Bootstrap & Infrared effects in small x evolution equations

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March 20th, 2012

LHC-Phenonet Annual Meeting, DURHAM

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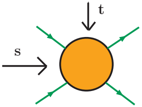
Outline

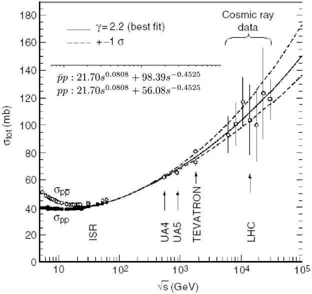
- High energy limit: Regge theory & QCD
  - Regge theory and the soft Pomeron
  - gluon reggeization
  - the hard Pomeron
- Bootstrap equation & running of the coupling
  - A model for the running
  - Gluon Green's function and MC
- Power corrections & renormalons
- Plots
- Comments

The low x limit: Regge theory

Regge (high energy) limit:



$$s >> |t|\,,\; Q^2 >> \Lambda^2_{QCD}$$
$$\alpha_s(Q^2) << 1$$
$$x \simeq |t|/s << 1$$

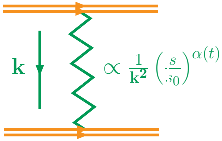


Regge theory prediction:

$$A(s,t) \sim s^{\alpha(t)} \;\Rightarrow\; \sigma(s) \sim s^{\alpha(0)-1}$$

Explanation by Regge principles:

Reggeization



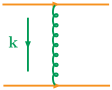
- Exchange of "something" in the t-channel carrying the Q.N. of the vacuum:

  SOFT POMERON: $\lambda_P \equiv \alpha(0) - 1 = 0.08.$

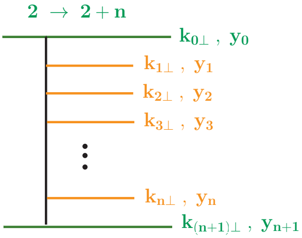
- particles with such propagator: reggeized particles

QCD and the Regge limit: $s >> |t|$

t-channel exchange:



Ladder-type diagrams:



- Multi Regge Kinematics:
  Impose strong ordering in rapidity:

$$y_1 >> y_2 >> \ldots >> y_n$$

- Dominated by: $\bar{\alpha}_s \log(s/s_0)$
- What happens with pert. theory?

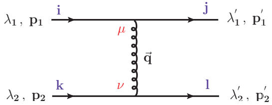
$$\sum_{n=0}^{\infty} \bar{\alpha}_s^n \log^n(s/s_0) \quad , \quad \bar{\alpha}_s \log(s/s_0) \sim 1$$

Need of resummation of logs of energy:

GLUON REGGEIZATION

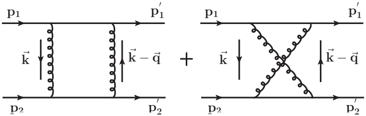
Gluon reggeization

Take into account all possible t-channel virtual contributions leading in MRK:



- First order in p-theory:

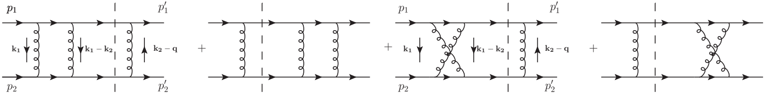
$$A_0(s,t) = 8\pi\alpha_s\, t^{\alpha}_{ij}\, t^{\alpha}_{kl}\, \frac{s}{t}$$



- Second order in p-theory:

$$A_8^{(1)} = A_0(s,t)\log(s/|t|)\,\epsilon(t) \;+\; \dots$$

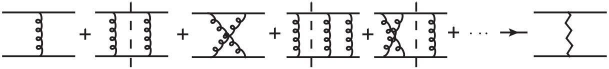
- Third order in p-theory: $A_8^{(2)} = A_0(s,t)\frac{1}{2}\log^2\left(\frac{s}{|t|}\right)\epsilon^2(t) \;+\;$ extra terms



Gluon reggeization & Bootstrap

- Ansatz for higher orders: gluon reggeization

$$A_8(s,t) = A^{(0)}(s,t)\left[1 + \log\left(\frac{s}{|t|}\right)\,\epsilon(t) + \frac{1}{2!}\log^2\left(\frac{s}{|t|}\right)\,\epsilon^2(t) + \dots\right] = A^{(0)}(s,t)\left(\frac{s}{|t|}\right)^{\epsilon(t)}$$



Bootstrap equation: proves this ansatz to all orders in p-theory

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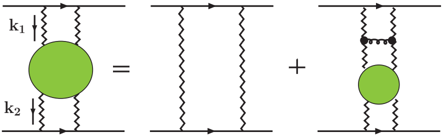
The integral equation for the t-channel exchange of two reggeized gluons in the color octet has a solution with a Regge pole in Mellin space of the form $\omega = \epsilon(t)$, ensuring a power-like growth of the total hadronic cross section and justifying reggeization.

BFKL evolution eq. and hard Pomeron

- Mellin transform of the imaginary part of the amplitude



$$\omega f(\omega, \mathbf{k}_1, \mathbf{k}_2) = \delta^2(\mathbf{k}_1 - \mathbf{k}_2) + \mathcal{K}^{\mathrm{BFKL}} \otimes f(\omega, \mathbf{k}_1, \mathbf{k}_2)$$

- Solution: $\mathcal{K}\ \phi_i(\mathbf{k}) = \lambda_i\ \phi_i(\mathbf{k})\ ,\ \ \mathcal{K}\,|n,\nu\rangle = \omega\,|n,\nu\rangle$
- Using the Saddle Point Approx...

$$\omega = \bar{\alpha}_s\ \chi_0(\nu = 0)$$

$$\chi_0(\nu) = 4\log(2) - 14\zeta(3)\nu^2 + \dots$$

$$\lambda_P^{\mathrm{QCD}} \simeq 4\bar{\alpha}_s \log(2) \simeq 0.5 \quad >> \quad \lambda_P^{\mathrm{soft}} \simeq 0.08$$

Bootstrap & running (I/III)

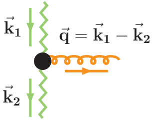
- The idea: introduce the running coupling in the BFKL eq. in a way consistent with bootstrap
- Why running coupling? Motivation:

Gluon Green's function

	Fixed coupling	Running coupling	Regge theory
Solution:	cut (branch point)	pole-like	pole-like
Pomeron intercept:	0.5	0.3	0.08

Natural appearance of renormalons $\Longleftrightarrow$ approach to the infrared.

Bootstrap & running (II/III)



- Bootstrap equation:

$$\omega - \epsilon(\mathbf{k}_1) - \epsilon(\mathbf{k}_1 - \mathbf{q}) = \frac{\bar{\alpha}_s}{4\pi} \int d^2\mathbf{l}\, \mathcal{K}(\mathbf{l}, \mathbf{k}_1, \mathbf{k}_1 - \mathbf{q})$$

BFKL eq. with fixed coupling when imposing a pole-like solution:

$$\phi_\omega(\mathbf{q}^2, \mathbf{k}^2) = \frac{A}{\omega - \epsilon(-\mathbf{q}^2)}$$

[Braun (1994) hep-ph/9408261]
[E. Levin (1995) hep-ph/9412345]
[Kovchegov, Weigert (2006) hep-ph/0609090]

Bootstrap & running (III/III)

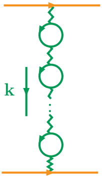
Fixed coupling:

Propagator: $\frac{\bar{\alpha}_s(\mu^2)}{\mathbf{k}^2}\left(\frac{s}{s_0}\right)^{\epsilon(-\mathbf{k}^2)}$

Trajectory: $\epsilon(-\mathbf{k}^2) = -\frac{\bar{\alpha}_s(\mu^2)}{4\pi}\int \frac{d^2\mathbf{k}'}{\mathbf{k}'^2}\frac{\mathbf{k}^2}{(\mathbf{k}-\mathbf{k}')^2}$

Impose bootstrap condition to trajectory $\Longleftrightarrow$ $\mathcal{K}^{\mathrm{LO}}_{\mathrm{BFKL}}(\mathbf{q},\mathbf{k},\mathbf{k}')$

Running coupling: by insertion of a gluon chain.



Propagator: $\frac{\bar{\alpha}_s(\mathbf{k}^2)}{\mathbf{k}^2}\left(\frac{s}{s_0}\right)^{\tilde{\epsilon}(-\mathbf{k}^2)}$

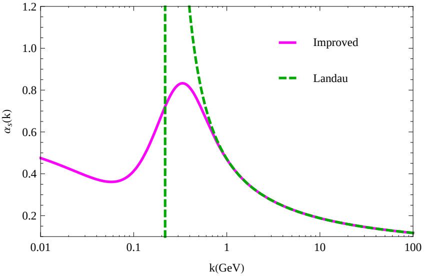
Trajectory: $\tilde{\epsilon}(-\mathbf{k}^2) = -\frac{1}{4\pi}\int \frac{d^2\mathbf{k}'}{\eta(bk')}\frac{\eta(\mathbf{k})}{\eta(\mathbf{k}-\mathbf{k}')}$ with $\eta(\mathbf{k}) \equiv \frac{\mathbf{k}^2}{\bar{\alpha}_s(\mathbf{k}^2)}$

Take $\tilde{\epsilon}(-\mathbf{k}^2)$ and insert it into bootstrap eq. $\Longrightarrow$ $\mathcal{K}^{\mathrm{running}}_{\mathrm{BFKL}}(\mathbf{q},\mathbf{k},\mathbf{k}')$

A model for the running

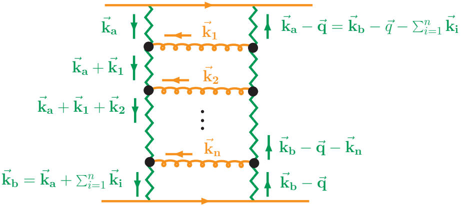
Running coupling analytical in the infrared and compatible with power corrections to jet observables:

$$\bar{\alpha}_s\left(\mathbf{k}^2\right) = \frac{4N_c}{\beta_0}\left(\frac{1}{\ln\frac{\mathbf{k}^2}{\Lambda^2_{QCD}}} + 125\frac{\left(\Lambda^2_{QCD}+4\mathbf{k}^2\right)}{\left(\Lambda^2_{QCD}-\mathbf{k}^2\right)\left(4+\frac{\mathbf{k}^2}{\Lambda^2_{QCD}}\right)^4}\right)$$



[B. Webber (1998) hep-ph/9805484]

Non-forward GGF: iteration and MC



Non-forward:

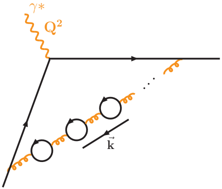
- $t = -\mathbf{q}^2 \neq 0$
- diffractive
- hard scale: $|t| >> \Lambda^2_{QCD}$

$$\mathcal{F}(k_1,k_2,q,x) = \Bigg\{ \delta^{(2)}(k_1-k_2) + \sum_{n=1}^{\infty} \prod_{i=1}^{n} \int d^2q_i \mathcal{K}\left(k_1 + \sum_{l=1}^{i-1} q_l, k_1 + \sum_{l=1}^{i} q_l; q\right) \theta(q_i^2 - \lambda^2)$$
$$\times\, \delta^{(2)}\left(k_1 + \sum_{l=1}^{n} q_l - k_2\right) \int_{x_{i-1}}^{1} \frac{dx_i}{x_i} x_i^{\left(\epsilon_\lambda(k_1+\sum_{l=1}^{i-1} q_l) - \epsilon_\lambda(k_1+\sum_{l=1}^{i} q_l)\right)}$$
$$\times\, x_i^{\left(\epsilon_\lambda(k_1-q+\sum_{l=1}^{i-1} q_l) - \epsilon_\lambda(k_1-q+\sum_{l=1}^{i} q_l)\right)} \Bigg\} x^{-\epsilon_\lambda(k_1)-\epsilon_\lambda(k_1-q)}$$

[Andersen, Sabio Vera, hep-ph/0309337]

Power corrections & renormalons

Running: $$\alpha_s(\mathbf{k}^2) = \frac{\alpha_s(\mu^2)}{1 + \frac{\beta_0}{4\pi}\alpha_s(\mu^2)\log(\mathbf{k}^2/\mu^2)} = \sum_{n=0}^{\infty} \alpha_s(\mu^2)\left(-\frac{\beta_0}{4\pi}\right)^n \log^n\left(\frac{\mathbf{k}^2}{\mu^2}\right)$$



Perturbative series: $$\mathcal{R} \simeq \sum_{n=0}^{\infty} r_n\, \alpha_s^{n+1}$$

Borel transform: $$\mathcal{B}[\mathcal{R}](u) \equiv \sum_{m=0}^{\infty} u^m \frac{r_m}{m!} \Rightarrow \mathcal{R} = \int_0^{\infty} \mathrm{d}u\, \mathrm{e}^{-u/\alpha_s}\, \mathcal{B}[\mathcal{R}](u)$$

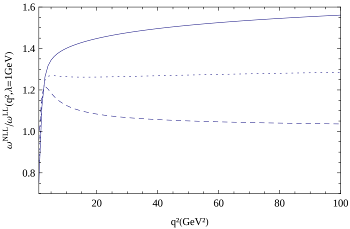
Our case: $$\mathcal{A}(Q^2) = \int \frac{\mathrm{d}\mathbf{k}^2}{\mathbf{k}^2} f(\mathbf{k}^2, Q^2)\,\alpha_s(\mathbf{k}^2)\left[\theta(\lambda^2 - \mathbf{k}^2) + \theta(\mathbf{k}^2 - \lambda^2)\right]$$

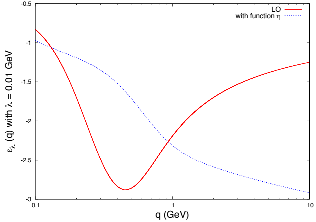
$$\mathcal{A}_{IR} \sim \sum_{n=0}^{\infty}\left(\frac{\beta_0}{m}\right)^n n!\,\alpha_s^{n+1}(Q) \Rightarrow \mathcal{B}[\mathcal{A}](u) = \frac{m/\beta_0}{m/\beta_0 - u} \Rightarrow u = \frac{m}{\beta_0},\ m \in \mathbb{N}$$

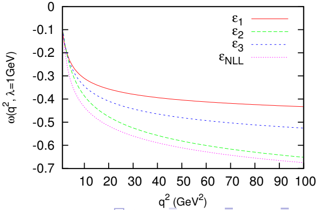
Plots: the trajectory

$$\epsilon_\lambda^{\rm run}(\mathbf{q}^2) \quad = \quad -\frac{\mathbf{q}^2}{\bar\alpha_s(\mathbf{q}^2)} \int \frac{d^2\mathbf{k}}{2\pi\,\mathbf{k}^2} \frac{\bar\alpha_s(\mathbf{k}^2)\,\bar\alpha_s(\mathbf{k}+\mathbf{q})}{\mathbf{k}^2+(\mathbf{k}+\mathbf{q})^2}\,\theta(\mathbf{k}^2-\lambda^2)$$

$$\begin{aligned}\epsilon_\lambda^{\rm LO}(\mathbf{q}^2) \quad &= -\bar\alpha_s(\mathbf{q}^2)\,\mathbf{q}^2 \int \frac{d^2\mathbf{k}^2}{2\pi\,\mathbf{k}^2} \frac{\theta(\mathbf{k}^2-\lambda^2)}{\mathbf{k}^2+(\mathbf{k}+\mathbf{q})^2} \\ &\simeq -\frac{\bar\alpha_s(\mathbf{q}^2)}{2}\ln\frac{\mathbf{q}^2}{\lambda^2}\end{aligned}$$



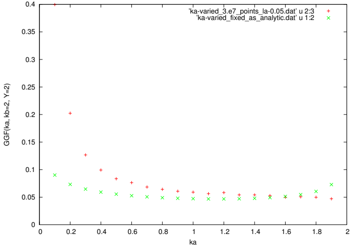


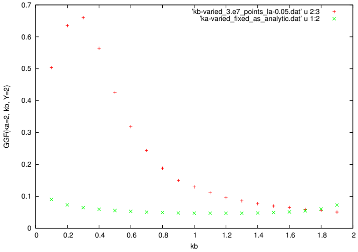


Solid line: $\bar\alpha_s(\mathbf{q}^2)$, dotted: $\bar\alpha_s(\mathbf{k}^2)$,
dashed: $\sqrt{\bar\alpha_s(\mathbf{q}^2)}\,\sqrt{\bar\alpha_s(\mathbf{k}^2)}$

Plots: gluon Green's function

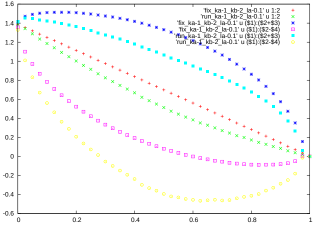
$\lambda = 0.05$



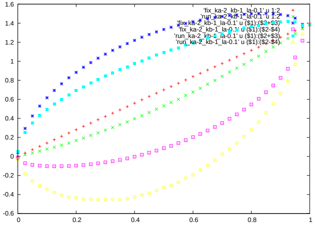


$\lambda = 0.1$

$k_a = 1\,\text{GeV},\ k_b = 2\,\text{GeV}$



$k_a = 2\,\text{GeV},\ k_b = 1\,\text{GeV}$



Comments and conclusions

- The insertion of the running coupling...
  - provides smooth transition between the hard and soft Pomeron
  - leads to the appearance of renormalons $\Rightarrow$ sensitivity to the infrared
- The model used for the running coupling...
  - is analytic at the Landau pole, freezing in the IR
  - compatible with power corrections to jet observables
- BFKL eq. solved numerically using an iterative expression for the running case
- We are ready to implement this formalism for the study of LHC-like processes

THANK YOU

FOR YOUR ATTENTION,

specially for being the very last talk of the journey...