Infrared Singularities of Massless Non-abelian Gauge Theories

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- Massless perturbative QCD is of great use for phenomenology at high-energy experiments (Tevatron, LHC)
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  - $k^\pm \to 0$, $k^\mp$ fixed, $\vec{k}_\perp \to 0$ gives collinear poles

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- Theoretical Motivation:
  - Are present and have not been determined to all orders
  - Potential to make all-orders statements about a QFT

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- Now sum over all permutations of emissions

$$\sum_{\text{all perm. }\pi} \frac{1}{p\cdot k_{\pi(1)}}\,\frac{1}{p\cdot(k_{\pi(1)}+k_{\pi(2)})}\cdots\frac{1}{p\cdot(k_{\pi(1)}+\ldots+k_{\pi(n)})} = \frac{1}{p\cdot k_1}\cdots\frac{1}{p\cdot k_n}$$

Eikonal Approximation (in abelian theory) II

- Introduce a second leg with momentum p_2 to make loop diagrams:

$$\overline{u}(p_1)(i\mathcal{M}_{\text{hard}})u(p_2)\left(\frac{p_1}{p_1 \cdot k_1}\frac{p_2}{-p_2 \cdot k_1}\right)\ldots$$

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- Recalling propagator and integrating loop

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- Exponentiation and factorization!

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- In practice, factorization involves 'simulating' soft emission by replacing hard partons with Wilson lines

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- $\mathcal{F}_{D'}$ becomes difficult after two loops
- Summary of current state-of-the-art:
  - Know NLO and NNLO poles. Need N^3LO poles
  - Need an algorithm for NNLO subtraction. Can be done for NLO (Catani-Seymore Dipole Subtraction)