Double real radiation antenna functions for heavy quark pair production

Oliver Dekkers

Institut für Theoretische Teilchenphysik und Kosmologie,
RWTH Aachen University

Institut für Theoretische Physik,
ETH Zürich

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Motivation

The exploration of heavy quark production, in particular $t\bar{t}$ and or single top production, is a central issue at high energy colliders

- By studying the top quark in detail, it is hoped to gain inside into the origin of particle masses and the mechanism of electroweak symmetry breaking
- Important background to a number of new physics searches, including (non-SM) Higgs boson(s) and SUSY particles.
- New heavy resoances R may exist that decay into heavy quark pairs $R \to Q\bar{Q}X$, $Q = t, b$. Investigation of the properties of R also requires predictions of distributions.
- ...

$\Rightarrow$ Next-to-next-to-leading order predictions for heavy quark pair production cross sections are desirable.

As a step towards a full NNLO treatment of $t\bar{t}$ production at the LHC, investigate

$$S \to Q\bar{Q} + X \quad \text{at NNLO QCD}$$

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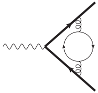
with uncolored initial state S, e.g. $e^+e^- \to \gamma^*, Z \to Q\bar{Q}X$ or $H \to Q\bar{Q}X$.

NNLO ingredients

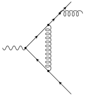
$$d\sigma_{\mathrm{NNLO}} = \int_{d\Phi_2} d\sigma^{VV}_{\mathrm{NNLO}} + \int_{d\Phi_3} d\sigma^{RV}_{\mathrm{NNLO}} + \int_{d\Phi_4} d\sigma^{RR}_{\mathrm{NNLO}}$$

- VV: Two-loop corrections to $S \to Q\bar{Q}$



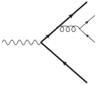
explicit infrared poles from loop integrals

- RV: One-loop correction to $S \to Q\bar{Q}g$



explicit infrared poles from loop integral and
implicit infrared poles due to single unresolved radiation

- RR: Tree-level double real radiation correction: $S \to Q\bar{Q}Q\bar{Q}$, $Q\bar{Q}gg$, and $Q\bar{Q}q\bar{q}$



implicit infrared poles due to double unresolved radiation

Infrared poles only cancel in the sum.

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NNLO infrared subtraction

Generic structure of NNLO cross section with subtraction terms:

$$
\begin{aligned}
d\sigma_{\text{NNLO}} = & \int_{d\Phi_4} \left(d\sigma^{RR}_{\text{NNLO}} - d\sigma^{S}_{\text{NNLO}} \right) \\
& + \int_{d\Phi_3} \left(d\sigma^{VR}_{\text{NNLO}} - d\sigma^{V,S}_{\text{NNLO}} \right) \\
& + \int_{d\Phi_2} d\sigma^{VV}_{\text{NNLO}} + \int_{d\Phi_4} d\sigma^{S}_{\text{NNLO}} + \int_{d\Phi_3} d\sigma^{V,S}_{\text{NNLO}}
\end{aligned}
$$

- $d\sigma^{S}_{\text{NNLO}}$ coincides with $d\sigma^{RR}_{\text{NNLO}}$ in all singular limits.
- $d\sigma^{V,S}_{\text{NNLO}}$ coincides with $d\sigma^{VR}_{\text{NNLO}}$ in all singular limits.

Each line above is finite and free of IR-poles.

Several methods for constructing subtraction terms have been proposed at NNLO.

$\rightarrow$ In this talk: Antenna subtraction

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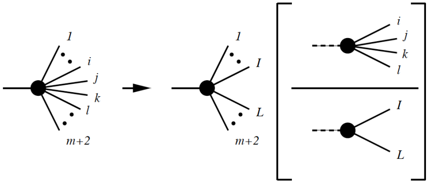
A Short Glance at Antenna Subtraction at NNLO

[Campbell, Cullen, Glover, Kosover, Gehrmann-De Ridder, Gehrmann,...]

The main building blocks for constructing subtraction terms are antenna functions.



All antenna functions are derived from physical color-ordered squared matrix elements, normalized to two-parton matrix elements.

Tree-level four-parton antenna functions for $S \to Q\bar{Q} + X$:

- A_4^0, $\tilde{A}_4^0$ from $\gamma^* \to Q\bar{Q}gg$
- B_4^0 from $\gamma^* \to Q\bar{Q}q\bar{q}$

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Integrated Subtraction Terms

Introduction of subtraction terms must be counterbalanced by adding their integrated form:

- Phase space factorisation

$$d\Phi_{m+2}(p_1,\ldots,p_{m+2};q) = d\Phi_m(p_1,\ldots,\tilde{p}_L,\tilde{p}_K,\ldots,p_{m+2};q)\, d\Phi_{X_{ijkl}}(p_i,p_j,p_k,p_l;\tilde{p}_L+\tilde{p}_K)$$

- Integrated antenna functions

$$\mathcal{X}^0_{ijkl} = \left(8\pi^2\,(4\pi)^{-\varepsilon}\, e^{\varepsilon\gamma_E}\right)^2 \int d\Phi_{X_{ijkl}}\, X^0_{4,ijkl} \;\propto\; \int d\Phi_4 |M^0_{ijkl}|^2 \,.$$

Integration-by-parts reduction: [Chetyrkin, Tkachov '81, Laporta '00]

- Introduce Cut propagators [Cutkosky '60]

$$2\pi i\delta^+\left(p_i^2 - m_i^2\right) = \frac{1}{p_i^2 - m_i^2 + i0} - \frac{1}{p_i^2 - m_i^2 - i0} =: \frac{1}{D_i}, \quad i = 1,\ldots,4$$

with $m_1 = m_2 = m$ and $m_3 = m_4 = 0$.

$$\Rightarrow\; d\Phi_4\,(p_1,\,p_2,\,p_3,\,p_4,\,q) = \frac{\mu^{12-3d}}{i^4(2\pi)^{3d}} \prod_{i=1}^{4} \frac{d^d p_i}{D_i}\, \delta^{(d)}\left(q - \sum_{i=1}^{4} p_i\right)$$

- Carry out IBP reduction as for loop integrals [Anastasiou, Melnikov '02]

$\longrightarrow$ `AIR` [Anastasiou, Lazopoulos '02], `FIRE` [Smirnov '08], `REDUZE` [Studerus, v. Manteuffel '08, '12]

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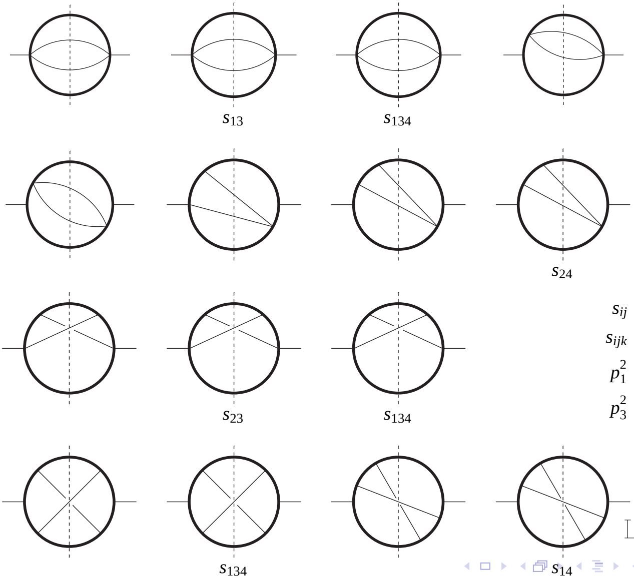
The Master Integrals

$\mathcal{A}^0_{Qgg\bar{Q}}$, $\tilde{\mathcal{A}}^0_{Qgg\bar{Q}}$, $\mathcal{B}^0_{Qq\bar{q}\bar{Q}}$ can be expressed in terms of 15 master integrals:



$$s_{ij} = 2p_ip_j$$
$$s_{ijk} = s_{ij} + s_{ik} + s_{jk}$$
$$p_1^2 = p_2^2 = m^2$$
$$p_3^2 = p_4^2 = 0$$

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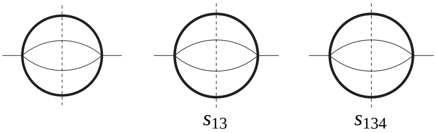
Computation of MIs



Rewrite phase space measure:

$$d\Phi_4(p_1,p_2,p_3,p_4;q) = \frac{1}{2\pi}\int_{4m^2}^{q^2} dM^2 \, d\Phi_2(p_4,k;q)\, d\Phi_3(p_1,p_2,p_3;k), \qquad k^2 = M^2$$

$\rightarrow$ We find compact expressions in terms of hypergeometric functions ${}_3F_2$, e.g.

$$ {}_3F_2\left(-\frac{1}{2}+\epsilon, -2+3\epsilon, -3+4\epsilon; \epsilon, -1+2\epsilon; z\right), \quad z = \frac{4m^2}{q^2}$$

Expansion around $d = 4$:

- algorithms of Moch, Uwer, Weinzierl ‘01, Weinzierl ‘04
- implementations: `NestedSums` Weinzierl ‘02, `XSummer` Moch, Uwer ‘05
- alternative, mixed approach: `HypExp`, `HypExp2` Huber, Maitre ‘05, ‘07

$\rightarrow$ expansion yields harmonic polylogarithms (HPLs) of argument $y = \frac{1-\sqrt{1-z}}{1+\sqrt{1-z}}$

Computation of MIs

Differential equations method [Kotikov '91; Remiddi '97; Gehrmann, Remiddi '00]

- We differentiate the MIs with respect to $y = \frac{1-\sqrt{1-z}}{1+\sqrt{1-z}}$ $(z = \frac{4m^2}{q^2})$ and q^2.
- Reduce resulting expressions to MIs via IBP identities.

$\Rightarrow$ Linear, inhomogeneous, first order differential equations for each MI.

- Solve differential equation order by order in ϵ
  - Exploit properties of HPLs
  - HPL-Package [Maitre '06]
- Fixing constants of integration:
  - phase space vanishes at threshold $y = 1$ $(z = 1)$
  - match to massless limit $y = 0$ $(z = 0)$

    [Gehrmann-De Ridder, Gehrmann, Heinrich '04]

Final Result:

$\rightarrow$ Master integrals are obtained analytically to all relevant orders in terms of HPLs.

Application and Check of $\mathcal{B}_4^0$: The $\alpha_s^2 e_Q^2 N_f$ Correction to the Ratio R

$$R = \frac{\sigma(e^+(k_1)\, e^-(k_2) \;\to \gamma^* \;\to\; Q\bar{Q} + X)}{\sigma_{\text{pt}}}\,, \qquad \sigma_{\text{pt}} = \frac{e_Q^4}{12\pi q^2}$$

- Consider one heavy quark (charge e_Q, mass m) and N_f massless quark flavors
- $R_{\alpha_s^2 e_Q^2 N_f}$ gauge-invariant and IR finite.

$$R_{\alpha_s^2 e_Q^2 N_f} = 6\pi e_Q^2\,(1-\epsilon)\left(\Pi^{Q\bar{Q}q\bar{q}}_{\alpha_s^2 N_f} + \Pi^{Q\bar{Q}}_{\alpha_s^2 N_f} + \Pi^{Q\bar{Q}g}_{\alpha_s^2 N_f}\right).$$

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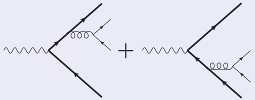
$Q\bar{Q}q\bar{q}$ final state

$$\Pi^{Q\bar{Q}q\bar{q}}_{\alpha_s^2 N_f} \leftrightarrow \int d\Phi_4 \left| \quad \right|^2$$



→ Directly related to the integrated antenna function:

$$\Pi^{Q\bar{Q}q\bar{q}}_{\alpha_s^2 N_f} = \frac{(4\pi\alpha_s)^2 4 C_F N_c T_R N_f}{q^2(3-2\epsilon)}\,\frac{P_2(q^2,m)\,|\mathcal{M}_{Q\bar{Q}}|^2}{(8\pi^2(4\pi)^{-\epsilon}e^{\gamma_E\epsilon})^2}\,\mathcal{B}^0_{Qq\bar{q}\bar{Q}}\,,$$

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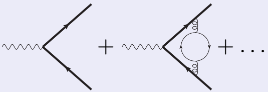
$Q\bar{Q}$ final state

$$\Pi^{Q\bar{Q}}_{\alpha_s^2 N_f} \propto \int d\Phi_2 \left| \cdots + \cdots + \ldots \right|^2$$



$\to$ Heavy quark from factors can be obtained within dim-reg from

[Bernreuther, Bonciani, Gehrmann, Heinesch, Leineweber, Mastrolia, Remiddi, ‘05]
[Gluza, Mitov, Moch, Riemann ‘09]

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Application and Check of $\mathcal{B}_4^0$: The $\alpha_s^2 e_Q^2 N_f$ Correction to the Ratio R

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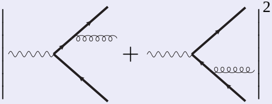
Counterterm contribution from $Q\bar{Q}g$ final state

Apply on-shell renormalization for external states $\Rightarrow$ $\delta Z_{Q\bar{Q}g,\alpha_s N_f} = \frac{\alpha_s N_f T_R (4\pi)^\epsilon}{6\pi\epsilon}\Gamma(1+\epsilon)$

$$\Pi^{Q\bar{Q}g}_{\alpha_s^2 N_f} \propto \int d\Phi_3\, \delta Z_{Q\bar{Q}g,\alpha_s N_f} \left| \quad + \quad \right|^2$$



[Gehrmann-de Ridder, Ritzmann '09]

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$\to$ exact cancellation of IR poles provides a strong check for the IR poles structure of $\mathcal{B}_4^0$.

- For $d = 4$ the coefficient $R_{\alpha_s^2 e_Q^2 N_f}$ can be obtained from [Hoang, Kühn, Teubner '95]

  Technical Remark: Their result is expressed in terms of the integrals

$$T_2(\eta,\,\xi) = \int_0^1 dx\,\frac{\arctan(\xi\,x)}{x^2+\eta^2} \qquad T_2^\star(\eta,\,\xi) = \int_0^1 dx\,\frac{\ln(x^2+\xi^2)}{x^2+\eta^2} \qquad T_3(\eta,\,\xi,\,\chi) = \int_0^1 dx\,\frac{\ln(x^2+\xi^2)\arctan(\chi x)}{x^2+\eta^2}$$

  We had to solve these expression in terms of polylogarithms for our comparison.

$\to$ analytical agreement with our result

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Summary and Outlook

A step towards antenna subtraction at NNLO QCD with massive quarks:

- Calculation of the integrated double-real antenna functions for $S \to Q\bar{Q}q\bar{q}, Q\bar{Q}gg$
  - $\rightarrow$ analytic results in terms of HPLs
  - $\rightarrow$ analytic integration of double-real antenna subtraction terms is feasible!
- Comparison of $\mathcal{B}_4^0$ with results in the literature

Antenna functions are universal

$\rightarrow$ will enter the computation of $t\bar{t}$ production at hadron colliders at NNLO

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Antenna functions are universal

$\to$ will enter the computation of $t\bar{t}$ production at hadron colliders at NNLO

Next steps:

- (Integrated) real-virtual antenna functions for $S \to Q\bar{Q}g$ (1-loop)
  - $\to$ work in progress with W. Bernreuther and C. Bogner
  - $\to$ apply method to $H \to b\bar{b}$, $e^+e^- \to Q\bar{Q}$ at NNLO
- Missing building blocks for $pp(p\bar{p}) \to t\bar{t}$ at NNLO
  - $\to$ work in progress with A. Gehrmann-de Ridder, G. Abelof: Integration of initial-final double-real antenna functions.

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Thanks!