# Gluon fragmentation into charmonium at NLO

Pierre Artoisenet
NIKHEF

In collaboration with
Eric Braaten (OSU)

LHCPhenoNet Annual Workshop
21 March 2012

# Outline

Part I. Charmonium production in hadron collisions

Part II. Gluon fragmentation into charmonium at NLO

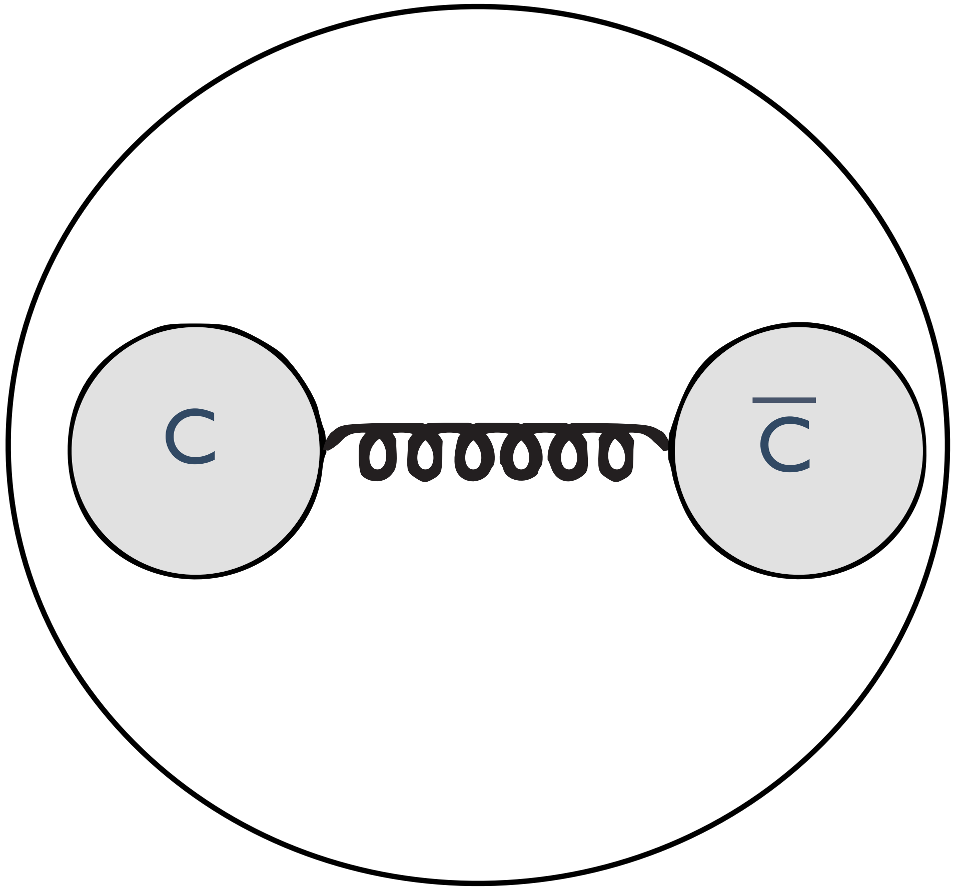
# The charmonium system

- $c\bar{c}$ state bound by strong interactions
- S-wave ground states:

  $\eta_c \ \ = c\bar{c}[^1S_0]$ (spin 0)

  $J/\psi = c\bar{c}[^3S_1]$ (spin 1)

- approx. non relativistic (v~0.4)



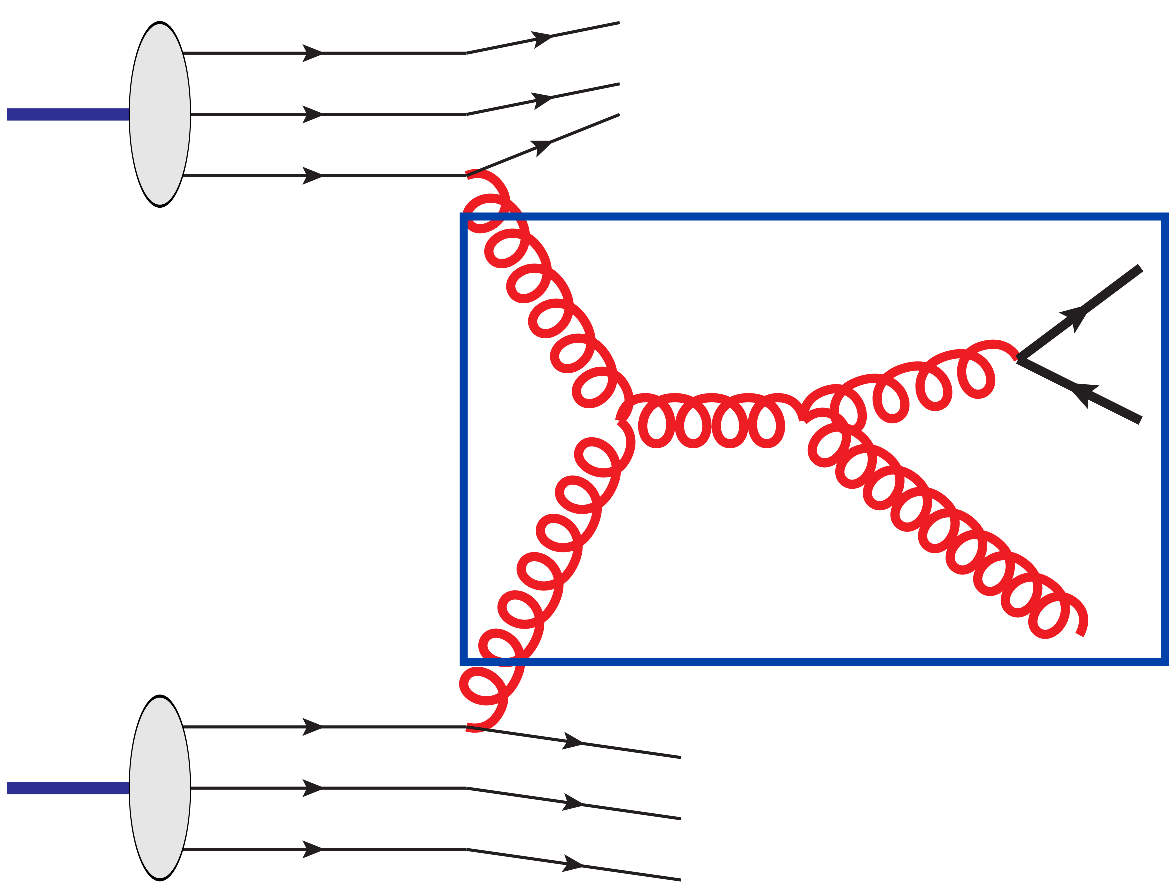

$1/m_c \ll$ size $\sim 1/(m_c v) \sim 0.3$ fm

typical distance for $c\bar{c}$ creation

# Charmonium production in hadron collisions

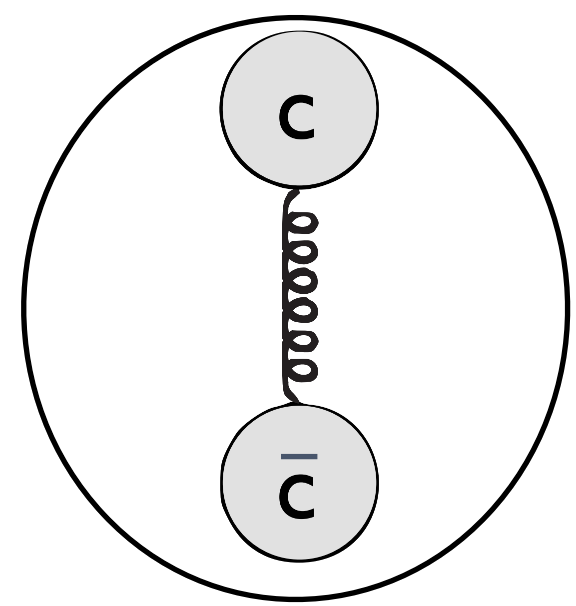
- involves both

- perturbative effects (creation of a charm quark pair)



momentum scales: $m_c$ or higher

- non-perturbative effects (evolution into charmonium)




momentum scales: $m_c v$

# NRQCD factorization

Bodwin, Braaten & Lepage 1995

- Factorized expression of cross sections:

$$\sigma[H] = \sum_n \hat{\sigma}_\Lambda[c\bar{c}(n)] \langle \mathcal{O}^H(n) \rangle_\Lambda$$

**parton cross sections**
probabilities to create a $c\bar{c}$ state in the quantum state n, expanded in powers of $\alpha_s$

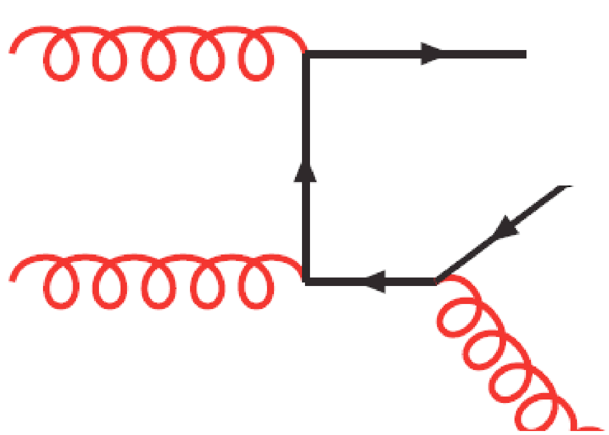
**LD matrix elements**
probabilities for the $c\bar{c}$ state to bind into a charmonium,
- non-perturbative
- universal
- scale as powers of v

# NRQCD factorization

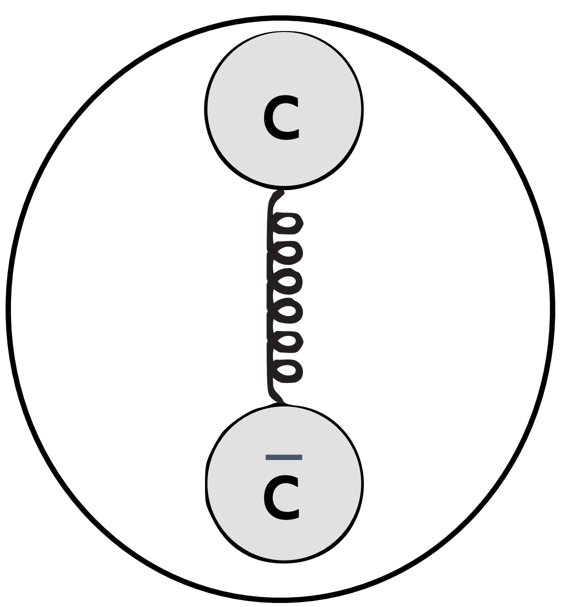
- At leading order in v: (Color-Singlet Model)

$$\sigma[J/\psi] = \hat{\sigma}[c\bar{c}(^3S_1)]\,|\psi(0)|^2$$

**parton cross section**
=probability to create a charm quark pair in a S=1, L=0, c=1 state with 0 relative momentum



**Wave function at origin**
=probability to find the cc quarks at the same point in the J/ψ

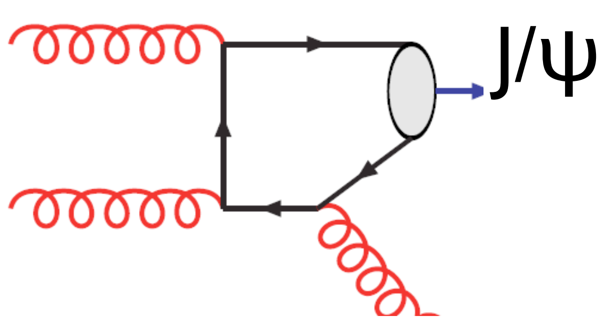


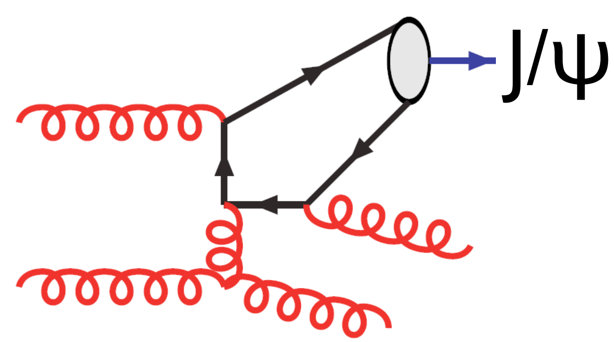
- Let us focus on the production at leading order in v: (Color-Singlet Model)

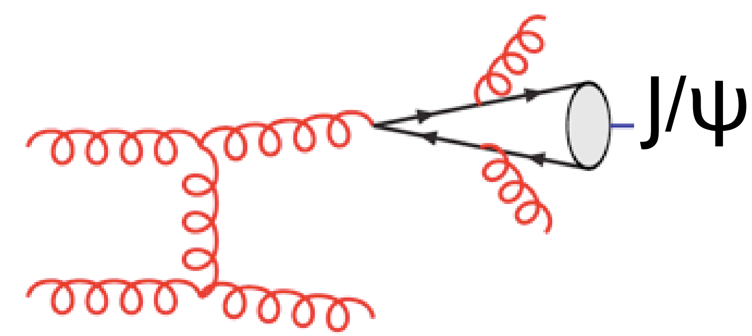
$$\sigma[J/\psi] = \hat{\sigma}[c\bar{c}(^3S_1)]\,|\psi(0)|^2$$

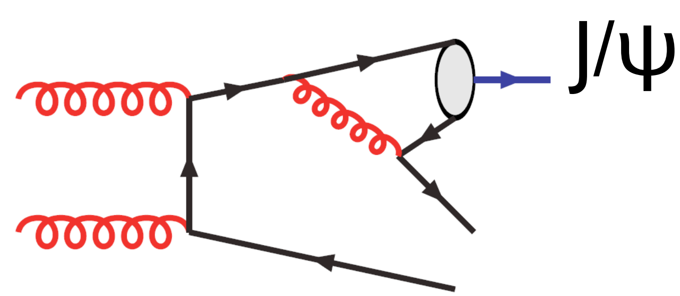
expansion in $\alpha_s$:

$$\hat{\sigma}[c\bar{c}(^3S_1)] = A_1\,\alpha_s^3 \quad + \quad A_2\,\alpha_s^4 \quad + \quad A_3\,\alpha_s^5 \quad + \quad \ldots$$



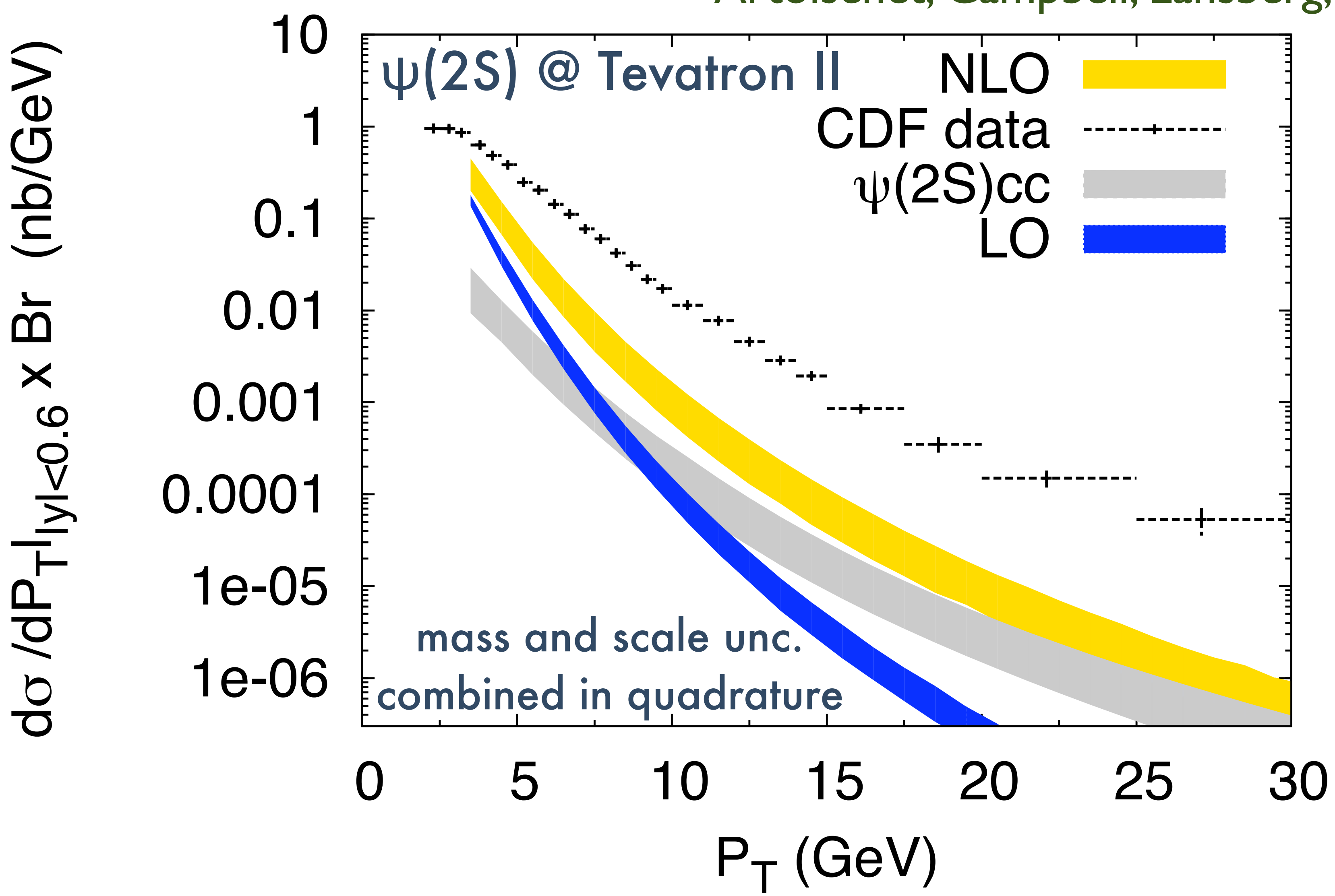



# Theoretical uncertainties

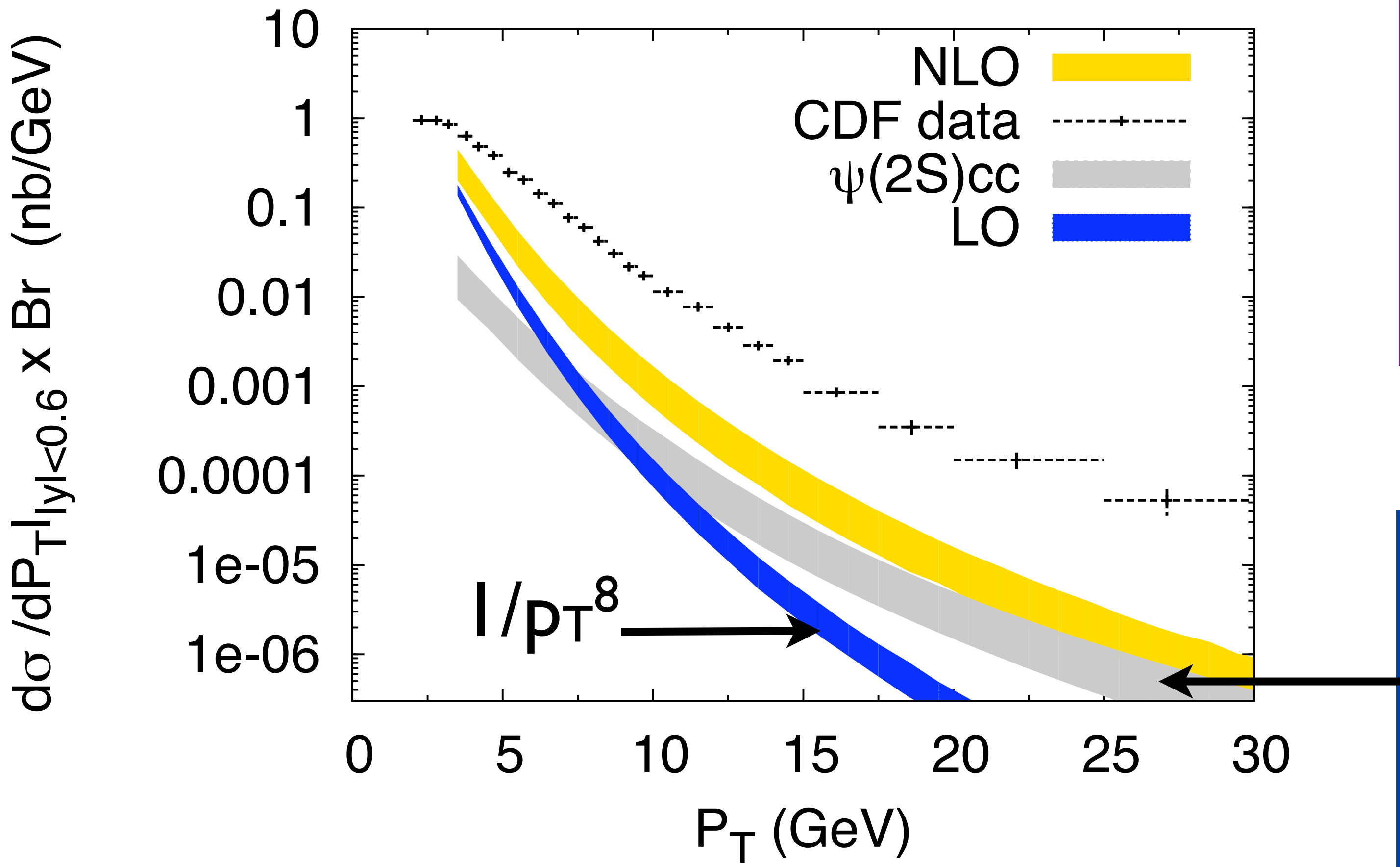
- contributions at LO in v suffer from large QCD corrections at high $p_T$

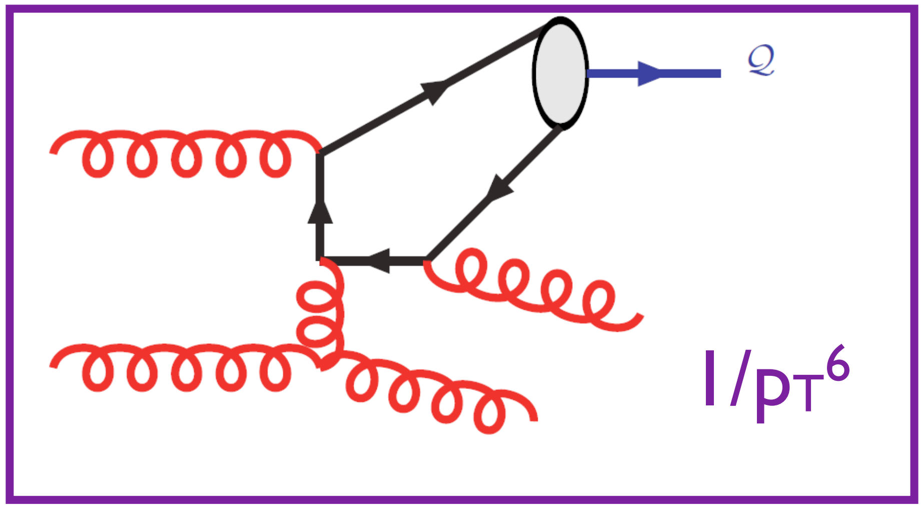
Artoisenet, Campbell, Lansberg, Maltoni, Tramontano (2008)

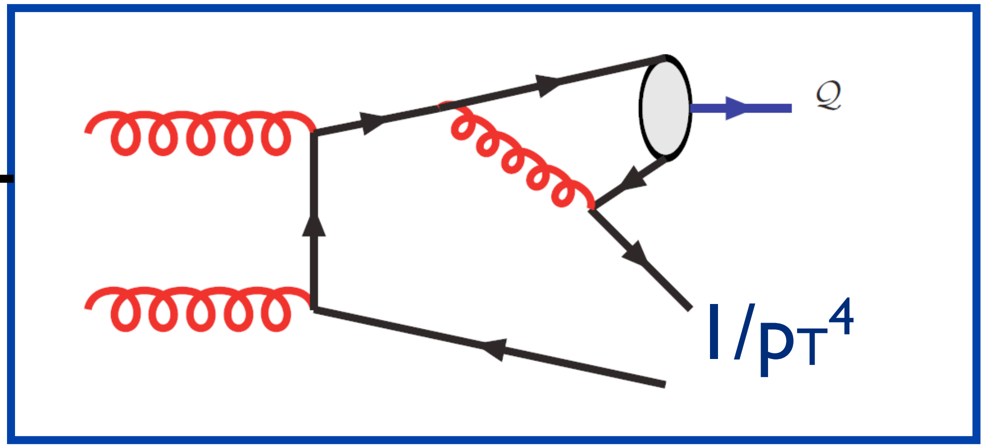



- new channels at $\alpha_s^4$ give rise to a huge enhancement **at large $p_T$**
- large th. unc., mainly from variations of the scales

# Theoretical uncertainties




- $1/p_T^4$ and $1/p_T^6$ contributions only occur at order $\alpha_s^4$ (or higher)
  → only leading-order accuracy at $\alpha_s^4$
- can these contributions be calculated at NLO accuracy ?

A. Can the $1/p_T^4$ contribution be calculated at NLO accuracy ?

YES

# PQCD Factorization Theorem

Collins & Soper 1982

production of a single hadron
with large transverse momentum
is dominated by fragmentation

- hard scattering produces parton with larger momentum
- parton hadronizes into a jet that includes the hadron
- factorization formula: proved rigorously to all orders in $\alpha_s$

# PQCD Factorization Theorem

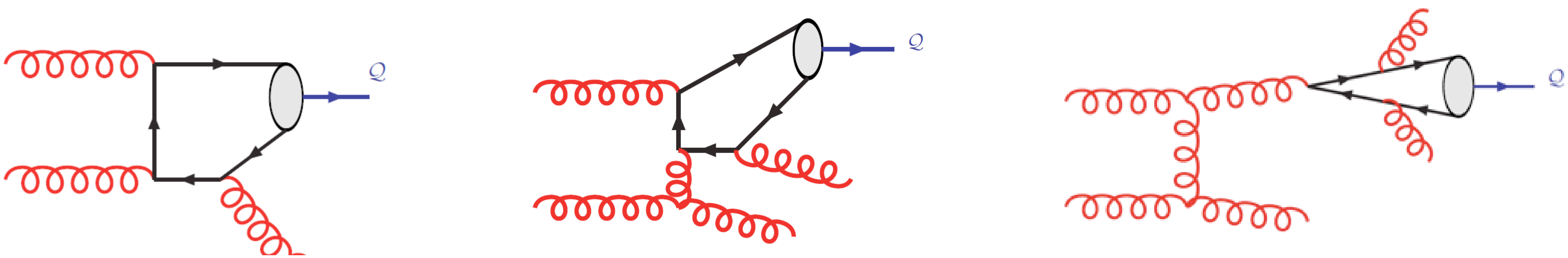
Collins & Soper 1982

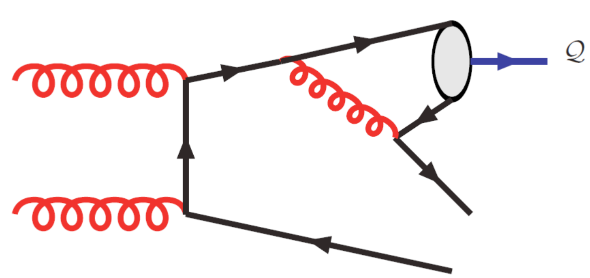
$$d\sigma[H(P)] \;=\; \sum_i \int_0^1 dz \; d\hat{\sigma}[i(P/z)] \; D_{i\to H}(z) \\ +\mathcal{O}(\Lambda^2_{\rm QCD}/p_T^2)$$

- sum over partons *i*
  integral over momentum fraction *z*
- cross section $d\hat{\sigma}$ for parton with larger momentum *P/z*
  calculate using PQCD as power series in $\alpha_s(p_T/z)$
- fragmentation function $D_{i\to H}(z)$
  probability for hadron to carry fraction *z* of parton momentum
  nonperturbative function, but logarithmic evolution with $p_T$ is perturbative

# Application to charmonium

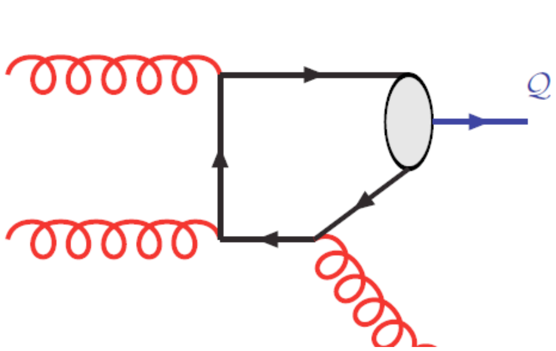
$\hat{\sigma}[c\bar{c}(^3S_1)]$ $= A_1 \, \alpha_s^3 \quad + \quad A_2 \, \alpha_s^4 \quad + \quad A_3 \, \alpha_s^5 \quad + \quad ...$

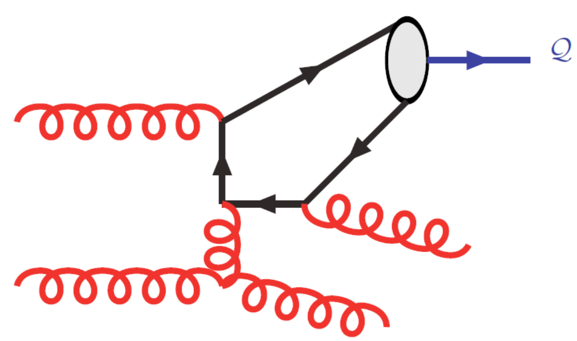


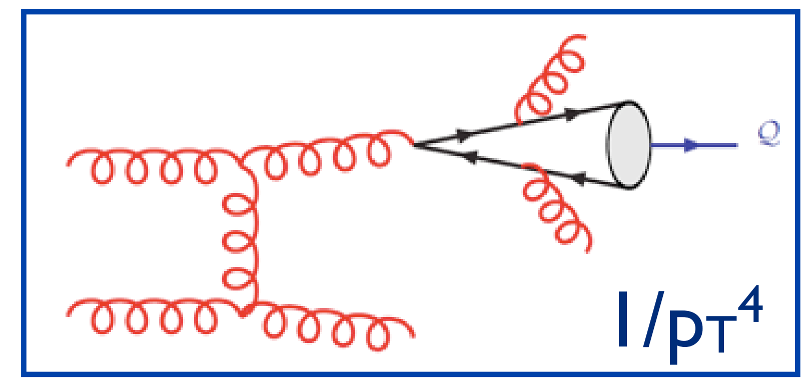



# Application to charmonium

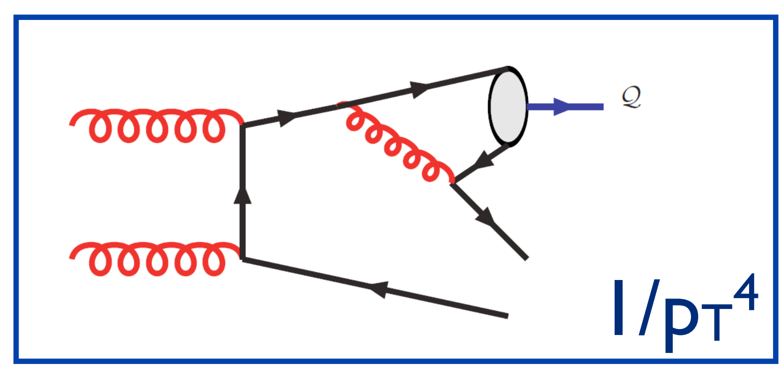
$$\hat{\sigma}[c\bar{c}(^3S_1)] = A_1\,\alpha_s^3 + A_2\,\alpha_s^4 + A_3\,\alpha_s^5 + \ldots$$








gluon fragmentation

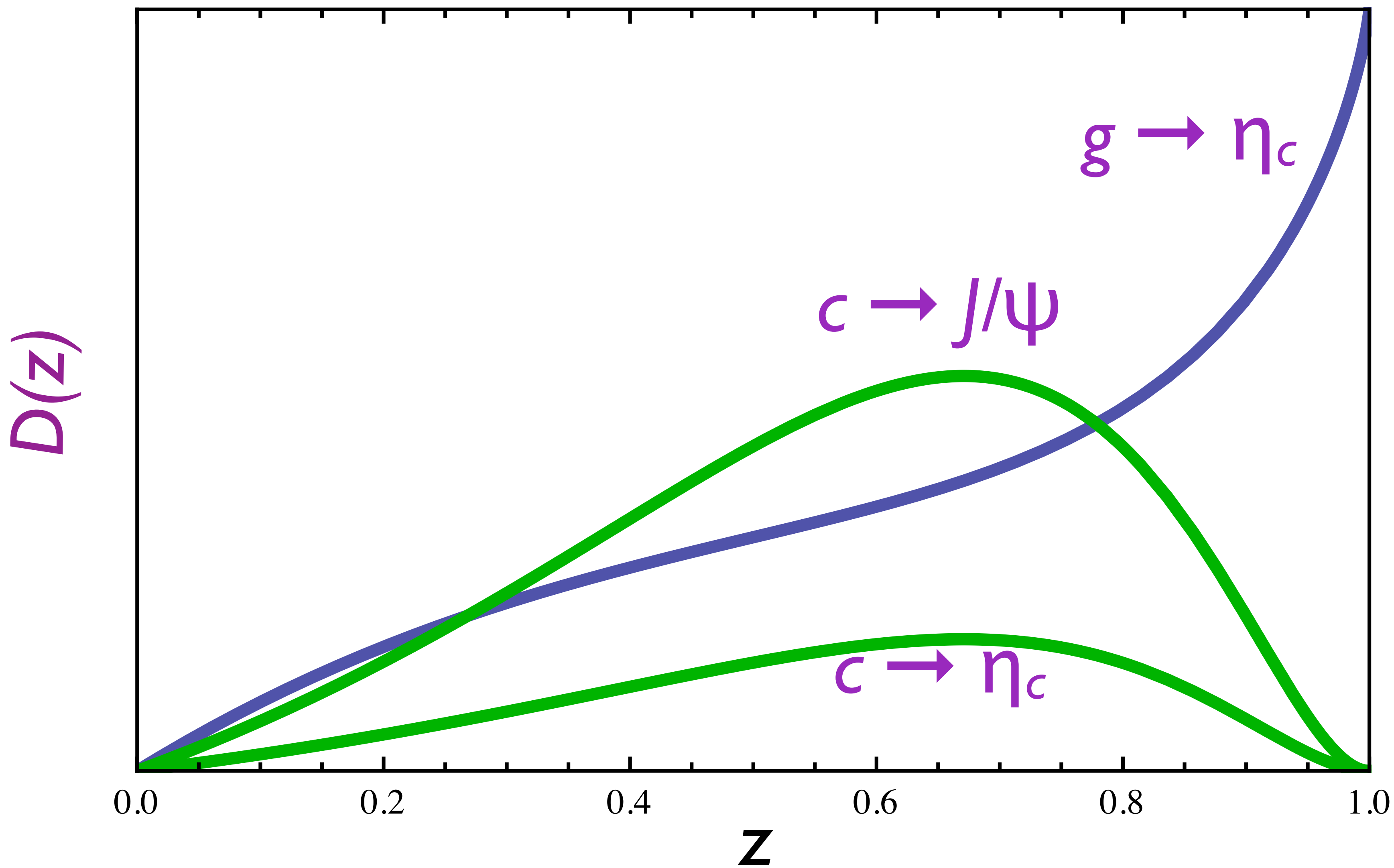



charm quark fragmentation

# Parton fragmentation

- fragmentation functions $D_{i\to H}(z)$ for S-wave charmonium
  can be calculated using PQCD in Color-Singlet Model

Braaten, Cheung, and Yuan 1993




- reduces nonperturbative functions $D_{i\to H}(z)$
  to nonperturbative constants $|\psi(0)|^2$

B. Can the $1/p_T^6$ contribution be calculated at NLO accuracy ?

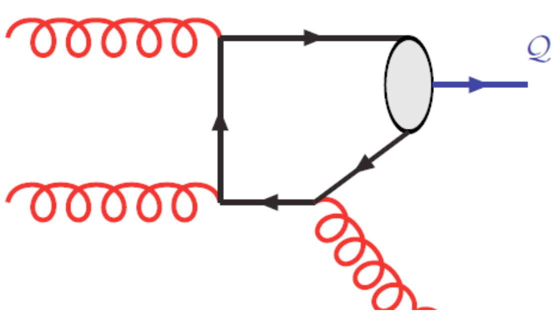
YES: the next-to-leading power $1/p_T^6$ can be expressed as the fragmentation of a $c\bar{c}$ pair into charmonium

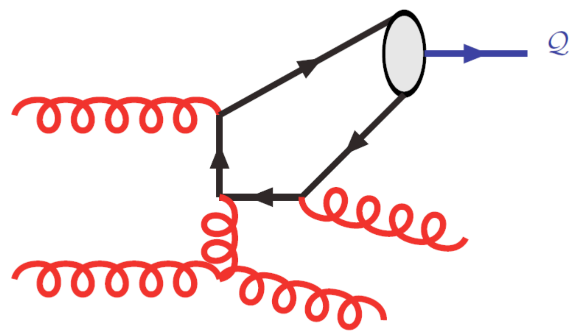
Kang, Qiu, Sterman 2011

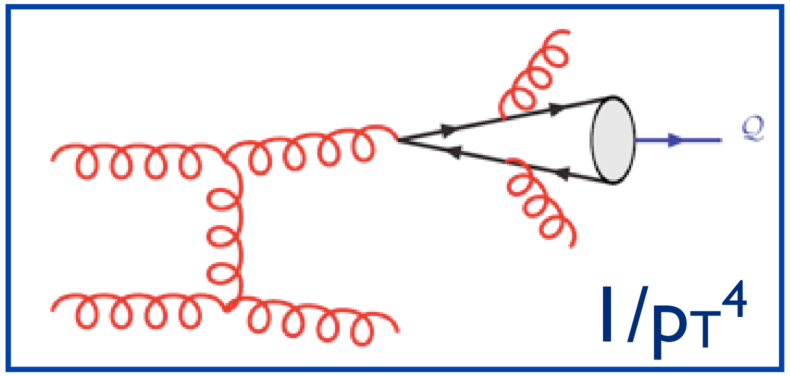
$$d\hat{\sigma}[Q\bar{Q}_m] \otimes D[Q\bar{Q}_m \to H]$$

# Application to J/ψ production (CSM)

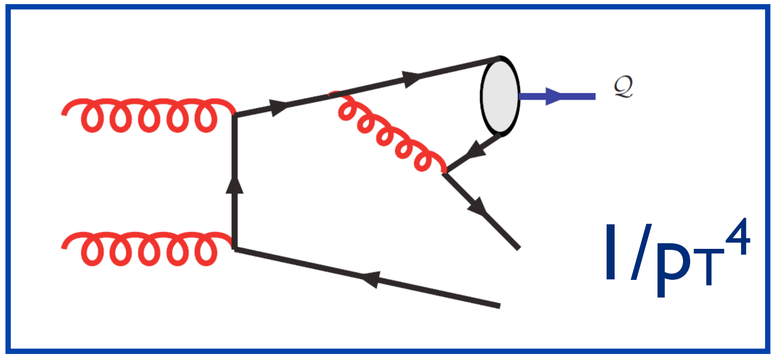
$$\hat{\sigma}[c\bar{c}(^3S_1)] = A_1\,\alpha_s^3 + A_2\,\alpha_s^4 + A_3\,\alpha_s^5 + \ldots$$







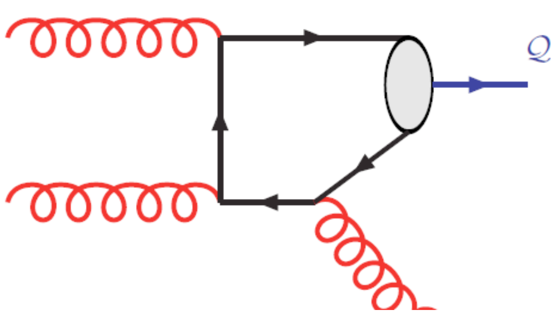

gluon fragmentation



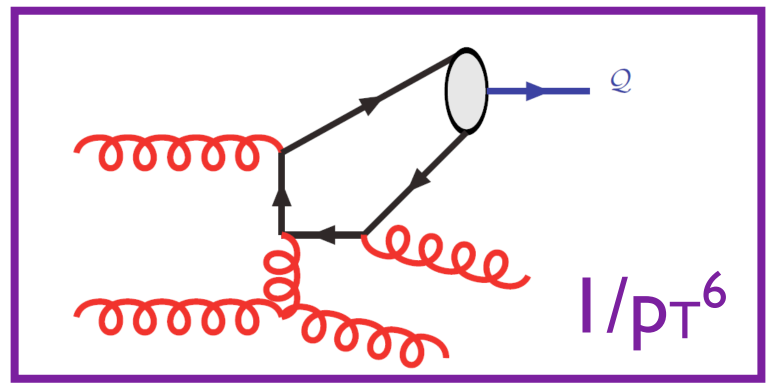

charm quark fragmentation

# Application to J/ψ production (CSM)

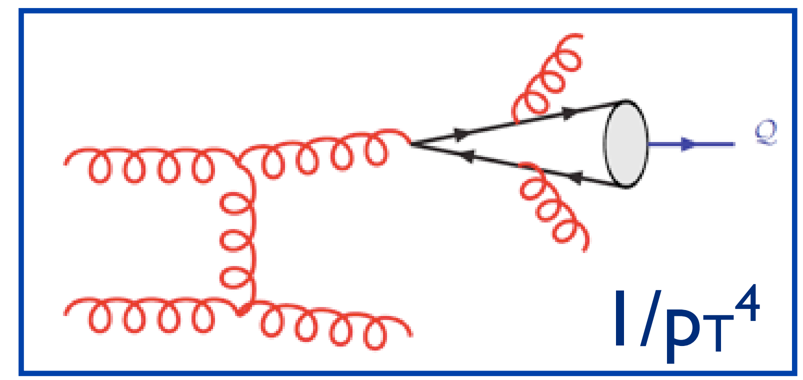
$\hat{\sigma}[c\bar{c}(^3S_1)]$ $= A_1\ \alpha_s^3 \quad + \quad A_2\ \alpha_s^4 \quad + \quad A_3\ \alpha_s^5 \quad + \quad \ldots$



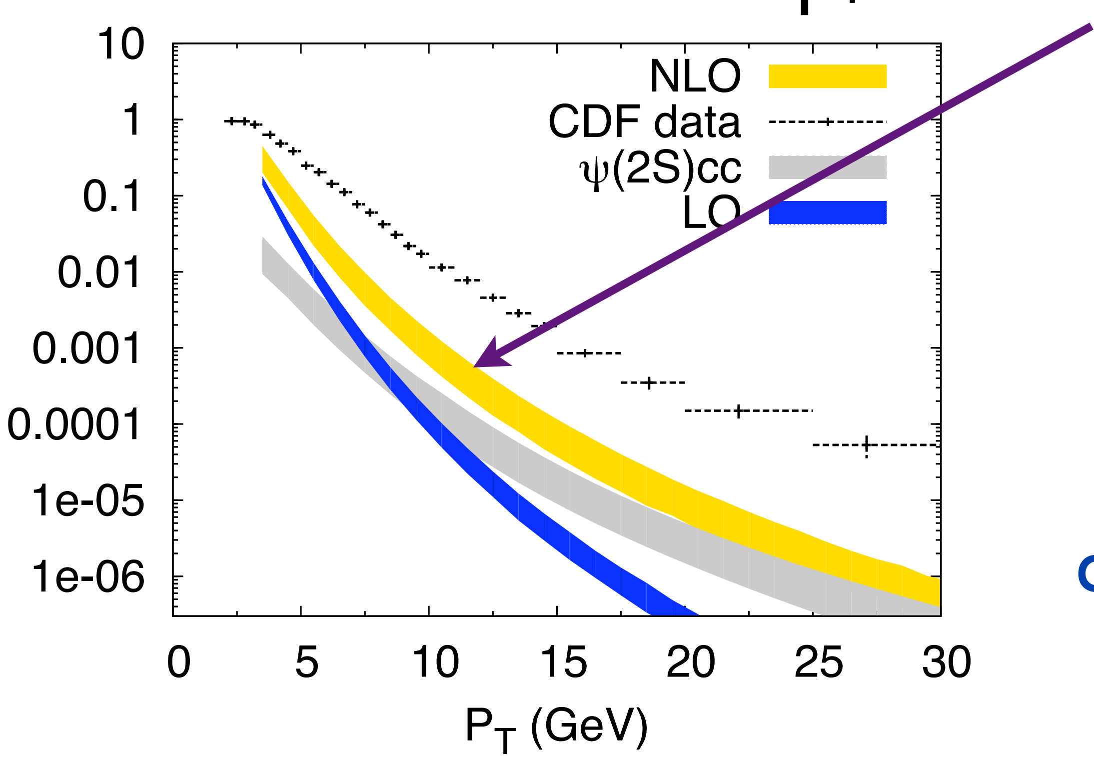

$1/p_T^8$

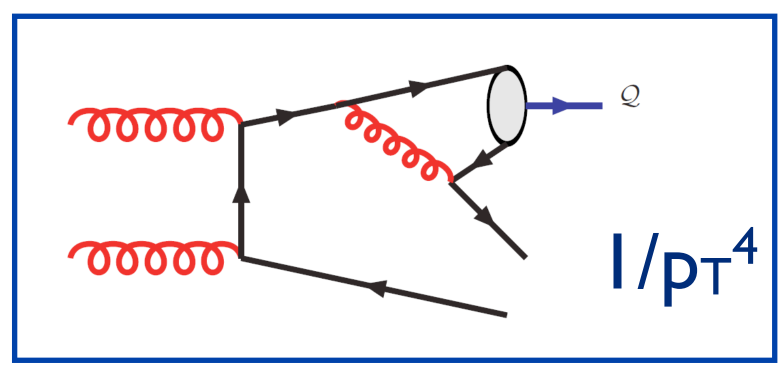



charm quark pair frag.




gluon fragmentation




charm quark fragmentation

# Fragmentation revisited

New factorization formula

$$d\sigma[H] = \sum_i d\hat{\sigma}[i] \otimes D[i \to H]$$

LO in $m_c/p_T$

$$+ \sum_m d\hat{\sigma}[Q\bar{Q}_m] \otimes D[Q\bar{Q}_m \to H]$$

order $m_c^2/p_T^2$

$$+ d\sigma_{\text{direct}}[H]$$

order $m_c^4/p_T^4$

To make predictions with LO (NLO) accuracy at all $p_T$,
cross sections and fragmentation functions
should all be calculated to LO (NLO) in $\alpha_s$

# Fragmentation functions: current status

- parton fragmentation functions

  LO in $\alpha_s$:

  S-waves Braaten, Cheung, & Yuan 1993; Braaten and Yuan 1993,1995

  P-waves Braaten and Yuan 1994; Yuan 1994; Chen 1994; Ma 1995; Hao, Zuo & Qiao 2009

  D-waves Cho & Wise 1995; Cheung & Yuan 1996; Qiao, Yuan & Chao 1997

  NLO:

  $g \to \underline{8}^3S_1$ Braaten & Lee 2004

  $c \to \underline{1}^3S_1$ Gong, Li & Wang 2011

- $c\bar{c}$ fragmentation functions

  LO in $\alpha_s$: Kang, Qiu & Sterman

  NLO?

# Fragmentation function: formal definition

- fragmentation functions can be defined formally as matrix elements for non-local gauge-invariant operators

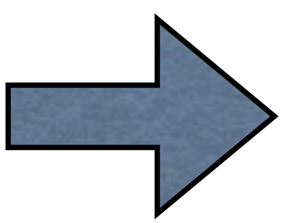
Collins & Soper 1982

$$D_{g\to H}(z,\mu) = \frac{-z^{d-3}}{16\pi(d-2)k^+}\int dx^- e^{-ik^+.x^-}$$

$$\times\langle 0|G_c(0)^{+\mu}\mathcal{E}^\dagger(0^-)_{cb}\mathcal{P}_{H(zk^+,0_\perp)}\mathcal{E}(0^-)_{ba}G_a(0^+,x^-,0_\perp)^+_\mu|0\rangle$$

with the line-integral defined as

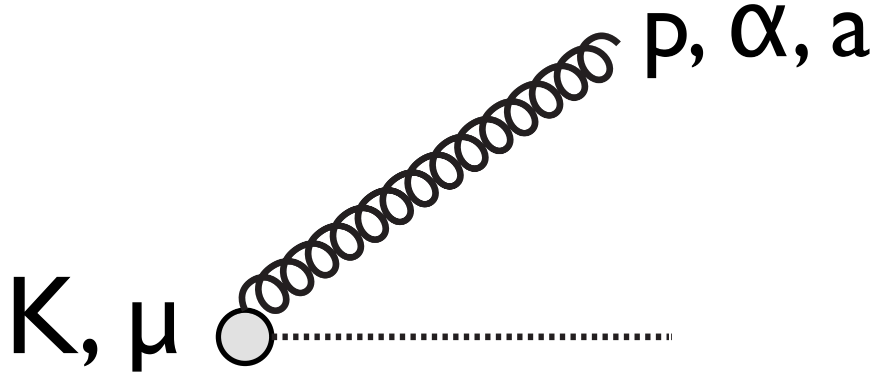
$$\mathcal{E}^\dagger(0^-)_{ba} = Pexp\left[ig\int_{x^-}^{\infty}dz^- A^+(0^+,z^-,0_\perp)\right]_{ba}$$



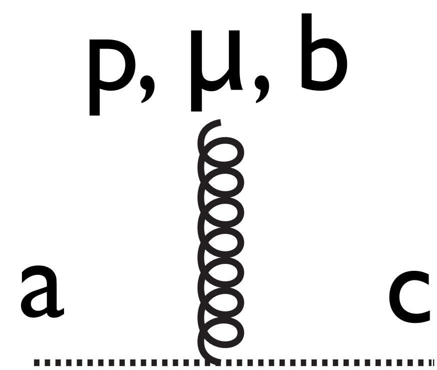
the calculation of radiative corrections can be simplified by using the Feynman gauge

# Fragmentation function: formal definition

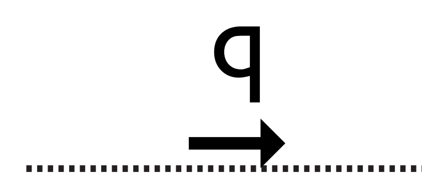
- The perturbative expansion of this definition in powers of $\alpha_s$ leads to a simple set of Feynman rules




$$i(K.n g^{\mu\alpha} - p^{\mu} n^{\alpha})\delta_{ab}$$




$$g n^{\mu} f_{abc}$$



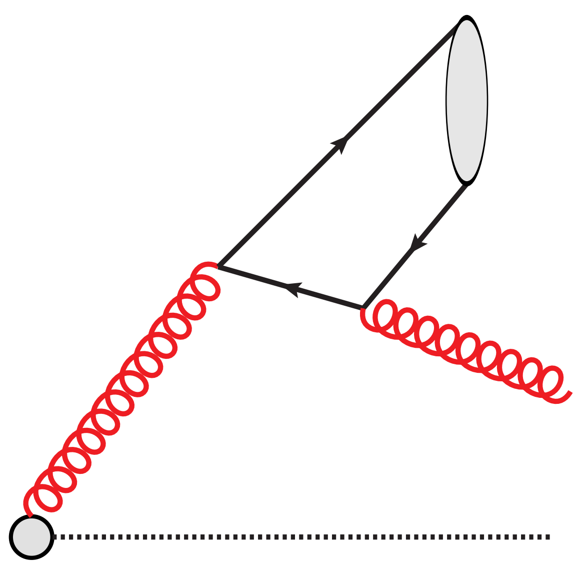

$$\frac{i}{q.n + i\epsilon}$$

# Gluon fragmentation into $\eta_c$ at NLO

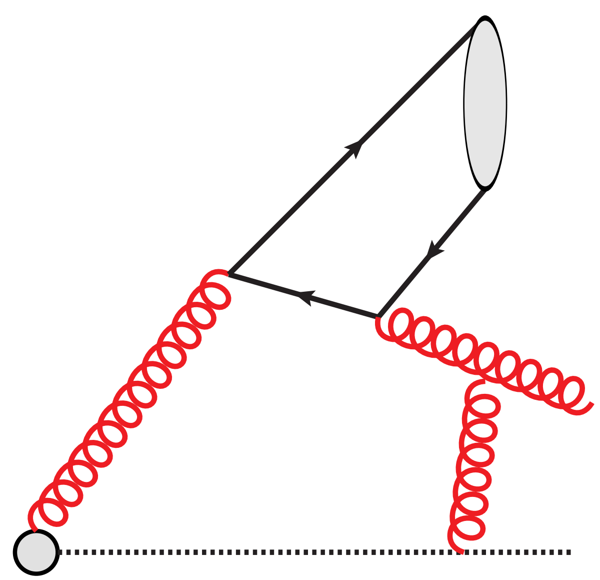
$$D(g \to \eta_c(z) + X) =$$

$$N \int d\phi_B |M_B|^2 + N \int d\phi_B |M_V|^2 + N \int d\phi_R |M_R|^2$$

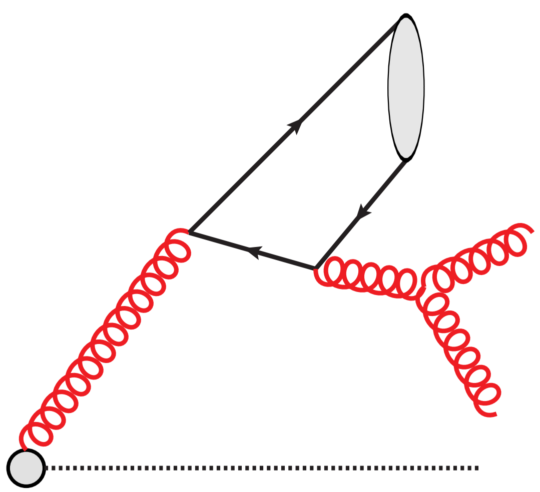
Born — Virtual — Real



+ ... (2 diagrams)



+ ... (36 diagrams)



+ ... (24 diagrams)

# Gluon fragmentation into $\eta_c$ at NLO

$$D(g \to \eta_c(z) + X) =$$

$$N \int d\phi_B |M_B|^2 + N \int d\phi_B |M_V|^2 + N \int d\phi_R |M_R|^2$$

Born Virtual Real

$$d\phi_B = \frac{d^{D-1}q}{(2\pi)^{D-1} 2q^0} 2\pi\delta[K.n - P.n - q.n]$$

$$d\phi_R = \frac{d^{D-1}q_1}{(2\pi)^{D-1} 2q_1^0} \frac{d^{D-1}q_2}{(2\pi)^{D-1} 2q_2^0} 2\pi\delta[K.n - P.n - q.n]$$

# Projection onto $^1S_0$ [1]

- Dimensional regularization (D=4-2ε)
- Projection method: the projector onto $\eta_c$ involves the Dirac matrix $\gamma_5$ → anti-commutation rules in D dimension ?
- Instead, express the amplitude in terms of antisymmetric combination of γ matrices

$$|M_B|^2 = (C_{ggg} g_{\alpha_1,\beta_1} g_{\alpha_2,\beta_2} g_{\alpha_3,\beta_3} + C_{nngg} n_{\alpha_1} n_{\beta_1} g_{\alpha_2,\beta_2} g_{\alpha_3,\beta_3})$$

$$\times \bar{u}[\gamma^{\alpha_1}, \gamma^{\alpha_2}, \gamma^{\alpha_3}] v \bar{v} [\gamma^{\beta_1}, \gamma^{\beta_2}, \gamma^{\beta_3}] u$$

# Strategy for the virtual

- Reduction to a minimal set of one-loop scalar integrals (with the help of FeynCalc)

- Basis: 6 bubbles, 5 triangles, 1 box, +

$$\int d^D l \frac{1}{D_1 D_2 (l.n + i\epsilon)}$$ (x12) poles: $\frac{1}{\epsilon_{\mathrm{IR}}\,\epsilon_{\mathrm{UV}}}$, $\frac{1}{\epsilon_{\mathrm{UV}}}$

$$\int d^D l \frac{1}{D_1 D_2 D_3 \,(l.n + i\epsilon)}$$ (x8) poles: $\frac{1}{\epsilon^2_{\mathrm{IR}}}$, $\frac{1}{\epsilon_{\mathrm{IR}}}$

Analytic expression in D=4-2ε in terms of hypergeometric and Appell functions, expanded in ε using HypExp (Mathematica) or XSummer (FORM)

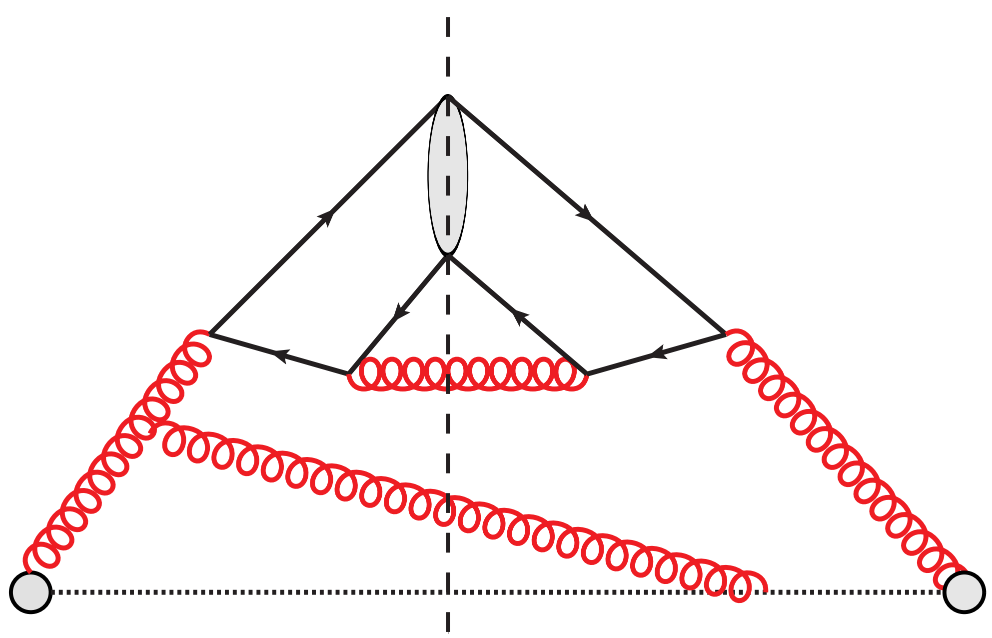
# Strategy for the real

- Simplification of the amplitude with the use of FeynCalc

- Extraction of the UV/IR poles

Analytic expression in D=4-2ε in terms of hypergeometric and Appell functions, expanded in ε using HypExp (Mathematica) or XSummer (FORM)

ex:



$$\frac{1}{\epsilon_{\text{IR}}\epsilon_{\text{UV}}}\int d\phi_B|M_B|^2$$

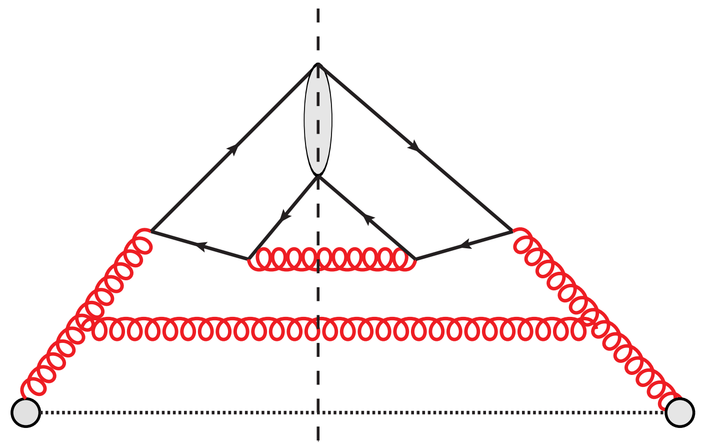
# Cancellation of the poles

- $\dfrac{1}{\epsilon_{\rm IR}\,\epsilon_{\rm UV}}\,,\ \dfrac{1}{\epsilon_{\rm IR}^2},\ \dfrac{1}{\epsilon_{\rm IR}}$

cancellation among the real and virtual contributions

- $\dfrac{1}{\epsilon_{\rm UV}}$

cancelled by renormalization of the non-local operator, the coupling constant and the heavy quark mass.



# Conclusion

- Cross section for charmonium production at large $p_T$ can be calculated in a (generalized) fragmentation framework
- Most of the fragmentation functions are known only at leading order accuracy in $\alpha_s$
- I presented work in progress for $D(g \to \eta_c + X)$ at NLO

# backup slides

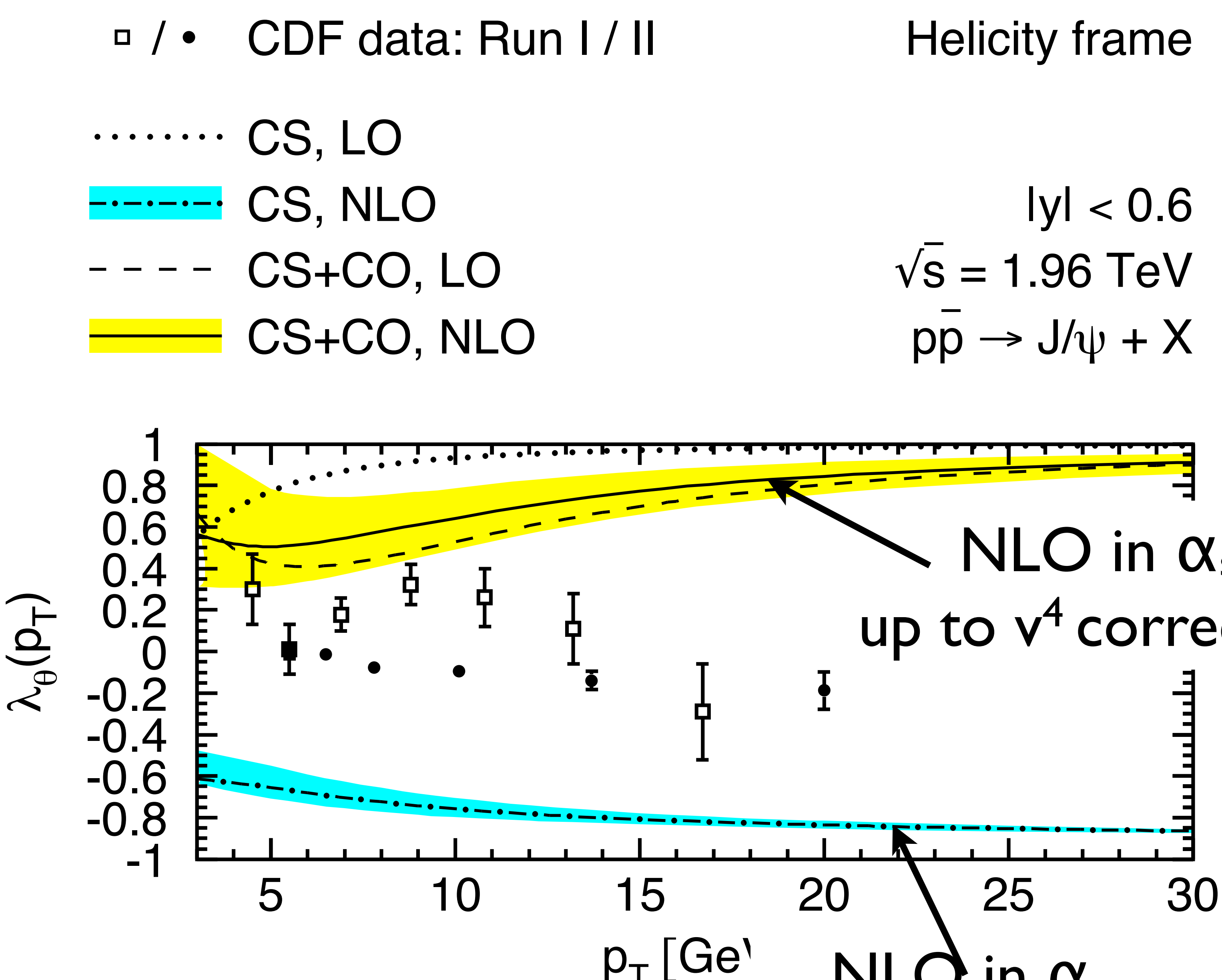
# Progress on parton cross sections

$$\sigma[\mathcal{Q}] = \sum_n \hat{\sigma}_\Lambda[Q\bar{Q}(n)] \langle \mathcal{O}^{\mathcal{Q}}(n) \rangle_\Lambda$$

- photoproduction — Kramer, Zunft, Steegborn, Zerwas 1995; Kramer 1996
  Artoisenet, Campbell, Maltoni, Tramontano 2009
  Chang, Li, Wang 2009; Li, Chao 2009
  Butenschoen, Kniehl 2009
- γγ collisions — Klasen, Kniehl, Mihaila, Steinhauser 2005
- e+e- ➔ double charmonium — Zhang, Gao, Chao 2005; Zhang, Ma, Chao 2008
  Gong, Wang 2008
- e+e- ➔ charmonium + X — Zhang, Chao 2006; Ma, Zhang, Chao 2008
  Gong, Wang 2008, 2009
  Zhang, Ma, Wang, Chao 2009
- hadron collisions — Petrelli, Cacciari, Greco, Maltoni, Mangano 1998
  Campbell, Maltoni, Tramontano 2008; Artoisenet, Lansberg, Maltoni, 2008
  Li, Wang 2008; Gong, Wang 2008; Gong, Li, Wang 2009,
  Butenschoen, Kniehl 2010, Butenschoen, Kniehl 2012

# J/ψ polarization at the Tevatron

Butenschoen, Kniehl 2012




NLO in $\alpha_s$ , up to $v^4$ corrections

NLO in $\alpha_s$ , leading order in v

distribution of the muons in the J/ψ rest frame:

$$\propto 1 + \lambda_\theta \cos\theta$$

$\lambda_\theta = 1$ : transverse J/ψ

$\lambda_\theta = -1$ : longitudinal J/ψ

flagrant discrepancy between NRQCD prediction and the data