Quantum Gravity on the Lattice

Ben Galloway

in collaboration with J. Laiho & D. Coumbe

School of Physics and Astronomy
University of Glasgow

BUSSTEPP 2012, Durham

Background

- Quantum gravity is perhaps *the* biggest unsolved problem in theoretical physics to date.
- SM is a low-energy effective theory valid to a scale $\Lambda << m_{Planck}$. Gravity is not included!
- At high energies $\Lambda \approx m_{Planck}$ — e.g. black holes, beginning of the Universe — we need a quantum field theory of gravity.

Renormalization

- G_N has negative mass dimension

$\rightarrow$ infinite number of counterterms required at higher and higher orders in a perturbative expansion (confirmed by explicit calculation at two loops [Goroff & Sagnotti, 1986])

$\rightarrow$ a straightforward quantization of GR is perturbatively non-renormalizable.

Renormalization

- G_N has negative mass dimension

$\rightarrow$ infinite number of counterterms required at higher and higher orders in a perturbative expansion (confirmed by explicit calculation at two loops [Goroff & Sagnotti, 1986])

$\rightarrow$ a straightforward quantization of GR is perturbatively non-renormalizable.

- Perhaps a non-perturbative formulation is required.

Asymptotic Safety

Two classes of Wilsonian-renormalizable QFTs:

[F. Saueressig, International Europhysics Conference for High Energy Physics, 2011]

Asymptotic Safety

Two classes of Wilsonian-renormalizable QFTs:

[F. Saueressig, International Europhysics Conference for High Energy Physics, 2011]

- Gaussian Fixed Point
  - perturbatively renormalizable field theories
  - UV-limit: free theory
  - asymptotic freedom (e.g. QCD)

Asymptotic Safety

Two classes of Wilsonian-renormalizable QFTs:

[F. Saueressig, International Europhysics Conference for High Energy Physics, 2011]

- Gaussian Fixed Point
  - perturbatively renormalizable field theories
  - UV-limit: free theory
  - asymptotic freedom (e.g. QCD)
- Non-Gaussian Fixed Point
  - non-perturbatively renormalizable field theories
  - UV-limit: interacting theory
  - asymptotic safety (e.g. gravity in $2 + \epsilon$ dimensions)

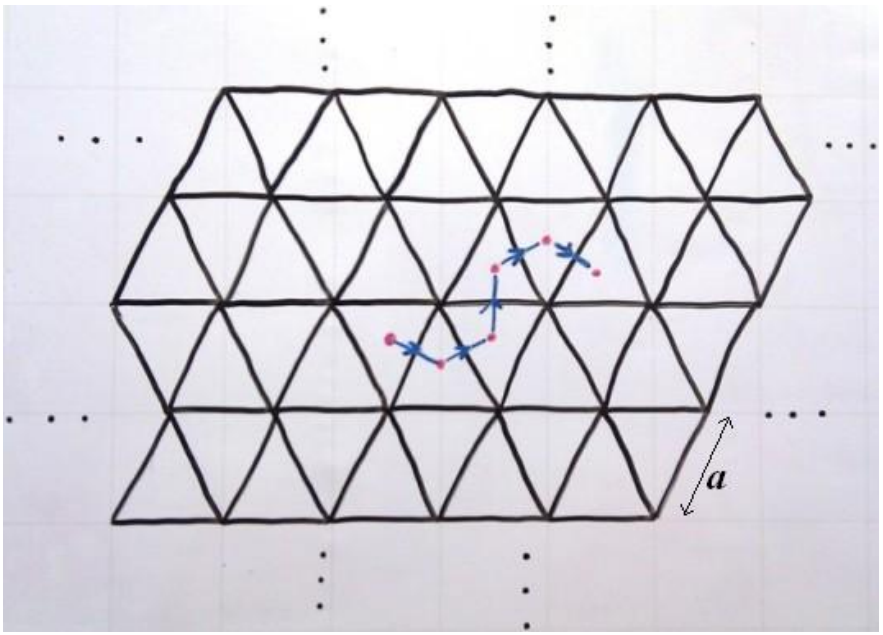
Asymptotic Safety

Weinberg's asymptotic safety conjecture (1979):

Gravity has a non-Gaussian UV fixed point in 4 dimensions, with finite-dimensional UV critical surface.

Euclidean Dynamical Triangulations

- An analogy to path integral methods in QFT
- A path integral for gravity must be a superposition of all possible evolving shapes of spacetime
- Numerical simulations possible — build our spacetimes out of discrete blocks and use lattice methods:



- **Not** a discrete theory though.

Euclidean Dynamical Triangulations

- Two distinct phases in 4-d formulation — unfortunately neither resembles 4-dimensional GR.
- Introduce a non-trivial measure term with accompanying free parameter — now a total of three — in the action.
  - c.f. CDT
- $\rightarrow$ Two-dimensional parameter space in κ_2 and β.
- In a lattice formulation of an asymptotically-safe field theory, the fixed point would appear as a second-order critical point, the approach to which would define a continuum limit.

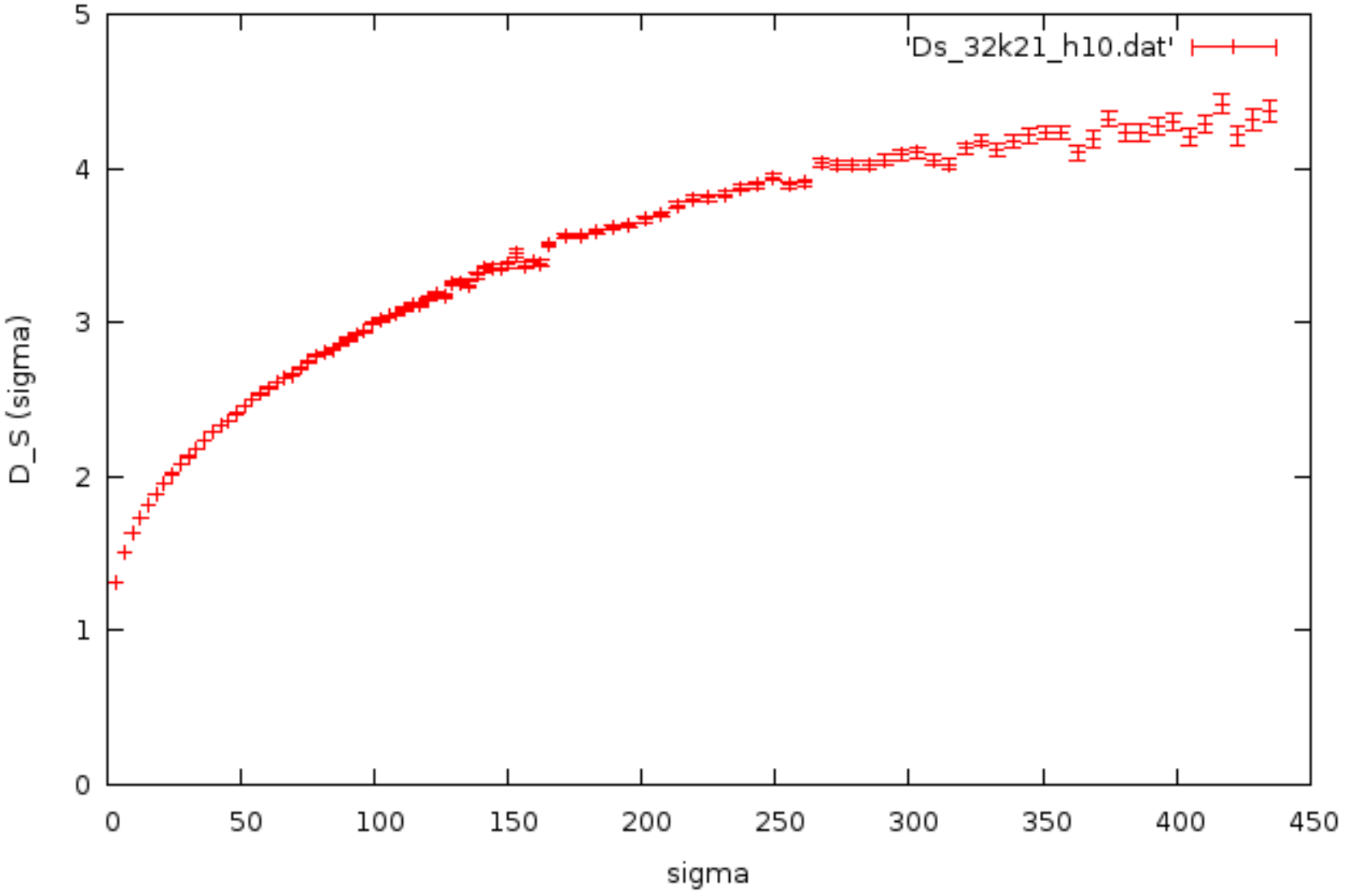
Definitions of Fractal Dimension

- Spectral Dimension
  - Consideration of the time evolution of a diffusion process
  - Can directly access via simulations:

$$D_S = -2\frac{\mathrm{d}logP(\sigma)}{\mathrm{d}log\sigma}$$

  - $P(\sigma)$ is the probability that a random walk will return to its origin after σ steps.
  - We can get our simulation to determine this!

Spectral Dimension

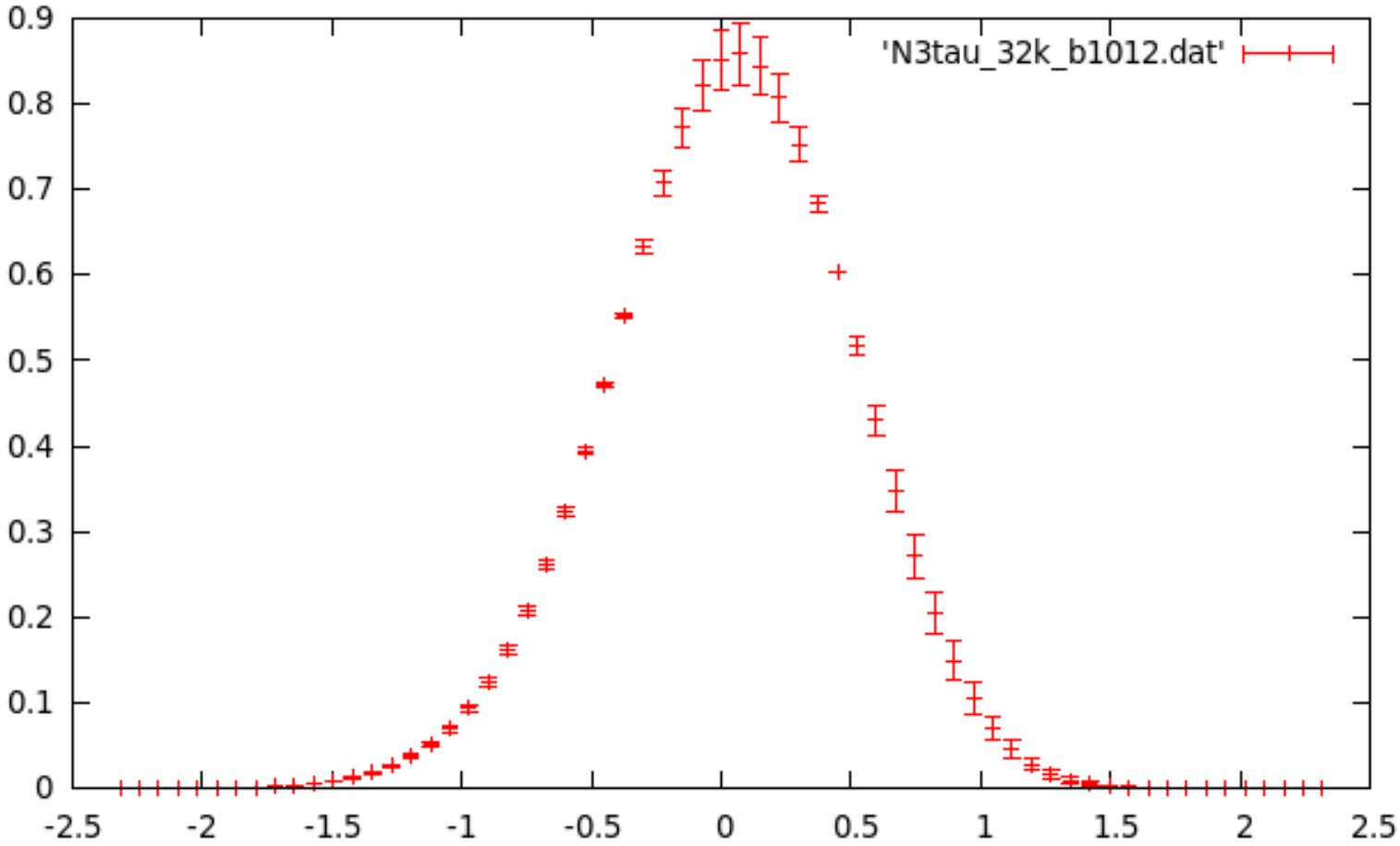


Shape of Universe

The average shape of the sliced universe, normalised so the centre of volume is at zero:

Shape of Universe

The average shape of the sliced universe, normalised so the centre of volume is at zero:



Summary

- Gravity is not perturbatively renormalizable — perhaps we can find a non-perturbative formulation that is.
- Euclidean Dynamical Triangulations is one such possibility, with a third parameter added to the action.
- Positive result on the running of the spectral dimension.
- Results for the shape of the universe provide strong evidence that it has a de Sitter geometry in the region of phase space under consideration.
- Much further work to be done!

References

- Evidence for Asymptotic Safety from Lattice Quantum Gravity (J. Laiho, D. Coumbe) — arXiv: 1104.5505 [hep-lat]
- Exploring the Phase Diagram of Lattice Quantum Gravity (D. Coumbe, J. Laiho) — arXiv: 1201.2864 [hep-lat]
- Another paper currently in preparation.

- *A 'quantum gravity expert' is presumably someone well acquainted with the details of our immense ignorance of the subject. I suppose I count.* — John Baez