Two-loop corrections to $Z\,\gamma$ and $W\,\gamma$ production at hadron colliders

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talk based on a work together with *Thomas Gehrmann*:
JHEP 1202 (2012) 004 [arXiv:1112.1531]
[*Two-loop QCD helicity amplitudes for* $q\bar{q} \to W^{\pm}\gamma$ *and* $q\bar{q} \to Z^0\gamma$]

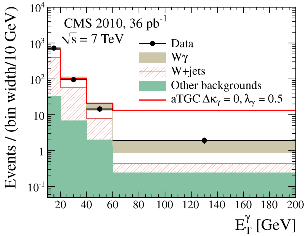
Motivations

Why study vector boson pair production?

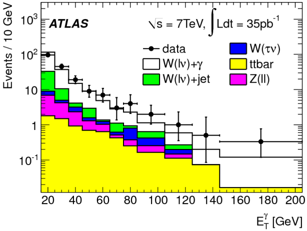
- Check of **anomalous coupling** $WW\gamma$, WWZ
- **Background estimate for Higgs production** at LHC: Most promising decays are:
  1. $H \to \gamma\,\gamma$ *for a light Higgs*
  2. $H \to Z\,Z$, $H \to W^+W^-$ *for heavier (not only...) Higgs*
- Study of Higgs mechanism, **unitarization** of $W_L W_L$ scattering amplitudes.

Why two-loop precision is needed?

- **LHC is running !!**
  Already many events collected on $(Z\,\gamma, W^{\pm}\,\gamma, W^{+}W^{-}, \cdots)$ production:



arXiv:1105.2758



arXiv:1106.1592

Plots with data collected in 2010. In 2011 data for $\approx 1 fb^{-1}$ of integrated luminosity.
The NLO theoretical precision will become soon the main source of uncertainties.

- We are interested in the following processes

$$q\,\bar{q} \rightarrow V_1 V_2 \quad \text{where} \quad V_i = (\gamma, Z, W^{\pm})$$

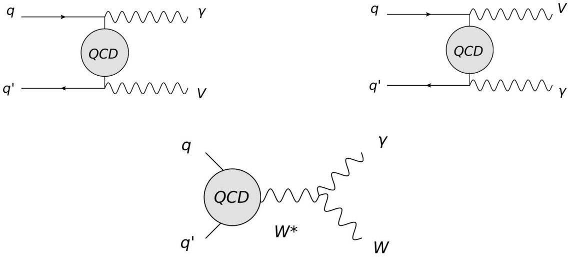
- **We can group them as follows:**
  1. $q\,\bar{q} \rightarrow \gamma\gamma$
     4-point function with 4 legs on-shell
     already known at NNLO *(Catani et al. arXiv:1110.2375)*.
  2. $q\,\bar{q} \rightarrow Z\,\gamma, \quad q\,\bar{q} \rightarrow W^{\pm}\,\gamma$
     4-point function with 3 legs on-shell, and 1 leg off-shell.
  3. $q\,\bar{q} \rightarrow Z\,Z, \quad q\,\bar{q} \rightarrow W^{\pm}\,W^{\mp}$
     4-point function with 2 legs on-shell, and 2 legs off-shell with the same invariant mass.
  4. $q\,\bar{q} \rightarrow Z\,W^{\pm}$
     4-point function with 2 legs on-shell, and 2 legs off-shell with two different invariant masses.

Let's **focus** on two-loop corrections to

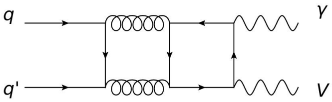
$$q\,\bar{q} \to Z\,\gamma \qquad q\,\bar{q} \to W^{\pm}\gamma$$

In order to treat both $V = Z, W^{\pm}$ cases in the same framework we need to divide the two-loop diagrams into **six different classes**.

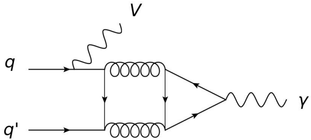
- The first three appear also at the **tree-level**:

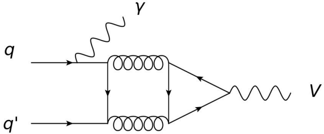


- At the **two-loop** level though, new contributions arise:



Plus Diagrams null for **Furry's Theorem**:

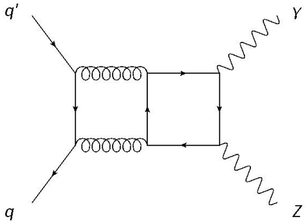




Remarks on the calculation:

Main task in a two-loop calculation is the **evaluation of the loop-integrals**.

Typical two-loop planar diagram:



$$\int_k \int_l \frac{\mathcal{N}_i}{k^2 l^2 (k-l)^2 (k-p_1)^2 (k-p_1-p_2)^2 (l-p_1-p_2)^2 (l-p_1-p_2+p_3)^2}$$

- We work in massless **QCD**.
- UV and IR divergences regularized in **Dimensional Regularization**.
- The couplings of vector bosons to fermions are **spin-dependent**. Computing the **Helicity Amplitudes (HA)** for the process allows to account trivially for the spin-correlations with the leptonic decay products

$$q'\,\bar{q} \to \gamma\,Z^0 \to \gamma\,l^+\,l^-\,,$$

$$q'\,\bar{q} \to \gamma\,W^- \to \gamma\,\bar{\nu}\,l^-\,.$$

1. Notice that the processes

$$q'\,\bar{q} \to \gamma^*/Z^* \to \gamma\,l^+\,l^-\,,$$
$$q'\,\bar{q} \to W^* \to \gamma\,\bar{\nu}\,l^-\,,$$

must be taken into account for **NNLO implementation**.

- The HA are extracted by applying **d-dimensional projection operators** to the most general tensor structure of the amplitude.

1. **Tensor Structure** for $q(p_1) + \bar{q}(p_2) \to \gamma(-p_3) + V(p_4)$

$$\begin{aligned} S_\mu(q;\gamma;\bar{q}) = & \; A_{11}\; T_{11\mu} + A_{12}\; T_{12\mu} + A_{13}\; T_{13\mu} \\ & + A_{21}\; T_{21\mu} + A_{22}\; T_{22\mu} + A_{23}\; T_{23\mu} \\ & + B\; T_\mu . \end{aligned}$$

2. Where the **tensors** are defined as:

$$\begin{aligned} T_{1j\mu} &= \bar{u}(p_1)\left[\not{p}_3\; \epsilon_3 \cdot p_1\; p_{j\mu} - \frac{s_{13}}{2} \not{\epsilon}_3\; p_{j\mu} + \frac{s_{j4}}{4} \not{\epsilon}_3 \not{p}_3 \gamma_\mu \right] v(p_2), \\ T_{2j\mu} &= \bar{u}(p_1)\left[\not{p}_3\; \epsilon_3 \cdot p_2\; p_{j\mu} - \frac{s_{23}}{2} \not{\epsilon}_3\; p_{j\mu} + \frac{s_{j4}}{4} \gamma_\mu \not{p}_3 \not{\epsilon}_3 \right] v(p_2), \\ T_\mu &= \bar{u}(p_1)\left[s_{23} \left(\gamma_\mu \epsilon_3 \cdot p_1 + \frac{1}{2} \not{\epsilon}_3 \not{p}_3 \gamma_\mu \right) \right. \\ & \qquad \left. s_{13} \left(\gamma_\mu \epsilon_3 \cdot p_2 + \frac{1}{2} \gamma_\mu \not{p}_3 \not{\epsilon}_3 \right) \right] v(p_2). \end{aligned}$$

Fixing the helicities of the external particles, the contraction of the hadronic tensor structure with the leptonic current can be easily written in **Dixon notation** as:

$$A^{+}_{RR}(p_5, p_6; p_1, p_3, p_2) = L^{\mu}_{R}(p_5^-; p_6^+) S_{R,\mu}(p_1^-; p_3^+; p_2^+)$$
$$= -2\sqrt{2}\left[\frac{\langle 25\rangle\langle 12\rangle[16]}{\langle 13\rangle\langle 23\rangle}\,\alpha(u,v) - \frac{\langle 25\rangle[36]}{\langle 13\rangle}\,\beta(u,v) + \frac{\langle 15\rangle[13][36]}{\langle 13\rangle[23]}\,\gamma(u,v) \right]$$

where the **new coefficients** have been introduced

$$\alpha(u,v) = \frac{s_{13}s_{23}}{4}\Big(2B + A_{12} - A_{11}\Big),$$
$$\beta(u,v) = \frac{s_{13}}{4}\Big(2s_{23}B + 2(s_{12}+s_{13})A_{11} + s_{23}(A_{12}+A_{13})\Big),$$
$$\gamma(u,v) = \frac{s_{13}s_{23}}{4}\Big(A_{11} - A_{13}\Big),$$

where

$$u = -\frac{y}{x} \qquad v = \frac{1}{x}$$

and

$$x = \frac{s_{12}}{s_{123}}, \quad y = \frac{s_{13}}{s_{123}}, \quad z = \frac{s_{23}}{s_{123}}, \quad s_{ij} = (p_i + p_j)^2.$$

- After applying the projectors, the HA are expressed in terms of thousands of Planar and Non-Planar scalar integrals of the general form

$$I = \int \Pi_i d^d k_i \, \frac{S_1^{j_1} \cdots S_m^{j_m}}{D_1^{r_1} \cdots D_n^{r_n}}$$

For Example: For the case $q\,\bar{q} \to \gamma\,Z$ we had

1. **143** Feynman Diagrams
2. **1051** *PLANAR* Integrals
3. **933** *NON-PLANAR* Integrals

- Exploiting *Integration By Parts Identities* (**IBPs**)

$$\int \Pi_i d^d k_i \left(\frac{\partial}{\partial k_i^\mu} v_\mu \frac{S_1^{j_1} \cdots S_m^{j_m}}{D_1^{r_1} \cdots D_n^{r_n}} \right) = 0, \qquad v^\mu = (k_i^\mu, p_i^\mu)$$

one can express them as linear combination of small number of **irreducible Master Integrals (MIs)**

- The **MIs** are computed deriving and solving **differential equations** in the external invariants (i.e. $s_{ij} = (p_i + p_j)^2$).

- Using the techniques listed above we obtained an explicit expression for the coefficients $\Omega_j = (\alpha_j, \beta_j, \gamma_j)$, independently for the different groups $j = 1, 2, 3, 4, 5, 6$, expressed in terms of 1- and 2-dim **Harmonic Polylogarithms (HPLs)**, up to weight 4.

- With these, one can easily **reconstruct** the correct **two-loop amplitudes** for $V = Z, W^{\pm}$ summing up the different contributions with **appropriate weights**:

$$\Omega^{(2)}_{W^\pm} = U_{qq'} \left(e_{q'}\, \Omega^{(2)}_1 + e_q\, \Omega^{(2)}_2 + \Omega^{(2)}_3 \right)$$

$$\Omega^{(2)}_{Z} = e_q \left(\Omega^{(2)}_1 + \Omega^{(2)}_2 \right) + N_{F,Z}\, \Omega^{(2)}_4$$

U_{ij} are the **CKM matrix elements**,
$N_{F,Z}$ is the charge weighted sum of quark flavours in the loop.

- Different **non-trivial checks** have been performed

1. Symmetry relations for coefficients A_{ij} and B under $p_1 \leftrightarrow p_2$:

$$\begin{aligned} A_{21}(s_{13}, s_{23}, s_{123}) &= -A_{12}(s_{23}, s_{13}, s_{123}), \\ A_{22}(s_{13}, s_{23}, s_{123}) &= -A_{11}(s_{23}, s_{13}, s_{123}), \\ A_{23}(s_{13}, s_{23}, s_{123}) &= -A_{13}(s_{23}, s_{13}, s_{123}), \\ B(s_{13}, s_{23}, s_{123}) &= B(s_{23}, s_{13}, s_{123}). \end{aligned}$$

2. We verified the validity of **Furry's Theorem** on the sum of the diagrams in classes 5) and 6).

3. Match of the **IR singularities** structure with **Catani's Ansatz**.

4. We decomposed the amplitudes according to their **colour structure**, and verified that the **subleading colour** part coincides with the one for **3-jet** production.

Future Developments:

- $q\,\bar{q} \to Z\,\gamma\,, \qquad q\,\bar{q} \to W^{\pm}\gamma$
  1. Implementation of the fully differential **NNLO cross section** for $V\gamma$ production at LHC $\longrightarrow$ q_T subtraction method.
  2. Study of two loop corrections to $g\,g \to Z\,\gamma$, relative weight of N^{3}LO compared to NNLO from $q\,\bar{q} \to Z\,\gamma$

- $q\,\bar{q} \to Z\,Z\,, \qquad q\,\bar{q} \to W\,W$
  1. Reduction to MIs has been performed with `Reduze 2`.
  2. The MIs have been classified into **2 PL** and **1 NPL** topologies.
  3. We are studying the differential equations for the MIs.
  4. **Goal:** Computation of **helicity amplitudes**.

Thanks !!

- How to reconstruct the full amplitudes?
  1. **Right-handed leptonic current**

$$L_R^{\mu}(p_5^-, p_6^+) = [6\,|\gamma^{\mu}|\,5\rangle. \tag{1}$$

  2. Contraction with the **right-handed hadronic current**:

$$A_{RR}^{+}(p_5, p_6; p_1, p_3, p_2) = L_R^{\mu}(p_5^-; p_6^+) S_{R,\mu}(p_1^-; p_3^+; p_2^+).$$

  3. All the other **helicity configurations** can be obtained from this one:

$$A_{RR}^{-}(p_5, p_6; p_1, p_3, p_2) = [-A_{RR}^{+}(p_6, p_5; p_2, p_3, p_1)]^*,$$
$$A_{RL}^{+}(p_5, p_6; p_1, p_3, p_2) = -A_{RR}^{+}(p_5, p_6; p_2, p_3, p_1),$$
$$A_{RL}^{-}(p_5, p_6; p_1, p_3, p_2) = [A_{RR}^{+}(p_6, p_5; p_1, p_3, p_2)]^*,$$

etc...

- The explicit form of the **gauge boson coupling** to fermions depends on the gauge boson, on the type of fermions, and on their helicities

$$\mathcal{V}_\mu^{V,f_1 f_2} = -i\, e\, \Gamma_\mu^{V,f_1 f_2} \qquad \text{with} \qquad e = \sqrt{4\pi\alpha},$$

and

$$\Gamma_\mu^{V,f_1 f_2} = L_{f_1 f_2}^V\, \gamma_\mu \left(\frac{1-\gamma_5}{2}\right) + R_{f_1 f_2}^V\, \gamma_\mu \left(\frac{1+\gamma_5}{2}\right).$$

- The **vector boson propagator** can be written as:

$$P_{\mu\nu}^V(q,\xi) = \frac{i\,\Delta_{\mu\nu}^V(q,\xi)}{D_V(q)},$$

where $\Delta_{\mu\nu}^V(q,\xi)$ and $D_V(q)$ are, respectively, the numerator and the denominator in the R_ξ gauge.

With this notation we can reconstruct the **full amplitude** in the different cases $V = (Z, W^{\pm})$ as:

$$\mathcal{M}_V(p_5^-, p_6^+; p_1^-, p_3^+, p_2^+) = -i\,(4\pi\alpha)\,\frac{R^V_{q_1 q_2}\, R^V_{f_5 f_6}}{D_V(p_{56})}\; A^+_{RR}(p_5, p_6; p_1, p_3, p_2),$$

$$\mathcal{M}_V(p_5^-, p_6^+; p_1^+, p_3^-, p_2^-) = -i\,(4\pi\alpha)\,\frac{L^V_{q_1 q_2}\, R^V_{f_5 f_6}}{D_V(p_{56})}\; [A^+_{RR}(p_6, p_5; p_1, p_3, p_2)]^*,$$

$$\mathcal{M}_V(p_5^-, p_6^+; p_1^+, p_3^+, p_2^-) = -i\,(4\pi\alpha)\,\frac{L^V_{q_1 q_2}\, R^V_{f_5 f_6}}{D_V(p_{56})}\; [-A^+_{RR}(p_5, p_6; p_2, p_3, p_1)],$$

$$\mathcal{M}_V(p_5^-, p_6^+; p_1^-, p_3^-, p_2^+) = -i\,(4\pi\alpha)\,\frac{R^V_{q_1 q_2}\, R^V_{f_5 f_6}}{D_V(p_{56})}\; [-A^+_{RR}(p_6, p_5; p_2, p_3, p_1)]^*.$$

with the obvious notation $p_{ij} = p_i + p_j$.

The corresponding amplitudes for **left-handed leptonic current** can be obtained simply interchanging $p_5 \leftrightarrow p_6$ and $R^V_{f_5 f_6} \longrightarrow L^V_{f_5 f_6}$.