^2$$

Experimental Limits [PDG 2011, MEG 2011]

$$\mathrm{Br}(\mu \to e\gamma) < 2.4 \cdot 10^{-12}$$

$$\mathrm{Br}(\tau \to e\gamma) < 1.1 \cdot 10^{-7}$$

$$\mathrm{Br}(\tau \to \mu\gamma) < 4.5 \cdot 10^{-8}$$

Solutions

- $m_{RR}/g \gtrsim 10$ TeV
- $g_e \ll g_\mu$ (or $g_\mu \ll g_e$) (allowed by flavour structure)

Anomalous Magnetic Moment of Muon

$$\delta(g-2)_\mu/2 \sim 10^{-11} \left(\frac{300\ \ \mathrm{GeV}}{m_{RR}}\right)^4 |g_\mu|^2 \lesssim \text{exp. uncertainty}$$

Electroweak Precision Tests

Fermionic Doublets[Maekawa (1995); Cynolter, Lendvai (2008)]

$$\hat{S} \simeq 0 \qquad \hat{T} \simeq 0$$

$$W \simeq \frac{g^2_{\mathrm{SU}(2)}}{120\pi^2}\frac{m_W^2}{m_{RR}^2} \qquad Y \simeq \frac{g^2_{\mathrm{U}(1)}}{120\pi^2}\frac{m_W^2}{m_{RR}^2}$$

Higgs Triplet

$$\hat{S} \simeq \frac{g^2_{\mathrm{SU}(2)}}{24\pi^2}\xi \qquad \hat{T} \simeq \frac{25 g^2_{\mathrm{SU}(2)}}{576\pi^2}\frac{m_\Delta^2}{m_W^2}\xi^2$$

$$W \simeq -\frac{7 g^2_{\mathrm{SU}(2)}}{720\pi^2}\frac{m_W^2}{m_\Delta^2} \qquad Y \simeq -\frac{7 g^2_{\mathrm{U}(1)}}{480\pi^2}\frac{m_W^2}{m_\Delta^2}$$

with $\xi := (m^2_{\Delta^{++}} - m^2_\Delta)/m^2_\Delta$ and $m^2_{\Delta^{++}} = m^2_\Delta + 2m^2_{\Delta^+}$

	$10^3\,\hat{S}$	$10^3\,\hat{T}$	$10^3\,W$	$10^3\,Y$
$m_h = 115$ GeV	0.0 ± 24.0	0.6 ± 5.0	-2.2 ± 4.4	6.1 ± 73.6

[Barbieri, Pomarol, Rattazi, Strumia (2004); Cacciapaglia, Csaki, Marandella, Strumia (2006)]

Invisible Z Decay Width

If DM particle δ_1 couples to Z-boson and $M_1 + M_2 < m_Z$:

$$\Gamma(Z \to \delta_1 \delta_2) = \frac{G_F \sin^2 \alpha_1 \sin^2 \alpha_2}{6\sqrt{2}\pi} m_Z^3$$

$\Rightarrow$ Bound on mixing angles: $\sin \alpha_1 \sin \alpha_2 \lesssim 0.07$

Collider Physics

Higgs Search

- Higgs might decay dominantly invisibly if $2\,M_1 < m_h$

$$H \to \delta_1 \delta_1\,, \quad H \to \delta_2 \delta_2 \to (\delta_1 \nu \bar{\nu})(\delta_1 \nu \bar{\nu})$$

Collider Physics

Higgs Search

- Higgs might decay dominantly invisibly if $2\,M_1 < m_h$

$$H \to \delta_1 \delta_1\,, \quad H \to \delta_2 \delta_2 \to (\delta_1 \nu \bar{\nu})(\delta_1 \nu \bar{\nu})$$

- Displaced vertex if there is a large mass splitting between $\delta_{1,2}$:
  For $\delta_2 \to \delta_1 \mu^+ \mu^-$ in ATLAS Muon detector: $M_2 - M_1 \gtrsim 480\,\mathrm{MeV} (\gamma v)^{1/5}$

Collider Physics

Higgs Search

- Higgs might decay dominantly invisibly if $2\,M_1 < m_h$

$$H \to \delta_1\delta_1\,, \quad H \to \delta_2\delta_2 \to (\delta_1\nu\bar{\nu})(\delta_1\nu\bar{\nu})$$

- Displaced vertex if there is a large mass splitting between $\delta_{1,2}$:
  For $\delta_2 \to \delta_1\mu^+\mu^-$ in ATLAS Muon detector: $M_2 - M_1 \gtrsim 480\,\mathrm{MeV}(\gamma v)^{1/5}$

New Particles

- New particles couple dominantly to leptons
  $\Rightarrow$ although accessible, more difficult to produce at the LHC

Collider Physics

Higgs Search

- Higgs might decay dominantly invisibly if $2\,M_1 < m_h$

$$H \to \delta_1\delta_1\,, \quad H \to \delta_2\delta_2 \to (\delta_1\nu\bar{\nu})(\delta_1\nu\bar{\nu})$$

- Displaced vertex if there is a large mass splitting between $\delta_{1,2}$:
  For $\delta_2 \to \delta_1\mu^+\mu^-$ in ATLAS Muon detector: $M_2 - M_1 \gtrsim 480\,\mathrm{MeV}(\gamma v)^{1/5}$

New Particles

- New particles couple dominantly to leptons
  $\Rightarrow$ although accessible, more difficult to produce at the LHC
- They decay into the SM particles and DM $\Rightarrow$ missing energy

Collider Physics

Higgs Search

- Higgs might decay dominantly invisibly if $2\,M_1 < m_h$

$$H \to \delta_1\delta_1\,, \quad H \to \delta_2\delta_2 \to (\delta_1\nu\bar{\nu})(\delta_1\nu\bar{\nu})$$

- Displaced vertex if there is a large mass splitting between $\delta_{1,2}$:
  For $\delta_2 \to \delta_1\mu^+\mu^-$ in ATLAS Muon detector: $M_2 - M_1 \gtrsim 480\,\mathrm{MeV}(\gamma v)^{1/5}$

New Particles

- New particles couple dominantly to leptons
  $\Rightarrow$ although accessible, more difficult to produce at the LHC
- They decay into the SM particles and DM $\Rightarrow$ missing energy
- Measurement of Yukawa couplings via decays of charged particles, e.g.
  $\Gamma(E_R^- \to \ell_\alpha^-\delta_{1,2}) \propto |g_\alpha|^2$ and $\Gamma(\Delta^{++} \to \ell_\alpha^+\ell_\beta^+\delta_{1,2}) \propto |(\tilde{g}_\Delta)_\alpha g_\beta + (\tilde{g}_\Delta)_\beta g_\alpha|^2$

Collider Physics

Higgs Search

- Higgs might decay dominantly invisibly if $2\,M_1 < m_h$

$$H \to \delta_1\delta_1\,, \quad H \to \delta_2\delta_2 \to (\delta_1\nu\bar{\nu})(\delta_1\nu\bar{\nu})$$

- Displaced vertex if there is a large mass splitting between $\delta_{1,2}$:
For $\delta_2 \to \delta_1\mu^+\mu^-$ in ATLAS Muon detector: $M_2 - M_1 \gtrsim 480\,\mathrm{MeV}(\gamma v)^{1/5}$

New Particles

- New particles couple dominantly to leptons
$\Rightarrow$ although accessible, more difficult to produce at the LHC
- They decay into the SM particles and DM $\Rightarrow$ missing energy
- Measurement of Yukawa couplings via decays of charged particles, e.g.
$\Gamma(E_R^- \to \ell_\alpha^-\delta_{1,2}) \propto |g_\alpha|^2$ and $\Gamma(\Delta^{++} \to \ell_\alpha^+\ell_\beta^+\delta_{1,2}) \propto |(\tilde{g}_\Delta)_\alpha g_\beta + (\tilde{g}_\Delta)_\beta g_\alpha|^2$
- Mass relation of triplet $2m_{\Delta^+}^2 = m_{\Delta^{++}}^2 + m_\Delta^2$ testable

Outline

Conclusions

- Origin of neutrino masses from TeV scale physics and linked to dark matter.

Conclusions

- Origin of neutrino masses from TeV scale physics and linked to dark matter.
- This usually implies a discrete symmetry which leads DM candidate.

Conclusions

- Origin of neutrino masses from TeV scale physics and linked to dark matter.
- This usually implies a discrete symmetry which leads DM candidate.
- These models are testable.

Conclusions

- Origin of neutrino masses from TeV scale physics and linked to dark matter.
- This usually implies a discrete symmetry which leads DM candidate.
- These models are testable.
- The interplay of different experiments imposes strong constraints: DM detection, LFV, collider, . . .

Conclusions

- Origin of neutrino masses from TeV scale physics and linked to dark matter.
- This usually implies a discrete symmetry which leads DM candidate.
- These models are testable.
- The interplay of different experiments imposes strong constraints: DM detection, LFV, collider, . . .
- At the moment, the strongest ones are from LFV rare decays.

Conclusions

- Origin of neutrino masses from TeV scale physics and linked to dark matter.
- This usually implies a discrete symmetry which leads DM candidate.
- These models are testable.
- The interplay of different experiments imposes strong constraints: DM detection, LFV, collider, . . .
- At the moment, the strongest ones are from LFV rare decays.
- This mechanism also works with a fourth generation.
  So, the talk might be justified in this context:

  Flavour and the Fourth Family

Conclusions

- Origin of neutrino masses from TeV scale physics and linked to dark matter.
- This usually implies a discrete symmetry which leads DM candidate.
- These models are testable.
- The interplay of different experiments imposes strong constraints: DM detection, LFV, collider, . . .
- At the moment, the strongest ones are from LFV rare decays.
- This mechanism also works with a fourth generation.
  So, the talk might be justified in this context:

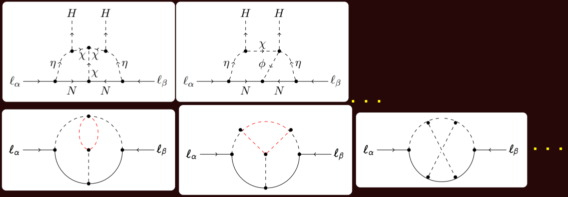
  > Flavour and the Fourth Family

  Neutrino Masses and a Fourth Generation of Fermions

  [M. Lindner, MS, A. Smirnov (Sep. 2011)]

Outlook

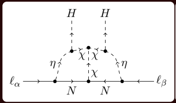
- Generalization of results [with Y. Farzan, S. Pascoli]
  $\rightarrow$ different groups, gauged group, higher loop orders, particle content

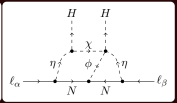


- Symmetry is larger: $\mathbb{Z}_m$, $m > 2$, U(1), non-Abelian
- Possibly multi-component DM [see e.g. Batell(2010); Adulpravitchai, Batell, Pradler (2011)]
- DM annihilation more involved: e.g. $\chi\chi \rightarrow \chi h$ [D'Eramo, Thaler (2010)]
- LFV rare decays still at one loop level $\Rightarrow$ flavour symmetry needed

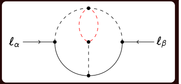
Outlook

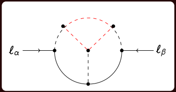
- Generalization of results [with Y. Farzan, S. Pascoli]
  $\to$ different groups, gauged group, higher loop orders, particle content

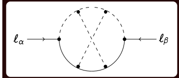




...







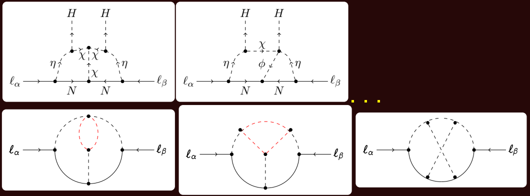
...

- Symmetry is larger: $\mathbb{Z}_m$, $m > 2$, U(1), non-Abelian
- Possibly multi-component DM [see e.g. Batell(2010); Adulpravitchai, Batell, Pradler (2011)]
- DM annihilation more involved: e.g. $\chi\chi \to \chi h$ [D'Eramo, Thaler (2010)]
- LFV rare decays still at one loop level $\Rightarrow$ flavour symmetry needed

- Flavour symmetries [with M. Holthausen]

Outlook

- Generalization of results [with Y. Farzan, S. Pascoli]
  $\rightarrow$ different groups, gauged group, higher loop orders, particle content



. . .

. . .

- Symmetry is larger: $\mathbb{Z}_m$, $m > 2$, U(1), non-Abelian
- Possibly multi-component DM [see e.g. Batell(2010); Adulpravitchai, Batell, Pradler (2011)]
- DM annihilation more involved: e.g. $\chi\chi \rightarrow \chi h$ [D'Eramo, Thaler (2010)]
- LFV rare decays still at one loop level $\Rightarrow$ flavour symmetry needed

- Flavour symmetries [with M. Holthausen]
- Study of collider signatures

Discussion: Experimental Tests

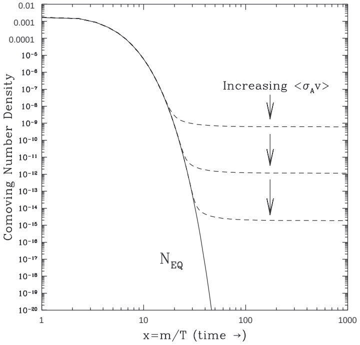
1. LFV processes
   - At one loop level unless there is a flavour symmetry
   - Strongest bound today from $\mu \to e\gamma$
   - What are the experimental prospects for further constraints?
   - What about other LFV processes like τ decays?
2. DM experiments
   - Direct detection experiments: future prospects? Can the current low mass DM hints be explained?
   - In future: constraints from indirect detection experiments
   - High-energy neutrino flux from sun. Further improvements?
3. Collider searches:
   - What branching fraction is acceptable for invisible Higgs decay?
   - New particles couple to EW gauge bosons
   - Decay channels: directly into leptons plus missing energy or via EW gauge bosons
   - What is the potential reach of the LHC for these particles?
   - Can the LHC exclude these models in combination with the other experiments?

Thank you very much for your attention.

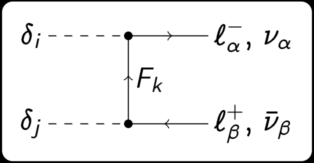
Basics of Dark Matter Freeze Out

- Assumption: thermal production after inflation
- Annihilation rate related to production rate
- Quasi-degenerate scalar masses because of approximate $U(1)_X$ $\Rightarrow$ both species have to be considered
- If $\sigma_{12} \ll \sigma_{11}, \sigma_{22} \Rightarrow \delta_1$ and δ_2 produced and later $\delta_2 \to \delta_1 \nu \bar{\nu}$
- $\sum_{i=1}^{2} \langle \sigma(\delta_i \delta_i \to \ldots) v \rangle = 3 \cdot 10^{-26} \ \frac{\mathrm{cm}^3}{\mathrm{sec}}$

Thermal freezeout one species



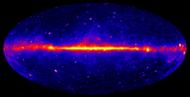
Dark Matter Annihilation (Fermion Exchange)



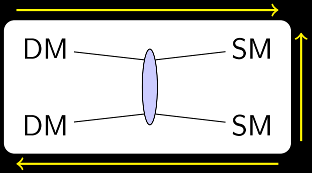
$$\left\langle \sigma(\delta_i\delta_j \to \ell_\alpha^- \ell_\beta^+, \nu_\alpha \bar{\nu}_\beta) v \right\rangle \simeq \frac{|g_\alpha g_\beta|^2}{32\pi} \frac{(m_\alpha^2 + m_\beta^2)}{(M_i M_j + m_F^2)^2} \frac{(M_i + M_j)^2}{M_i M_j}$$

- $g_{\alpha,\beta}$ – Yukawa couplings
- $m_{\alpha,\beta}$ – final state fermion masses
- Dominant annihilation into $\tau^+\tau^-$ pair (for similar Yukawa couplings)
- One possibility to obtain a dominant annihilation into leptons (PAMELA, FERMI, ...)

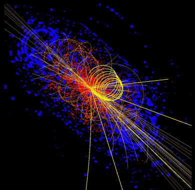
Detection of WIMP Dark Matter

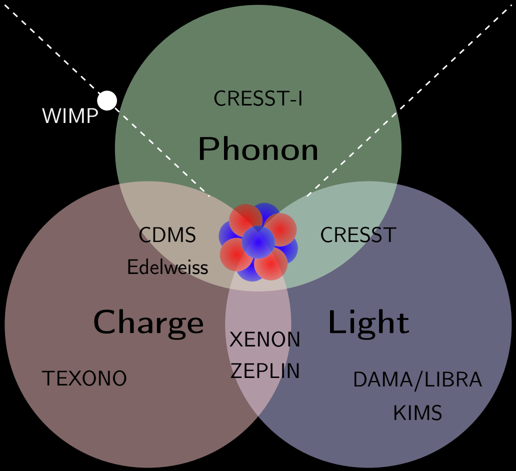


indirect detection: $\gamma, \nu, e^+, \ldots$



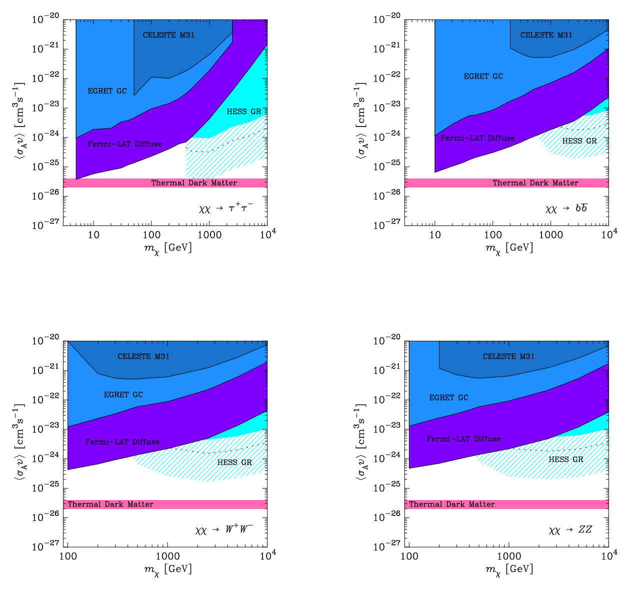
collider searches



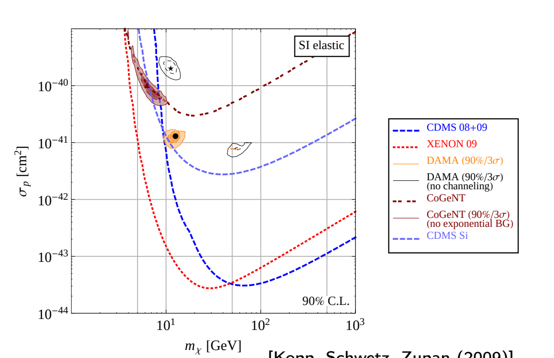


direct detection

Constraints from Isotropic Diffuse γ-ray Background[Abazajian, Agrawal, Chacko, Kilic (2010)]



Elastic SI Scattering – CoGeNT/DAMA claim



[Kopp, Schwetz, Zupan (2009)]

- $7\,\mathrm{GeV} \lesssim M_1 \lesssim 11\,\mathrm{GeV}$ with $\sigma_n \sim 10^{-41}\ \mathrm{cm}^2$–$10^{-40}\ \mathrm{cm}^2$

$$\sigma_n \approx 1.3 \times 10^{-40} \left(\frac{f}{0.3}\right)^2 \left(\frac{8\,\mathrm{GeV}}{M_1}\right)^2 \mathrm{cm}^2$$

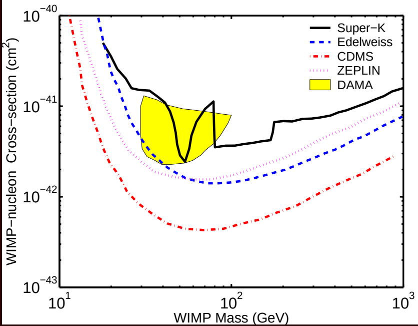
- Might also explain DAMA for intermediate channelling

Disclaimer: A proper analysis is required to make definite statements!

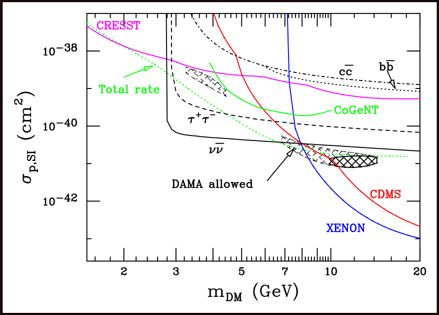
Light WIMPs – Neutrinos From Sun

- Number of WIMPs: $\dot{N} = C - AN^2 - EN$
- Capture rate
  $C(\rho_{DM}, \bar{v}, m_{DM}, \sigma) \simeq 1.3 \cdot 10^{25}\mathrm{sec}^{-1}$" $\propto \rho_{DM}\sigma\,\bar{v}^{-1}\, m_{DM}^{-1}$"
- Annihilation Rate $A = \langle \sigma v \rangle / V_{eff}$
- Evaporation rate $E \approx 10^{-(\frac{7}{2}(m_{DM}/\,\mathrm{GeV})+4)} \left(\frac{\sigma_H}{5 \cdot 10^{39}\mathrm{cm}^2} \right) \mathrm{sec}^{-1}$

Bounds



[SuperKamiokande (2004)]



[Hooper,Petriello, Zurek, Kamionkowski (2008)]

Particle Content and Symmetries

	$SU(3)_c$	$SU(2)_L$	$U(1)_Y$	$U(1)_X[\mathbb{Z}_2]$	$U(1)_{L1}$	$U(1)_{L2}$	$U(1)_{L3}$
$Q_L^{(i)}$	3	2	1/6				
$u_R^{(i)}$	3	1	2/3				
$d_R^{(i)}$	3	1	-1/3				
$\ell_L^{(i)}$	1	2	-1/2	0[+]	+1	-1	+1
$e_R^{(i)}$	1	1	-1	0[+]	-1	+1	-1
H	1	2	1/2				
R_R	1	2	-1/2	1[-]	+1	+1	+1
R'_R	1	2	1/2	-1[-]	-1	-1	-1
Δ	1	3	1	1[-]	0	0	-2
ϕ	1	1	0	-1[-]	0	0	0

Yukawa Couplings

Particle Content

	$\mathrm{U}(1)_X$	$\mathbb{Z}_2$	$\mathrm{U}(1)_{L1}$	$\mathrm{U}(1)_{L2}$	$\mathrm{U}(1)_{L3}$
$\ell_L^{(i)}$	0	+	+1	-1	+1
R_R	1	-	+1	+1	+1
R'_R	-1	-	-1	-1	-1
Δ	1	-	0	0	-2
ϕ	-1	-	0	0	0

- Dirac mass $m_{RR} \gtrsim 100\,\mathrm{GeV}$

$$-m_{RR}(R'^C)^\dagger \cdot R - g_\alpha \phi^\dagger R^\dagger \ell_{L\alpha} - \tilde{g}_\alpha \phi R^\dagger \ell_{L\alpha} - (\tilde{g}_\Delta)_\alpha R'^\dagger \cdot \Delta \cdot \ell_{L\alpha} + \mathrm{h.\,c.}$$

- $\not{L}_2$
- $\not{L}_2$, $U(1)_X$ breaking
- $\not{L}_1$, $U(1)_X$ breaking

Higgs Potential

Particle Content

	$U(1)_X$	$\mathbb{Z}_2$	$U(1)_{L1}$	$U(1)_{L2}$	$U(1)_{L3}$
Δ	1	-	0	0	-2
ϕ	-1	-	0	0	0

- DM coannihilation into SM Higgs boson h
- Direct mass terms

$$\mathcal{V} \supset m_\Delta^2 \, \mathrm{Tr}\, \Delta^\dagger \Delta + m_\phi^2 \phi^\dagger \phi + \lambda_L \frac{v_H}{\sqrt{2}} h \left(\delta_1^2 + \delta_2^2\right)$$

$$+ \frac{2 m_{\phi\Delta}^2}{v_H^2} H^T \, \mathrm{i}\, \sigma_2 \Delta^\dagger H \phi^\dagger + \frac{2 \tilde{m}_{\phi\Delta}^2}{v_H^2} H^T \, \mathrm{i}\, \sigma_2 \Delta^\dagger H \phi + \tilde{m}_\phi^2 \phi^2 + \mathrm{h.\,c.}$$

- $\not{L}_3$, $U(1)_\phi \times U(1)_\Delta \to U(1)_X$
- $\not{L}_3$, $U(1)_X \to \mathbb{Z}_2$, mixing ϕ and Δ
- $U(1)_X \to \mathbb{Z}_2$, mass splitting of $\mathrm{Re}(\phi)$ and $\mathrm{Im}(\phi)$

Neutral Scalar Masses

$$\phi = (\phi_1 + \mathrm{i}\ \phi_2)/\sqrt{2} \qquad\qquad \Delta^0 = (\Delta_1 + \mathrm{i}\ \Delta_2)/\sqrt{2}$$

$$\begin{pmatrix} \delta_1 \\ \delta_2 \\ \delta_3 \\ \delta_4 \end{pmatrix} = \begin{pmatrix} \cos\alpha_1 & 0 & \sin\alpha_1 & 0 \\ 0 & \cos\alpha_2 & 0 & \sin\alpha_2 \\ -\sin\alpha_1 & 0 & \cos\alpha_1 & 0 \\ 0 & -\sin\alpha_2 & 0 & \cos\alpha_2 \end{pmatrix} \begin{pmatrix} \phi_1 \\ \phi_2 \\ \Delta_1 \\ \Delta_2 \end{pmatrix}$$

$$M_1^2 \simeq m_\phi^2 - \frac{m_{\phi\Delta}^4}{m_\Delta^2 - m_\phi^2} - \tilde{m}_\phi^2 - 2\frac{m_{\phi\Delta}^2}{m_\Delta^2 - m_\phi^2}\tilde{m}_{\phi\Delta}^2$$

$$M_2^2 \simeq m_\phi^2 - \frac{m_{\phi\Delta}^4}{m_\Delta^2 - m_\phi^2} + \tilde{m}_\phi^2 + 2\frac{m_{\phi\Delta}^2}{m_\Delta^2 - m_\phi^2}\tilde{m}_{\phi\Delta}^2$$

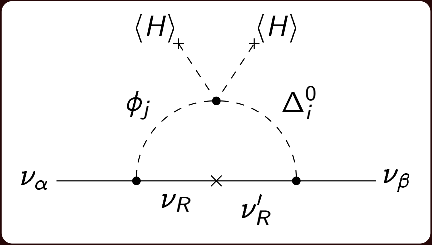
$$M_3^2 \simeq m_\Delta^2 + 2\frac{m_{\phi\Delta}^2}{m_\Delta^2 - m_\phi^2}\tilde{m}_{\phi\Delta}^2$$

$$M_4^2 \simeq m_\Delta^2 - 2\frac{m_{\phi\Delta}^2}{m_\Delta^2 - m_\phi^2}\tilde{m}_{\phi\Delta}^2$$

mixing angles: $|\tan 2\alpha_1| \simeq |\tan 2\alpha_2| \simeq 2m_{\phi\Delta}^2/(m_\Delta^2 - m_\phi^2)$

Neutrino Masses (Technical Details)

One Loop Diagram Generating Neutrino Masses



- neutral scalar mass eigenstates δ_i
- with scalar masses M_i
- α_1 mixing between $\delta_{1,3}$
- α_2 mixing between $\delta_{2,4}$

back

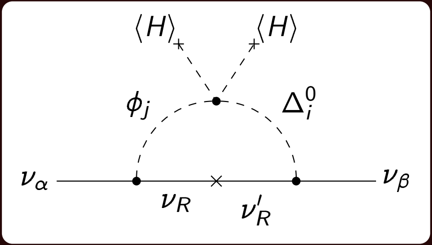
$$(m_\nu)_{\alpha\beta} = g_\alpha (\tilde{g}_\Delta)_\beta \tilde{\eta} + \tilde{g}_\alpha (\tilde{g}_\Delta)_\beta \eta + (\alpha \leftrightarrow \beta)$$

$$\eta = \eta(m_{RR}, M_i, \alpha_i)$$
$$\tilde{\eta} = \tilde{\eta}(m_{RR}, M_i, \alpha_i)$$

$$\tilde{\eta} = \frac{m_{RR}}{64\pi^2} \left(\frac{M_3^2}{m_{RR}^2 - M_3^2} \ln \frac{m_{RR}^2}{M_3^2} - \frac{M_1^2}{m_{RR}^2 - M_1^2} \ln \frac{m_{RR}^2}{M_1^2} \right) \sin 2\alpha_1 + [(1,\ 3) \to (2,\ 4)]$$

Neutrino Masses (Technical Details)

One Loop Diagram Generating Neutrino Masses



- neutral scalar mass eigenstates δ_i
- with scalar masses M_i
- α_1 mixing between $\delta_{1,3}$
- α_2 mixing between $\delta_{2,4}$

back

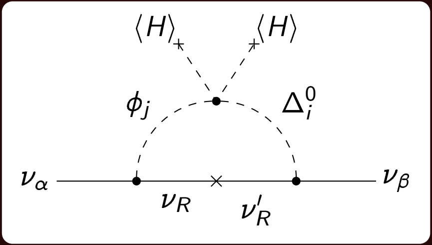
$$(m_\nu)_{\alpha\beta} = g_\alpha (\tilde{g}_\Delta)_\beta \tilde{\eta} + \tilde{g}_\alpha (\tilde{g}_\Delta)_\beta \eta + (\alpha \leftrightarrow \beta)$$

$$\tilde{\eta} \simeq \frac{m_{RR}}{16\pi^2} \left(\frac{\tilde{m}_\phi^2 m_{\phi\Delta}^2}{m_{RR}^2 m_\Delta^2} \left(\frac{m_{RR}^2}{m_{RR}^2 - m_\Delta^2} \ln \frac{m_{RR}^2}{m_\Delta^2} + 1 - \ln \frac{m_{RR}^2}{M_1^2} \right) - \frac{\tilde{m}_{\phi\Delta}^2}{m_{RR}^2 - m_\Delta^2} \ln \frac{m_{RR}^2}{m_\Delta^2} \right)$$

$$\eta \simeq - \frac{m_{RR}}{16\pi^2} \frac{m_{\phi\Delta}^2}{m_{RR}^2 - m_\Delta^2} \ln \frac{m_{RR}^2}{m_\Delta^2}$$

Neutrino Masses (Technical Details)

One Loop Diagram Generating Neutrino Masses



- neutral scalar mass eigenstates δ_i
- with scalar masses M_i
- α_1 mixing between $\delta_{1,3}$
- α_2 mixing between $\delta_{2,4}$

back

$$(m_\nu)_{\alpha\beta} = g_\alpha (\tilde{g}_\Delta)_\beta \tilde{\eta} + \tilde{g}_\alpha (\tilde{g}_\Delta)_\beta \eta + (\alpha \leftrightarrow \beta)$$

$$g\tilde{g}_\Delta \simeq 4.0 \times 10^{-6} \frac{m_\nu}{0.05\ \text{eV}} \frac{70\ \text{GeV}}{M_1} \frac{50\ \text{MeV}}{\delta} \frac{m_{RR}}{300\ \text{GeV}} \frac{0.1}{|\sin\alpha_1|} \left(\frac{m_{RR}^2}{m_{RR}^2 - m_\Delta^2} \cdots \right)^{-1}$$

$$g\tilde{g}_\Delta \simeq 4.5 \times 10^{-6} \frac{m_\nu}{0.05\ \text{eV}} \frac{300\ \text{GeV}}{m_{RR}} \frac{1\ \text{GeV}^2}{\tilde{m}_{\phi\Delta}^2} \left(\frac{m_\Delta}{500\ \text{GeV}} \right)^2 \frac{m_{RR}^2 - m_\Delta^2}{m_\Delta^2} \left(\log \frac{m_{RR}^2}{m_\Delta^2} \right)^{-1}$$

$$\tilde{g}\tilde{g}_\Delta \simeq 1.8 \times 10^{-10} \frac{m_\nu}{0.05\ \text{eV}} \frac{300\ \text{GeV}}{m_{RR}} \frac{0.1}{\sin\alpha_1} \frac{m_{RR}^2 - m_\Delta^2}{m_\Delta^2} \left(\log \frac{m_{RR}^2}{m_\Delta^2} \right)^{-1}$$