The anomalous magnetic moment of τ lepton and its leptonic radiative decays

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work in progress in collaboration with:

S. Eidelman, D. Epifanov, L. Mercolli, C. Ng, M. Passera.

Outline

1. The SM prediction of the tau $g-2$.
2. The tau $g-2$: experimental bounds.
3. Tau Radiative Decays and $g-2$

Anomalous Magnetic Moment

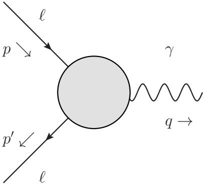
The magnetic moment for a particle of spin $\vec{S}$ is

$$\vec{\mu} = g\frac{e}{2m}\vec{S}.$$

For a spin-1/2 fermion:

$$(i\partial_\mu - eA_\mu)\gamma^\mu\psi = m\psi \Longrightarrow g = 2.$$ P. Dirac '28

The g-factor is modified by quantum correction: $a = (g-2)/2$.



$$\bar{u}(p')\Gamma^\mu u(p) =$$

$$\bar{u}(p')\left\{F_1(q^2)\gamma^\mu + F_2(q^2)\frac{i\sigma^{\mu\nu}}{2m_\ell}q_\nu + \ldots\right\}u(p)$$

$$a_\ell = F_2|_{\text{on-shell}}$$

The leading contribution $a_\ell = \alpha/(2\pi)$

J. Schwinger '48

Leptons Anomalous Magnetic Moment

The present experimental values:

$$a_e = 1\,159\,652\,180.\,73\,(28) \cdot 10^{-12}$$

0.24 parts per billion! Hanneke et al, PRL100 (2008) 120801

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$$-0.052 \le a_\tau \le 0.013$$

DELPHI - EPJC35 (2004) 159

This is not even a test of the leading contribution!

$$a_\tau = \frac{\alpha}{2\pi} + \mathcal{O}(\alpha^2) \approx 0.00116\ldots$$

J. Schwinger '48

The SM Prediction

The Standard Model prediction of the tau $g-2$ is:

$$
\begin{aligned}
a_\tau^{\mathrm{SM}} = {} & 117\,324\,(2) && \times 10^{-8} && \text{QED} \\
& + 47.4\,(5) && \times 10^{-8} && \text{EW} \\
& + 337.5\,(3.7) && \times 10^{-8} && \text{HLO} \\
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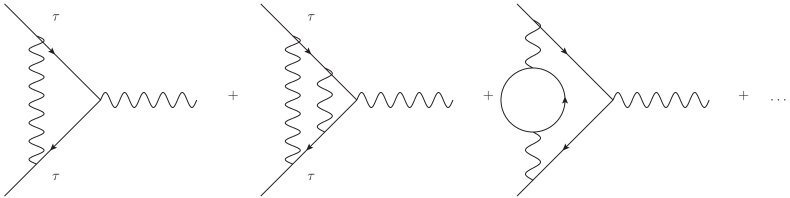

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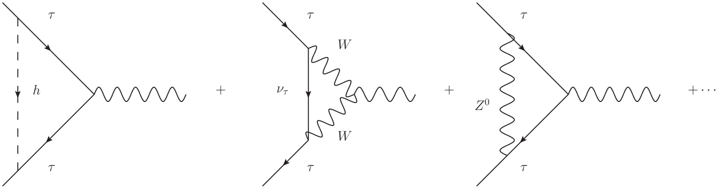

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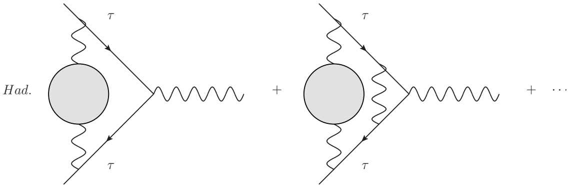

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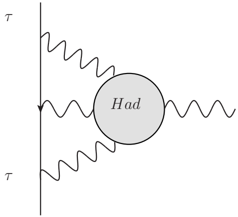

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\end{aligned}
$$

$$a_\tau^{\rm SM} = 117\,721\,(5) \; \times 10^{-8}$$

S. Eidelman, M. Passera, MPLA 22 (2007) 159

Why is a_τ important?

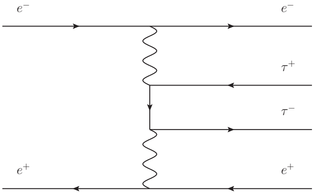
- The tau $g-2$ is a property of an elementary particle.
- NP associated with a mass scale Λ is expected to modify the $g-2$ of a lepton ℓ by a contribution $a_\ell^{\rm NP} \approx m_\ell^2/\Lambda^2$.
- Given $m_\tau^2/m_\mu^2 \sim 283$, a_τ is much more sensitive than the muon one to EW and NP loop effects which give contributions $\sim m_\ell^2$.
- If an NP effect were of the same order of magnitude as that of the EW, a_τ would provide a "clean" NP test.

	Electron	Muon	Tau
$a^{\rm EW}/a^{\rm HAD}$	1/56	1/45	1/7
$a^{\rm EW}/\delta a^{\rm HAD}$	1.6	3	10

...if only we could measure it!

Experimental Bounds

- The very short lifetime of the tau ($2.9 \cdot 10^{-13}$ s) makes it impossible to put τ in a storage ring to measure a_τ via its spin precession.
- DELPHI's result, from $e^+e^- \to e^+e^-\tau^+\tau^-$ total cross-section measurements at LEP 2 (the PDG value):



$$-0.052 \leq a_\tau \leq 0.013$$ DELPHI - EPJC35 (2004) 159

- González-Sprinberg et al. analysis of LEP1/2 and SLC $e^+e^- \to \tau^+\tau^-$ data, using effective Lagrangian approach:

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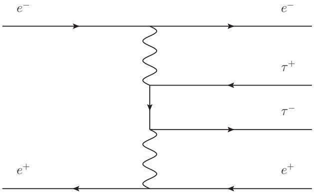

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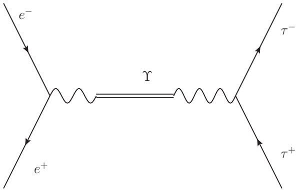
Is it possible to do better?

Bernabéu et al. propose the measurement of $F_2(q^2 = M_\Upsilon^2)$ from $e^+e^- \to \tau^+\tau^-$ production at B factories.

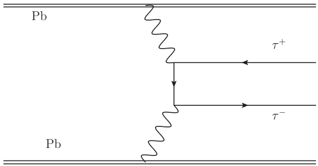


$\Longrightarrow$ bounds on $F_2(q^2 = M_\Upsilon^2)$

Expected sensitivity for $F_2(q^2 = M_\Upsilon^2)$ at Super B: $\sim 10^{-6}$

Bernabéu et al. NPB 790 (2008) 160.

Possibility to use heavy ion collision at LHC: Pb Pb $\to$ Pb Pb $\tau^+\tau^-$.



Advantage: Pb Pb $\gamma\gamma \approx$ on-shell.
Expected sensitivity $\sim 10^{-5}$.

F. del Aguila et al, PLB 271 (1991) 256

Radiative Decays of the Tau

What about $\tau^- \to \ell^- \bar{\nu}_\ell \nu_\tau \gamma$?

It was suggested to search for the a_τ in radiative leptonic τ decays using the phenomenon of radiation zero: the tree-level amplitude (which corresponds to $a_\tau = 0$) vanishes in the phase space region

$$\cos(\ell,\gamma) = -1, \quad x = \frac{2E_\ell}{m_\tau} = 1 + \frac{m_\ell^2}{m_\tau^2} \qquad \text{(in the } \tau \text{ r.f).}$$

M. L. Laursen et al. PRD 29 (1984) 2652

Our aim is to:

- revisit τ radiative decays,
- study the dependence of the pol. diff. decay rate on a_τ,
- probe a_τ at $\mathcal{O}(10^{-3})$ with minimal assumption,

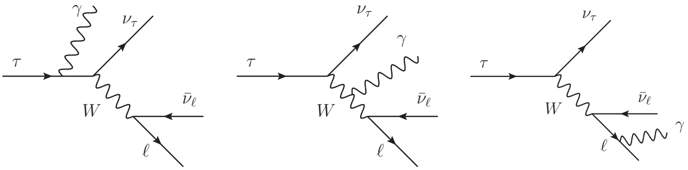
No search for BSM physics!

Discrepancy of a_μ scaled as $\frac{m_\tau^2}{m_\mu^2}$: $\sim 10^{-6}$ needed.

Effective Lagrangian Approach

Add a minimal $g-2$ coupling:

$$\mathcal{L}_{\rm eff} = e\frac{\tilde{a}}{4m_\tau}\bar{\tau}\sigma^{\mu\nu}\tau F_{\mu\nu}.$$



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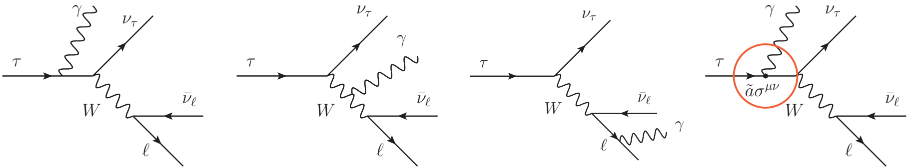


The new term contributes to the tree level decay rate:

$$d\Gamma(\tau^- \to \ell^- \bar{\nu}_\ell \nu_\tau \gamma) = d\Gamma_{\text{SM}} + \tilde{a}\, d\Gamma_a.$$

Terms $\mathcal{O}(\tilde{a}^2)$ are neglected since of order 10^{-6}.

The $\tilde{a}$ Parameter

- Since we want to probe a_τ at $\mathcal{O}(10^{-3})$, we cannot neglect terms of the same order in the decay rate.
- We need $d\Gamma$ at order α^2, i. e. we must include the QED NLO corrections

What does $\tilde{a}$ parametrize?

The introduction of the new effective coupling shifts the on-shell form factor F_2:

$$F_2\Big|_{\text{on-shell}} = F_2^{\text{SM}}\Big|_{\text{on-shell}} + \tilde{a}.$$

Therefore the tau $g-2$ at leading order is now given by:

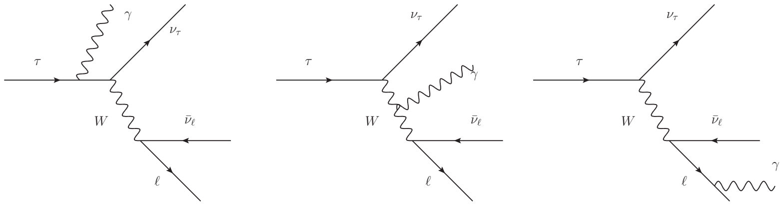
$$a_\tau = F_2^{(1)\text{QED}} + \tilde{a} + \mathcal{O}(\alpha^2) = \frac{\alpha}{2\pi} + \tilde{a} + \mathcal{O}(\alpha^2)$$

After the inclusion of QED one-loop corrections in $d\Gamma$,
$\tilde{a}$ parametrizes all non-QED contributions to the $g-2$ of $\mathcal{O}(\alpha)$.

QED one-loop Corrections to Tau Radiative Decay

We computed the polarized decay rate up to order α^2, including the effective coupling of $\tilde{a}$.

$$d\Gamma = d\Gamma_0 + \frac{m_\tau^2}{m_W^2} d\Gamma_W + \tilde{a}\, d\Gamma_a + \frac{\alpha}{\pi} d\Gamma_1$$



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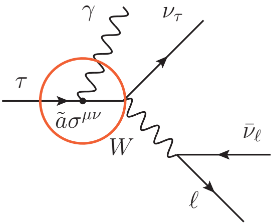

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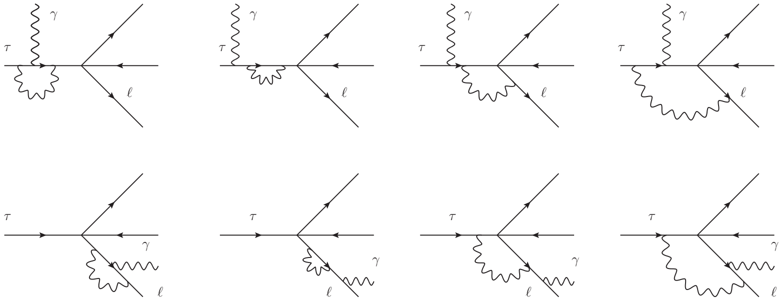

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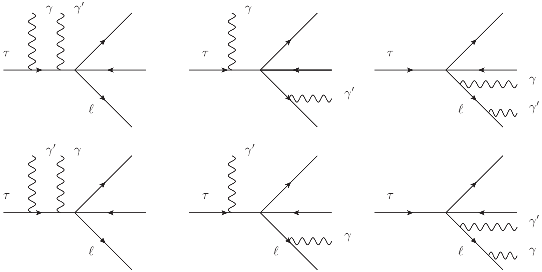

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- Neutrinos momenta integrated over the phase space.
- W propagator effects can be neglected in $d\Gamma_1$ since $\mathcal{O}\left(\alpha^2 \frac{m_\tau^2}{m_W^2}\right)$.
- Fermi four-fermion interaction used in the computation of $d\Gamma_1$.
- Agree with previous results: T. Kinoshita, A. Sirlin PRL2(1959)177; Y. Kuno, Y. Okada, RMP73(2001)151; A. Fischer et al. PRD49(1994)3426; A. B. Arbuzov PLB597(2004)285.

Opportunities and Challenges

- High statistics is needed.
- Phase space integration kills $d\Gamma_a$, since $d\Gamma_0$ has phase space singularities (E_γ and collinear photon):

$$Br(\tau^- \to \mu^- \bar{\nu}_\mu \nu_\tau \gamma)\Big|_{E\gamma > 10\text{MeV}} = 0.361\,(38)\% \qquad \left[0.367 + (8.6 \cdot 10^{-4})\tilde{a}\right]\%$$

CLEO coll. PRL84(2000)830, our results

$$Br(\tau^- \to e^- \bar{\nu}_e \nu_\tau \gamma)\Big|_{E\gamma > 10\text{MeV}} = 1.75\,(18)\% \qquad \left[1.837 + (8.2 \cdot 10^{-4})\tilde{a}\right]\%$$

- A full phase space analysis is needed.
- Feasibility for the search of the tau $g-2$ at Belle and future Belle-II is currently under study.
- What can LHC tell us about tau $g-2$?

Summary and Outlook

- It is desirable to get a more precise measurement of anomalous magnetic moment of the τ.
- There are proposals to improve the PDG value, through the study of the production cross section and decay rate, e.g. radiative decays.
- The possibility to measure a_τ in radiative decays at Belle is under investigation.
- A feasibility study for the search of a_τ at LHC is needed.

Thanks!

Decay Rate Formula

The total differential decay for a polarized τ lepton in the tau r.f. is

$$\frac{d^3\Gamma}{dx\, dy\, d\cos\theta} = \frac{\alpha\, G_F^2 m_\tau^5}{2(4\pi)^6} y\sqrt{x^2-4r^2}$$
$$\times\Big\{ G(x,y,\theta_{\ell\gamma}) + \cos\theta_\ell \sqrt{x^2-4r^2} J(x,y,\theta_{\ell\gamma}) + y\cos\theta_\gamma K(x,y,\theta_{\ell\gamma}) \Big\}\,,$$

where

$$x = \frac{2E_\ell}{m_\tau}, \quad y = \frac{2E_\gamma}{m_\tau}, \quad r = \frac{m_\ell^2}{m_\tau^2},$$

θ_ℓ (θ_γ) is the angle between the lepton (photon) momentum and the polarization vector.

$$G = G_0 + \frac{m_\tau^2}{m_W^2} G_W + \tilde{a}\, G_a + \frac{\alpha}{\pi} G_{\rm RC}$$

The functions J and K have the same structure.

$$G_0 = -\frac{64\pi^2}{3y^2z^2}\left[r^4\left(6xy^2+6y^3-6y^2z-8y^2\right)+r^2\left(-4x^2y^2-6x^2yz\right.\right.$$
$$-8xy^3+2xy^2z+6xy^2+6xyz^2+8xyz+6xz^2-4y^4+5y^3z+6y^3$$
$$\left.-2y^2z^2-6y^2z-3yz^3+6yz^2-6z^3-8z^2\right)+4x^3yz+8x^2y^2z$$
$$-8x^2yz^2-6x^2yz-4x^2z^2+6xy^3z-8xy^2z^2-6xy^2z+6xyz^3$$
$$-2xyz^2+8xz^3+6xz^2+2y^4z-2y^3z^2-3y^3z+2y^2z^3-2y^2z^2$$
$$\left.-2yz^4+5yz^3+6yz^2-6z^3\right]$$

Kinoshita & Sirlin ('59), Kuno & Okada ('01)

$$G_a = -\frac{64\pi^2}{3yz}\left[xyz+2xz^2+y^2z+yz^2-yz-2z^3-2z^2\right.$$
$$\left.+r^2\left(-y^2+yz-3z^2\right)\right]$$

$$G_{aa} = -\frac{16\pi^2}{3}\left[2x^2+2xy-2xz-3x-yz+2z+r^2\left(-3x-2y+4\right)\right]$$