Progress at NNLO in QCD

– *hard scattering, cross sections, PDFs* –

Sven-Olaf Moch

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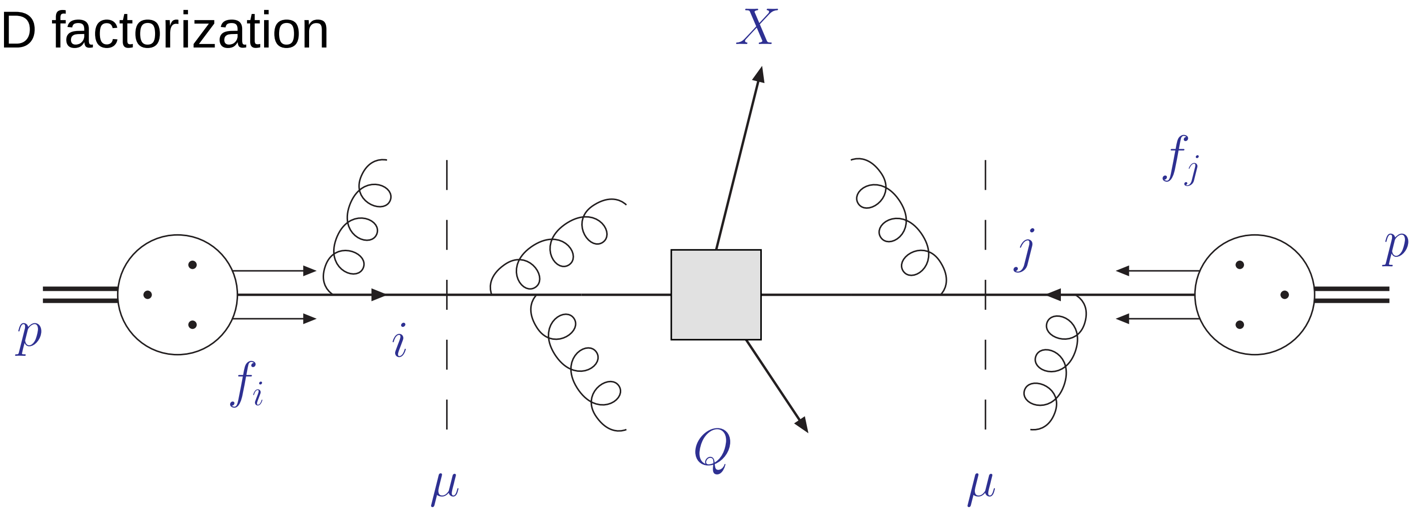
DESY, Zeuthen

– *Annual meeting of the LHCPhenoNet Initial Training Network*, Chester le Street, Mar 22, 2012 –

Introduction

- QCD factorization

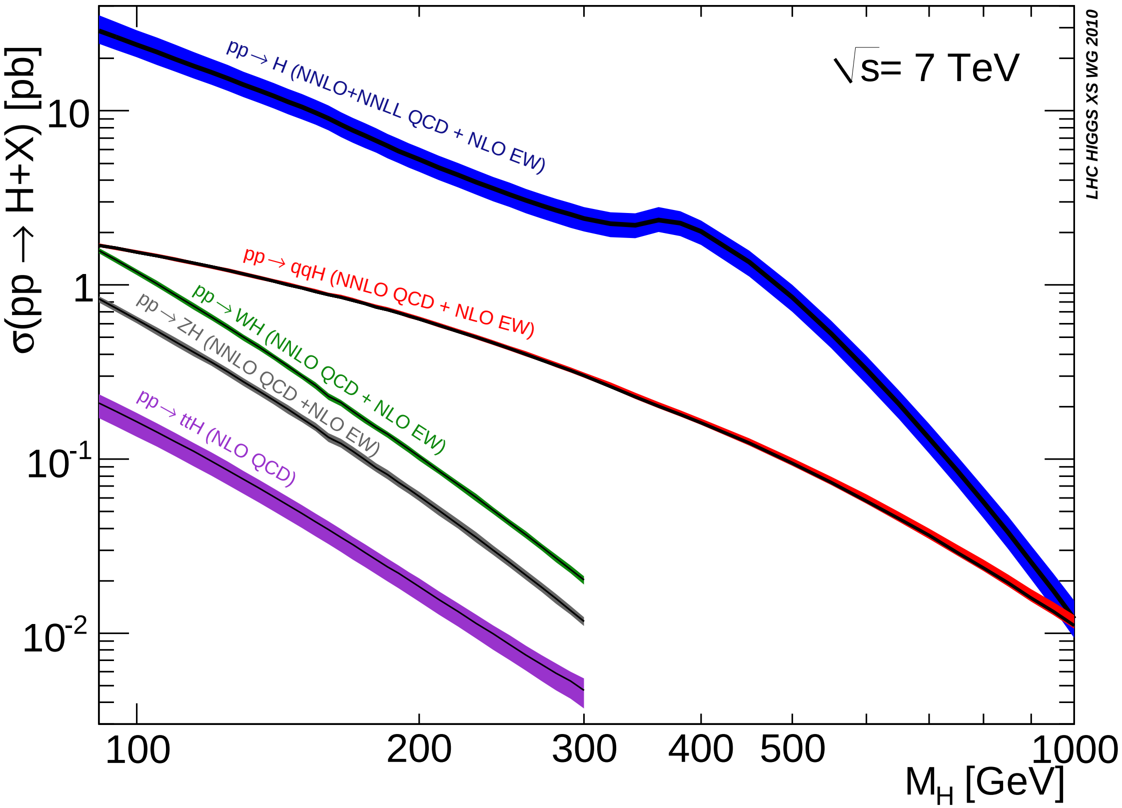


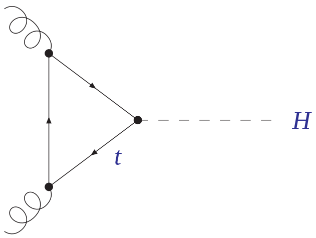
$$\sigma_{pp\to X} = \sum_{ij} f_i(\mu^2) \otimes f_j(\mu^2) \otimes \hat{\sigma}_{ij\to X} \left(\alpha_s(\mu^2), Q^2, \mu^2, m_X^2\right)$$

- Hard parton cross section $\hat{\sigma}_{ij\to X}$ calculable in perturbation theory
  - known to NLO, NNLO, . . . ($\mathcal{O}(\text{few}\%)$ theory uncertainty)
- Non-perturbative parameters: parton distribution functions f_i, strong coupling α_s, particle masses m_X
  - known from global fits to exp. data, lattice computations, . . .

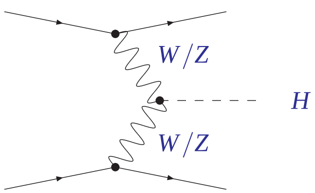
Cross section for Higgs production

- Dominant channels for Higgs boson production LHC Higgs XS WG '10

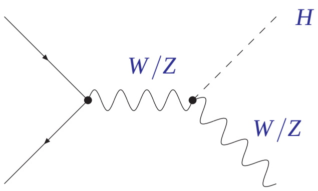




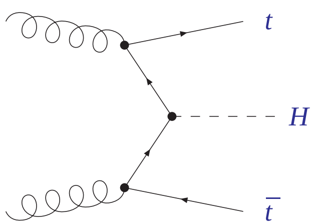
gg-fusion



weak boson fusion

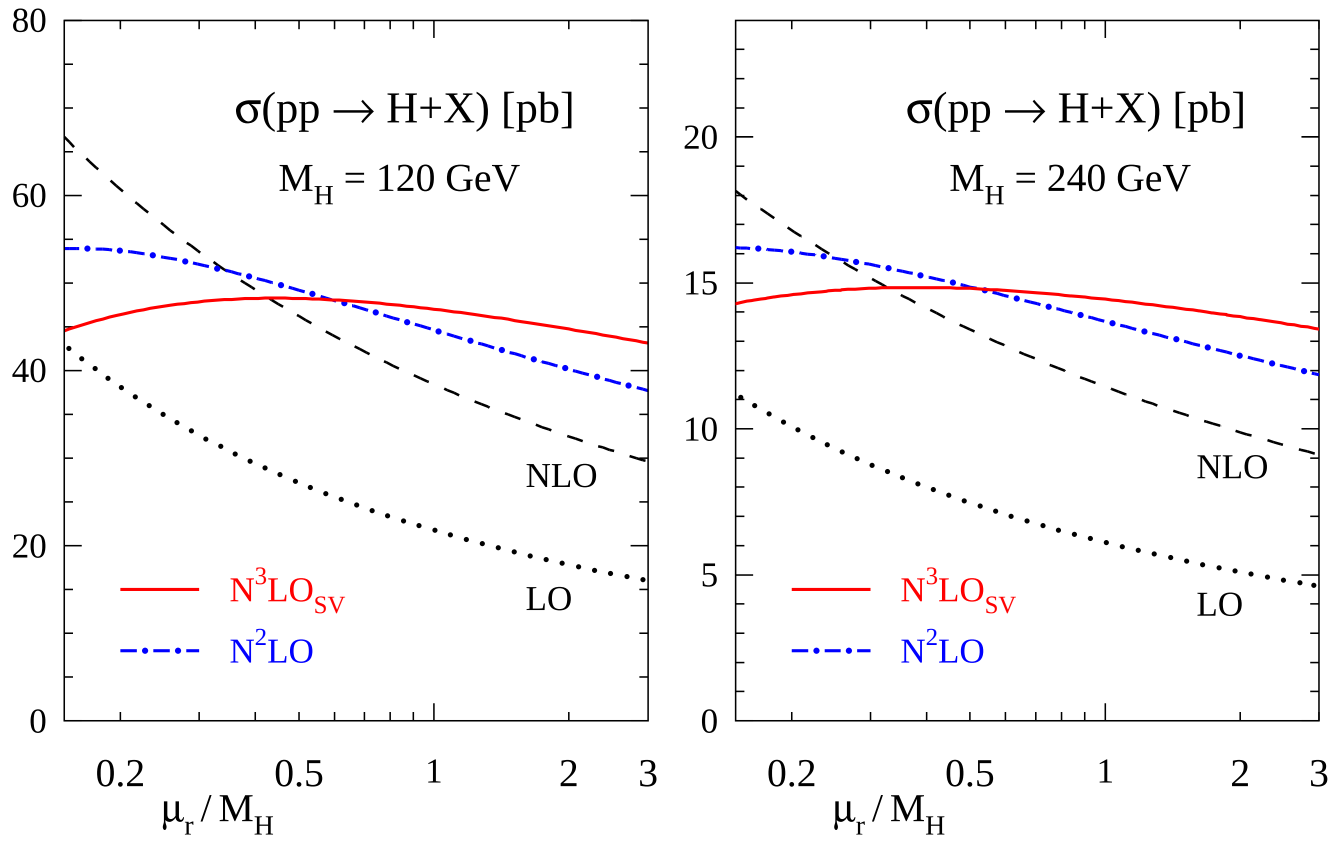


Higgs strahlung



$t\bar{t}H$-channel

Perturbation theory at work



- Apparent convergence of perturbative expansion
  - NNLO corrections still large
    Harlander, Kilgore '02; Anastasiou, Melnikov '02; Ravindran, Smith, van Neerven '03
  - improvement through complete soft N^3LO corrections S.M., Vogt '05 or NNLL resummtion Catani, de Florian, Grazzini, Nason '03, Ahrens et al. '10
- Perturbative stability under renormalizaion scale variation

Non-perturbative parameters

Input for collider phenomenology

- Non-perturbative parameters are universal
- Determination from comparision to experimental data
  - masses of heavy quarks m_c, m_b, m_t
  - parton distribution functions $f_i(x, \mu^2)$
  - strong coupling constant $\alpha_s(M_Z)$

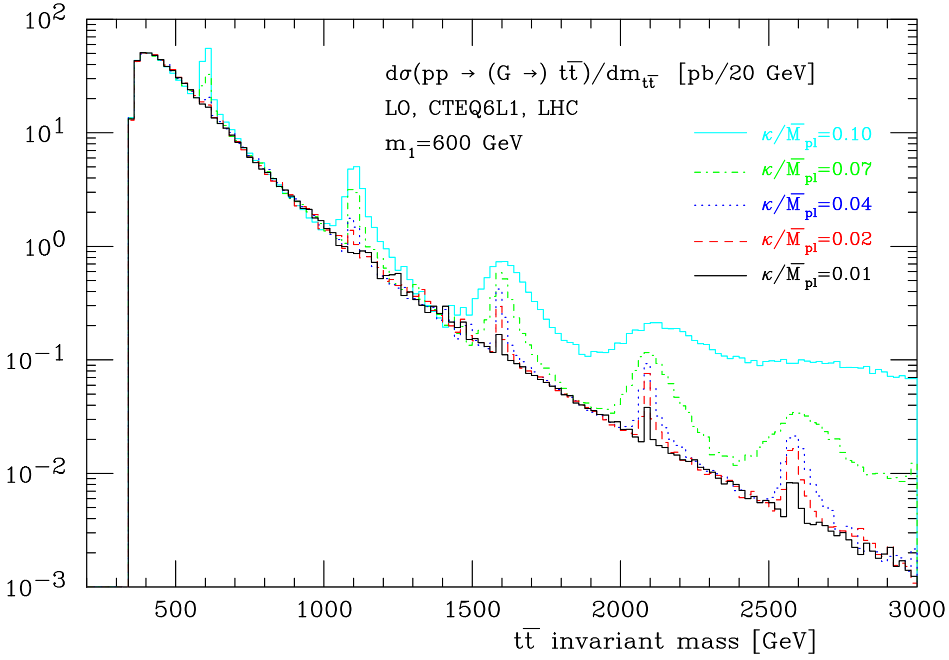
Interplay with perturbation theory

- Accuracy of determination driven by precision of theory predictions
- Non-perturbative parameters sensitive to
  - radiative corrections at higher orders
  - renormalization and factorization scales μ_R, μ_F
  - chosen scheme (e.g. ($\overline{MS}$ scheme)
  - ...

New physics discoveries

- Suppose we observe . . .
  - . . . e.g. Kaluza-Klein resonances
    (s-channel graviton in $t\bar{t}$ invariant mass spectrum at LHC)

Frederix, Maltoni '07



- Which non-perturbative parameters ?

New physics discoveries

- Suppose we observe ...
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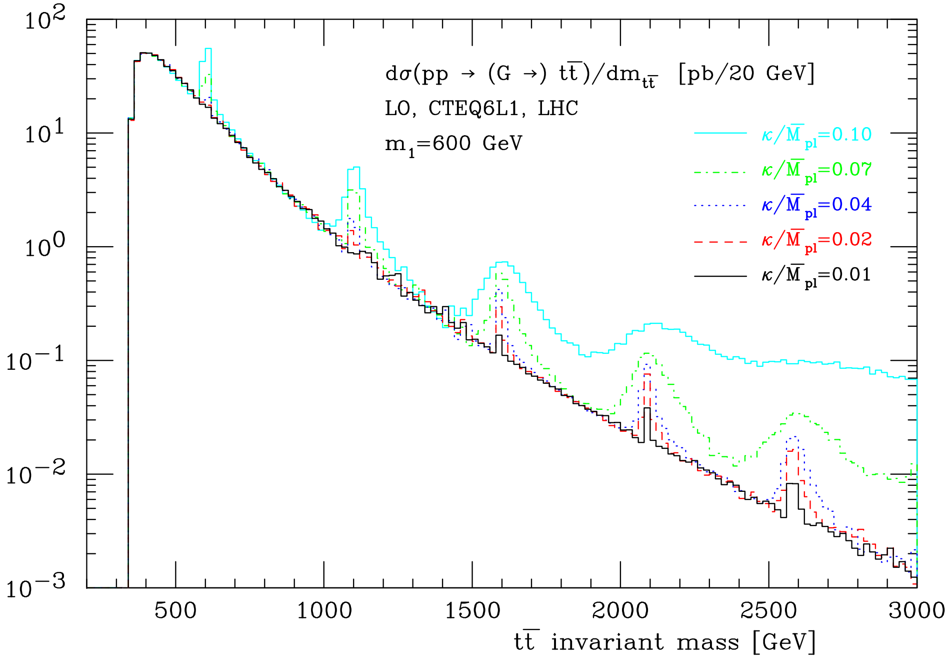
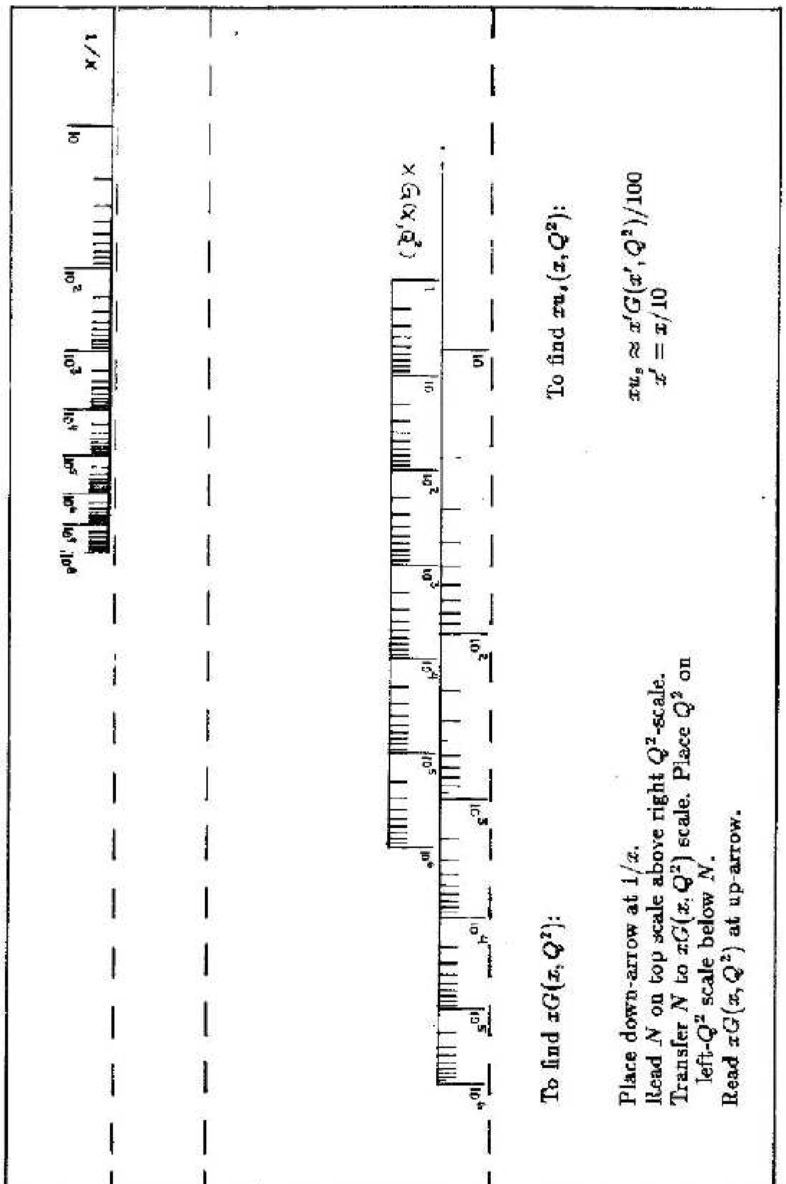


- Which non-perturbative parameters ?
  - $\alpha_s(M_Z) = 0.13$, $m_c = 1.5$ GeV, $m_b = 4.5$ GeV, ...
  - any PDF set

J. Ralston (Phys.Lett. B172 (1986) 430)

Pocket partonometer



for t- or heavier particle distributions one must model thresholds numerically such as done in ref. [4] [‡4]. However, departures from a symmetrically distributed sea, which complicate the boundary conditions, can be reproduced by the ratios $u_s \approx d_s \approx s_s \approx 2c_s \approx 2b_s$.

The analytic gluon solution (3), boundary conditions included, is calculated by the partonometer (fig. 2). The scales automate the logarithms of certain functions of $1/x$ and Q^2 left to the reader. In systematic testing the accuracy of the gizmo is at the 10–20% level depending on the operator's ability to read logarithmic scales. It is much better than interpolating between graphs such as fig. 1a. The speed is even faster than adding a new card [‡4] to an existing program that runs.

Gluon distributions are read off directly; see the example below. Quark sea distributions can be evaluated using the identity

$$xu_s(x,Q^2) = (2/b)\partial xG(x,Q^2)/\partial y, \qquad (7)$$

and evaluating the derivative numerically. But wait! To minimize reading errors, one finds that the derivative above and the normalization change are roughly represented by

$$xu_s(x,Q^2) \approx x'G(x',Q^2)/100, \quad x' = x/10. \qquad (8)$$

This estimate is actually quite close to the re-scaled $xu_s(x,Q^2)$ of ref. [5] and is not too bad a match to

[‡4] Private communication with well known phenomenologist.

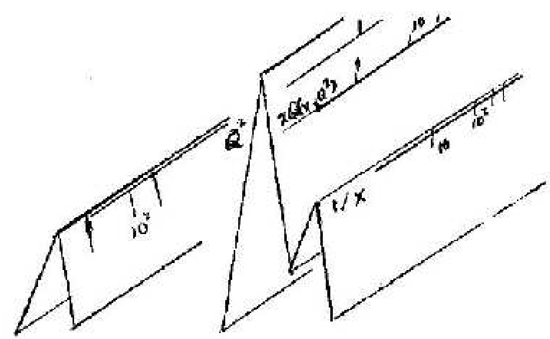
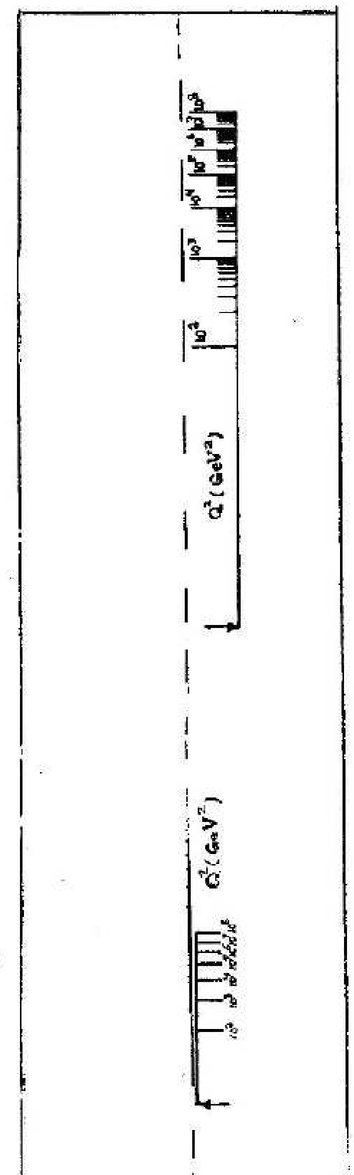
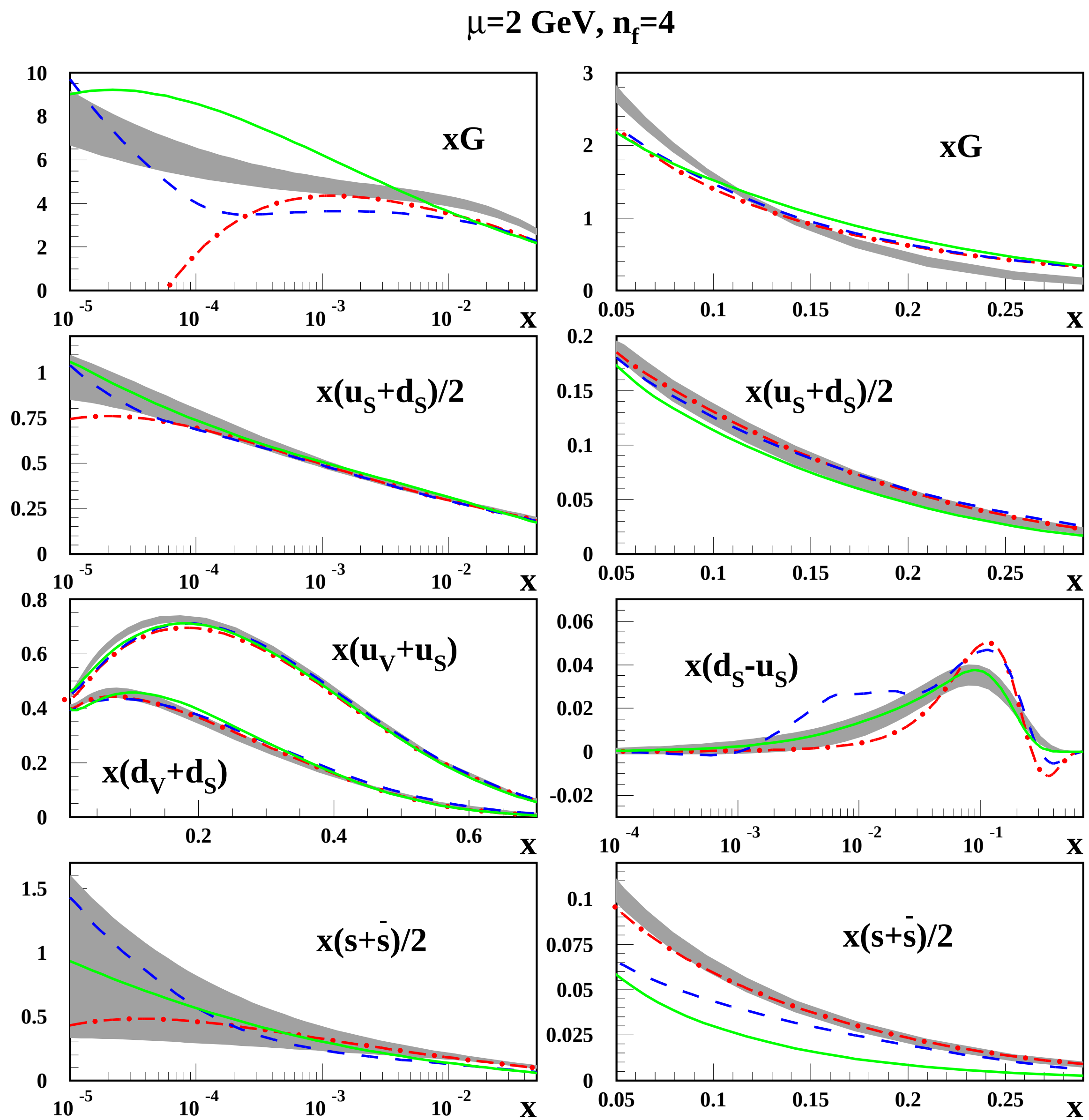


Fig. 2. The partonometer. To assemble: cut on solid lines, fold on dotted lines.



PDFs



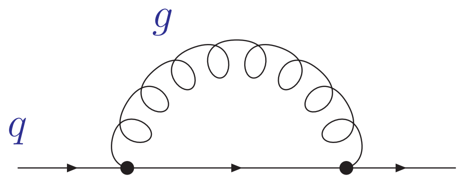
- 1σ band for ABM11 PDFs (NNLO, 4-flavors) at $\mu = 2$ GeV
- comparison with: JR09 (solid lines), MSTW (dashed dots) and NN21 (dashes)
- Some interesting observations to be made . . .

Heavy-quark masses

Pole mass

- Based on (unphysical) concept of top-quark being a free parton

$$\not{p} - m_q - \Sigma(p, m_q)\Big|_{p^2=m_q^2}$$



  - heavy-quark self-energy $\Sigma(p, m_q)$ receives contributions from regions of all loop momenta – also from momenta of $\mathcal{O}(\Lambda_{QCD})$
- Definition of pole mass ambiguous up to corrections $\mathcal{O}(\Lambda_{QCD})$

Running quark masses

- $\overline{MS}$ mass definition $m(\mu_R)$ realizes running mass (scale dependence)
  - short distance mass probes at scale of hard scattering
    $m_{\rm pole} = m_{\rm short\ distance} + \delta m$
  - conversion between pole mass and $\overline{MS}$ mass definition in perturbation theory: $m = m(\mu_R)\left(1 + a_s(\mu_R)d^{(1)} + a_s(\mu_R)^2 d^{(2)}\right)$

Quark masses in PDF fits

- Choice of value for heavy-quark masses part of uncertainty
- PDF fits assume pole mass scheme for heavy-quarks
  - numerical values systematically lower than those from PDG (2-loop conversion to pole mass)

[GeV]	PDG	ABKM	GJR	HERAPDF	MSTW	CT10	NNPDF2.1
m_c	$1.66\ ^{+0.09}_{-0.15}$	$1.5\ ^{+0.25}_{-0.25}$	1.3	$1.4\ ^{+0.25}_{-0.05}$	1.3	1.3	1.41
m_b	$4.79\ ^{+0.19}_{-0.08}$	$4.5\ ^{+0.5}_{-0.5}$	4.2	$4.75\ ^{+0.25}_{-0.45}$	4.75	4.75	4.75

PDG

- PDG quotes running masses:
  charm: $m_c(m_c) = 1.27^{+0.07}_{-0.11}$ GeV, bottom: $m_b(m_b) = 4.20^{+0.17}_{-0.07}$ GeV
- ABM11 uses running masses:
  charm: $m_c(m_c) = 1.27^{+0.08}_{-0.08}$ GeV, bottom: $m_b(m_b) = 4.19^{+0.13}_{-0.13}$ GeV

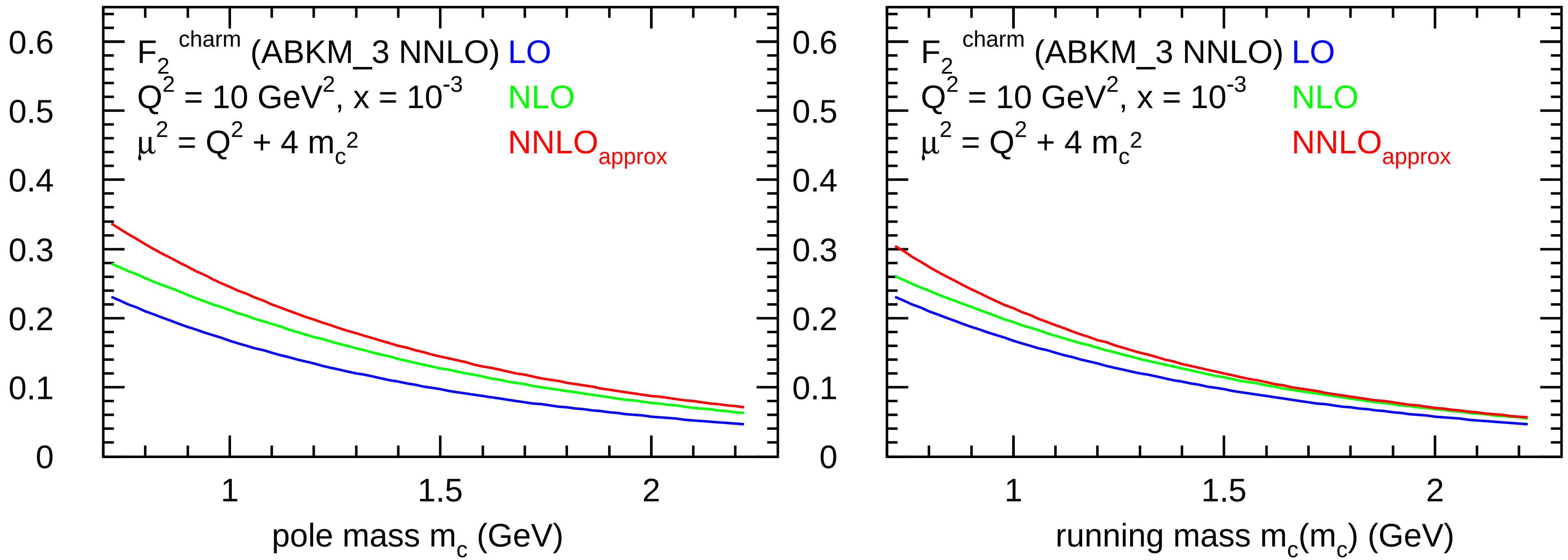
Impact on LHC cross sections

- $W^{\pm}$ and Z cross sections at LHC
- Uncertainties due to choice of pole mass value sizable
  - $\Delta\sigma_{W^{\pm}/Z} \simeq 4\%$ for $m_c = \pm 0.35$ GeV MSTW arXiv:1007.2624

Variable $\alpha_S(M_Z^2)$		Tevatron ($\sqrt{s} = 1.96$ TeV)			LHC ($\sqrt{s} = 7$ TeV)			LHC ($\sqrt{s} = 14$ TeV)		
m_c (GeV)	m_b (GeV)	$\delta\sigma^W$	$\delta\sigma^Z$	$\delta\sigma^H$	$\delta\sigma^W$	$\delta\sigma^Z$	$\delta\sigma^H$	$\delta\sigma^W$	$\delta\sigma^Z$	$\delta\sigma^H$
1.05		-2.6	-2.8	+0.4	-4.1	-4.6	-2.4	-5.1	-5.5	-3.8
1.10		-2.2	-2.4	+0.2	-3.5	-3.9	-2.1	-4.3	-4.7	-3.3
1.15		-1.8	-1.9	+0.1	-2.9	-3.3	-1.8	-3.6	-3.9	-2.8
1.20		-1.4	-1.5	+0.1	-2.3	-2.6	-1.5	-2.8	-3.1	-2.3
1.25		-1.0	-1.1	0.0	-1.7	-1.9	-1.2	-2.1	-2.3	-1.7
1.30		-0.7	-0.7	0.0	-1.1	-1.3	-0.8	-1.4	-1.5	-1.2
1.35		-0.3	-0.4	0.0	-0.6	-0.6	-0.4	-0.7	-0.8	-0.6
1.40	**4.75**	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
1.45		+0.3	+0.3	0.0	+0.6	+0.6	+0.4	+0.7	+0.8	+0.6
1.50		+0.6	+0.6	0.0	+1.1	+1.3	+0.8	+1.3	+1.5	+1.2
1.55		+0.8	+0.9	+0.1	+1.6	+1.9	+1.2	+2.0	+2.3	+1.8
1.60		+1.1	+1.2	+0.2	+2.1	+2.5	+1.8	+2.6	+3.0	+2.5
1.65		+1.3	+1.5	+0.1	+2.6	+3.0	+2.0	+3.2	+3.7	+2.9
1.70		+1.5	+1.8	+0.2	+3.1	+3.6	+2.5	+3.8	+4.4	+3.6
1.75		+1.8	+2.0	+0.3	+3.5	+4.2	+2.9	+4.3	+5.1	+4.1

Running quark masses in DIS

- Charm structure function



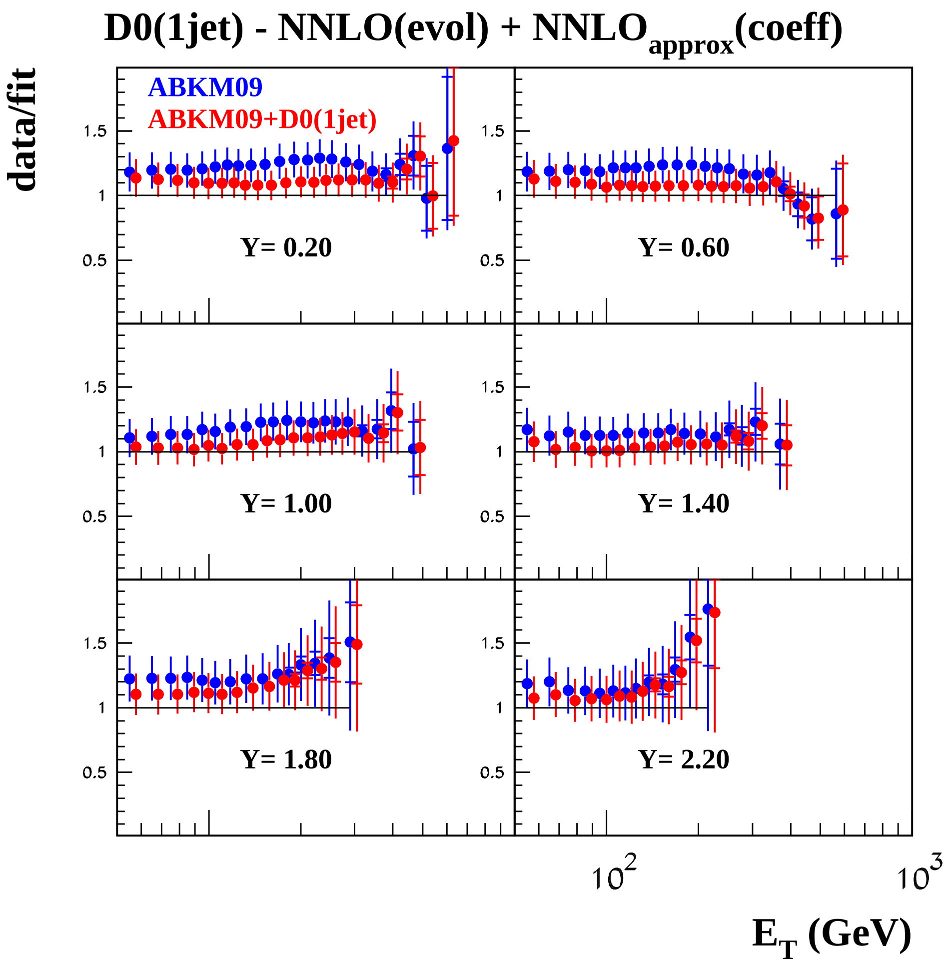
- Running quark masses in DIS
  - improved convergence and reduced scale dependence
- Comparison with pole mass scheme
- Consistent description of heavy quark DIS allows to control correlation of g, α_s and m_c

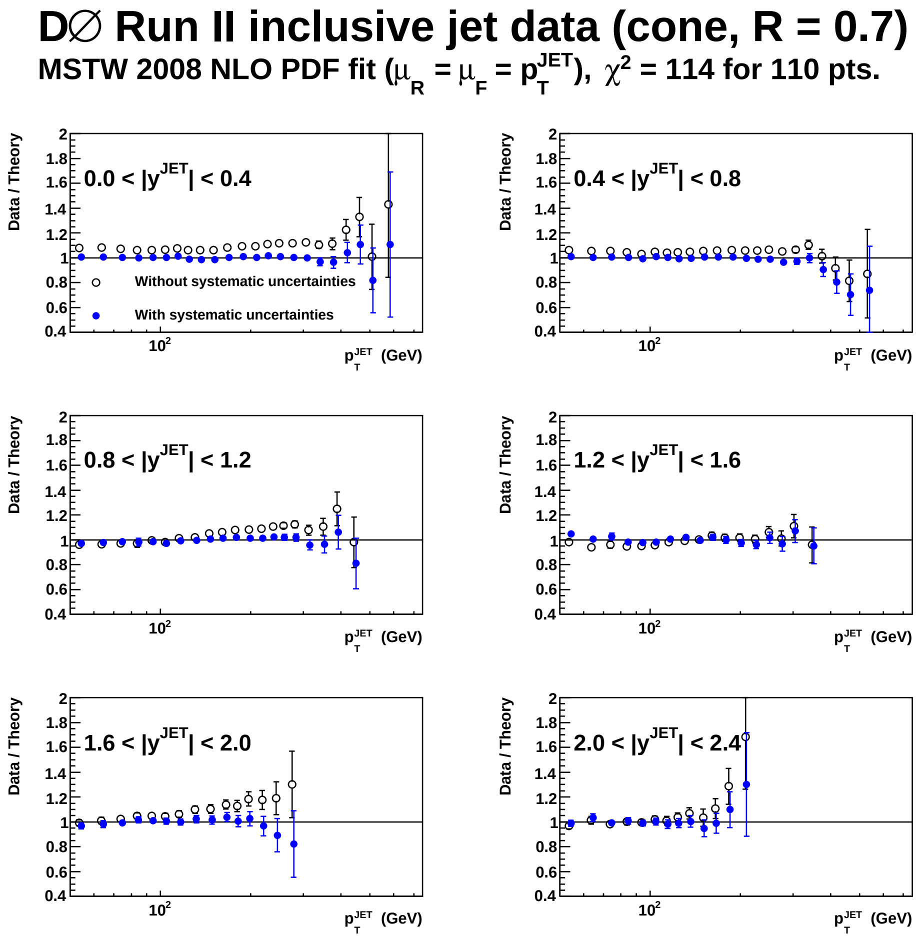
Jet data from Tevatron and LHC

General remarks

- QCD corrections important
  - only NLO known exactly
  - soft logarithms for 1-jet inclusive distributions define $\text{NNLO}_{\text{approx}}$ Kidonakis, Owens '01
  - ongoing effort towards NNLO Gehrmann, Glover, . . . (many others)
- PDF fits with 3-flavors for DIS, 5-flavors for jets (matching from 3 to 5-flavors)
- QCD evolution over large range
- Possible impact of jet definition and algorithm

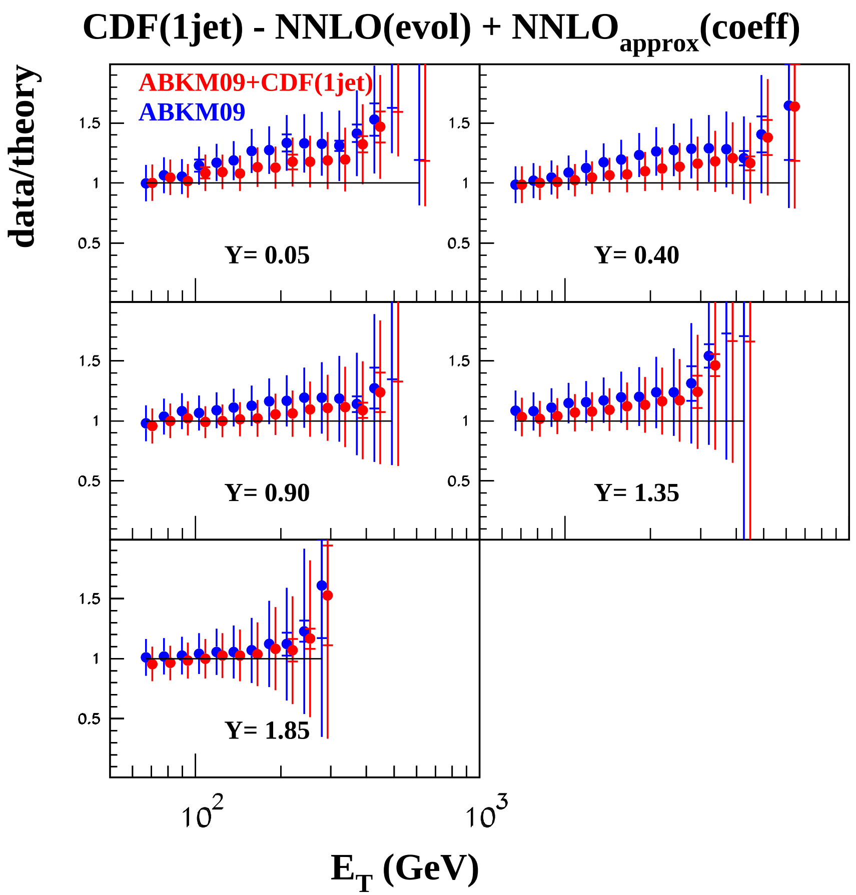
Tevatron jet data (D0) – 1-jet inclusive

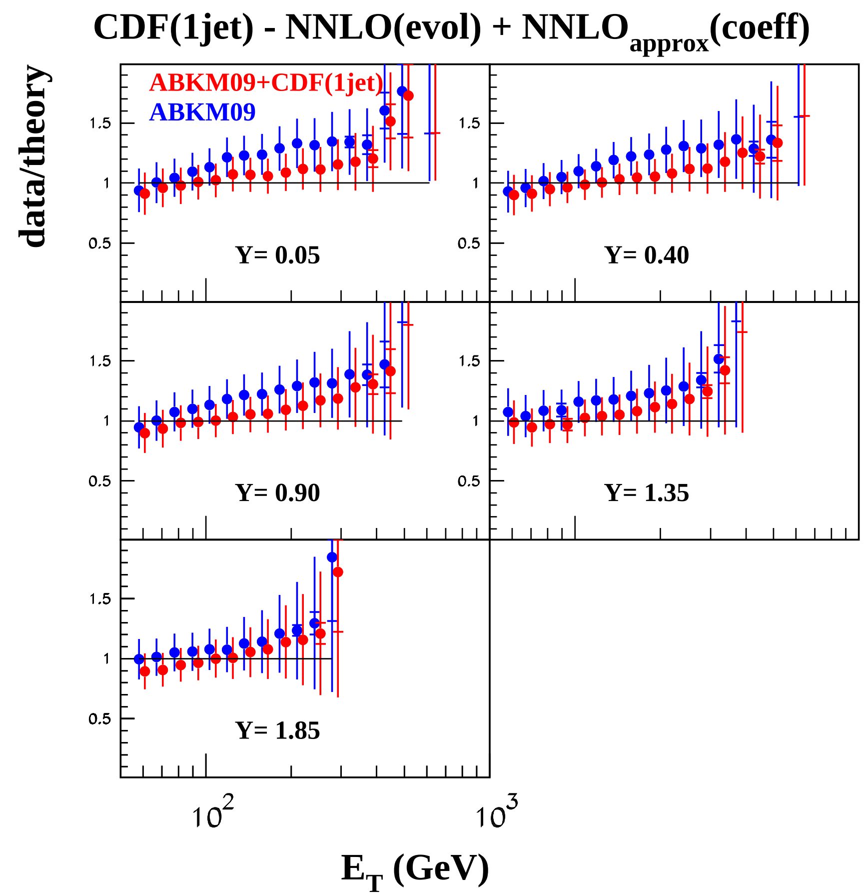




- PDF fits to Tevatron jet data (with NNLO$_{\rm approx}$ corr. Kidonakis, Owens '01) Alekhin, Blümlein, S.M. '11 (left); MSTW `arXiv:0901.0002` (right)
- 3-flavor PDFs for DIS, 5-flavor PDFs for jets, scale $\mu_r = \mu_f = E_T$

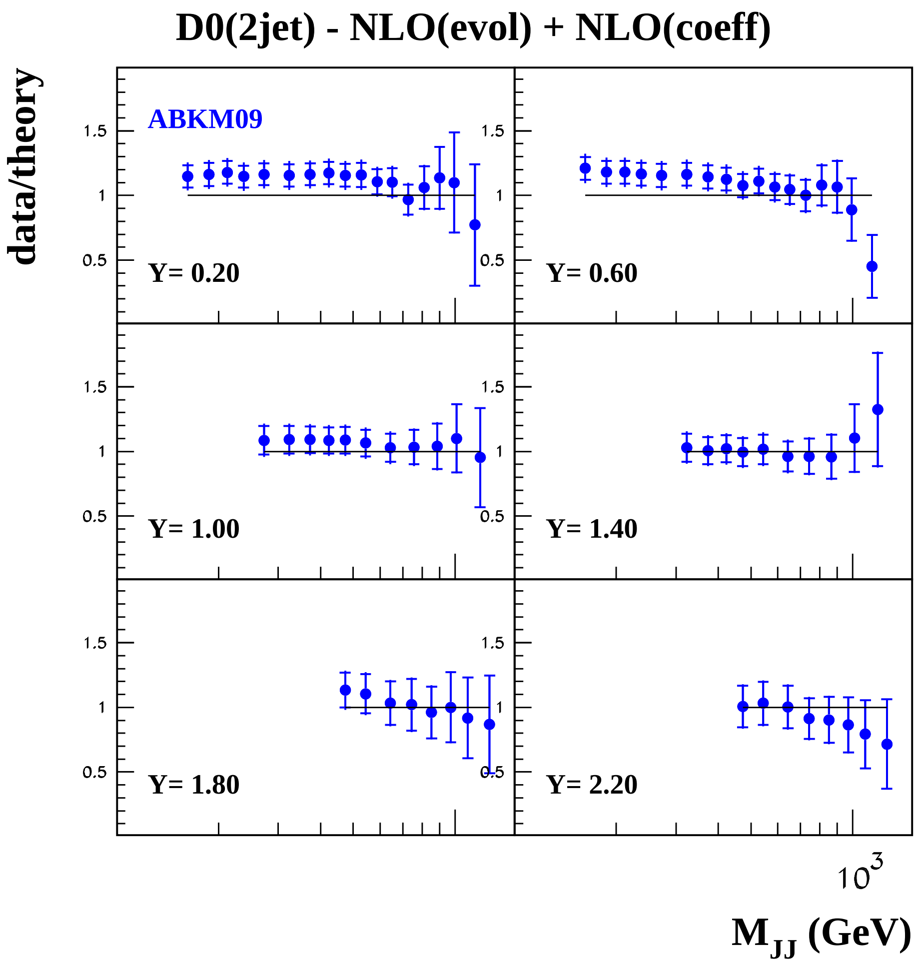
Tevatron jet data (CDF) – 1-jet inclusive

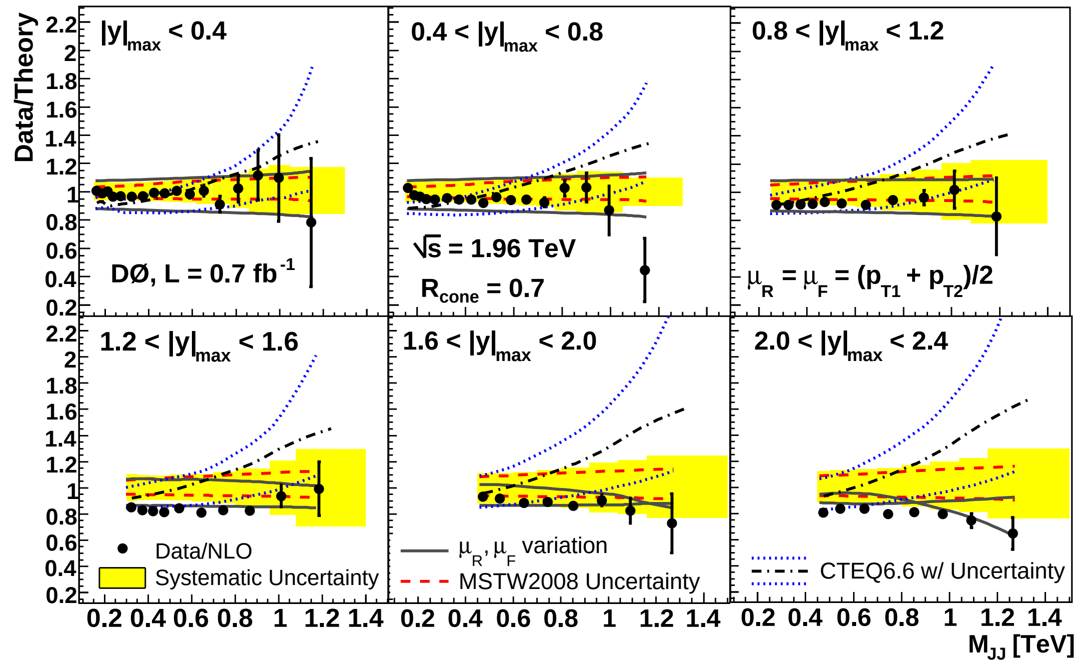




- Cone algorithm (left); k_T algorithm (right); scale $\mu_r = \mu_f = p_T$ s
- Disagreement in slope at large E_T can hardly be improved
  - large E_T is dominated by quark-quark scattering; PDFs well constrained

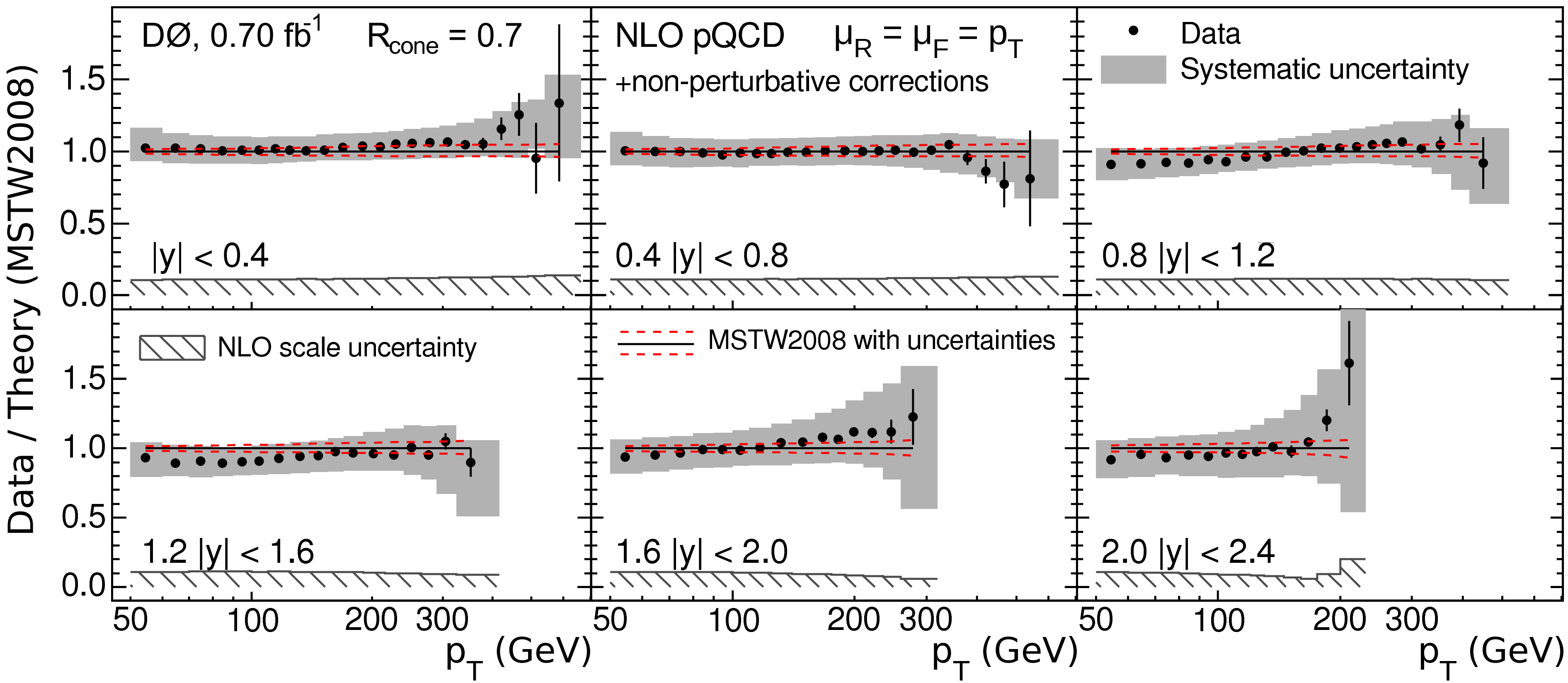
Tevatron jet data (D0) – di-jet invariant mass





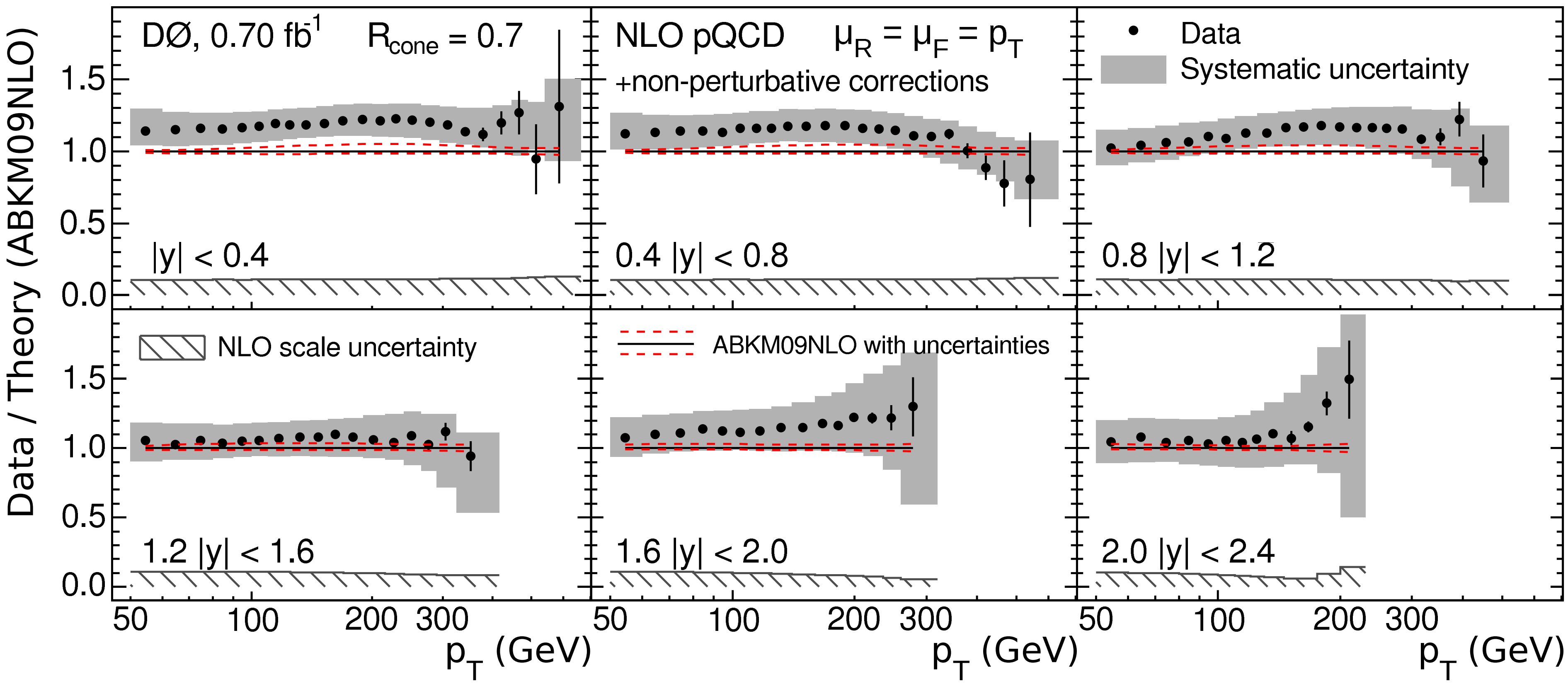
- Predictions for Tevatron di-jet data (no NNLO corrections known)
  Alekhin, Blümlein, S.M. '11 (left); D0 coll. arXiv:1002.4594 (right)
- Uncertainty due to missing NNLO corrections; scale $\mu_r = \mu_f = M_{JJ}$

New analysis (D0) – 1-jet inclusive



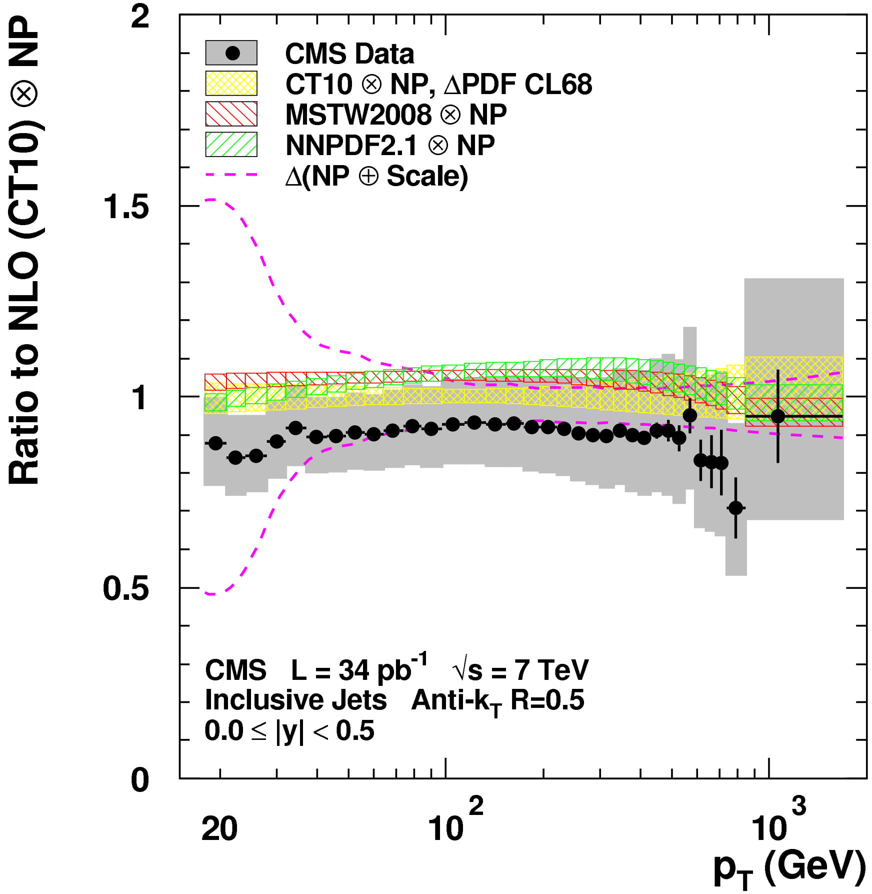
- New analysis of 1-jet inclusive data D0 coll. arXiv:1110.3771
  - MSTW PDF set with PDF (red) and theory (shaded) uncertainty

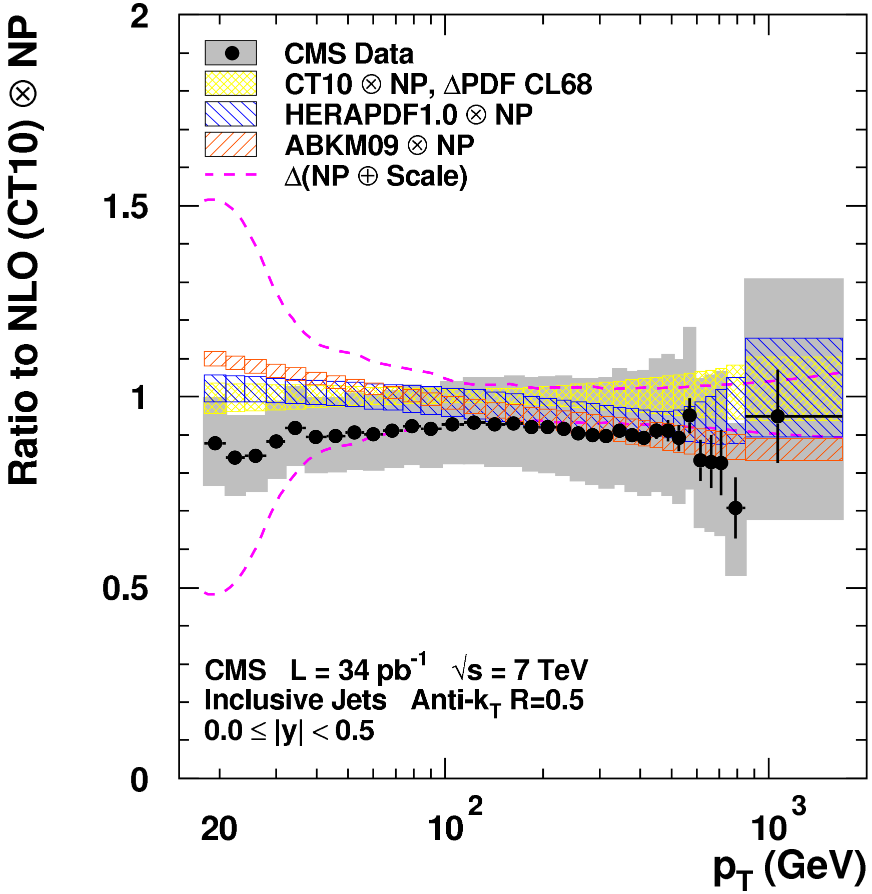
New analysis (D0) – 1-jet inclusive



- New analysis of 1-jet inclusive data D0 coll. arXiv:1110.3771
  - ABKM PDF set with PDF (red) and theory (shaded) uncertainty

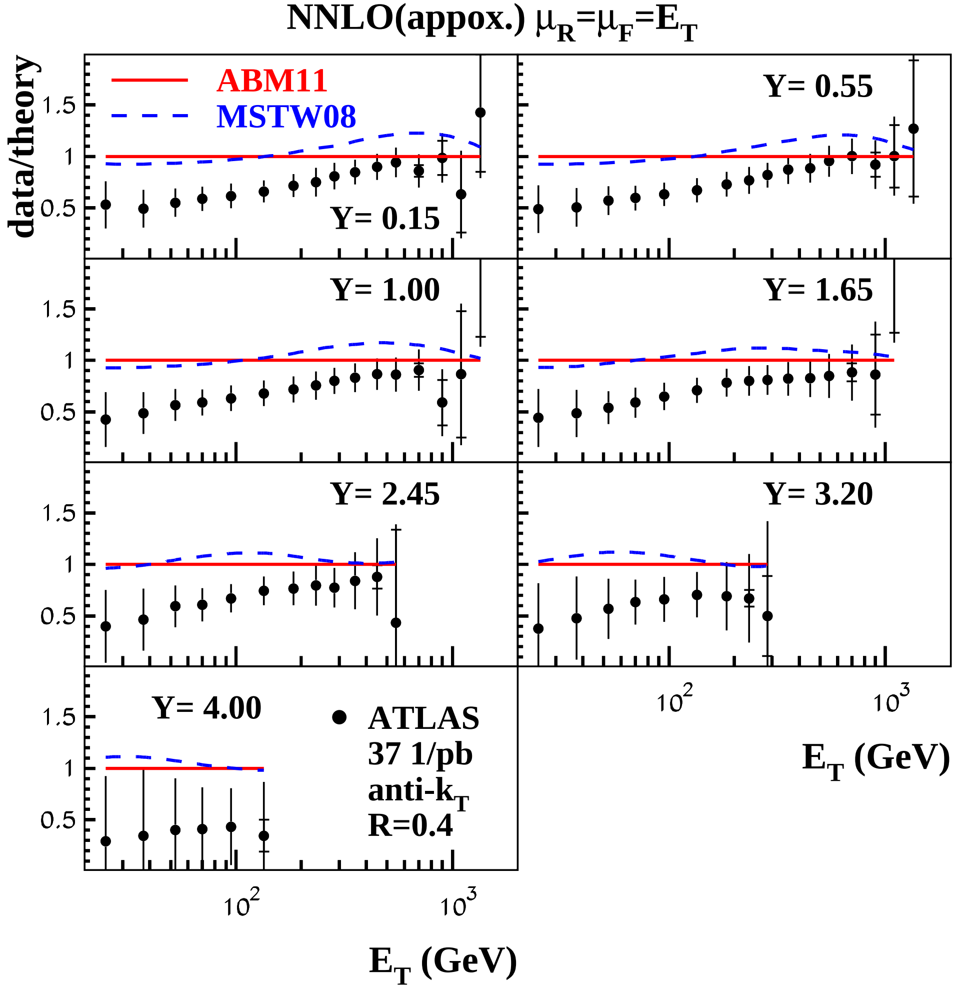
LHC jet data (CMS) – 1-jet inclusive

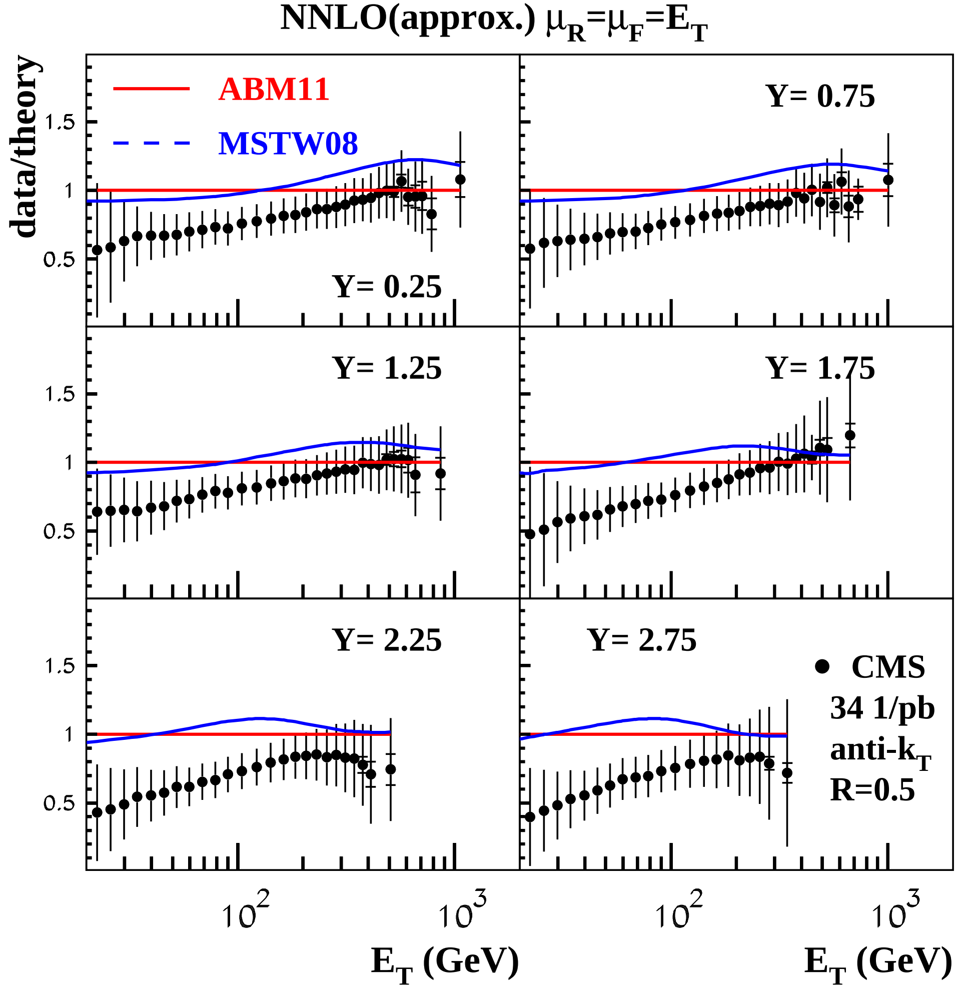




- Analysis of 1-jet inclusive data CMS coll. `CMS NOTE 2011/004`
  - Comparisions of various PDF sets courtesy K. Rabbertz

LHC jet data





- Comparision to LHC data: ATLAS coll. (left) and CMS coll. (right) in good agreement
- LHC jet data prefers small gluon PDF at large x

Strong coupling constant

Essential facts

- $\alpha_s(M_Z)$ from e^+e^- data high
- $\alpha_s(M_Z)$ from DIS data low
- World average 1992
  $\alpha_s(M_Z) = 0.117 \pm 0.004$

Bethke, Catani CERN TH-6484/92

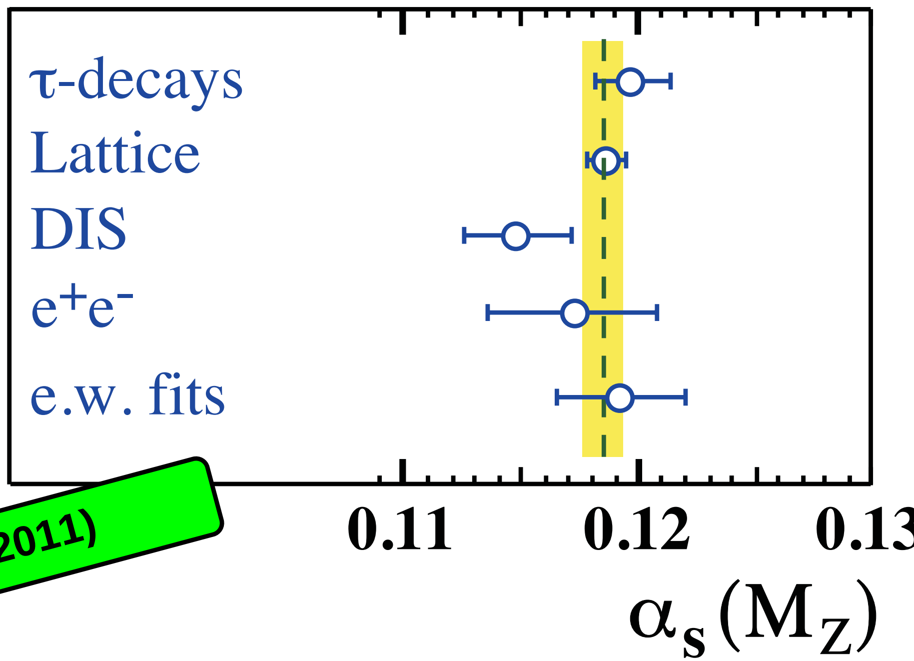
	Process	Ref.	Q [GeV]	$\alpha_s(Q)$	$\alpha_s(M_{Z^0})$	$\Delta\alpha_s(M_{Z^0})$ exp.	$\Delta\alpha_s(M_{Z^0})$ theor.	order of perturb.
1	R_τ [LEP]	[7-10]	1.78	$0.318\,^{+0.048}_{-0.039}$	$0.117\,^{+0.006}_{-0.005}$	$^{+0.003}_{-0.004}$	$^{+0.005}_{-0.004}$	NNLO
2	R_τ [world]	[2]	1.78	0.32 ± 0.04	$0.118\,^{+0.004}_{-0.006}$	–	–	NNLO
3	DIS [ν]	[3]	5.0	$0.193\,^{+0.019}_{-0.018}$	$0.111\,^{+0.006}_{-0.007}$	$^{+0.004}_{-0.006}$	0.004	NLO
4	DIS [μ]	[12]	7.1	0.180 ± 0.014	0.113 ± 0.005	0.003	0.004	NLO
5	$J/\Psi, \Upsilon$ decay	[4]	10.0	$0.167\,^{+0.015}_{-0.011}$	$0.113\,^{+0.007}_{-0.005}$	–	–	NLO
6	e^+e^- [σ_{had}]	[14]	34.0	0.163 ± 0.022	0.135 ± 0.015	–	–	NNLO
7	e^+e^- [shapes]	[15]	35.0	0.14 ± 0.02	0.119 ± 0.014	–	–	NLO
8	$p\bar{p} \to b\bar{b}X$	[11]	20.0	$0.136\,^{+0.025}_{-0.024}$	$0.108\,^{+0.015}_{-0.014}$	0.006	$^{+0.014}_{-0.013}$	NLO
9	$p\bar{p} \to W\ jets$	[13]	80.6	0.123 ± 0.027	0.121 ± 0.026	0.018	0.020	NLO
10	$\Gamma(Z^0 \to \text{had.})$	[5]	91.2	0.133 ± 0.012	0.133 ± 0.012	0.012	$^{+0.003}_{-0.001}$	NNLO
11	Z^0 ev. shapes							
	ALEPH	[7]	91.2	$0.119\,^{+0.008}_{-0.010}$		–	–	NLO
	DELPHI	[8]	91.2	0.113 ± 0.007		0.002	0.007	NLO
	L3	[9]	91.2	0.118 ± 0.010		–	–	NLO
	OPAL	[10]	91.2	$0.122\,^{+0.006}_{-0.005}$		0.001	$^{+0.006}_{-0.005}$	NLO
	SLD	[6]	91.2	$0.120\,^{+0.015}_{-0.013}$		0.009	$^{+0.012}_{-0.009}$	NLO
	Average	[6-10]	91.2		0.119 ± 0.006	0.001	0.006	NLO
12	Z^0 ev. shapes							
	ALEPH	[7]	91.2	0.125 ± 0.005		0.002	0.004	resum.
	DELPHI	[8]	91.2	0.122 ± 0.006		0.002	0.006	resum.
	L3	[9]	91.2	0.126 ± 0.009		0.003	0.008	resum.
	OPAL	[10]	91.2	$0.122\,^{+0.003}_{-0.006}$		0.001	$^{+0.003}_{-0.006}$	resum.
	Average	[7-10]	91.2		0.123 ± 0.005	0.001	0.005	resum.

Table 1: Summary of measurements of α_s. For details see text.

2

α_s 2011

World Summary of α_s 2011:



$\alpha_s(M_Z)$= 0.1197 ± 0.0016

$\alpha_s(M_Z)$= 0.1186 ± 0.0008

$\alpha_s(M_Z)$= 0.1148 ± 0.0024

$\alpha_s(M_Z)$= 0.1172 ± 0.0037

$\alpha_s(M_Z)$= 0.1193 ± 0.0028

S. Bethke (Ringberg 2011)

–> $\alpha_s(M_Z)$= 0.1185 ± 0.0008

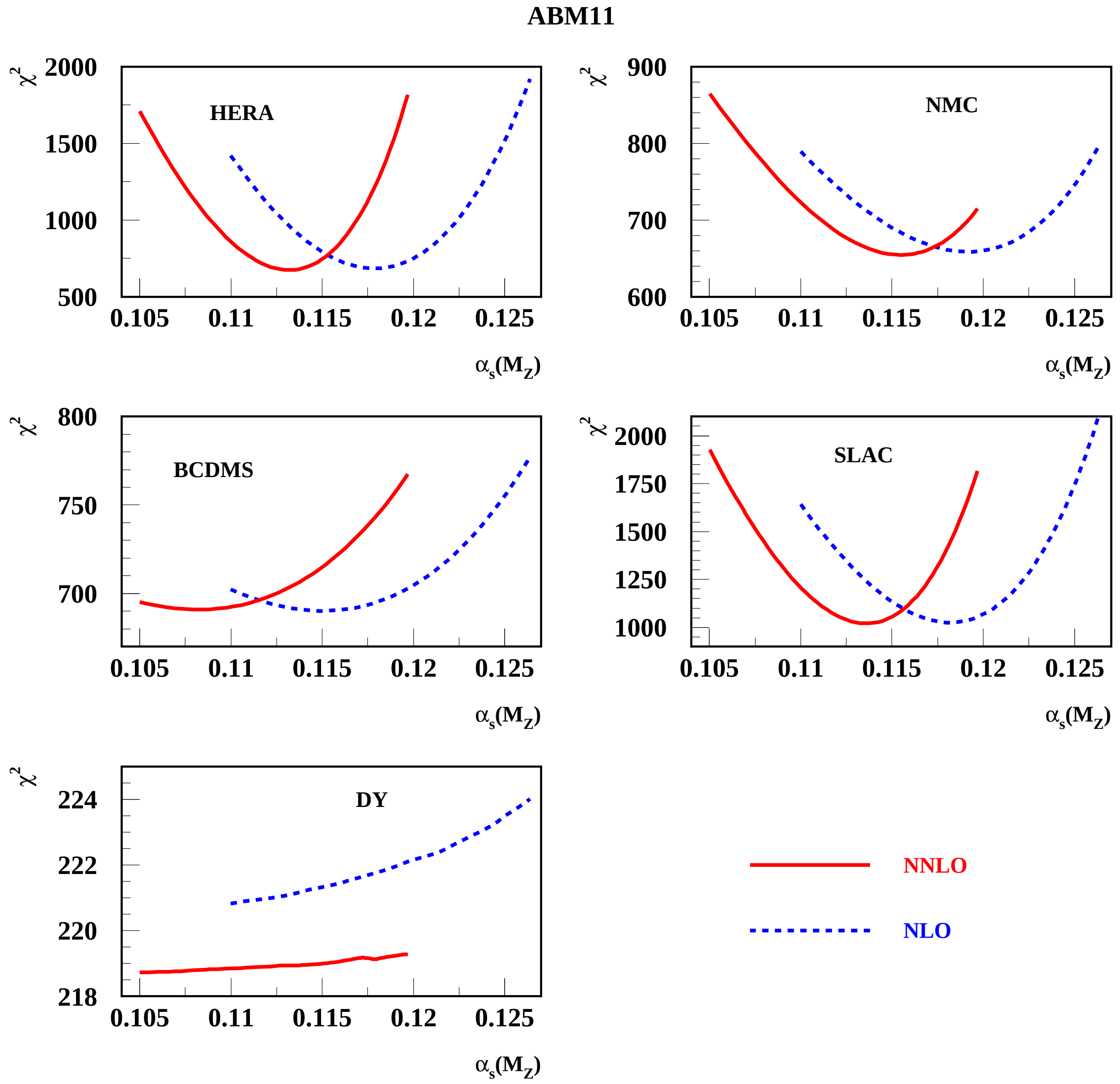
(–> RPP 2012)

$\Lambda_{\overline{MS}}^{(5)}$ = (214 ±10) MeV

$\Lambda_{\overline{MS}}^{(4)}$ = (298 ±12) MeV

S. Bethke: α_s (2011) summary | *Ringberg workshop on HERA physics* | *Sep. 26, 2011* | 10

α_s from DIS and PDFs



- Profile of χ^2 for different data sets

α_s from DIS and PDFs

- Comparison of α_s values
  - effects for differences between ABM and MSTW understood

	α_s at NNLO	target mass corr.	higher twist	error correl.
ABM11	0.1134 ± 0.0011	**yes**	**yes**	**yes**
NNPDF2.1	0.1166 ± 0.0008	**yes**	**no**	**yes**
MSTW	0.1171 ± 0.0014	**no**	**no**	**no**

Theory issues

Benchmark processes

- Theory improvements needed
  - QCD corrections to NNLO
- Deep-inelastic scattering
  - Heavy-quark structure functions for neutral and charged current
  - $ep \to 2 + 1$ jets inclusive production
- Hadron colliders
  - production of $pp \to 1$ jet $+ X$ inclusive, $pp \to 2$ jets, ...
  - $pp \to W/Z + 1$ jet production
  - top-quark production ($t\bar{t}$ and single-t)
  - ...

Summary

Parton distributions, $\alpha_S(M_Z)$ and all that

- Currently source of largest differences for Higgs cross section predictions
- Recent improvements are mainly theory driven
- Continuous benchmarking mandatory

Experimental perspectives

- Need for high precision data ($\mathcal{O}(\mathrm{few}\%)$ uncertainty) for benchmark processes
  - structure functions from HERA (final run II analysis)
  - (differential) $W^{\pm}/Z$ production at LHC
  - jet data from LHC (Tevatron)

Theoretical perspectives

- Need for improved predictions at NNLO QCD for Standard Model processes