Thermal Quantum Field Theory in Real and Imaginary Time

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What is Thermal QFT?

Ordinary "vacuum" QFT

- few *in* and *out* particles: $\langle 0| \phi\phi \ldots \phi |0\rangle$
- $S_{fi} = \langle f, +\infty | i, -\infty \rangle$: no temporal information about the evolution of the system

Equilibrium Thermal QFT

- huge amount of particles, stationary state
- expectation values, not asymptotic amplitudes

Non-equilibrium Thermal QFT

- huge amount of particles
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- time evolution of the system

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Why Thermal QFT?

Quantum Field Theory at Finite Temperature

- Collider Physics: quark-gluon plasma, nuclear matter
- Early Universe: electroweak phase transition, nucleosynthesis

Nonequilibrium Quantum Field Theory

- Baryogenesis - third Sakharov's condition
  - Quantum effects in Leptogenesis
- Reheating at the end of inflation
- Heavy-ion collisions

Imaginary-Time Formalism

Canonical Ensemble

$\rho = \frac{1}{Z} e^{-\beta H} = \frac{1}{Z} e^{-i(-i\beta)H}$ evolution operator in imaginary-time

$$\mathrm{Tr}\,\rho = \frac{1}{Z} \sum_{\phi(\mathbf{x})} \langle \phi(\mathbf{x}) |\, e^{-\beta H} \,| \phi(\mathbf{x}) \rangle = \frac{1}{Z} \sum_{\phi(\mathbf{x})} \langle \phi(\mathbf{x}), t - i\beta | \phi(\mathbf{x}), t \rangle$$

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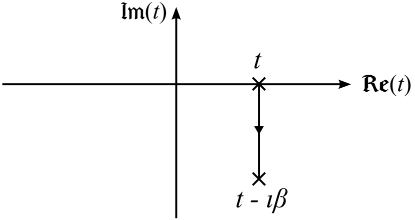

Imaginary-Time Free Propagator

- imaginary time $\longrightarrow$ euclidean theory
- periodicity $\longrightarrow$ discrete frequencies

Matsubara Propagator

$$\bar{\Delta}^0(i\omega_n, \mathbf{k}) = \frac{1}{\omega_n^2 + \omega_{\mathbf{k}}^2}$$

$$\omega_n = 2\pi n/\beta \qquad \omega_{\mathbf{k}} = \sqrt{\mathbf{k}^2 + m^2}$$

- Same diagrams as in the T=0 theory
- Discrete summations to evaluate loop corrections
- Analytic continuation to obtain real-time physical functions

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Real-Time Formalism

Unitarity

$$\mathrm{Tr}\,\rho = \mathrm{Tr}\{\rho\, S^\dagger\, S\} = \frac{1}{Z}\,\mathrm{Tr}\{e^{-i(-i\beta)H}\, S^\dagger\, S\}$$

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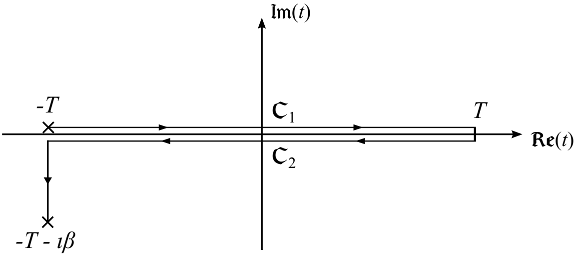


$T \to \infty \quad \Longrightarrow$ | can be factorized

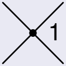
Real-Time Free Propagator

Propagator

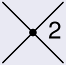
contour-ordered propagator $\longrightarrow$ 2×2 matrix

$$i\boldsymbol{\Delta}(x-y) = i \begin{pmatrix} \Delta_F(x-y) & \Delta_<(x-y) \\ \Delta_>(x-y) & \Delta_D(x-y) \end{pmatrix}$$

Vertices



$= -i\lambda$



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- Generalizable to non-equilibrium

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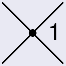
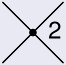

1 $= -i\lambda$


2 $= +i\lambda$

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Real-Time Free Propagator

Free scalar field

$$\boldsymbol{\Delta}^{\mathbf{0}}(k) = \begin{pmatrix} \frac{1}{k^2-m^2+i\epsilon} - i\,n(k)2\pi\delta(k^2-m^2) & -i\,[n(k)+\theta(-k_0)]2\pi\delta(k^2-m^2) \\ -i\,[n(k)+\theta(k_0)]2\pi\delta(k^2-m^2) & \frac{-1}{k^2-m^2-i\epsilon} - i\,n(k)2\pi\delta(k^2-m^2) \end{pmatrix}$$

- Equilibrium boundary condition $\Longrightarrow$ $n(k) = \frac{1}{e^{\beta|k_0|}-1}$

Self-Energy

Imaginary-Time resummed propagator

$$\bar{\Delta}(i\omega_n, \mathbf{k}) = \frac{1}{\bar{\Delta}^0(i\omega_n,\mathbf{k})^{-1} + \bar{\Pi}(i\omega_n,\mathbf{k})}$$

Real-Time resummed propagators

$$\Delta_F(k) = \frac{k^2 - m^2 + \Pi(k)^*}{[k^2 - m^2 + \Re\Pi_R(k)]^2 + [\Im\Pi_R(k)]^2}$$

$$\Delta_{R/A}(k) = \frac{1}{k^2 - m^2 + \Pi_{R/A}(k)}$$

$$\mathbf{\Pi} = \begin{pmatrix} \Pi & -\Pi_< \\ -\Pi_> & -\Pi^* \end{pmatrix}$$

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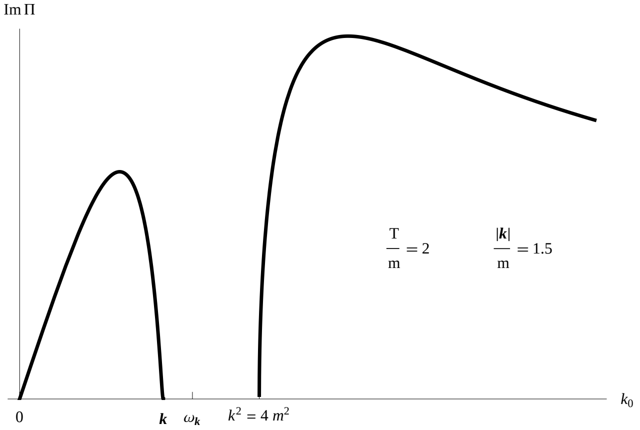
QUASIPARTICLES

$$M^2(k) = m^2 - \Re\Pi_R(k)$$

$$\Gamma = |\Im\Pi_R|/M$$

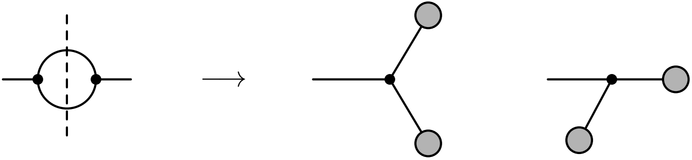
ϕ^3 Theory - $\Im\Pi_R$



Thermal Cutting Rules

$\Im\Pi_R$ related to thermal disappearance of particles

ϕ^3 Theory



WHAT I COULDN'T TALK ABOUT

- fermions, gauge interactions
- IR problems: Hard-Thermal-Loop techniques
- phenomenology: quark-gluon plasma, early universe, ...
- CJT formalism (2PI effective action): "resummed" self-consistent equations for the propagator
  - Goldstone theorem not satisfied[1]...
- Non-equilibrium "resummed" formalism (based on CJT): Kadanoff-Baym equations
- Non-equilibrium applications: thermalization of quantum fields, resonant leptogenesis, ...
- Non-equilibrium perturbation theory

[1] D. Teresi and A. Pilaftsis, *Goldstone-symmetry improved CJT formalism*, in preparation