

A critical appraisal of NLO+PS methods

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[arXiv:1111.1220](#), [arXiv:1201.5882](#)



LHCphenOnet

General NLO+PS matching

- NLO calculation with subtraction methods

$$\sigma^{\text{NLO}} = \int d\Phi_B \left[B(\Phi_B) + V(\Phi_B) + I(\Phi_B) \right] \\ + \int d\Phi_R \left[R(\Phi_R) - \sum_i D_i^{(S)}(\Phi_R) \right]$$

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- $D_i^{(A)}$ and $D_i^{(S)}$ need to have same momentum maps and IR limit

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General NLO+PS matching

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- use $D_i^{(A)}$ as resummation kernels
- resummation phase space limited by $\mu_Q^2 = t_{\text{max}}$
 - starting scale of parton shower evolution
 - should be of the order of the hard resummation scale
- choice of coupling scales:
 - fixed order parts: μ_R
 - resummation: needs $\mu_{\text{exp}} \rightarrow p_{\perp}$ as $p_{\perp} \rightarrow 0$, otherwise arbitrary

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POWHEG

Special choices:

Nason JHEP11(2004)040, Frixione et.al. JHEP11(2007)070

- exponentiation kernel $D_i^{(A)} = \rho_i \cdot R$ with $\rho_i = D_i^{(S)} / \sum_i D_i^{(S)}$
 → each R_i contains only one divergence structure as defined by $D_i^{(S)}$

Consequences:

- no H-events, resummation scale μ_Q^2 at kinematic limit $\frac{1}{2} s_{\text{had}}$
- in CS-subtraction instabilities in ρ_i due to different cuts on R and $D_i^{(S)}$
- exponentiation of R through matrix element corrected parton shower
 NLO accuracy depends crucially on presence of exact same terms in subtraction and parton shower

Modifications:

- introduce suppression function $f(p_\perp) = h^2 / (p_\perp^2 + h^2)$ Alioli et.al. JHEP04(2009)002
 → $D_i^{(A)} = R_i \cdot f(p_\perp)$
 → continuous dampening of resummation kernel at large p_\perp

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MC@NLO – traditional scheme

Special choices:

Fraxione, Webber JHEP06(2002)029

- exponentiation kernel $D_i^{(A)} = B \cdot \mathcal{K}_i$ with \mathcal{K}_i parton shower kernels

Consequences:

- resummation scale $\mu_Q^2 = t_{\max}$ parton shower starting scale
- in general, $D_i^{(A)}$ only leading colour approximation
NLO accuracy depends crucially on correctness of IR-limit

Modifications:

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- introduce soft modification function $f(p_\perp)$ such that

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MC@NLO – $D_i^{(A)} = D_i^{(S)}$ scheme

Special choices:

SH, FK, MS, FS arXiv:1111.1220

- exponentiation kernel $D_i^{(A)} = D_i^{(S)}$

Consequences:

- simplification of $\bar{B}^{(A)}$ -integral
- resummation scale $\mu_Q^2 = t_{\max}$ set by phase space limitation of subtraction terms
 - subtraction constrained in parton shower t needed for physical resummation
 - exemplary/temporarily use α_{cut} to explore effects Nagy PRD68(2003)094002
- trivially NLO correct independent of the process without arbitrary parameter choices

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MC@NLO – $D_i^{(A)} = D_i^{(S)}$ scheme

Implemented in SHERPA – full-colour first parton shower emission

Tricky point: $D_i^{(A)} < 0$ e.g. for subleading colour dipoles

Use modified Sudakov veto algorithm

SH, FK, MS, FS arXiv:1111.1220

- Assume $f(t)$ as function to be generated, and overestimate $g(t)$
Standard probability for *one* acceptance with n rejections

$$\frac{f(t)}{g(t)} g(t) \exp \left\{ - \int_t^{t_1} d\bar{t} g(\bar{t}) \right\} \prod_{i=1}^n \left[\int_{t_{i-1}}^{t_{n+1}} dt_i \left(1 - \frac{f(t_i)}{g(t_i)} \right) g(t_i) \exp \left\{ - \int_{t_i}^{t_{i+1}} d\bar{t} g(\bar{t}) \right\} \right]$$

- Can split weight into MC and **analytic** part using auxiliary function $h(t)$

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$$w(t, t_1, \dots, t_n) = \frac{g(t)}{h(t)} \prod_{i=1}^n \frac{g(t_i)}{h(t_i)} \frac{h(t_i)}{g(t_i)} \frac{f(t_i)}{g(t_i)}$$

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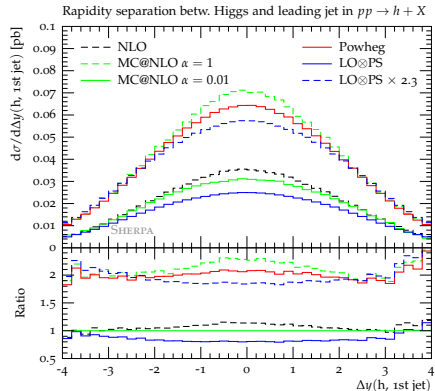
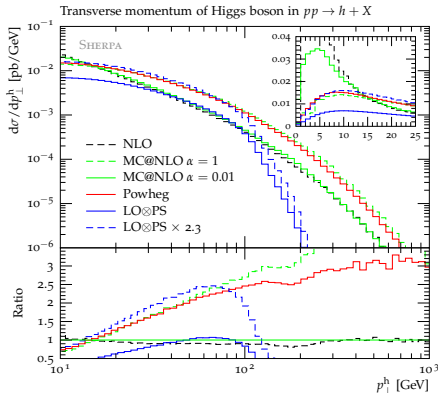
Identify $f(t)$, $g(t)$, $h(t)$:

- $f(t)$ determined by MC@NLO $\Rightarrow D_i^{(A)}$
- $h(t)$ determined by parton shower $\Rightarrow D_i^{(PS)}$
- $g(t)$ **can be chosen freely** $\Rightarrow \text{const.} \cdot f$
constraints: $\text{sign}(f) = \text{sign}(g)$, $|f| \leq |g|$

Automation in SHERPA framework

- easy to automate and process independent
only virtual correction V needs to be supplied
- leading order pieces and phase space generation taken care of by well-tested automated tree-level matrix element generators AMEGIC++ and/or COMIX
- based on Catani-Seymour subtraction
- using dipole-like parton shower
- parton showers are easy to correct with matrix elements
 $D_i^{(A)}/B$ always non-zero and close to pure parton shower result
deviations in soft-gluon regime limited by parton shower cut-off

Results – $h + X$ production in gluon fusion

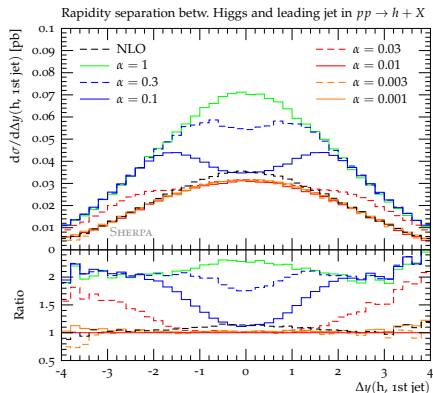
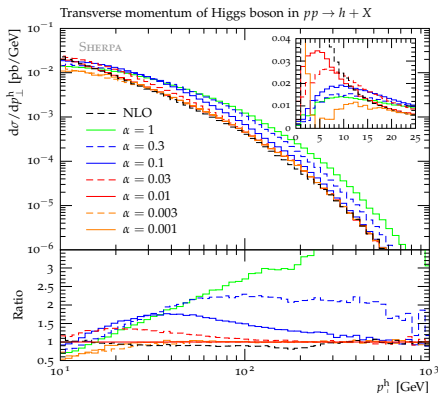


Sanity checks:

- Sudakov shape of LO \otimes PS result for $\alpha_{\text{cut}} \rightarrow 1$
- High- p_{\perp} tail of NLO reproduced for $\alpha_{\text{cut}} \ll 1$

Ideally both simultaneously, as in traditional MC@NLO,
but currently limited by inappropriate choice of μ_Q^2 through α_{cut}

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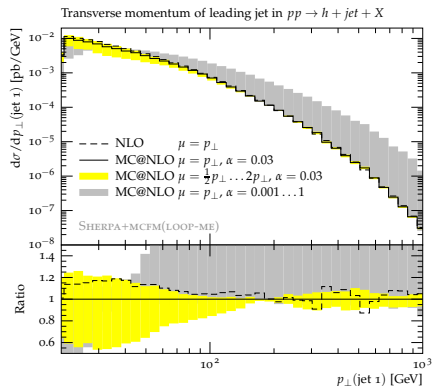
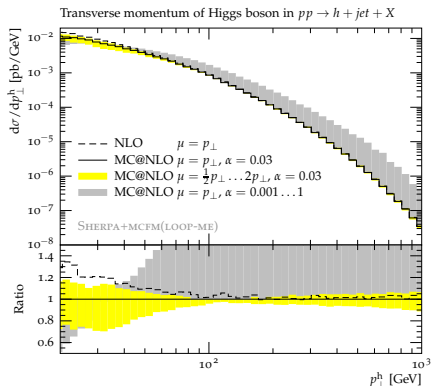


Essential features of [Alioli et.al. JHEP04\(2009\)002](#) reproduced

- hardness of POWHEG/MC@NLO $\alpha_{cut} = 1$
- dip in Δy for intermediate α_{cut}

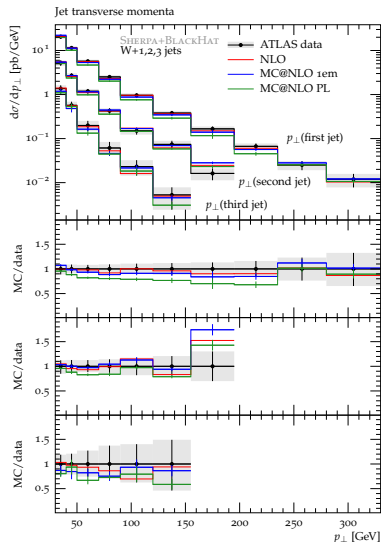
Purely academic, rather define $D^{(A)}$ properly \Rightarrow **phase space separation in t**

Results – $h + j + X$ production



- increased parton multiplicity worsens problems
- large dependence on exponentiated phase space
- unphysical results for $\mu_Q^2 \rightarrow \frac{1}{2} s_{\text{had}}$

Results – $W + n$ jet production



$W + 1, 2, 3$ jets at LHC (ATLAS data)

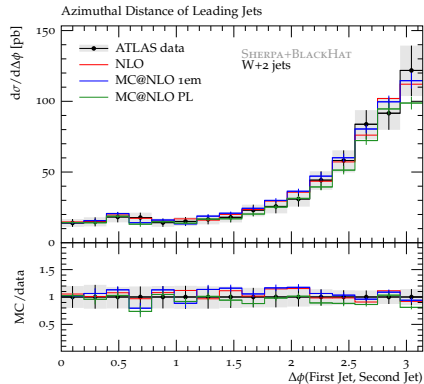
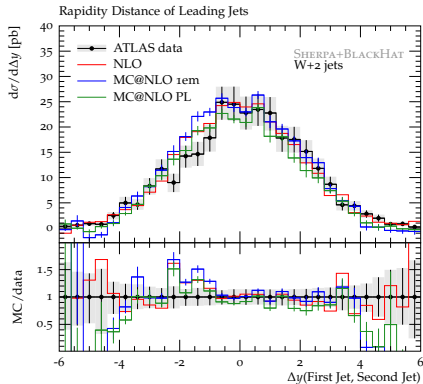
- complexity not a problem
- speed limited by the virtual amplitude in $W + 3j$
- scales:

$$\mu_R = \mu_F = \frac{1}{2} \hat{H}'_T$$

$$\mu_{\text{exp}} = \frac{1}{(1/p_{\perp}^2 + 1/\mu_R^2)^{\frac{1}{2}}}$$

- fixed order behaviour at high p_{\perp}
 \rightarrow smoother transition to $\overline{\text{MS}}$ -events

Results – $W + n$ jet production



Conclusions

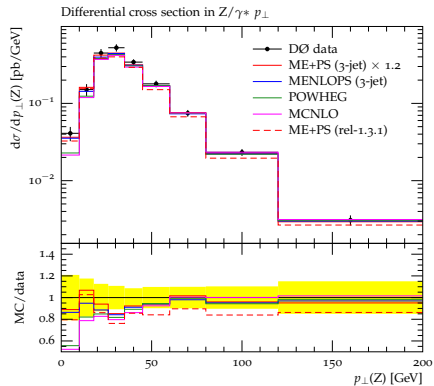
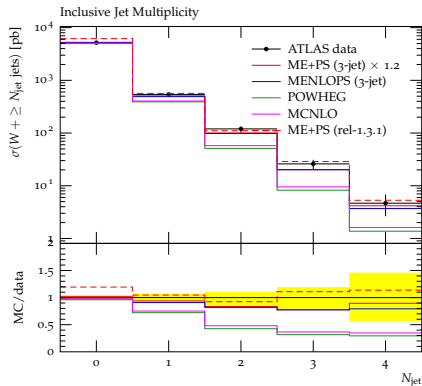
- care must be taken to preserve the correct resummation properties as not to introduce large spurious subleading logarithms

Banfi, Salam, Zanderighi JHEP08(2004)062

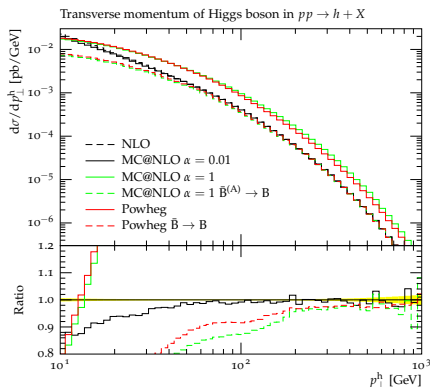
- MC@NLO $D^{(A)} = D^{(S)}$ scheme facilitates implementation of process independent correct soft-gluon limit needed for NLO accuracy
- no conceptual problems for high-multiplicity processes
- independent check of POWHEG \leftrightarrow MC@NLO differences
- both MC@NLO and POWHEG can be trivially combined with ME+PS merging \Rightarrow MENLOPS

SHERPA-1.4.0

MENLOPS with MC@NLO



Additional Sudakov suppression by inappropriately sized resummation phase space



- first emission only
- $\mu_{\text{exp}}^2 = \min(p_{\perp}^2, \mu_R^2)$ and $\mu_R = m_h$, such that $\mu_{\text{exp}} = \mu_R$ for $p_{\perp} > m_h$
- already 10% suppression at m_h