Domenico Bonocore

NIKHEF

Next to Eikonal Webs

LHCphenonet Annual Meeting 2012

LHCphenonet

Why Resummation?

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- *Understanding infrared divergences and large logarithms*

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example: Drell-Yan $z \to 1$

$$\frac{d\sigma}{dQ^2} = \sigma_B C_F \left[\delta(1-z) \left(\frac{2\pi^2}{3} - 8 \right) + 4(1+z^2) \left(\frac{\ln(1-z)}{1-z} \right)_+ - 2 \left(\frac{1+z^2}{1-z} \right) \ln z \right]$$

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Fixed order calculation near the threashold converges poorly

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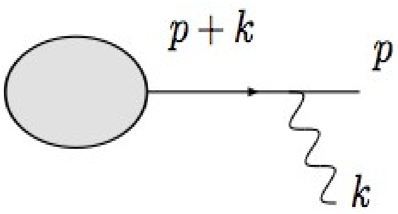
$\longrightarrow$ *Let's focus on soft divergences*

Eikonal Approximation and exponentiation



$$
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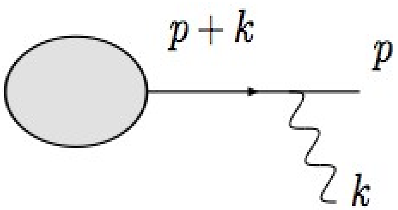

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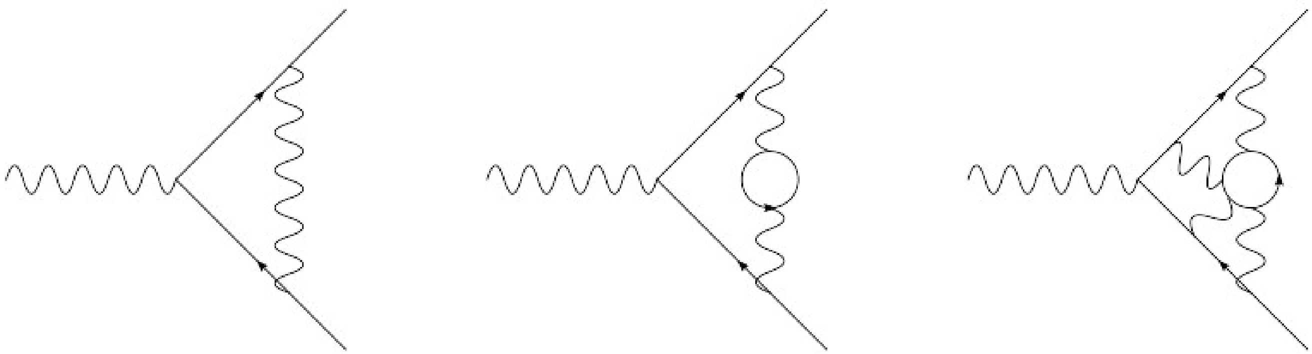
Abelian ⟶ **Connected subdiagrams** (~ '60 – Yennie, Frautschi, Suura)



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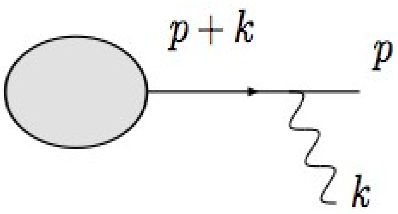

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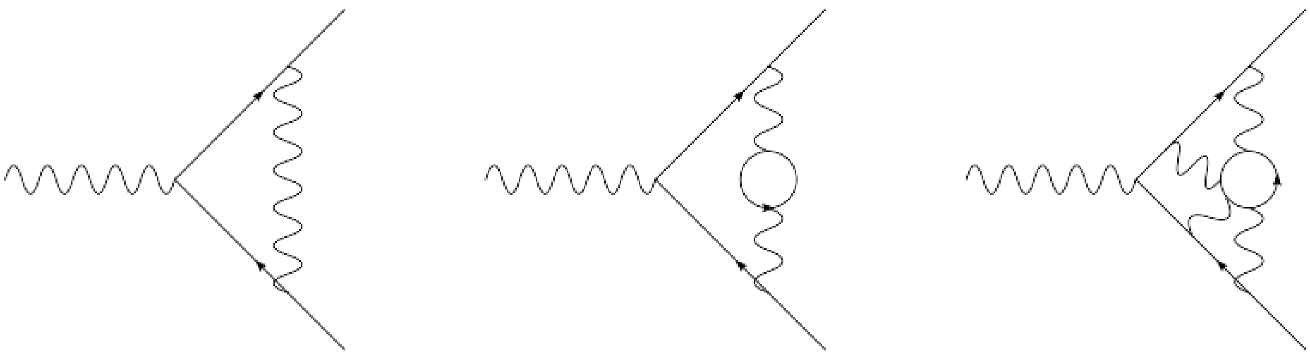

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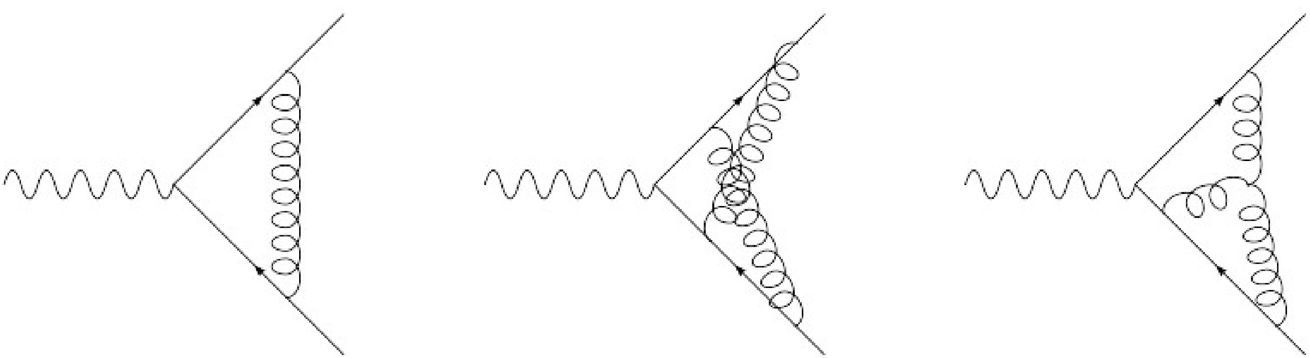
Non abelian ⟶ **Webs** (~ '80 Gatheral, Frenkel, Taylor, Sterman)



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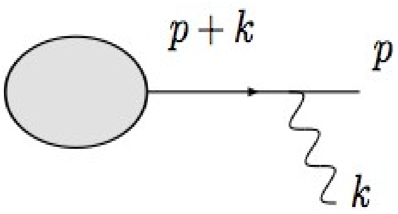

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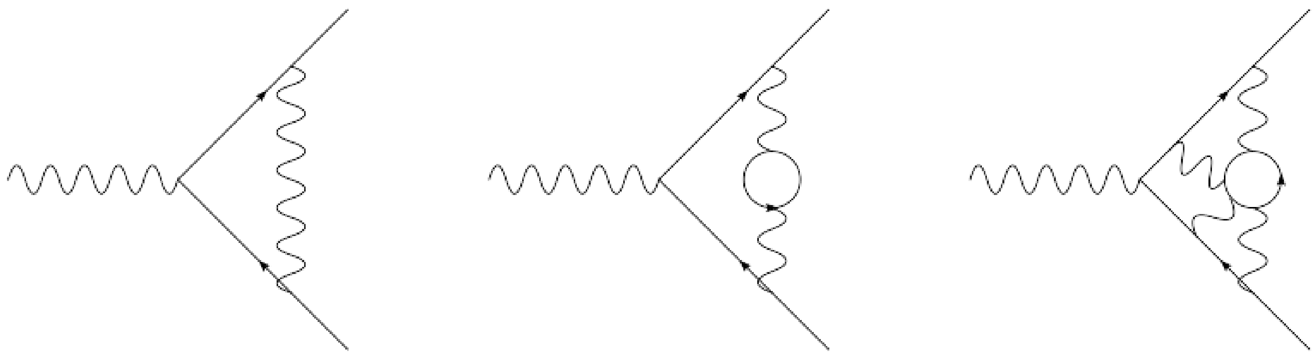

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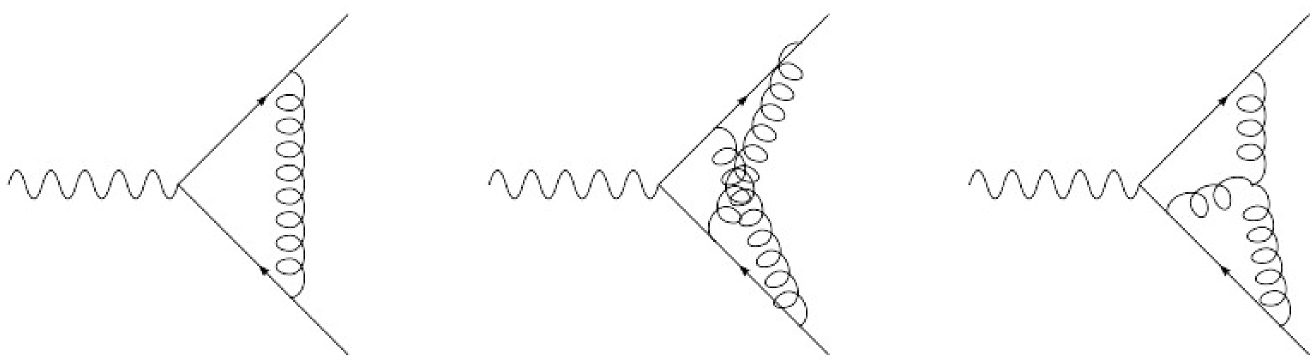

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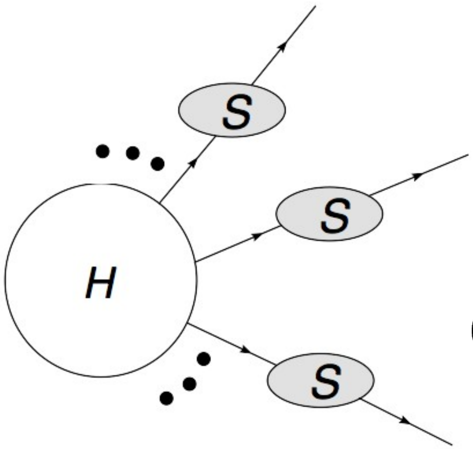
(2008: Laenen,Stavenga,White - 0811.2067) *new approach via path integral...*

A path integral approach: QED case



$$\int \mathcal{D}A^\mu \equiv \int \mathcal{D}A_s^\mu \mathcal{D}A_h^\mu$$

Factorization of hard and soft modes

$$G(p_1,\ldots,p_n) = \int \mathcal{D}A_s^\mu H(x_1,\ldots,x_n;A_s^\mu) S(p_1,x_1)\ldots S(p_n,x_n) e^{iS[A_s]}$$

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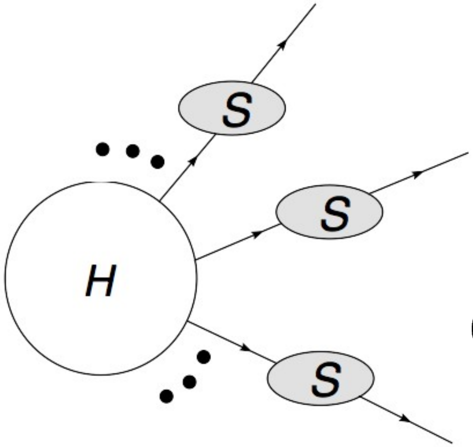

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external propagators S(p,x) as first quantized path integral

$$S(p,x) = \int \mathcal{D}p\mathcal{D}x \;\; exp[i\int_0^T dt(p\dot{x} - \frac{1}{2}p^2 + (p_f + p)\cdot A_s(x_i + p_f t + x) + \frac{1}{2}\partial \cdot A_s(x_i + p_f t + x) - A_s^2(x_i - p_f t + x))]$$

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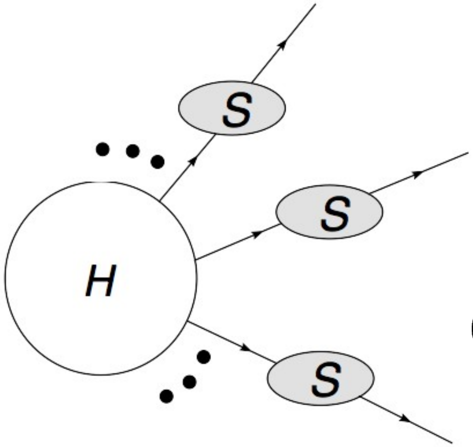

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going on with computation...

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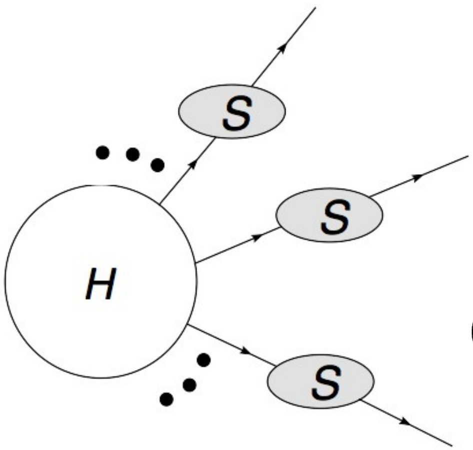

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a theory for the soft field where terms in the exponent work as sources ⟶ **Exponentiation like in usual textbook results!**

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→ ***first approx: neglect fluctuation*** ⟶ ***classical trajectory***

$$f(\infty) \propto e^{i\int dx\cdot A(x)}$$

Wilson lines

This is exactly the eikonal limit !

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$p_j = \lambda n_j; n_j^2 = 0$ expanding in $1/\lambda$:

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For both E and NE approx one can find exponentiation and Feynman rules as usual in QFT

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Much more difficult: a key point for exponentiation in QED was that everything commute

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Also in the non abilian case exponentiation become possible at Eikonal and Next to Eikonal level

- 2 parton final legs (2008: Laenen,Stavenga,White - 0811.2067)
- multiparton case (2010: Gardi,Laenen,Stavenga,White - 1008.0098)

State of the art:

Evaluating martix elements at E and NE level

$$\mathcal{M}^{E} \qquad \mathcal{M}^{NE}$$

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It remains to compute a full cross section
(various attempts by means of other approaches)

$$\frac{d\sigma}{d\xi} = \int d\mathrm{PS}^{(\mathrm{E})}\, |\mathcal{M}^{(\mathrm{E})}|^2 + \left[\int d\mathrm{PS}^{(\mathrm{E})}\, |\mathcal{M}^{(\mathrm{NE})}|^2 + \int d\mathrm{PS}^{(\mathrm{NE})}\, |\mathcal{M}^{(\mathrm{E})}|^2\right] + \mathcal{O}(\mathrm{NNE})$$

Thank you!