An analogue of Gauge-String and Gauge-Gravity Duality

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The Hermitian Matrix Model

The Hermitian Matrix Model is defined by the convergent path integral,

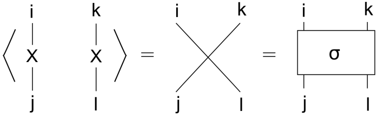
$$\langle \mathcal{O}(X) \rangle = \int DX \mathcal{O}(X) e^{-\frac{1}{2}\mathrm{tr}X^2}$$

- The integration is performed over Hermitian $N \times N$ matrices
- The integral is $U(N)$ invariant, $X \mapsto UXU^\dagger$
- The operators are made of products of traces, e.g.

$$\langle \mathrm{tr}X^2 \rangle, \quad \langle \mathrm{tr}X \mathrm{tr}X^3 \rangle, \quad \langle \mathrm{tr}X^{2n} \rangle$$

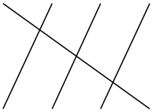
Combinatoric Methods

$$\langle X^i{}_j X^k{}_l \rangle = \delta^i_l \delta^k_j \qquad \longleftrightarrow$$



- $\sigma = (12) \in S_2$ is a permutation of two elements
- Multitrace correlators and Wick contractions can be represented by permutations on the Matrix indices

$$\mathrm{tr} X^4 = X^i{}_j X^j{}_k X^k{}_l X^l{}_i \quad \longleftrightarrow$$



$$\longleftrightarrow \quad (1234) \in S_4$$

In general, we denote

$$\mathrm{tr}(\sigma \mathbf{X}) = \mathrm{tr} X^{a_1} \dots \mathrm{tr} X^{a_k},$$

where

$$\sigma \in S_{2n}, \quad \sum_{i=1}^{k} a_i = 2n, \quad \sigma \in [a_1 a_2 \dots a_k].$$

Using Wick's theorem, we can write the correlator as a sum over the conjugacy class of permutations $[2^n]$, where e.g.

$$[2^2] = \{(12)(34), (13)(24), (14)(23)\} \subset S_4$$

$$\langle \mathrm{tr}(\sigma \mathbf{X}) \rangle = \sum_{\tau \in [2^n]} N^{C_{\sigma\tau}},$$

where $C_{\sigma\tau}$ is the number of disjoint cycles in the permutation $\sigma\tau$. Define the delta function

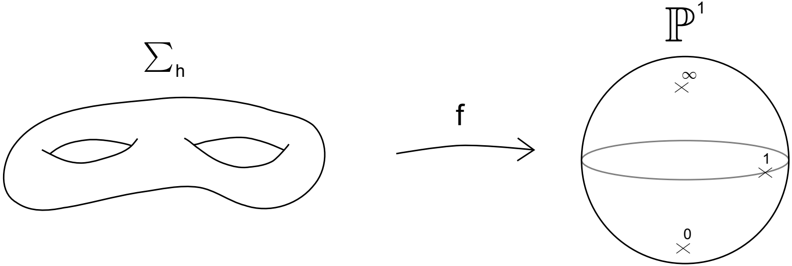
$$\delta(\sigma) = \begin{cases} 1 & \text{if } \sigma \text{ is the identity} \\ 0 & \text{otherwise.} \end{cases}$$

Therefore

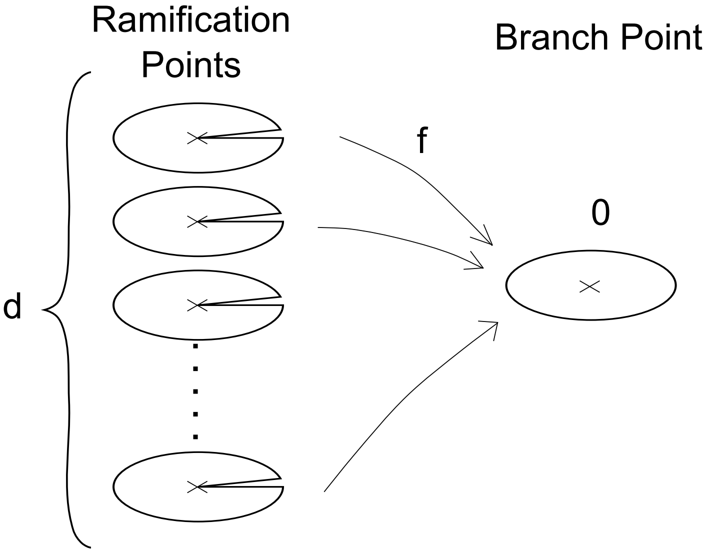
$$\langle \mathrm{tr}(\sigma \mathbf{X}) \rangle = \sum_{\tau \in [2^n]} \sum_{\gamma \in S_{2n}} N^{C_\gamma} \delta(\sigma\tau\gamma).$$

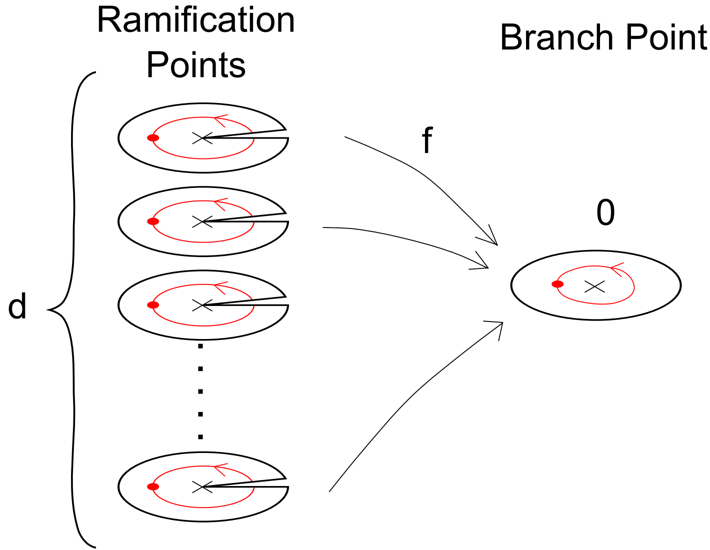
The correlators of the matrix model are sums over triples of permutations which multiply to the identity.

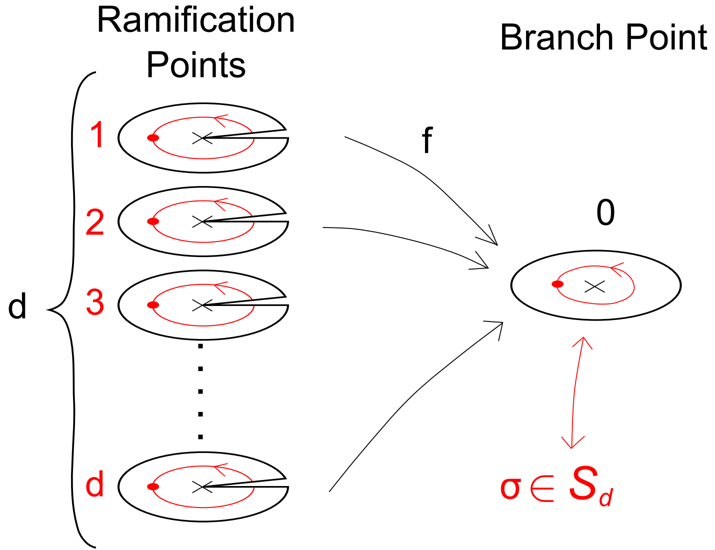
Branched Covers of Holomorphic Maps

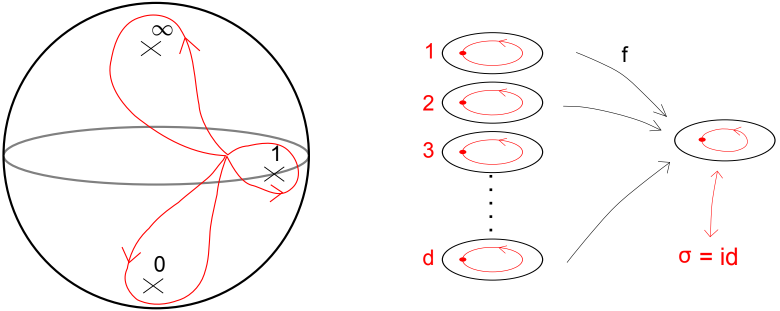


- Consider the holomorphic covering maps from a Riemann surface Σ_h of genus h onto the complex sphere $\mathbb{P}^1$, with branch points $0, 1, \infty$
- The degree of the map is d
- The points that map to branch points are the ramification points of f


Ramification
Points
Branch Point
f
0
d


Ramification Points
Branch Point
f
0
d


Ramification
Points
Branch Point
f
0
d
1
2
3
d
σ ∈ S_d



- The gluing of the Riemann surface Σ_h at branch points can be specified by permutations in S_d
- The three permutations associated with the branch points multiply to the identity

The Riemann-Hurwitz formula for $f : \Sigma_h \to \mathbb{P}^1$ with ramification profile r_i,

$$2 - 2h = 2d + \sum_i (r_i - 1)$$

Specifying to maps with $d = 2n$, where the branching of 0 and 1 is σ and $\tau \in [2^n]$,

$$\frac{|[\sigma]|}{(2n)!} N^{C_\sigma - n} \langle \mathrm{tr}(\sigma \mathbf{X}) \rangle = \sum_{\substack{f:\Sigma_h \to \mathbb{P}^1 \\ ([\sigma],[2^n]) \text{ at } (0,1)}} \frac{1}{|\mathrm{Aut}\ f|} N^{2-2h}.$$