Next-to-Leading Order Scattering Amplitudes with Open Loops

Philipp Maierhöfer

Institute for Theoretical Physics
University of Zürich

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In Collaboration with
Fabio Cascioli and Stefano Pozzorini

Outline

State of the Art @ NLO

In the last few years, new theoretical developments lead to tremendous progress in the field of NLO calculations.

$pp \to W^+W^- b\bar{b}$	[Denner, Dittmaier, Kallweit, Pozzorini '10]
	[Bevilacqua, Czakon, van Hameren, Papadopoulos, Worek '11]
$pp \to t\bar{t}b\bar{b}$	[Bredenstein, Denner, Dittmaier, Pozzorini '08, '09, '10]
	[Bevilacqua, Czakon, Papadopoulos, Pittau, Worek '09]
$pp \to t\bar{t}jj$	[Bevilacqua, Czakon, Papadopoulos, Pittau, Worek '10]
$pp \to W^\pm W^\pm + 2j$	[Melia, Melnikov, Rontsch, Zanderighi '10]
	[Greiner, Heinrich, Mastrolia, Ossola, Reiter, Tramontano '12]
$pp \to W^\pm + 3j$	[Ellis, Melnikov, Zanderighi '09]
$pp \to \gamma^*/Z/W^\pm + 3j$	[Berger, Bern, Dixon, Febres Cordero, Forde, Gleisberg, Ita, Kosower, Maître '09]
$pp \to Z/W^\pm + 4j$	[—"— '09, '10]
$pp \to 4j$	[Bern, Diana, Dixon, Febres Cordero, Hoeche, Kosower, Ita, Maître, Ozeren '11]
$pp \to b\bar{b}b\bar{b}$	[Greiner, Guffanti, Reiter, Reuter '11]
$pp \to W\gamma\gamma j$	[Campanario, Englert, Rauch, Zeppenfeld '11]
$e^+e^- \to 7j$	[Becker, Goetz, Reuschle, Schwan, Weinzierl '11]

However, there's a trade-off between automation, flexibility, and speed.

Can the advantages of the different approaches be combined in a single algorithm?

Feynman Diagrams and Colour factorisation

Tree and one-loop amplitudes are handled as sums of Feynman diagrams

$$\mathcal{M} = \sum_{d} \mathcal{M}^{(d)}, \qquad \delta\mathcal{M} = \sum_{d'} \delta\mathcal{M}^{(d')}$$

Colour and helicity summed scattering probability densities

$$\mathcal{W} = \sum_{\mathrm{hel,col}} |\mathcal{M}|^2, \qquad \delta\mathcal{W} = \sum_{\mathrm{hel,col}} 2\,\mathrm{Re}(\mathcal{M}^*\delta\mathcal{M})$$

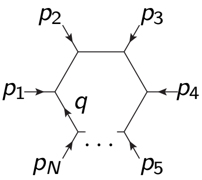
Diagrams factorise in colour factors and colour stripped amplitudes

$$\mathcal{M}^{(d)} = \mathcal{C}^{(d)}\mathcal{A}^{(d)}, \qquad \delta\mathcal{M}^{(d')} = \mathcal{C}^{(d')}\delta\mathcal{A}^{(d')}$$

Algebraic colour reduction and summation once per process

colour sums at zero cost

From amplitudes to scalar integrals



$$= \int d^d q \frac{\mathcal{N}(q)}{D_0 \; D_1 \; \dots \; D_{N-1}}, \qquad D_i = \left(q + \sum_{\ell=0}^{i} p_\ell\right)^2 - m_i^2$$

Tensor integral reduction

Reduce amplitude to a linear combination of scalar basis integrals

On-shell methods

$$\int d^d q \left[\sum_{i_1} \frac{a_{i_1}}{D_{i_1}} + \sum_{i_1, i_2} \frac{b_{i_1 i_2}}{D_{i_1} D_{i_2}} + \sum_{i_1, i_2, i_3} \frac{c_{i_1 i_2 i_3}}{D_{i_1} D_{i_2} D_{i_3}} + \sum_{i_1, i_2, i_3, i_4} \frac{d_{i_1 i_2 i_3 i_4}}{D_{i_1} D_{i_2} D_{i_3} D_{i_4}} \right]$$

Tensor integral reduction combined with off-shell current recursion can compete with OPP in *n*-gluon scattering with up to 10 gluons.

[van Hameren '09]

Tensor integral reduction

Separate tensor coefficients from tensor integrals.

$$\mathcal{A} = \sum_{r=0}^{R} \mathcal{N}_r^{\mu_1\cdots\mu_r} \cdot \int d^d q \, \frac{q_{\mu_1} \cdots q_{\mu_r}}{D_0 \; D_1 \; \ldots \; D_{N-1}}$$

- Covariant decomposition in tensor monomials built from $g^{\mu\nu}$ and p_i^μ.
- Reduce tensor integrals to scalar basis integrals.
  [Melrose], [Passarino, Veltman], [Denner, Dittmaier]
- Can be implemented in a numerically stable way (e. g. expansion in small Gram determinants):
  We use `Collier`, a private library by Denner and Dittmaier.

"Traditional" approach: construct $\mathcal{N}_r^{\mu_1\cdots\mu_r}$ analytically

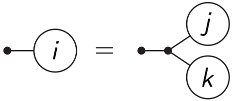
huge expressions & expensive algebraic simplifications limit applicability

OPP & Tree Generator

OPP directly extracts coefficients of the scalar basis integrals.

- Need multiple numerical evaluations of $\mathcal{N}(q)$ for complex q.
- Public implementations: `CutTools` [Ossola, Papadopoulos, Pittau]
  `Samurai` [Mastrolia, Ossola, Reiter, Tramontano]
- $\mathcal{N}(q)$ can be calculated by a generator for tree-level amplitudes.
  (diagrammatic like `MadLoop`, or current recursion like `HELAC-NLO`)

Wave functions w^α of "sub-trees" are 4-tuples
(for the spinor/Lorentz index) which are built by
recursivly connecting lower sub-trees with vertices and propagators.



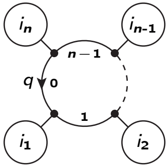
$$w^\beta(i) = \frac{X^\beta_{\gamma\delta}}{p_i^2 - m_i^2}\, w^\gamma(j)\, w^\delta(k)$$

external lines are not depicted

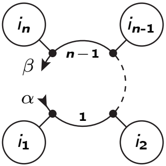
$X^\beta_{\gamma\delta}$ describes the interaction of i, j, k

Recursive Numerator Construction

A one-loop diagram is an ordered set of sub-trees $\mathcal{I}_n = \{i_1, \ldots, i_n\}$



$\xrightarrow{\text{cut } D_0}$ $\mathcal{N}^{\beta}_{\alpha}(\mathcal{I}_n; q) =$



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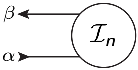
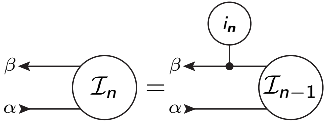


Connect sub-trees along the loop to build the numerator $\mathcal{N} = \mathcal{N}^{\alpha}_{\alpha}$



$$\mathcal{N}^{\beta}_{\alpha}(\mathcal{I}_n; q) = X^{\beta}_{\gamma\delta}\, \mathcal{N}^{\gamma}_{\alpha}(\mathcal{I}_{n-1}; q)\; w^{\delta}(i_n)$$

Separation of the loop momentum q ...

$$\sum_{r=0}^{n} \mathcal{N}^{\beta}_{\mu_1\ldots\mu_r;\alpha}(\mathcal{I}_n)\, q^{\mu_1} \ldots q^{\mu_r} = (Y^{\beta}_{\gamma\delta} + q^{\nu} Z^{\beta}_{\nu;\gamma\delta}) \sum_{r=0}^{n-1} \mathcal{N}^{\gamma}_{\mu_1\ldots\mu_r;\alpha}(\mathcal{I}_{n-1})\, q^{\mu_1} \ldots q^{\mu_r}\, w^{\delta}(i_n)$$

... leads to a recursion formula for the coefficients $\mathcal{N}^{\beta}_{\mu_1\ldots\mu_r;\alpha}$.

Open-Loops recursion

“Open loops” polynomials in q can be built recursively

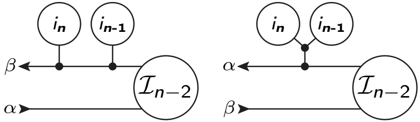
$$\mathcal{N}^{\beta}_{\mu_1\ldots\mu_r;\alpha}(\mathcal{I}_n) = \left[Y^{\beta}_{\gamma\delta}\, \mathcal{N}^{\gamma}_{\mu_1\ldots\mu_r;\alpha}(\mathcal{I}_{n-1}) + Z^{\beta}_{\mu_1;\gamma\delta}\, \mathcal{N}^{\gamma}_{\mu_2\ldots\mu_r;\alpha}(\mathcal{I}_{n-1}) \right] w^{\delta}(i_n)$$

$\Rightarrow$ **retains full loop momentum dependence.**

Once the polynomials are known, multiple evaluations of $\mathcal{N}(q) = \sum_{r=0}^{n} \mathcal{N}^{\alpha}_{\mu_1\ldots\mu_r;\alpha} q^{\mu_1}\ldots q^{\mu_r}$ are very fast. $\Rightarrow$ boosts OPP

On the other hand $\mathcal{N}^{\alpha}_{\mu_1\ldots\mu_r;\alpha}$ are the coefficients of the tensor integrals.

Open loops can be interfaced with both tensor integrals and OPP in a straight forward way.



Recycling:
Lower-point open-loops can be shared between diagrams if the cut it put appropriately.

Optimising OPP calls: efficient helicity summation

Once a tensor integral has been calculated, it can be reused in all diagrams with the same set of denominators.

The same degree of optimisation can be achieved for OPP reduction: Perform **interference** with the Born amplitude $\mathcal{M}$, **colour and helicity sums** and the sum over the set of diagrams Δ with identical denominator structure on the level of open-loop coefficients ***before* OPP reduction**.

$$\delta\mathcal{W}^{\Delta} = \sum_{\mathrm{hel,col}} 2\,\mathrm{Re}\left[\mathcal{M}^*\Big(\sum_{d'\in\Delta}\delta\mathcal{M}^{(d')}\Big)\right]$$

$$\delta\mathcal{W}^{\Delta}_{\mu_1\dots\mu_R} = \sum_{\mathrm{hel,col}} 2\times\left[\mathcal{M}^*\Big(\sum_{d'\in\Delta}\mathcal{C}^{(d')}\mathcal{N}^{(d')}_{\mu_1\dots\mu_R}\Big)\right]$$

Unpolarised, colour summed numerator $\mathcal{N}^{\Delta}(q) = \delta\mathcal{W}^{\Delta}_{\mu_1\dots\mu_R} q^{\mu_1}\dots q^{\mu_R}$

minimises the number of OPP calls
and leads to very efficient helicity sums.

Implementation

User input: process definition file

- `FeynArts` generates Feynman diagrams.
- `Mathematica` organises recursion and recycling, reduces colour factors and generates `Fortran 90` code.
- Numerical routines for QCD corrections to Standard Model processes implemented in `Fortran 90`.
- Symmetrising tensors keeps the number of components manageable.
- Rational terms R_2 are calculated using the tree generator. [Draggiotis, Garzelli, Papadopoulos, Pittau]
- Executable linked to `Collier`, `CutTools` and `Samurai`.

Consistency checks

- UV and IR cancellations
- Tensor integrals vs. OPP reduction
- "pseudo-tree": fix loop momentum and compare to tree generator

Speed and Flexibility

Performance studies for all non-trivial processes of the Les Houches priority list (Intel i5-750, single thread, compiled with ifort 10.1).

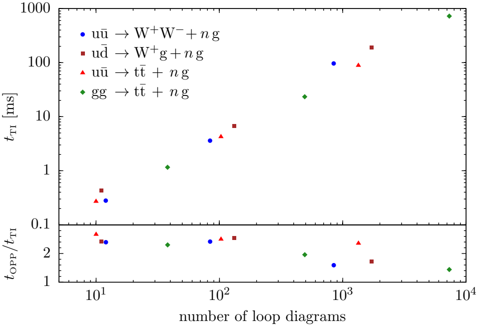
t_{code} = code generation & compilation (lots of room for improvement here);

size of the process library;

t_{TI} = time for a single phase space point using tensor integrals (unpolarised, single helicity for t/W);

t_{OPP} = the same with CutTools

Process	t_{code}/s	size/MB	t_{TI}/ms
$u\bar{u} \to t\bar{t}$	2.2	0.1	0.27
$u\bar{u} \to W^+W^-$	7.2	0.1	0.28
$u\bar{d} \to W^+g$	4.2	0.1	0.43
$gg \to t\bar{t}$	5.4	0.2	1.16
$u\bar{u} \to t\bar{t}g$	13	0.4	4.2
$u\bar{u} \to W^+W^-g$	40	0.4	3.6
$u\bar{d} \to W^+gg$	24	0.5	6.7
$gg \to t\bar{t}g$	53	1.2	23.4
$u\bar{u} \to t\bar{t}gg$	236	3.6	88.4
$u\bar{u} \to W^+W^-gg$	382	2.5	96.4
$u\bar{d} \to W^+ggg$	366	4.2	190.5
$gg \to t\bar{t}gg$	3005	16.0	725.0



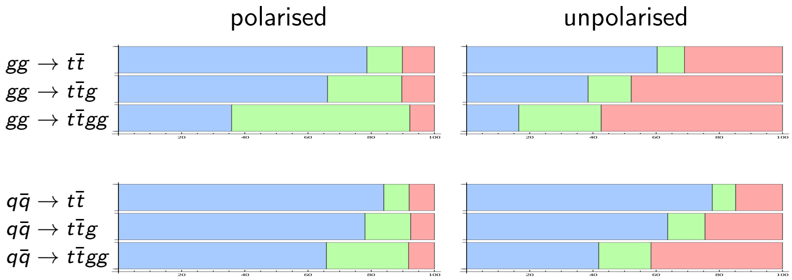
Timimg studies

single helicity: time for tensor reduction $\gg$ time for coefficients

full helicity sum: time for tensor reduction $\approx$ time for coefficients

For $2 \to 4$ processes

- full helicity sums cost only a factor ~ 2
- tensor integral reduction and OPP performance is similar



fractions of total runtime for scalar integrals, tensor reduction, coefficients

Numerical stability

The numerical precision can be estimated by a scaling test:

$$m_i \to \xi m_i,\ p_i^\mu \to \xi p_i^\mu \quad \text{leads to} \quad \delta\mathcal{W} \to \delta\mathcal{W}' = \xi^K \delta\mathcal{W}$$

$$\Rightarrow \text{ precision } \Delta = \left| \frac{\xi^{-K}\delta\mathcal{W}'}{\delta\mathcal{W}} - 1 \right|, \quad \text{rsp. } d = -\log_{10}\Delta \text{ decimal digits.}$$

Sample of 10^6 homogeniously distributed phase space points;

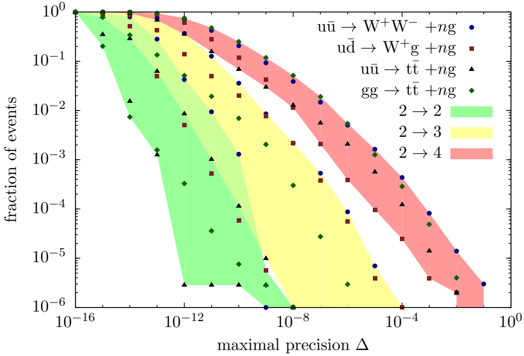
$\sqrt{s} = 1$ TeV

$p_T > 50$ GeV

$\Delta R_{ij} > 0.5$

using tensor integrals,

double precision



Summary

Open loops – A new algorithm for one-loop amplitudes

- Diagrammatic, tree-like recursion for loop momentum polynomials
- Implemented for QCD corrections to SM processes
- Fully automated:
  process definition $\rightarrow$ compact code within seconds/minutes
- Interfaced with tensor integral and OPP reduction
- Very fast: 0.1–1 s/PS-point for colour & helicity summed $2 \rightarrow 4$
- Numerically stable when using tensor integrals