Higgs + Multi-gluon Helicity Amplitudes

LHCphenonet Annual Meeting

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Introduction

- Recent data indicates that a discovery of a Higgs may be likely in 2012
- The key question is then what sort of Higgs is it?
- Measuring the properties of the Higgs - and hence identifying it - requires precise calculations
- Among different Higgs production channels gluon fusion to Higgs via quark loop is one of the most promising channels especially for Higgs mass around $120 - 130 GeV$

Analytical Matrix Elements

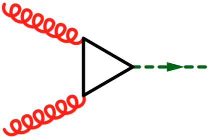
H+n gluons	tree level	1-loop	2-loop	3-loop
2	P.R.L.39,1304(1977) P.L.B.78,443(1978)	Nucl.Phys.B 359.283(1991) hep-ph/0608180 hep-ph/0611236	hep-ph/0207004 hep-ph/0302135 hep-ph/0201206	hep-ph/1004.3653
3	hep-th/0411092 hep-ph/0404013	hep-ph/0201114 hep-ph/9707448 hep-ph/9902483 hep-ph/0608180	hep-ph/1112.3554	✘
4	hep-th/0411092 hep-ph/0404013	hep-ph/0608180 hep-ph/0909.4475	✘	✘
5	hep-th/0411092 hep-ph/0404013	✘	✘	✘

Numerical Cross Sections

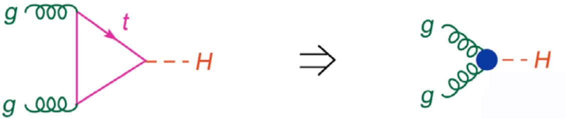
H+n gluons	tree level	1-loop	2-loop	3-loop
2	MadGraph Whizard Herwig++	MCFM MC@NLO aMC@NLO POWHEG	hep-ph/0207004 hep-ph/0302135 hep-ph/0201206 iHixs	✘
3	MadGraph Whizard Herwig++	MadLoop MCFM MC@NLO aMC@NLO POWHEG	Proceeding	✘
4	MadGraph Whizard Herwig++	MadLoop MCFM aMC@NLO POWHEG	✘	✘
5	MadGraph Whizard	✘	✘	✘
6	MadGraph Whizard	✘	✘	✘

The Higgs Model

- Higgs production via gluon fusion through a quark loop



- In the large top mass limit, we have the effective interaction



- The effective interaction term in Lagrangian

$$\mathcal{L}_H^{int} = \frac{C}{2} H \, \mathrm{Tr} G_{\mu\nu} G^{\mu\nu}, \qquad C = \frac{\alpha_s}{6\pi\mathcal{V}}(1 + \mathcal{O}(\alpha_s))$$

Wilczek, Shifman, Vainshtein, Zakharov 1970′s

The Higgs Model

- Inspired by the twister-space structure of Higgs+gluons amplitudes, consider H field as the real part of an complex field $\phi = \frac{1}{2}(H + iA)$ and divide $\mathcal{L}_H^{int}$ into two selfdual (SD) and anti-selfdual (ASD) gluon field strengths

$$\begin{aligned}\mathcal{L}_{H,A}^{int} &= \frac{C}{2}[H\ \mathrm{Tr}G_{\mu\nu}G^{\mu\nu} + iA\ \mathrm{Tr}G_{\mu\nu}{}^*G^{\mu\nu}] \\ &= C[\phi\, TrG_{SD\mu\nu}G_{SD}^{\mu\nu} + \phi^\dagger\, TrG_{ASD\mu\nu}G_{ASD}^{\mu\nu}]\end{aligned}$$

- The relations between different fields are

$$G_{SD\mu\nu} = \frac{1}{2}(G_{\mu\nu} + {}^*G_{\mu\nu}) \qquad G_{ASD\mu\nu} = \frac{1}{2}(G_{\mu\nu} - {}^*G_{\mu\nu})$$

$$H = \phi + \phi^\dagger \qquad A = \frac{\phi + \phi^\dagger}{i}$$

Dixon, Glover, Khoze, 2004

ϕ plus multi-gluon tree amplitudes

- The amplitudes for ϕ plus n gluons, and those for $\phi^\dagger$plus n gluons, separately have a simpler structure than the amplitudes for H
- The two-point vertex coupling ϕ to two off-shell gluons is

$$V^{\phi}_{\mu\nu}(k_1, k_2) = \eta_{\mu\nu} k_1 \cdot k_2 - k_{1\mu} k_{2\nu} + i\epsilon_{\mu\nu\rho\sigma} k_1^\rho k_2^\sigma$$

- Using the same technique for pure gluon helicity amplitudes (Berends-Giele currents/SUSY WI) the general $\phi(\phi^\dagger)$ plus n gluon amplitudes satisfy

$$A_n(\phi, 1^\pm, 2^+, 3^+, \cdots, n^+) = 0$$
$$A_n(\phi^\dagger, 1^\pm, 2^+, 3^+, \cdots, n^+) \neq 0$$
$$A_n(\phi^\dagger, 1^+, 2^+, \cdots, n^+) = \frac{m_H^4}{\langle 12\rangle\langle 23\rangle\cdots\langle n1\rangle}$$

Higgs plus multi-gluon tree amplitudes

- The first non-vanishing amplitudes for ϕ field is the $\phi - MHV$ amplitude with two negative helicities and similar structure in Parke-Taylor amplitudes (MHV amplitude)

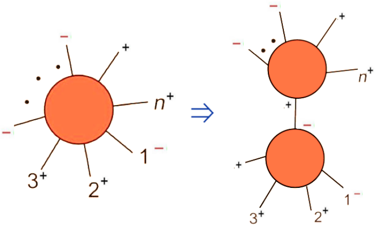
$$A_n(\phi, 1^+, \cdots, p^-, \cdots, q^-, \cdots, n^+) = \frac{\langle pq \rangle^4}{\langle 12 \rangle \langle 23 \rangle \cdots \langle n-1, n \rangle \langle n1 \rangle}$$

- The amplitudes for $\phi^\dagger$ field are related by parity, which changes $\langle i, j \rangle \leftrightarrow [j, i]$ in ϕ amplitudes
- Higgs plus multi-gluon tree amplitudes are the sum of ϕ and $\phi^\dagger$ amplitudes

$$A_n(H, 1, 2, \cdots, n) = \sum_{helicities} A_n(\phi, 1, 2, \cdots, n) + A_n(\phi^\dagger, 1, 2, \cdots, n)$$

CSW method for non-MHV helicity amplitudes

- The idea is to recycle the known MHV amplitude to form tree amplitudes with more than two negative helicities.



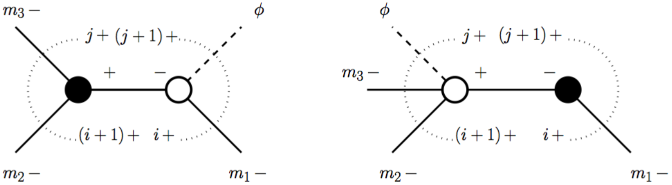
- The off-shell leg of the sub-MHV amplitudes is extended form on-shell spinor

$$p_{o\dot{o}} = \lambda_o \tilde{\lambda}_{\dot{o}} \rightarrow \xi_o = \lambda_o^{off-shell} = \frac{p_{o\dot{o}}\eta^{\dot{o}}}{[\tilde{\lambda}, \eta]}$$

Cashazo, Svrcek, Witten, 2004

NMHV amplitude for Higgs plus n gluons

- Using CSW method to connect $\phi-$MHV amplitudes with Parke-Taylor amplitudes



- One could derive the NMHV amplitude for Higgs plus n gluons

$$A_n^{(1)}(\phi, m_1, m_2, m_3) = \frac{\langle m_2, m_3\rangle^4}{\langle i+1, i+2\rangle \ldots \langle j-1, j\rangle \langle j, \xi\rangle \langle i+1, \xi\rangle p_{i+1.j}^2} \qquad A_{MHV}$$

$$\times \frac{\langle m_1, \xi\rangle^4}{\langle j+1, j+2\rangle \ldots \langle i-1, i\rangle \langle j+1, \xi\rangle \langle i, \xi\rangle} \qquad A_{\phi-MHV}$$

One-loop amplitude calculation method

- The unitarity of the scattering matrix implies

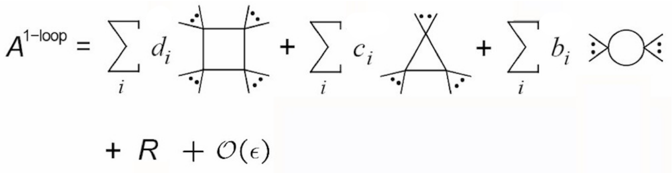
$$T^\dagger T = 2ImT$$

- To calculate the discontinuity of T one uses Cutkosky rules that

$$\frac{i}{p^2 + i\epsilon} \rightarrow 2\pi\delta^{(+)}(p^2)$$

R.Cutkosky 1960

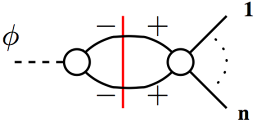
- In $D = 4 - 2\epsilon$ dimensions, one-loop integrals could be decomposed into a set of basis involving box, triangle and bubble (and tadpole) integrals



Bern, Dixon, Dunbar, Kosower 1994

One-loop amplitude calculation method

- Example of bubble coefficient of Higgs plus all-minus-helicity gluons



$$\int A_L \,\rangle\!\!\bigcirc\!\!\langle\, A_R = A^{(0)}(\phi; 1^-, \ldots, n^-) \int \frac{[l_1 l_2][1n]}{[l_2 1][n l_1]} \,\rangle\!\!\bigcirc\!\!\langle$$

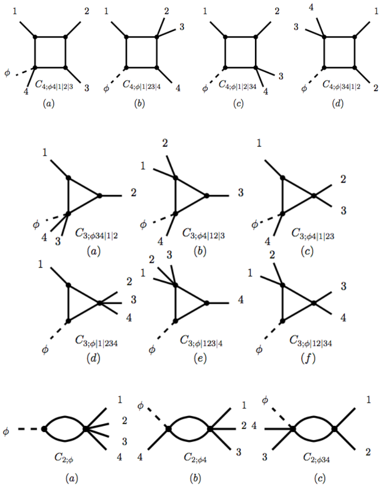
- Spinor algebra to simplify integrand and result in scalar loop integrals

$$\frac{[l_1 l_2][1n]}{[l_2 1][n l_1]} \to \frac{2P\cdot nP\cdot 1 - P^2 n\cdot 1}{(l_1+1)^2(l_2+n)^2} - \frac{P\cdot 1}{(l_1+1)^2} - \frac{P\cdot n}{(l_2+n)^2}$$

Badger, Glover 2006

One-loop amplitude calculation method

- All possible topologies of Higgs plus four gluons at one-loop



Badger, Glover, Mastrolia,Williams 2010

NNLO cross section structure

- The precise numerical cross section calculation for Higgs productions are evaluated at NNLO
- The general structure of NNLO calculation

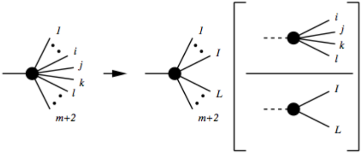
$$d\hat{\sigma}_{NNLO} = \int_{d\Phi_{m+2}} d\hat{\sigma}^{RR}_{NNLO} + \int_{d\Phi_{m+1}} d\hat{\sigma}^{RV}_{NNLO} + \int_{d\Phi_{m}} d\hat{\sigma}^{VV}_{NNLO}$$

- The general form for infrared subtraction terms

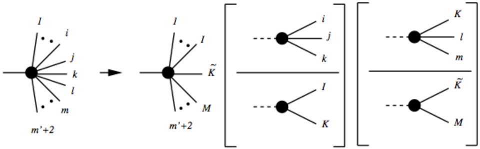
$$\begin{aligned} d\hat{\sigma}_{NNLO} &= \int_{d\Phi_{m+2}} (d\hat{\sigma}^{RR}_{NNLO} - d\hat{\sigma}^{RRS}_{NNLO}) + \int_{d\Phi_{m+2}} d\hat{\sigma}^{RRS}_{NNLO} \\ &+ \int_{d\Phi_{m+1}} (d\hat{\sigma}^{RV}_{NNLO} - d\hat{\sigma}^{RVS}_{NNLO}) + \int_{d\Phi_{m+1}} d\hat{\sigma}^{RVS}_{NNLO} \\ &+ \int_{d\Phi_{m}} d\hat{\sigma}^{VV}_{NNLO} + \int d\hat{\sigma}^{MF}_{NNLO} \end{aligned}$$

Antenna subtraction method

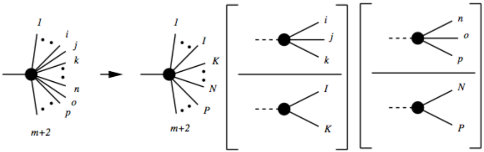
- Antenna subtraction method abstracts the infrared poles as universal factors (antenna function). At NNLO they are



(a) Color connect



(b) Color almost connect



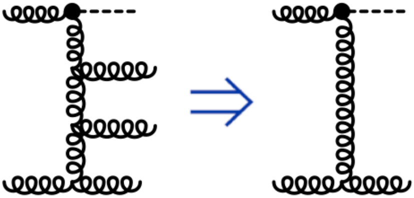
(c) Color not connect

Antenna subtraction application at NNLO and future work

- Successfully applied to $e^+e^- \to 3$ *jets* at NNLO

  Gehrmann-De Ridder, Gehrmann, Glover ,Heinrich 2007

- Now being applied to jet production in proton proton collisions
- The Higgs plus three gluon is in process at implementing the double real part numerically



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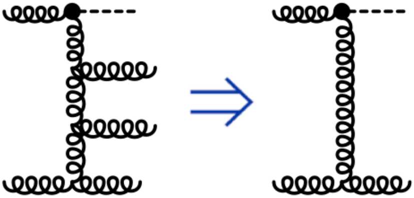

Thanks!