Two-loop QCD corrections to to the helicity amplitudes for H $\longrightarrow$ 3 partons

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Talk based on the work with T. Gehrmann, E.W.N.Glover and A. Koukoutsakis: JHEP **02** (2012) 056 [arXiv:1112.3554]

Motivation

Why study $h \to 3p$ ($h \to ggg$, $h \to q\bar{q}g$)?

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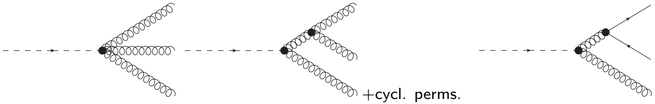

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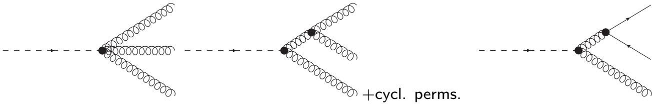

Tree diagrams for $h \to ggg$ and $h \to q\bar{q}g$

Crossed processes

$$gg \to hg$$
$$q\bar{q} \to hg$$
$$qg \to hq$$

provide valuable information about the $\mathrm{p_T}$ of the higgs and hence of the recoiling QCD radiation.

Discovery channels

For higgs masses below $2\mathrm{m_Z}$ a very important discovery channel at LHC is

$$h \rightarrow WW^{\star} \rightarrow l\bar{\nu}\bar{l}\nu$$

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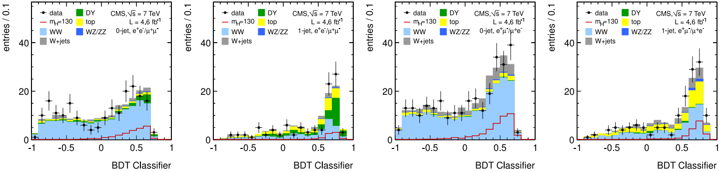
To enhance the signal, classify events according to their jet multiplicities. The different bins are sensitive to different higgs production processes and different backgrounds [Atlas collaboration 2011, CMS collaboration 2012]

Jet binning and jet veto

Differentiating between same- and opposite flavour leptons for 0- and 1-jet bins yield five categories.



[arXiv:1202.1489]

$h + 0j$ sensitive to LO $gg \to h$, bkgd from $pp \to WW$

$h + 1j$ additionaly sensitive to VH

$h + 2j$ sensitive to VBF, bkgd from top decays

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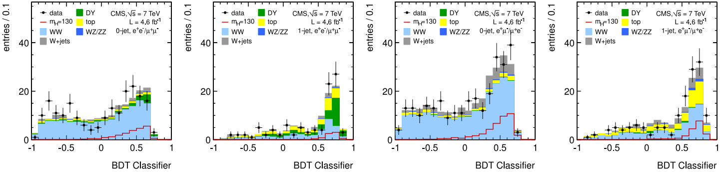

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\end{array}
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To get rid of $t\bar{t}$ background, apply cut on hard central jets [Dittmar, Dreiner 1997, S. Asai *et al* 2004].

Boosted higgs

The only way to see $h \longrightarrow b\bar{b}$ at the LHC is to use the substructure of the fat jet it yields to reconstruct the event [Butterworth, Davison, Rubin, Salam 2008]:

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Requires the higgs to be boosted against a V or a QCD jet.

Concepts of the calculation

- Higgs-gluon-interaction via top quark loop - integrating it out yields an effective field theory with a $H\mathcal{F}^{a}_{\mu\nu}\mathcal{F}^{\mu\nu a}$ interaction, resulting in Hgg, Hggg and Hgggg vertices. [Ellis, Gaillard, Nanopoulos, Sachrajda, 1979]

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- There are 328 $H \to q\bar{q}g$ and 1306 $H \to ggg$ diagrams at NNLO!
- Heavy use of computer programs (QGRAF [Nogueira 1993], FORM [Vermaseren 2008], REDUZE [Studerus 2010]).
- Use the spinor helicity formalism [Kleiss, Stirling 1985].

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The helicity tensors

For $H \longrightarrow ggg$:

$$
\begin{aligned}
T^{\mu\nu\rho} =& A_1 \left[\left(p_1^\rho - \frac{p_1 p_3}{p_2 p_3} p_2^\rho \right) \left(p_2^\mu p_{1\nu} - (p_1 p_2) g^{\mu\nu} \right) \right] \\
&+ A_2 \left[\left(p_1^\nu - \frac{p_1 p_2}{p_2 p_3} p_3^\nu \right) \left(p_3^\mu p_{1\rho} - (p_1 p_3) g^{\mu\rho} \right) \right] \\
&+ A_3 \left[\left(p_2^\mu - \frac{p_1 p_2}{p_1 p_3} p_3^\mu \right) \left(p_3^\nu p_{2\rho} - (p_2 p_3) g^{\nu\rho} \right) \right] \\
&+ A_4 \Big[p_3^\mu p_1^\nu p_2^\rho - p_2^\mu p_3^\nu p_1^\rho + g^{\nu\rho} \left((p_1 p_3) p_2^\mu - (p_1 p_2) p_3^\mu \right) \\
&\quad + g^{\mu\rho} \left((p_1 p_2) p_3^\nu - (p_2 p_3) p_1^\nu \right) + g^{\mu\nu} \left((p_1 p_3) p_2^\rho - (p_1 p_3) p_2^\rho \right) \Big] \\
=& A_1 T_1^{\mu\nu\rho} + A_2 T_2^{\mu\nu\rho} + A_3 T_3^{\mu\nu\rho} + A_4 T_4^{\mu\nu\rho}.
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=& A_1 T_1^{\mu\nu\rho} + A_2 T_2^{\mu\nu\rho} + A_3 T_3^{\mu\nu\rho} + A_4 T_4^{\mu\nu\rho} .
\end{aligned}
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For $H \longrightarrow q\bar{q}g$:

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\begin{aligned}
T_{i\bar{i}}^\mu =& A_1 \big(\overline{u(p_1)} \not{p}_3 v(p_2) p_2^\mu - (p_2 p_3) \overline{u(p_1)} \gamma^\mu v(p_2) \big) \\
&+ A_2 \big(\overline{u(p_1)} \not{p}_3 v(p_2) p_1^\mu - (p_1 p_3) \overline{u(p_1)} \gamma^\mu v(p_2) \big) \\
=& A_1 T_1^\mu + A_2 T_2^\mu .
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Tensorial part of the helicity amplitudes is the same at all orders.
Find their coefficients by means of projectors:

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  Yields 4 tensor components for $h \to ggg$ and 2 for $h \to q\bar{q}g$.
- Work out projectors on these tensor components.
- Construct out of them two independent $h \to ggg$ hel. amps. $|\mathcal{M}_{ggg}^{+++}\rangle$, $|\mathcal{M}_{ggg}^{-++}\rangle$ and one $h \to q\bar{q}g$ amp. $|\mathcal{M}_{q\bar{q}g}^{-++}\rangle$.

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- Use REDUZE to express the loop integrals by means of master integrals [Gehrmann, Remiddi 2001].
- Evaluate the master integrals in terms of 2-dimensional harmonic polylogarithms [Remiddi, Vermaseren 2000].

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Further steps:

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- Extract the IR-finite terms using Catani's dipole subtraction scheme [Catani 1998].
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The resulting expressions for the coefficients of the helicity amplitudes are still very long ($\mathcal{O}$(50kb)), but simplifications for $h \to ggg$ using symbols already exist [Duhr 2012].

Outlook

Build the amplitudes together with NLO $h \longrightarrow 4p$ and LO $h \longrightarrow 5p$ into a Monte Carlo program to determine the higgs plus one jet production cross section at NNLO.

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Thanks!

nothing

The effective couplings

$$\begin{aligned}
Hg^a(k_1,\epsilon_1^\mu)g^b(k_2,\epsilon_2^\nu) =& i\lambda\delta^{ab}(g^{\mu\nu}k_1k_2 - k_1^\nu k_2^\mu),\\
Hg^a(k_1,\epsilon_1^\mu)g^b(k_2,\epsilon_2^\nu)g^c(k_3,\epsilon_3^\rho) =& -g_s\lambda f^{abc}\Big[g^{\mu\nu}(k_1^\rho - k_2^\rho)\\
&+g^{\nu\rho}(k_1^\mu - k_2^\mu)\\
&+g^{\mu\rho}(k_3^\nu - k_1^\nu)\Big],\\
Hg^a(k_1,\epsilon_1^\mu)g^b(k_2,\epsilon_2^\nu)g^c(k_3,\epsilon_3^\rho)g^d(k_4,\epsilon_4^\sigma)&\\
=& -ig_s^2\lambda\Big[f^{abc}f^{cde}(g^{\mu\rho}g^{\nu\sigma} - g^{\mu\sigma}g^{\nu\rho})\\
&+f^{ade}f^{bce}(g^{\mu\nu}g^{\rho\sigma} - g^{\mu\rho}g^{\nu\sigma})\\
&+f^{ace}f^{dbe}(g^{\mu\sigma}g^{\nu\rho} - g^{\mu\nu}g^{\rho\sigma})\Big]\\
\lambda =& 2^{\frac{1}{4}}\sqrt{G_F}\Big(-\frac{4}{12}\frac{\alpha_s^{(6)}(\mu)}{\pi}\Big).
\end{aligned}$$

The LO helicity amplitudes

For $h \to ggg$:

$$\mathcal{M} = \frac{g_s}{2} f^{abc} m_0(p_1, \epsilon_1^{\mu}, p_2, \epsilon_2^{\nu}, p_3, \epsilon_3^{\rho}),$$

$$m_0(p_1^+, p_2^+, p_3^+) = \frac{-M_H^4}{\langle 12 \rangle \langle 23 \rangle \langle 31 \rangle},$$

$$m_0(p_1^-, p_2^+, p_3^+) = \frac{[23]^4}{\langle 12 \rangle \langle 23 \rangle \langle 31 \rangle}.$$

For $h \to q\bar{q}g$:

$$\mathcal{M} = \frac{g_s}{2} T^a_{i\bar{i}} m_0(p_1, \epsilon_1^{\mu}, p_2, h, p_3, \bar{h}),$$

$$m_0(p_1^+, p_2^-, p_3^+) = \frac{[13]^2}{[23]}.$$

The projectors (explicit)

Find projectors P such that

$$A_i = \sum_{\mu,\mu'} \sum_{\nu,\nu'} \sum_{\rho,\rho'} P_i^{\mu'\nu'\rho'} \epsilon_{1,\mu'} \epsilon_{2,\nu'} \epsilon_{3,\rho'} T^{\mu\nu\rho} \epsilon_{1,\mu} \epsilon_{2,\nu} \epsilon_{3,\rho},$$

$$P_i^{\mu'\nu'\rho'} = \alpha_1 T_1^{\dagger,\mu'\nu'\rho'} + \alpha_2 T_2^{\dagger,\mu'\nu'\rho'} + \alpha_3 T_3^{\dagger,\mu'\nu'\rho'} + \alpha_4 T_4^{\dagger,\mu'\nu'\rho'}.$$

These equations can be solved for the α_i in d dimensions.

The projectors (explicit)

This gives (for example for gluonic A_1:)

$$\alpha_1 = \frac{d p_2 p_3}{8(d-3)(p_1 p_2)^3 (p_1 p_3)},$$
$$\alpha_2 = \frac{(4-d) p_2 p_3}{8(d-3)(p_1 p_2)^2 (p_1 p_3)^2},$$
$$\alpha_3 = \frac{d-4}{8(d-3)(p_1 p_2)^3 (p_2 p_3)},$$
$$\alpha_4 = \frac{d-2}{8(d-3)(p_1 p_2)^2 (p_1 p_3)}.$$

The auxillary topologies

Planar topology

$$
\begin{aligned}
prp_1 &= k_1 \\
prp_2 &= k_2 \\
prp_3 &= k_1 - k_2 \\
prp_4 &= k_1 - p_1 \\
prp_5 &= k_2 - p_1 \\
prp_6 &= k_1 - p_1 - p_2 \\
prp_7 &= k_2 - p_1 - p_2 \\
prp_8 &= k_1 - p_1 - p_2 - p_3 \\
prp_9 &= k_2 - p_1 - p_2 - p_3
\end{aligned}
$$

Non-planar topology

$$
\begin{aligned}
prp_1 &= k_1 \\
prp_2 &= k_2 \\
prp_3 &= k_1 - k_2 \\
prp_4 &= k_1 - k_2 - p_3 \\
prp_5 &= k_2 - p_1 \\
prp_6 &= k_1 - p_1 - p_3 \\
prp_7 &= k_2 - p_1 - p_2 \\
prp_8 &= k_1 - p_1 - p_2 - p_3 \\
prp_9 &= k_2 - p_1 - p_2 - p_3
\end{aligned}
$$

Concepts of the shift finder

Only linear transformation in the loop momenta:

$$\begin{aligned} k_i &\rightarrow k_i \pm p_i, \\ k_i &\rightarrow k_i \pm k_{j\neq i}, \\ k_i &\rightarrow -k_i. \end{aligned}$$

Try only to subtract momenta from the loop momenta:

$$\begin{matrix} k_1 - k_2 - p_1 \\ k_1 \\ k_2 \end{matrix} \quad \overset{k_1 \rightarrow k_1 + p_1}{\not\rightarrow} \quad \begin{matrix} k_1 - k_2 \\ k_1 + p_1 \\ k_2 \end{matrix} \quad ,$$

$$\begin{matrix} k_1 - k_2 - p_1 \\ k_1 \\ k_2 \end{matrix} \quad \overset{k_2 \rightarrow k_2 - p_1}{\longrightarrow} \quad \begin{matrix} k_1 - k_2 \\ k_1 \\ k_2 - p_1 \end{matrix} \quad .$$

There are many more exceptional shifts to correct special situations. These operations are done inside a do-loop which terminates only if all momenta fit into the auxillary topology.