

# Broken Baby Skyrmons

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# Outline

- Solitons
- Skyrme Model
- Baby Skyrme Model
- Breaking the symmetry
- Three-colour systems
- Four-colour systems

# Solitons

Solitons are particle-like stable solutions to field theories which are homotopically distinct from the vacuum solution.

## Example: $\phi^4$ Kinks

- Consider a theory described by

$$\mathcal{L} = \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - \lambda (m^2 - \phi^2)^2$$

- Vacua of the system where  $\phi = \pm m$ , required at spatial infinity for finite energy. The choice of vacua at spatial infinities results in the possibility of solitons.
- Topological charge:

$$N = \frac{\phi_+ - \phi_-}{2m} = \frac{1}{2m} \int_{-\infty}^{\infty} \phi' dx$$

$N=0$  topologically vacua,  $N=1$  kink,  $N=-1$  anti-kink.

# Skyrme Model

- (3+1)-dimensional model of pions. The solitons of the theory model baryons.
- Model obtained from QCD as low energy effective field theory in the large colour limit (Witten 1983). Similarly obtained from holographic QCD (Saki, Sugimoto,2005).
- Lower dimensional analogue, baby Skyrme model, (2+1)-dimensional model. Itself is manifest in condensed matter physics (Rössler *et. al* 2006).

## Baby Skyrme Model

- The model is described by the Lagrangian

$$\mathcal{L} = \frac{1}{2} \partial_\mu \phi \cdot \partial^\mu \phi - \frac{\kappa^2}{4} (\partial_\mu \phi \times \partial_\nu \phi) \cdot (\partial^\mu \phi \times \partial^\nu \phi) - V(\phi),$$

where  $\phi$  is a three-component unit vector field.

- Associated topological charge

$$B = -\frac{1}{4\pi} \int \phi \cdot (\partial_1 \phi \times \partial_2 \phi) dx_1 dx_2,$$

- The model has an  $O(3)$  symmetry, but this can be broken by the choice of potential

## Breaking the baby Skyrme model

- We can break to the discrete symmetry group  $D_N$  by choice of potential

$$V(\phi) = m^2 \left| 1 - (\phi_1 + i\phi_2)^N \right|^2 (1 - \phi_3)$$

- The choice of  $N$  results in the energy density of a unit charge soliton having  $N$  partons, and is referred to as the  $N$  colour broken baby Skyrme model.
- This potential was introduced by Jäykkä, Speight and Sutcliffe (2011) where the three colour version was studied.

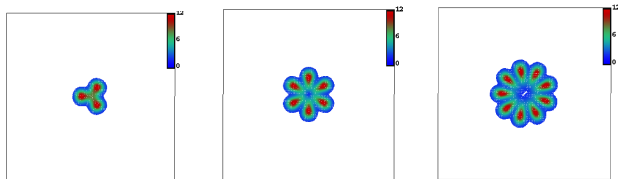
## Three colour system

- Starting with a field with topological charge  $B$  given by

$$\phi = (\sin(f) \cos(B\theta), \sin(f) \sin(B\theta), \cos(f)),$$

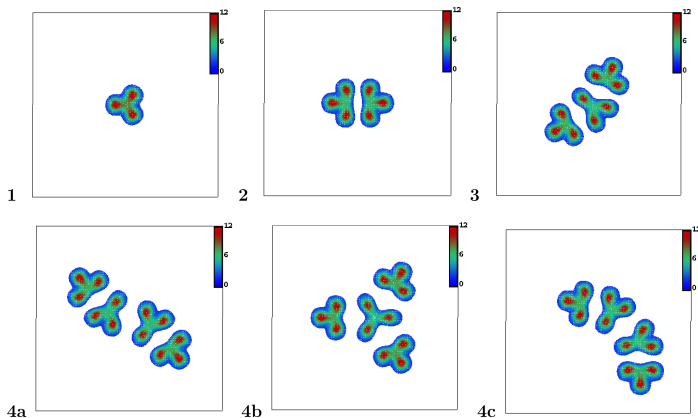
for  $f$  a monotonic radial profile function fulfilling specific boundary conditions.

- We then use an energy minimisation algorithm to find the static solitons which result in static ring solitons.



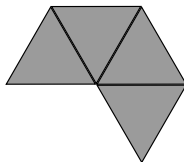
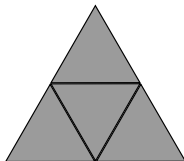
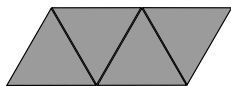
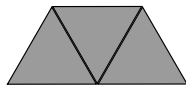
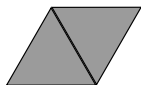


- Can perturb these ring solutions to get lower energy solutions



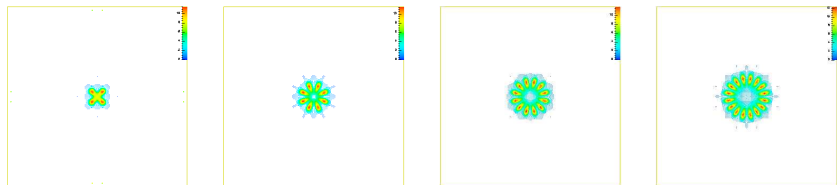
# Polyiamonds

- We can see that the structure of these form polyiamonds.



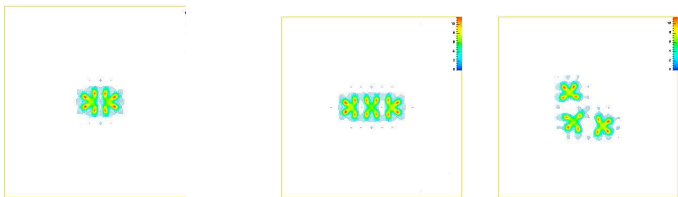
# Four colour system

- We now consider systems with  $N = 4$ .
- As before we begin with the initial field and relax the system to get static ring solutions

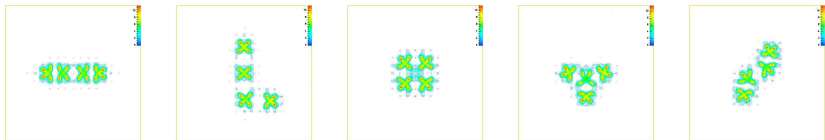


Higher charge ring solitons can be perturbed to get other static solitons

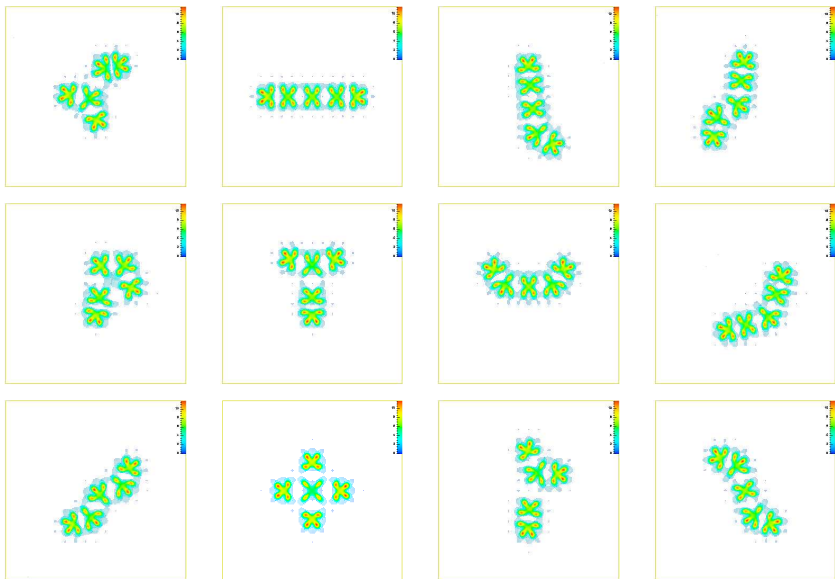
- $B = 2$  and  $B = 3$



- $B = 4$

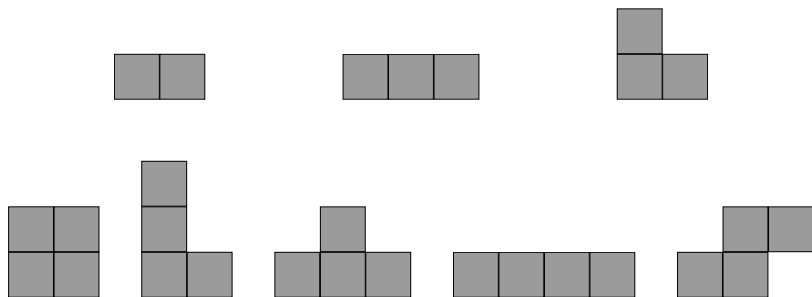


•  $B = 5$



# Polyominoes

Comparing with the above we see that these have the form of polyominoes, which can be seen below:



# Summary

- Solitons
- Introduced the broken baby Skyrme model
- Compared the structure of the energy density with polyforms.

Thank you!