Flav4G Workshop
IPPP - Durham

September 16th, 2011

Proton stability from a fourth family

Christopher Smith

- Outline

Introduction

I. Three generations

II. Four generations

Conclusion

Introduction

Introduction

- Baryon ($\mathcal{B}$) and lepton ($\mathcal{L}$) numbers are *accidentally conserved by* $\mathcal{L}_{SM}$.

  Leptons have $\mathcal{L}=\pm 1$, quarks have $\mathcal{B}=\pm 1/3$ $\leftarrow p^+ \sim \varepsilon^{\alpha\beta\gamma} u_\alpha u_\beta d_\gamma$.

  Lorentz + renormalizability $\Rightarrow \Delta\mathcal{B}, \Delta\mathcal{L} = 0,0$.

- *Not expected beyond the SM* (BAU, GUT, Majorana ν,...) → $\Delta\mathcal{L}=\pm 2$

- Exact conservation of $\mathcal{B}$ is *extremely well supported experimentally*:

  Proton decay: Lightest spin ½ baryon → must violate $\mathcal{B}$ and $\mathcal{L}$:

$$p^+ \to \pi^0 e^+ : 8.2\times 10^{33}\, yrs \Leftrightarrow \Gamma \sim 10^{-60}\, GeV$$
$$p^+ \to K^0 e^+ : 1.5\times 10^{32}\, yrs \Leftrightarrow \Gamma \sim 10^{-58}\, GeV$$
$$p^+ \to any : 2.1\times 10^{29}\, yrs \Leftrightarrow \Gamma \sim 10^{-55}\, GeV$$

  Neutron oscillations: Violates $\mathcal{B}$ by two units $\tau(n-\overline{n}) > 1.3\times 10^{8}\,\sec$.

  Many other channels: $Z \to p^+ e^-, \tau^- \to e^+ \pi^- \pi^-, K^+ \to \pi^- e^+ e^+, \ldots$

Introduction: The central question

Nikolidakis,CS '07, CS '11

> If non-zero, why are baryon ($\mathcal{B}$) and lepton ($\mathcal{L}$) number violations so small?

$\mathcal{B}$ and $\mathcal{L}$ are intrinsically flavored since they refer to quarks & leptons.

$$\text{Small } \mathcal{B} \text{ and } \mathcal{L} \text{ violation} \quad \overset{?}{\leftrightarrow} \quad \text{small NP effects in FCNC}$$

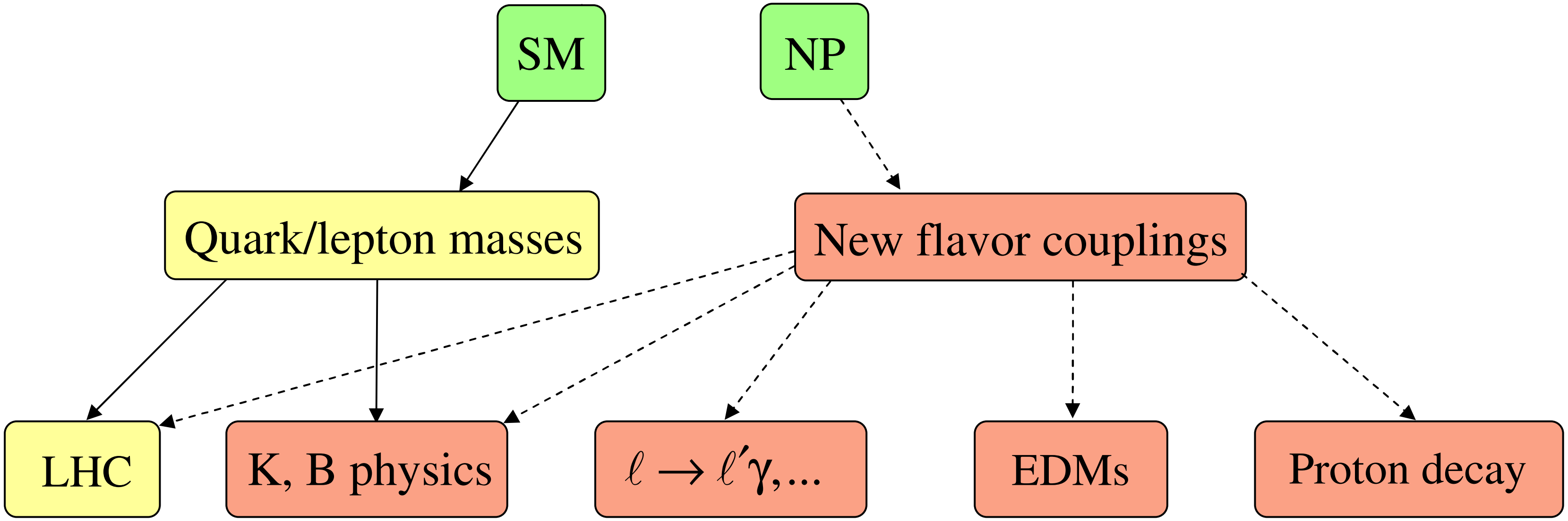
To answer this question, we will use the techniques of *Minimal Flavor Violation*.

> How large can $\mathcal{B}$ and $\mathcal{L}$ violation be in the absence of new flavor couplings?

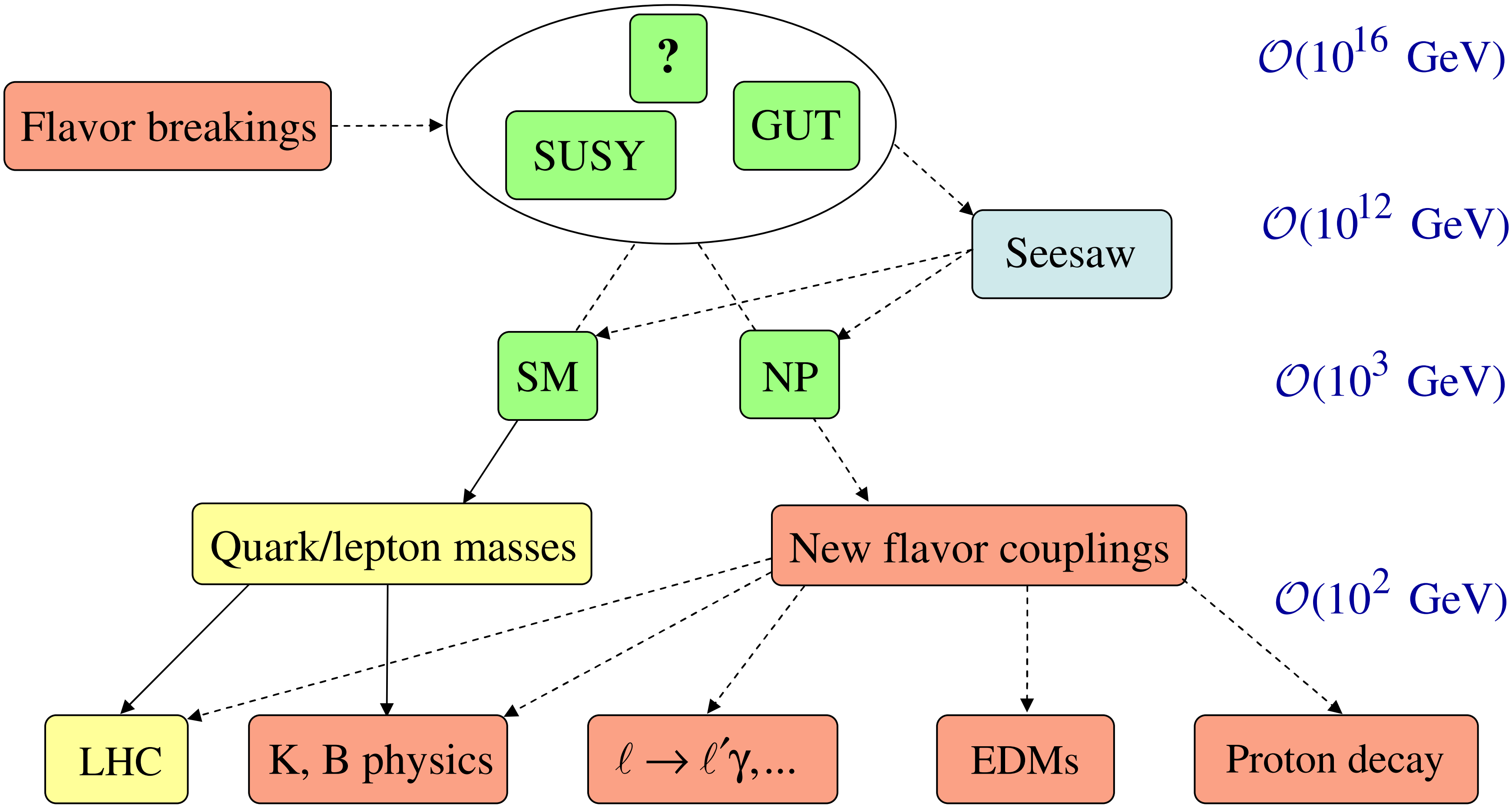
Introduction: Minimal Flavor Violation

Motivation: Flavor experiments require NP either to be very heavy, or to have a highly non-generic flavor sector.

MFV is a procedure to generate such a non-generic sector.

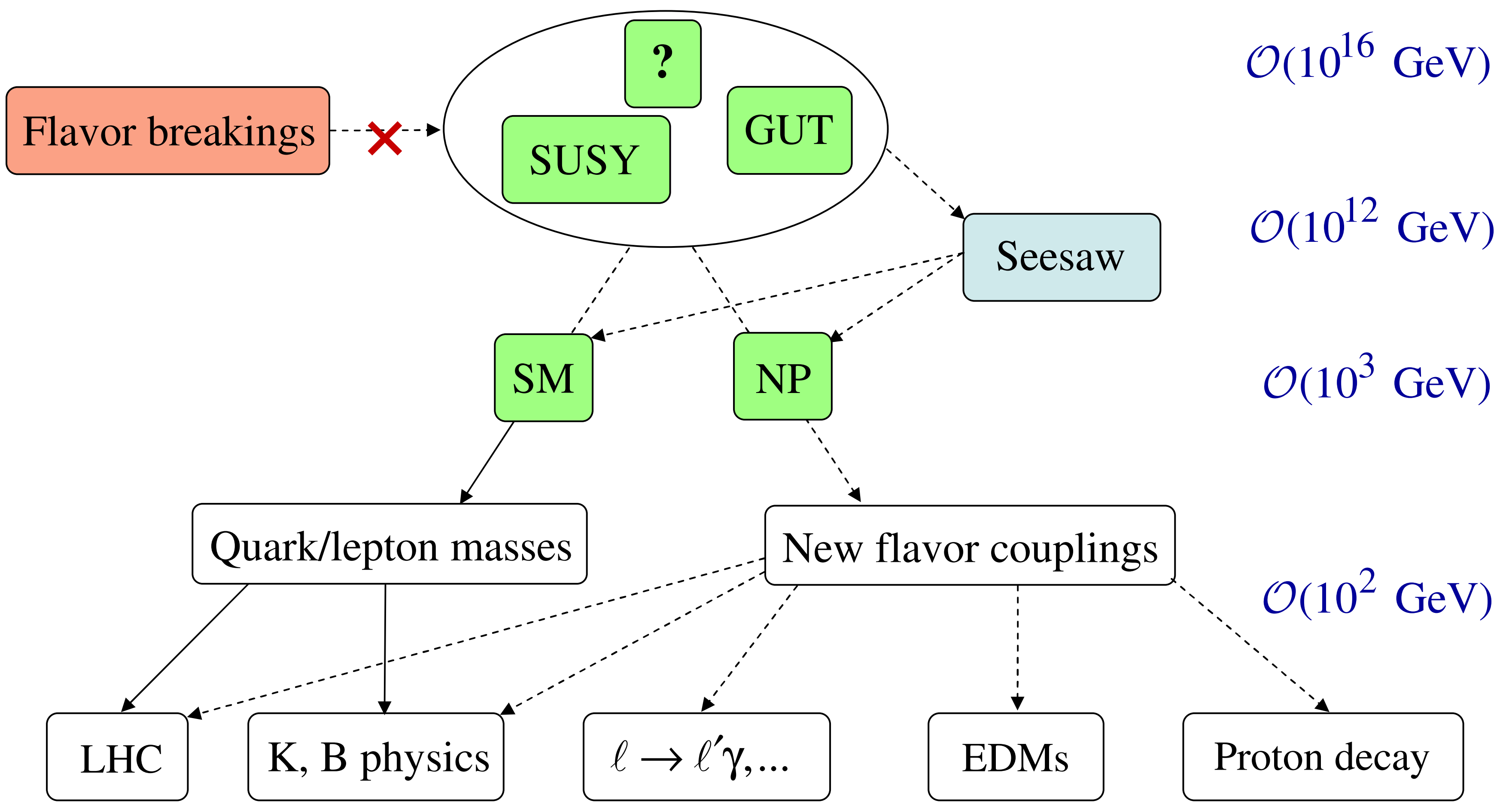


Introduction: Minimal Flavor Violation



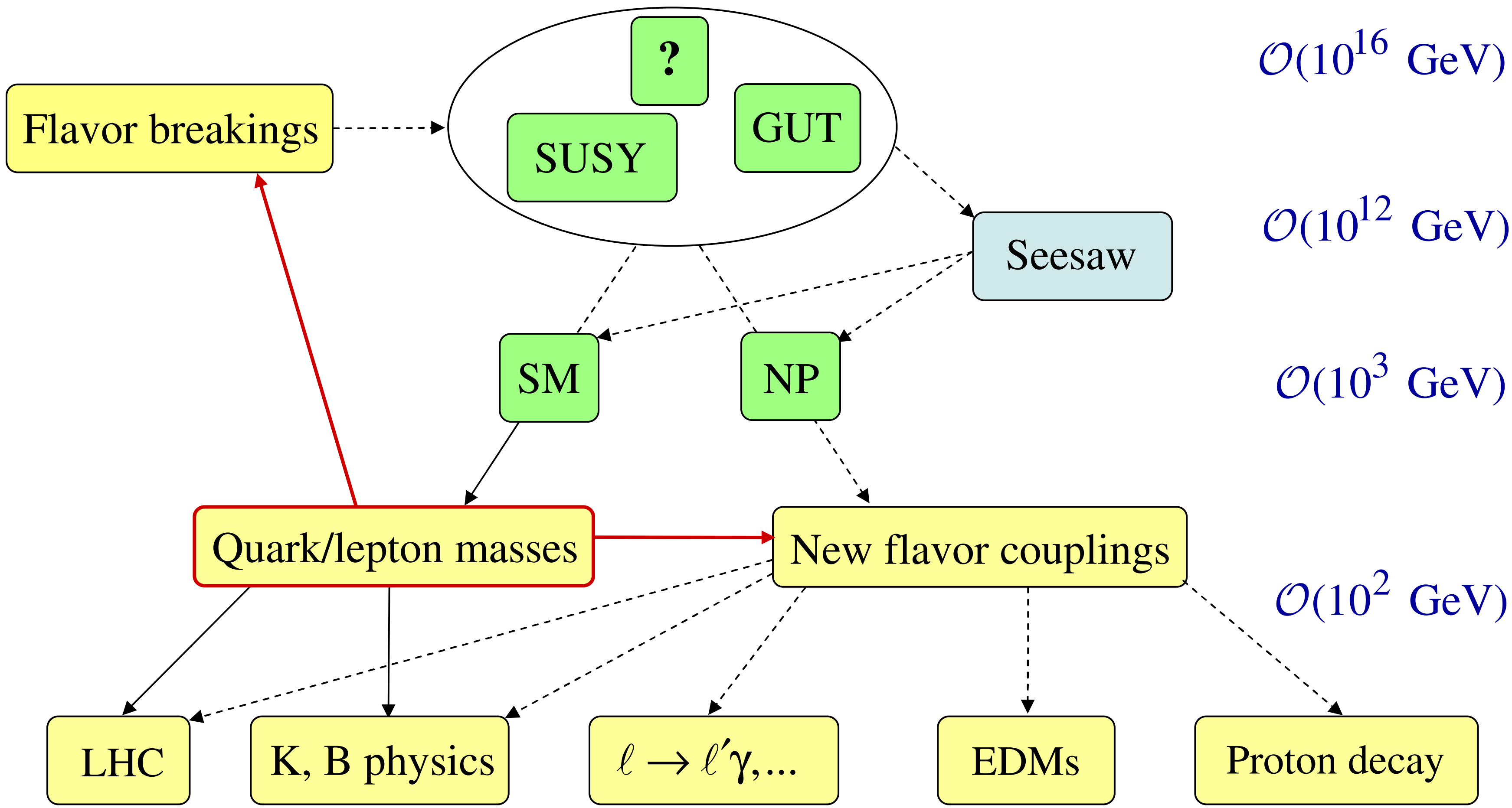
Some NP mechanism must explain the *origin of the flavor structures*.

Introduction: Minimal Flavor Violation



We assume a single mechanism is generating *all* the flavor structures.

Introduction: Minimal Flavor Violation



With MFV, all the flavor-breaking couplings are reconstructed in terms of the fermion masses and mixings, and become *naturally hierarchical*.

Introduction: Minimal Flavor Violation

- *Symmetry:* Gauge sector Lagrangian exhibit a $G_f = U(N_f)^5$ flavor symmetry.

- *Minimality:* Minimal spurion content allowing for the known fermion masses.
  (Yukawas $\mathrm{Y}_{u,d,e}$ plus a few seesaw spurions) ← *Central assumption*

- *Naturality*: All Lagrangian couplings written as formal G_f -invariants.
  If: $\mathrm{Y}_u \to g_U \mathrm{Y}_u g_Q^\dagger,\ \ \mathrm{Y}_d \to g_D \mathrm{Y}_d g_Q^\dagger,\ \ \mathrm{H} \to g_Q \mathrm{H} g_Q^\dagger$
  Then: $\mathrm{H} = a_0 1 + a_1 \mathrm{Y}_u^\dagger \mathrm{Y}_u + a_2 \mathrm{Y}_d^\dagger \mathrm{Y}_d + \ldots$ with $a_i \sim \mathcal{O}(1)$ ← *naturality*

- *Freezing of the spurions* at their physical values. With $N_f = 3$:

Hall,Randall '90
D'Ambrosio,Giudice, Isidori,Strumia '02

$$\mathrm{H} \approx \left(\begin{pmatrix} 1 & 10^{-4} & 10^{-3} \\ 10^{-4} & 1 & 10^{-2} \\ 10^{-3} & 10^{-2} & 1 \end{pmatrix} + i \begin{pmatrix} 0 & 10^{-4} & 10^{-3} \\ 10^{-4} & 0 & 10^{-4} \\ 10^{-3} & 10^{-4} & 0 \end{pmatrix} \right)$$

← *Inherited hierachies*

Introduction: Minimal Flavor Violation

Nikolidakis,CS '08
Colangelo,Nikolidakis,CS '08
CS, in preparation

Flavor breakings

Imagine there is a single fundamental spurion X at some high scale

A = Yukawas

SM

NP

B = Anything else

Introduction: Minimal Flavor Violation

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Flavor breakings

Imagine there is a single fundamental spurion X at some high scale

SM

$$\mathrm{A} = x_0 1 + x_1 \mathrm{X} + x_2 \mathrm{X}^2$$

$$x_i \sim \mathcal{O}(1)$$

NP

$$\mathrm{B} = x_0' 1 + x_1' \mathrm{X} + x_2' \mathrm{X}^2$$

$$x_i' \sim \mathcal{O}(1)$$

Remark: finite expansions thanks to Cayley-Hamilton identities:

$$\mathrm{X}^3 - \langle \mathrm{X} \rangle \mathrm{X}^2 + \tfrac{1}{2} \mathrm{X} (\langle \mathrm{X} \rangle^2 - \langle \mathrm{X}^2 \rangle) = \tfrac{1}{3} \langle \mathrm{X}^3 \rangle - \tfrac{1}{2} \langle \mathrm{X} \rangle \langle \mathrm{X}^2 \rangle + \tfrac{1}{6} \langle \mathrm{X} \rangle^3$$

(Similar for $N_f = 4$)

Introduction: Minimal Flavor Violation

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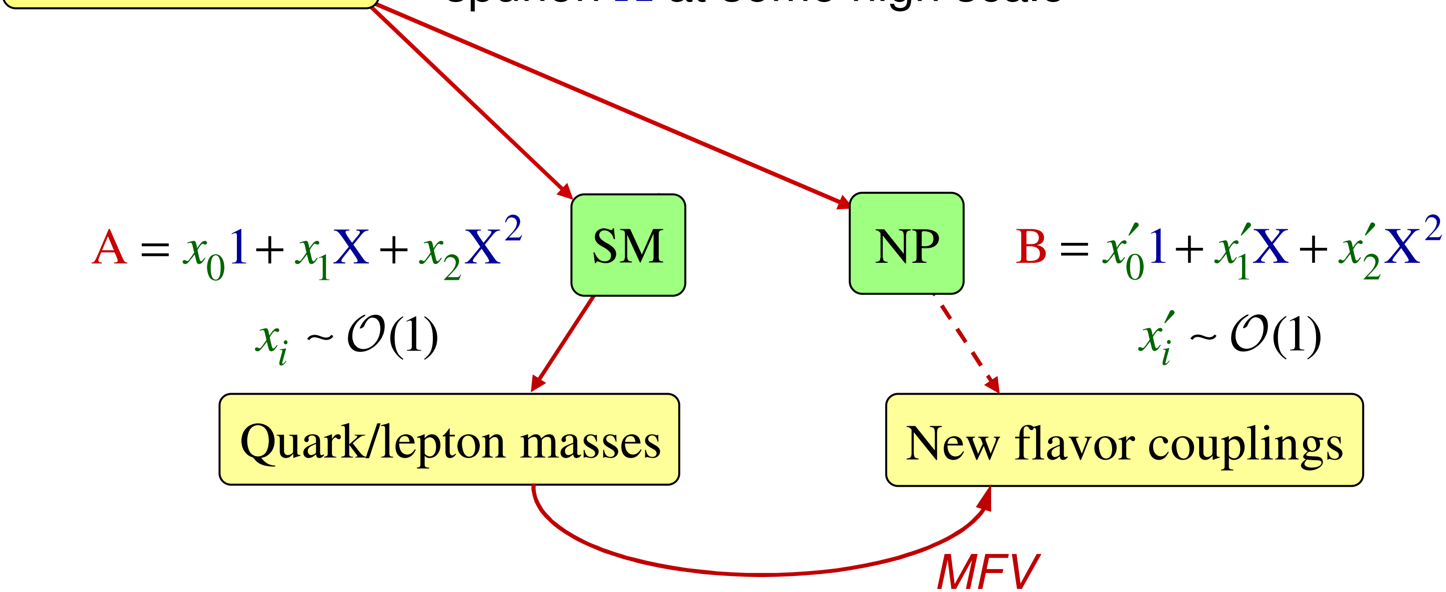
CS, in preparation

Imagine there is a single fundamental spurion X at some high scale



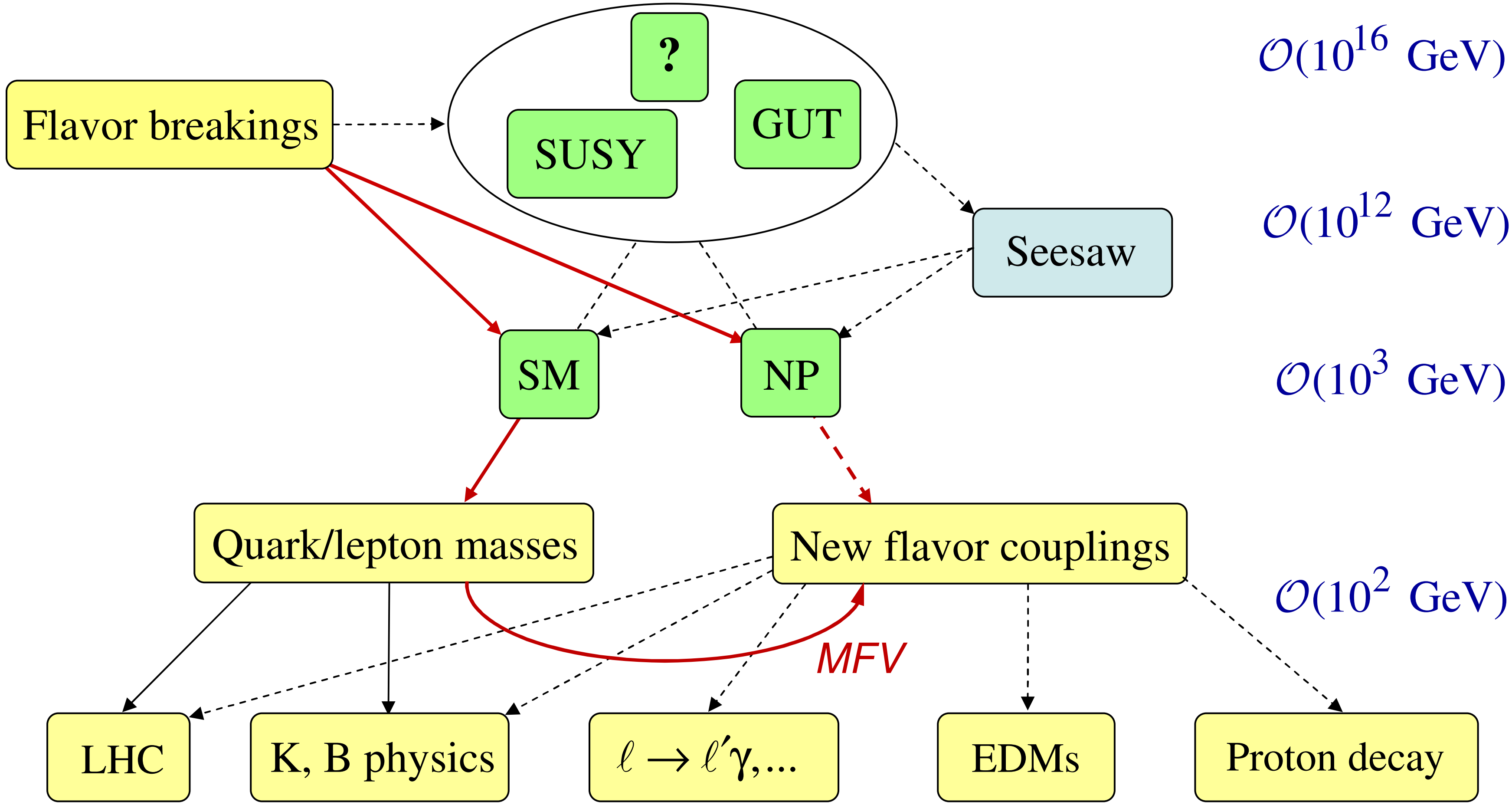
Then, all that is observable is an MFV relation between A and B:

$$\mathrm{A} = a_0 1 + a_1 \mathrm{B} + a_2 \mathrm{B}^2 \quad \text{or} \quad \mathrm{B} = b_0 1 + b_1 \mathrm{A} + b_2 \mathrm{A}^2 \quad \text{with} \quad a_i, b_i \sim \mathcal{O}(1)$$

The Yukawas are not given a special status when imposing MFV!

Introduction: Minimal Flavor Violation

Nikolidakis,CS '08
Colangelo,Nikolidakis,CS '08
CS, in preparation



The *flavor symmetry is always broken* (at all scale),

Low-energy MFV only reflects a high-scale redundancy of the flavor structures.

Introduction: The central question

Nikolidakis,CS '07, CS '11

How large can $\mathcal{B}$ and $\mathcal{L}$ violation be in the absence of new flavor couplings?

- $\mathcal{B}$ and $\mathcal{L}$ are combinations of the flavor $U(1)$'s:

$$G_f = SU(N_f)^5 \times U(1)_Q \times U(1)_U \times U(1)_D \times U(1)_L \times U(1)_E$$
$$= SU(N_f)^5 \times U(1)_{\mathcal{B}} \times U(1)_{\mathcal{L}} \times U(1)_Y \times U(1)_{PQ} \times U(1)_E$$

- We want $\mathcal{B}$ and $\mathcal{L}$ violating couplings that *break only* $U(1)_{\mathcal{B}} \times U(1)_{\mathcal{L}}$.

Any breaking of $SU(N_f)^5$ must be induced by the same spurions needed for fermion masses & mixings.

- *MFV under* $SU(N_f)^5$ instead of $U(N_f)^5$: *use ε–tensors* to form invariants.

II. Three generations

Step 1. Only the SM gauge interactions

What are the simplest operators breaking $U(1)_{\mathcal{B},\mathcal{L}}$ but not $SU(3)^5$ ...

...without any spurion (i.e., only the SM gauge interactions).

- Only *cubic monomials* are invariant under $SU(3)^5$ $(X = Q, U, D, L, E)$:

$$\varepsilon^{IJK} X^I X^J X^K \to \varepsilon^{IJK} (g_X X)^I (g_X X)^J (g_X X)^K = \det(g_X) \varepsilon^{IJK} X^I X^J X^K$$

- An even number of monomials is needed for *Lorentz invariance*.

- Since Q, U, D, L, E all have different *hypercharges*, at least four monomials:

$$\mathcal{L}_{\Delta\mathcal{B},\Delta\mathcal{L}} = \frac{1}{\Lambda^{14}} \left[c_1 (LQ^3)^3 + c_2 (EU^2 D)^3 + c_3 (EUQ^{\dagger 2})^3 + c_4 (LQD^\dagger U^\dagger)^3 \right]$$

- The c_i have complicated flavor structures, entwined with the epsilons of color,...

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t'Hooft '76

This is t'Hooft $\mathcal{B}+\mathcal{L}$ anomaly of the SM!

Step 2. Introducing the Yukawa couplings

What are the simplest operators breaking $U(1)_{\mathcal{B},\mathcal{L}}$ but not $SU(3)^5$ ...

...with only the Yukawas (i.e., massless neutrinos).

- The Yukawa spurions relate the flavor SU(3) spaces:

$$SU(3)_Q \overset{\mathbf{Y}_u}{\longleftrightarrow} SU(3)_U \qquad SU(3)_Q \overset{\mathbf{Y}_d}{\longleftrightarrow} SU(3)_D \qquad SU(3)_L \overset{\mathbf{Y}_e}{\longleftrightarrow} SU(3)_E$$

$$Q^{\dagger I} \sim (U\mathbf{Y}_u)^I \sim (D\mathbf{Y}_d)^I \qquad L^{\dagger I} \sim (E\mathbf{Y}_e)^I$$

- Only two epsilon contractions of three quarks or lepton fields are needed:

$$\mathcal{L}_{\Delta\mathcal{B},\Delta\mathcal{L}} = \frac{1}{\Lambda^5}\left[\underbrace{c_1 E L^{\dagger 2} U^3 + c_2 L^{\dagger 3} Q^{\dagger} U^2}_{\Delta\mathcal{B},\Delta\mathcal{L} = 1,3} + \underbrace{c_3 D^4 U^2 + c_4 D^3 U Q^{\dagger 2} + c_5 D^2 Q^{\dagger 4}}_{\Delta\mathcal{B},\Delta\mathcal{L} = 2,0}\right]$$

can induce proton decay! can induce neutron oscillations!

Step 2. Introducing the Yukawa couplings

What are the simplest operators breaking $U(1)_{\mathcal{B},\mathcal{L}}$ but not $SU(3)^5$ ...

...with only the Yukawas (i.e., massless neutrinos).

- $SU(3)^5$ *symmetry* and ε^{IJK} *antisymmetry* suppress the light-quark couplings:

$$c_2 L^{\dagger 3} Q^{\dagger} U^2 \to c_2 \times \varepsilon^{IJK} L^{\dagger I} L^{\dagger J} L^{\dagger K} \times \varepsilon^{LMN} Q^{\dagger L} (U\mathbf{Y}_u)^M (U\mathbf{Y}_u)^N + \ldots$$

$$\to c_2 \times \{\overline{\nu}_\mu e_L^c\}\{\overline{\nu}_\tau s_L^c\}\{\overline{u}_R u_R^c\} \underbrace{\frac{m_u^2}{v^2} V_{ub}}_{10^{-13}} + \ldots$$

- Exotic $\Delta\mathcal{L} = 3$ proton decay channels like $p^+ \to \overline{K}^0 e^+ \overline{\nu}_\mu \overline{\nu}_\tau$, not so constrained.

- For such channels, the bounds allow for $\Lambda \approx 1\ \mathrm{TeV}$ *or even below*!

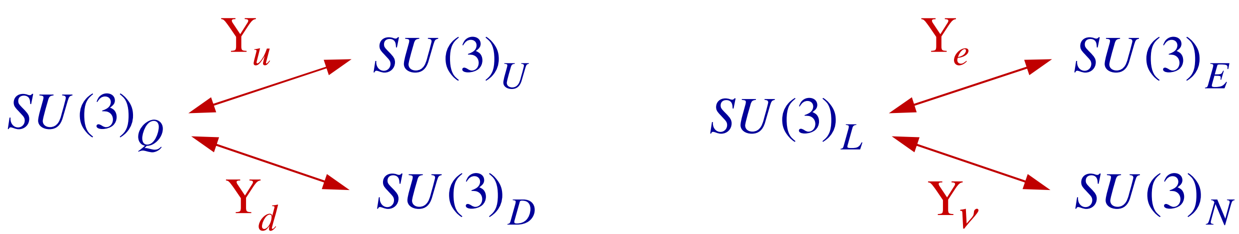
$$\Gamma \sim \frac{1}{(8\pi)^3} \frac{m_{p^+}^{11}}{\Lambda^{10}} (10^{-13})^2 \sim 10^{-60} GeV \times \left(\frac{1TeV}{\Lambda}\right)^{10}$$

Step 3. Introducing neutrino masses

What are the simplest operators breaking $U(1)_{\mathcal{B},\mathcal{L}}$ but not $SU(3)^5$ ...

...with Yukawas and neutrino masses.

- Introducing a *Dirac mass* term $N\mathbf{Y}_\nu L H$ would not change much:



$Q^{\dagger I} \sim (U\mathbf{Y}_u)^I \sim (D\mathbf{Y}_d)^I$ $L^{\dagger I} \sim (E\mathbf{Y}_e)^I \sim (N\mathbf{Y}_\nu)^I$

$\Rightarrow$ Simplest operators are still dimension nine $\Delta\mathcal{B},\Delta\mathcal{L} = 1,3$ or $2,0$.

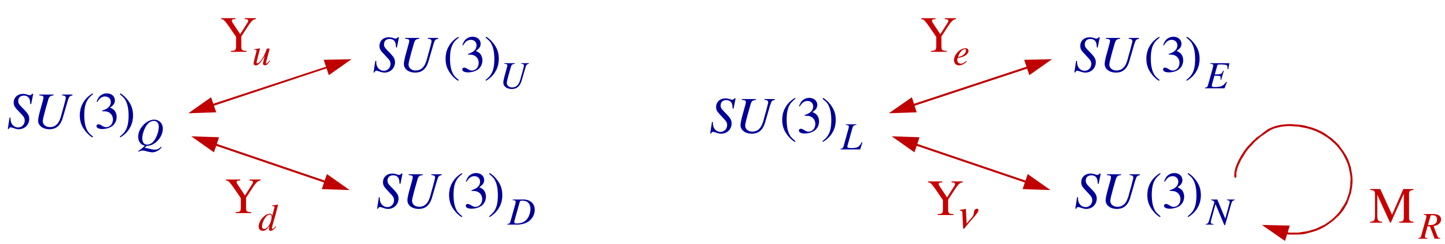
Lepton number still necessarily violated in steps of 3s.

Step 3. Introducing neutrino masses

What are the simplest operators breaking $U(1)_{\mathcal{B},\mathcal{L}}$ but not $SU(3)^5$ ...

...with Yukawas and neutrino masses.

- But with a *Majorana mass*, the spurion structure becomes very different:



$$Q^{\dagger I} \sim (U\mathrm{Y}_u)^I \sim (D\mathrm{Y}_d)^I \qquad L^{\dagger I} \sim (E\mathrm{Y}_e)^I$$

- A Majorana mass Υ_ν transforms as a symmetric 6 since $3 \otimes 3 = 6 \oplus \bar{3}$.

But $\bar{3} \subset 6 \otimes 8$, so $L^{\dagger I} \sim \varepsilon^{IJK} (\Upsilon_\nu \mathrm{Y}_e^\dagger \mathrm{Y}_e)^{JK} \sim \varepsilon^{IJK} (\Upsilon_\nu \mathrm{Y}_\nu^\dagger \mathrm{Y}_\nu)^{JK}$.

$\Delta\mathcal{L} = \pm 1$ is possible, but suppressed by $\Upsilon_\nu \equiv v_u \mathrm{Y}_\nu^T \mathrm{M}^{-1} \mathrm{Y}_\nu \sim \dfrac{m_\nu}{v} \sim 10^{-12}$!

Step 3. Introducing neutrino masses

What are the simplest operators breaking $U(1)_{\mathcal{B},\mathcal{L}}$ but not $SU(3)^5$ ...

...with Yukawas and neutrino masses.

- Dimension-six $\Delta\mathcal{B},\Delta\mathcal{L}=1,1$ Weinberg operators are finally allowed:

$$\mathcal{L}_{\Delta\mathcal{B},\Delta\mathcal{L}}=\frac{1}{\Lambda^2}\left[c_1 LQ^3+c_2 EU^2D+c_3 EUQ^{\dagger 2}+c_4 LQD^\dagger U^\dagger\right]$$

Weinberg '79

...but the c_i are suppressed by *neutrino, charged lepton, and quark masses!*

- The bounds are easily satisfied for $\Lambda\approx 5-10\ \mathrm{TeV}$.

- Could even be down to the EW scale if $c_i \sim g^2/4\pi$.

- Remark: The LQ^3 operator is the "cubic root" of the 't Hooft anomaly operator.

Step 4. What happens in supersymmetry?

What are the simplest operators breaking $U(1)_{\mathcal{B},\mathcal{L}}$ but not $SU(3)^5$ ...

...within the MSSM.

- Squarks/slepton carry $\mathcal{B}$ and $\mathcal{L}$ → Renormalizable $\Delta\mathcal{B},\Delta\mathcal{L}$ Yukawas exist:

$$\mathcal{W}_{RPV} = \underbrace{\mu'^I L^I H_d + \lambda^{IJK} L^I L^J E^K + \lambda'^{IJK} L^I Q^J D^K}_{\Delta\mathcal{L}=1} + \underbrace{\lambda''^{IJK} U^I D^J D^K}_{\Delta\mathcal{B}=1}$$

- The $\Delta\mathcal{B}=1$ *couplings can be constructed* using $\Delta\mathcal{B}=0$ quark Yukawas:

$$\lambda''^{IJK} = \varepsilon^{LJK} (\mathrm{Y}_u \mathrm{Y}_d^\dagger)^{IL} \quad \Rightarrow \lambda''^{IJK} U^I D^J D^K \to \det(g_D^\dagger)\lambda''^{IJK} U^I D^J D^K$$

$$\lambda''^{IJK} = \varepsilon^{LMN} \mathrm{Y}_u^{IL} \mathrm{Y}_d^{JM} \mathrm{Y}_d^{KN} \quad \Rightarrow \lambda''^{IJK} U^I D^J D^K \to \det(g_Q^\dagger)\lambda''^{IJK} U^I D^J D^K$$

$$\cdots$$

- But $\Delta\mathcal{L}=1$ *couplings are strictly forbidden* as long as $m_\nu = 0$ (or Dirac).

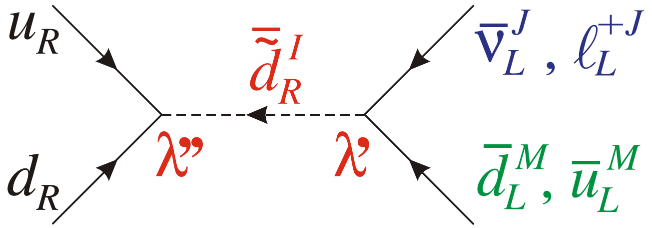
Step 4. What happens in supersymmetry?

Nikolidakis,C.S. '07

Structures ($\mathcal{W}_{RPV} = \mu' L H_d + \lambda LLE + \lambda' LQD + \lambda'' UDD$)		Scaling	Breaking
$\mu_1'^{I}$	$\mu\,\varepsilon^{STI}(\mathrm{Y}_e^\dagger \mathrm{Y}_e \Upsilon_\nu^\dagger)^{ST},\ldots$	$\tan^2\beta$	$U(1)_L$
λ_1^{IJK}	$\varepsilon^{STI}(\mathrm{Y}_e^\dagger \mathrm{Y}_e \Upsilon_\nu^\dagger)^{ST}(\mathrm{Y}_e)^{KJ},\ldots$	$\tan^3\beta$	$U(1)_L$
λ_2^{IJK}	$\varepsilon^{IMJ}(\mathrm{Y}_e \Upsilon_\nu^\dagger)^{KM},\ldots$	$\tan\beta$	$U(1)_L$
λ_3^{IJK}	$\varepsilon^{STI}(\mathrm{Y}_e^\dagger \mathrm{Y}_e \Upsilon_\nu^\dagger)^{ST}\varepsilon^{LMJ}\varepsilon^{ABK}(\mathrm{Y}_e^\dagger)^{LA}(\mathrm{Y}_e^\dagger)^{MB},\ldots$	$\tan^4\beta$	$U(1)_{L,E}$
$\lambda_1'^{IJK}$	$\varepsilon^{STI}(\mathrm{Y}_e^\dagger \mathrm{Y}_e \Upsilon_\nu^\dagger)^{ST}(\mathrm{Y}_d)^{KJ},\ldots$	$\tan^3\beta$	$U(1)_L$
$\lambda_2'^{IJK}$	$\varepsilon^{STI}(\mathrm{Y}_e^\dagger \mathrm{Y}_e \Upsilon_\nu^\dagger)^{ST}\varepsilon^{LMJ}\varepsilon^{ABK}(\mathrm{Y}_d^\dagger)^{LA}(\mathrm{Y}_d^\dagger)^{MB},\ldots$	$\tan^4\beta$	$U(1)_{L,D,Q}$
$\lambda_1''^{IJK}$	$\varepsilon^{LJK}(\mathrm{Y}_u \mathrm{Y}_d^\dagger)^{IL},\ldots$	$\tan\beta$	$U(1)_D$
$\lambda_2''^{IJK}$	$\varepsilon^{IMN}(\mathrm{Y}_d \mathrm{Y}_u^\dagger)^{JM}(\mathrm{Y}_d \mathrm{Y}_u^\dagger)^{KN},\ldots$	$\tan^2\beta$	$U(1)_U$
$\lambda_3''^{IJK}$	$\varepsilon^{LMN}(\mathrm{Y}_u)^{IL}(\mathrm{Y}_d)^{JM}(\mathrm{Y}_d)^{KN},\ldots$	$\tan^2\beta$	$U(1)_Q$
$\lambda_4''^{IJK}$	$\varepsilon^{LMN}\varepsilon^{ABI}\varepsilon^{CJK}(\mathrm{Y}_d^\dagger)^{LC}(\mathrm{Y}_u^\dagger)^{MA}(\mathrm{Y}_u^\dagger)^{NB},\ldots$	$\tan\beta$	$U(1)_{Q,U,D}$

(Similar expansions for R-parity violating soft-breaking terms)

Example of MFV suppression for a specific proton decay mechanism



$$\Rightarrow \left|\lambda'_{JMI}\lambda''^{*}_{11I}\right| < 10^{-26}\left(m[\tilde{d}_R^I]/300\,GeV\right)^2$$

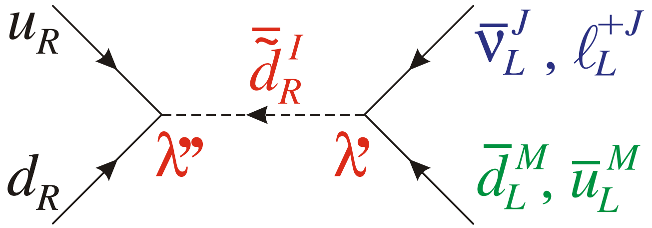
If the leading operators are: $\lambda'\ :\ (a_0\varepsilon^{LMI}(\mathrm{Y}_e^\dagger \mathrm{Y}_e \Upsilon_\nu^\dagger)^{LM}(\mathrm{Y}_d \mathrm{Y}_u^\dagger \mathrm{Y}_u)^{KJ})\ L^I Q^J D^K$

$$\lambda''\ :\ (a_1\varepsilon^{LJK}(\mathrm{Y}_u \mathrm{Y}_d^\dagger)^{IL} + a_2\varepsilon^{LMN}\mathrm{Y}_u^{IL}\mathrm{Y}_d^{JM}\mathrm{Y}_d^{KN})\ U^I D^J D^K$$

The MFV prediction is then

$$\left|\lambda'_{JMI}\lambda''^{*}_{11I}\right| \approx \frac{\Delta m_{31}}{v_u}\frac{m_\tau^2}{v_d^2}\ \lambda^3\ \frac{m_b m_t^2 m_u}{v_d v_u^3}\left(a_0 a_1\frac{m_s}{v_d} + a_0 a_2\frac{m_d m_b}{v_d^2}\right)$$

Example of MFV suppression for a specific proton decay mechanism



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If the leading operators are: λ' : $(a_0 \varepsilon^{LMI}(Y_e^\dagger Y_e \Upsilon_\nu^\dagger)^{LM}(Y_d Y_u^\dagger Y_u)^{KJ})\; L^I Q^J D^K$

$$\lambda'' : (a_1 \varepsilon^{LJK}(Y_u Y_d^\dagger)^{IL} + a_2 \varepsilon^{LMN} Y_u^{IL} Y_d^{JM} Y_d^{KN})\; U^I D^J D^K$$

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Neutrino mass, related to $\Delta m_{atm}^2 \approx 3\cdot 10^{-3}\,\mathrm{eV}^2$

Charged lepton mass

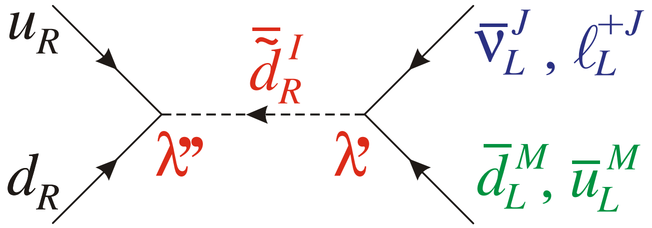
CKM factors

Light-quark masses

Symmetry of Υ_ν, the Majorana mass term

Antisymmetry of ε tensors

Example of MFV suppression for a specific proton decay mechanism



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$$\approx a_0 a_1 10^{-28}\tan^4\beta + a_0 a_2 10^{-31}\tan^5\beta \quad (for\; m_\nu^{lightest} = 0)$$

Conservatively, MFV can account for the necessary suppression.

- MFV coefficients of $\mathcal{O}(1)$, while $\mathcal{O}(\lambda)$ or $\mathcal{O}(g^2/4\pi)$ also natural,
- No GIM-like interferences, no cancellations among processes,

III. Four generations?

A. Introduction

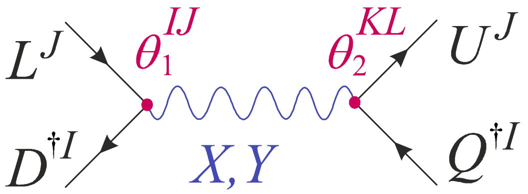
- With MFV, proton decay is around the corner → **Can we reinforce MFV?**

- In a GUT context, MFV is of no help:

Proton decay from *gauge interactions*

- Effective MFV spurions:

$$\frac{\theta^{IJ}}{M_{GUT}}(L^{\dagger I}D^J, U^{\dagger I}Q^J, E^I Q^{\dagger J})$$



- Background value $\theta^{IJ} = \delta^{IJ}$ protected by gauge symmetry.

- Proton decay is fine for

$$M_{GUT} > 10^{16}\,GeV$$

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- Effective MFV spurions:

$$\frac{\theta^{IJ}}{M_{GUT}}(L^{\dagger I}D^J, U^{\dagger I}Q^J, E^I Q^{\dagger J})$$

- Background value $\theta^{IJ} = \delta^{IJ}$ protected by gauge symmetry.

- Proton decay is fine for

$$M_{GUT} > 10^{16}\, GeV$$

Proton decay from *RPV interactions*

- If at the GUT scale:

$$\mathcal{W}_{RPV} = \Lambda^{IJK}\,\bar{5}^I\,\bar{5}^J\,10^K + \ldots$$

with $\Lambda \sim O(\mathrm{Y}_{\bar{5}}, \mathrm{Y}_{10})$ from MFV,

- Then $\Delta\mathcal{L}$ and $\Delta\mathcal{B}$ are much too large at EW scale:

Proton lifetime is tiny!

B. What changes with four generations, besides $SU(3)^5 \to SU(4)^5$?

1. *Mismatch between the number of colors and flavors.*

- The contractions $\varepsilon^{IJKL} X_1^I X_2^J X_3^K X_4^L \ (X_i = Q^\dagger, U, D)$ are not color singlets.

- The *only way* to break $\mathcal{B}$ is through twelve fields:

$$\varepsilon^{IJKL} X_1^I X_2^J X_3^K X_4^L \, \varepsilon^{MNOP} X_5^M X_6^N X_7^O X_8^P \, \varepsilon^{QRST} X_9^Q X_{10}^R X_{11}^S X_{12}^T$$

- Simplest effective interactions are $\Delta\mathcal{B}, \Delta\mathcal{L} = 4n, 4m \ \ (m, n = 0, 1, \ldots)$
($\Delta\mathcal{B}, \Delta\mathcal{L} = 4n, 2m$ possible but suppressed by the Majorana mass Υ_ν)

- No way to have $\Delta\mathcal{B} = 1$ with the νSM particle & spurion contents:

The proton is stable!

B. What changes with four generations, besides $SU(3)^5 \to SU(4)^5$?

2. *The SU(4) invariant tensors have an even number of indices.*

- Spurions also all have an even number of flavor indices...

...but RPV couplings involve an *odd number of flavored fields*!!!

→ *R-parity becomes exact in the MSSM and in GUT*.

- Higher dimensional R-parity conserving but $\mathcal{B}$, $\mathcal{L}$ violating superpotential terms are very suppressed since only $\Delta\mathcal{B}, \Delta\mathcal{L} = 4n, 4m$ is possible.

- So, again, the proton is stable (but for the usual GUT-induced processes)

- *Note*: This does not necessarily holds if the particle content is extended, e.g. to maintain perturbativity up to the GUT scale.

C. Comparison between 3G and 4G

Spurions		Three families		Four families	
		Dim.	$\pm(\Delta\mathcal{B},\Delta\mathcal{L})$	Dim.	$\pm(\Delta\mathcal{B},\Delta\mathcal{L})$
SM gauge	−	27	$(6,0)$, $(0,6)$, $(3,9)$, $(3,\pm3)$	24	$(4,4)^{\supset anom.}$
	−	18	$(3,3)^{\supset anom.}$	18	$(4,0)$, $(0,4)$
Yukawas	$\mathrm{Y}_{u,d,e,(\nu)}$	9	$(1,3)$, $(2,0)$	10	$(0,4)$
Seesaw	$\mathrm{Y}_{u,d,e,\nu}$, Υ_ν	6	$(1,1)$	7	$(0,2)$
MSSM	$\mathrm{Y}_{u,d,e,\nu}$, Υ_ν	4	$(1,1)$	5	$(0,4)$
	$\mathrm{Y}_{u,d,e,\nu}$, Υ_ν	3	$(1,0)$, $(0,1)$	4	$(0,2)$
SUSY GUT	$\mathrm{Y}_{\bar{5},10}$	3	$(1,0)$, $(0,1)$	3	−

Conclusion

B and L violating couplings are flavor couplings.

They may be directly related to the known flavor structures,
i.e. to the fermion masses and mixings.

With three generations:

In general allowed, but *very small because of the Yukawa hierarchies and of the small neutrino masses*

Though bounds are ok, the proton does ultimately decay.

With four generations:

Baryon number can be violated in steps of four only because:

- *Mismatch between the number of color and flavors*,
- *Flavor indices come in even numbers*.

In the absence of new flavor structures, the proton is absolutely stable.

(caution: GUT interactions do introduce new flavor structures.)