

# Collider Physics

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# Outline of the Lecture

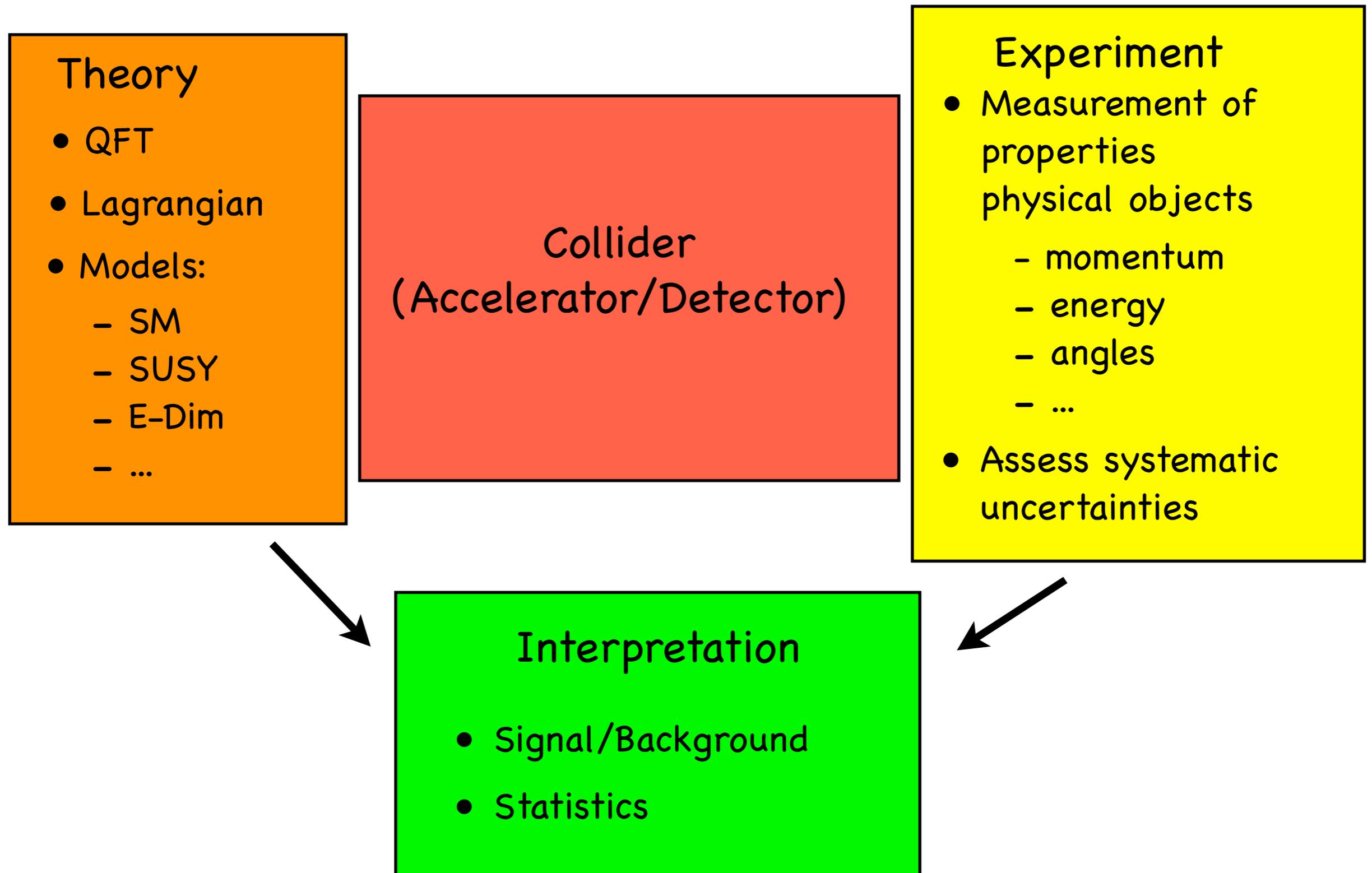
## Set the stage:

- Lecture 1
  - From theory to experiment
- Lecture 2
  - Reconstruction of physical objects
  - Jet physics

## Applications:

- Lecture 3
  - Higgs searches at the LHC
  - Measurement of Higgs couplings, Spin, CP
- Lecture 4
  - BSM physics at the LHC

The purpose of collider physics is to test theoretical predictions experimentally in a controllable environment



# High-energy colliders of the past, present and future (?)

Collider	Site	Initial State	Energy	Discovery / Target
SPEAR	SLAC	$e^+e^-$	4 GeV	charm quark, tau lepton
PETRA	DESY	$e^+e^-$	38 GeV	gluon
Sp $\bar{p}$ S	CERN	$p\bar{p}$	600 GeV	W, Z bosons
LEP	CERN	$e^+e^-$	210 GeV	SM: elw and QCD
SLC	SLAC	$e^+e^-$	90 GeV	elw SM
HERA	DESY	$ep$	320 GeV	quark/gluon structure of proton
Tevatron	FNAL	$p\bar{p}$	2 TeV	top quark
BaBar / Belle	SLAC / KEK	$e^+e^-$	10 GeV	quark mix / CP violation
LHC	CERN	$pp$	7/8/14 TeV	Higgs boson, elw. sb, New Physics
ILC		$e^+e^-$	> 200 GeV	hi. res of elw sb / Higgs couplings
CLIC		$e^+e^-$	3 - 5 TeV	hi. res of elw sb / Higgs couplings
VLHC		$pp$	200 TeV	disc. multi-TeV physics

# Physics ratio for collider facilities

$A + B \rightarrow M$  production in 2-particle collisions:  $M^2 = (p_1 + p_2)^2$

fixed target:  $p_1 \simeq (E, 0, 0, E)$  before after  
 $p_2 = (m, 0, 0, 0)$    
 $M \simeq \sqrt{2mE}$  root increase in M

- root  $E$  law: large energy loss in  $E_{\text{kin}}$
- dense target: large collision rate / luminosity

collider target:  $p_1 = (E, 0, 0, E)$  before after  
 $p_2 = (E, 0, 0, -E)$    
 $M \simeq 2E$

- linear  $E$  law: no energy loss
- less dense bunches: small collision rates

## Collider characteristics

Energy: ranges from a few GeV to several TeV (LHC)

Luminosity: measures the rate of particles in colliding bunches

$$\mathcal{L} = \frac{N_1 N_2 f}{A}$$

$N_i$  = number of particles in bunches

$A$  = transverse bunch area

$f$  = bunch collision rate

$\mathcal{L}\sigma$  = observed rate for process with cross section  $\sigma$

LHC (at moment):  $\mathcal{L} = 6.4 \cdot 10^{33} \text{ cm}^{-2} \text{ s}^{-1}$

LHC (targeted):  $\mathcal{L} = 10^{34} \text{ cm}^{-2} \text{ s}^{-1} \rightarrow 300 \text{ fb}^{-1}$  in 3 years

### circular vs linear collider:

charged particles in circular motion: permanently accelerated towards center  $\rightarrow$  emitting photons as synchrotron light  $\Delta E \sim E^4/R$

- large loss of energy [hypothetical TeV collider at LEP:  $\Delta E \simeq E$  per turn]
- no-more sharp initial state energy

# Connecting Theory and experiment: Scattering processes at hadron colliders

## Master formula:

General process at proton-proton collider  $\sigma(pp \rightarrow X)$

$$\sigma_{pp \rightarrow X} = \sum_{a,b} \int dx_1 dx_2 f_{a/p}(x_1, \mu_F) f_{b/p}(x_2, \mu_F) \hat{\sigma}_{a,b \rightarrow k}(\mu_F, \mu_R) \Theta(\text{Cuts}) D(k \rightarrow X)$$

where the partonic cross section is calculated by

$$\hat{\sigma}_{a,b \rightarrow k} = \frac{1}{2s} \int \left[ \prod_{i=1}^n \frac{d^3 \vec{q}_i}{(2\pi)^3 2E_i} \right] \left[ (2\pi)^4 \delta^4 \left( \sum_i q_i^\mu - (p_1 + p_2)^\mu \right) \right] |\mathcal{M}_{ab \rightarrow k}(\mu_F, \mu_R)|^2$$

↑
↑
↑
  
 [flux factor] ×      [phase space (LiPS)]      ×      [squared matrixelement]

Crucial pieces for the calculation of the hadronic cross section are the **parton distribution functions**  $f_{i/p}$  and the **squared matrix element**  $|\mathcal{M}|^2$

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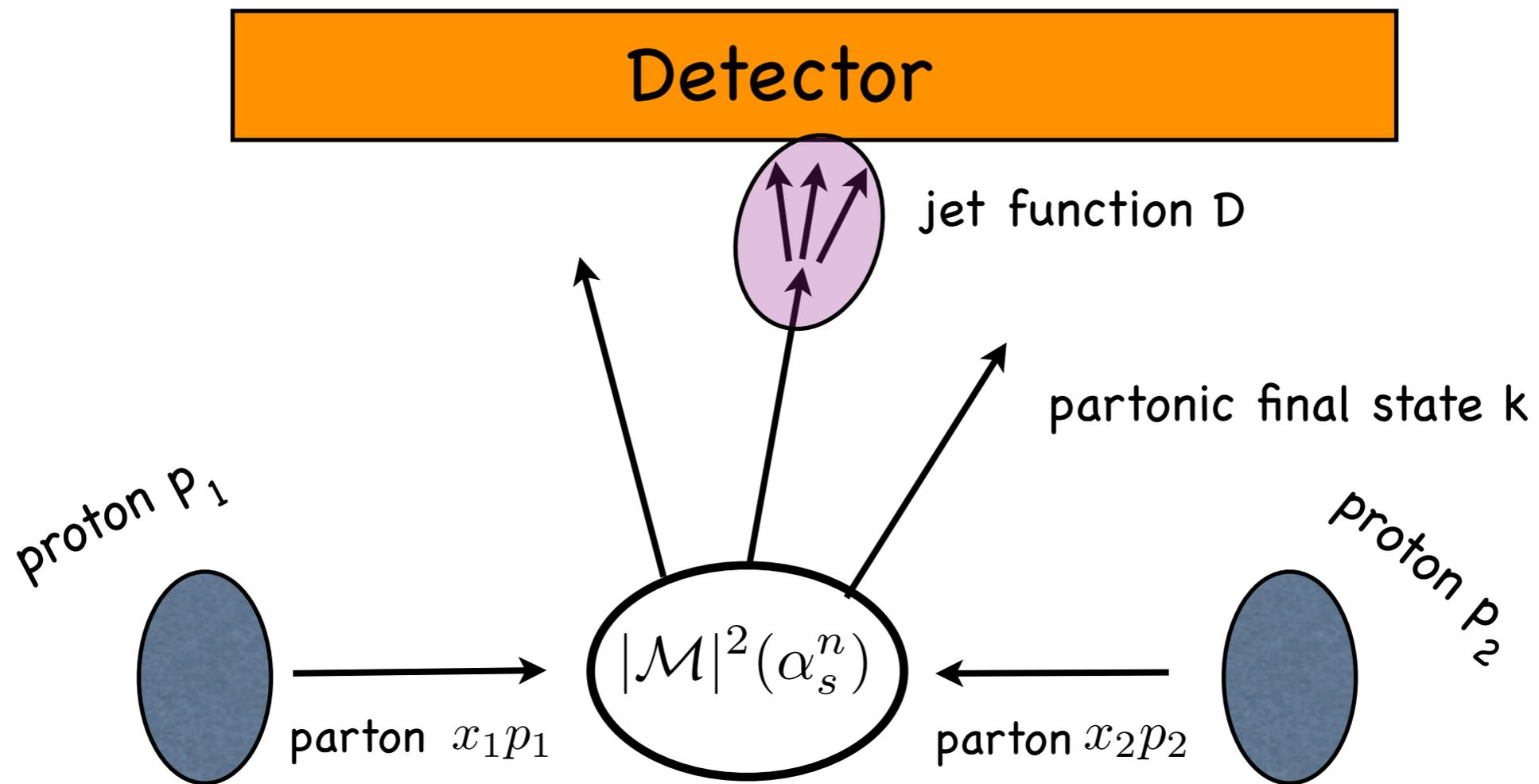
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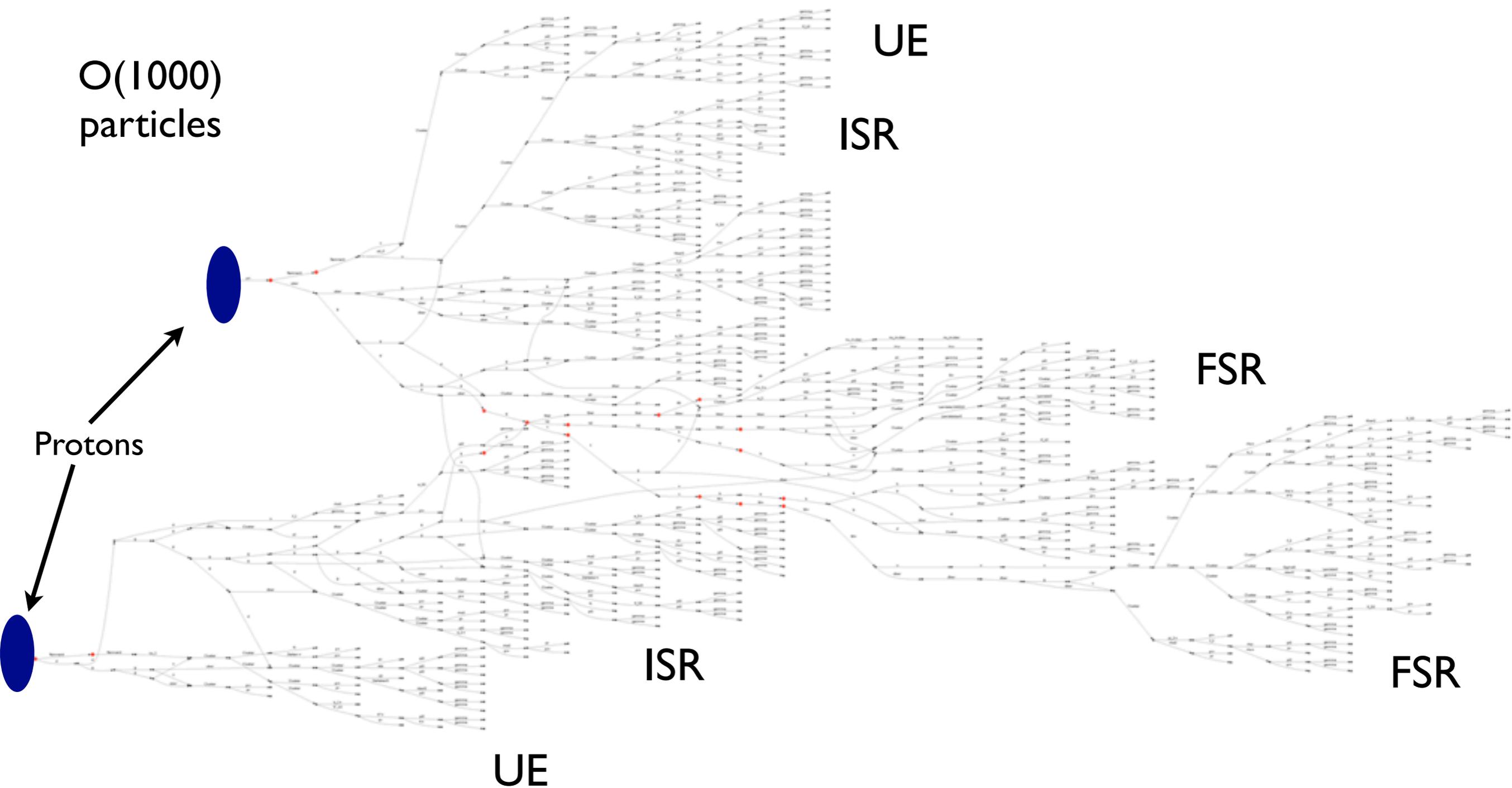
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# More detailed discussion of master formula

Pictorial description:

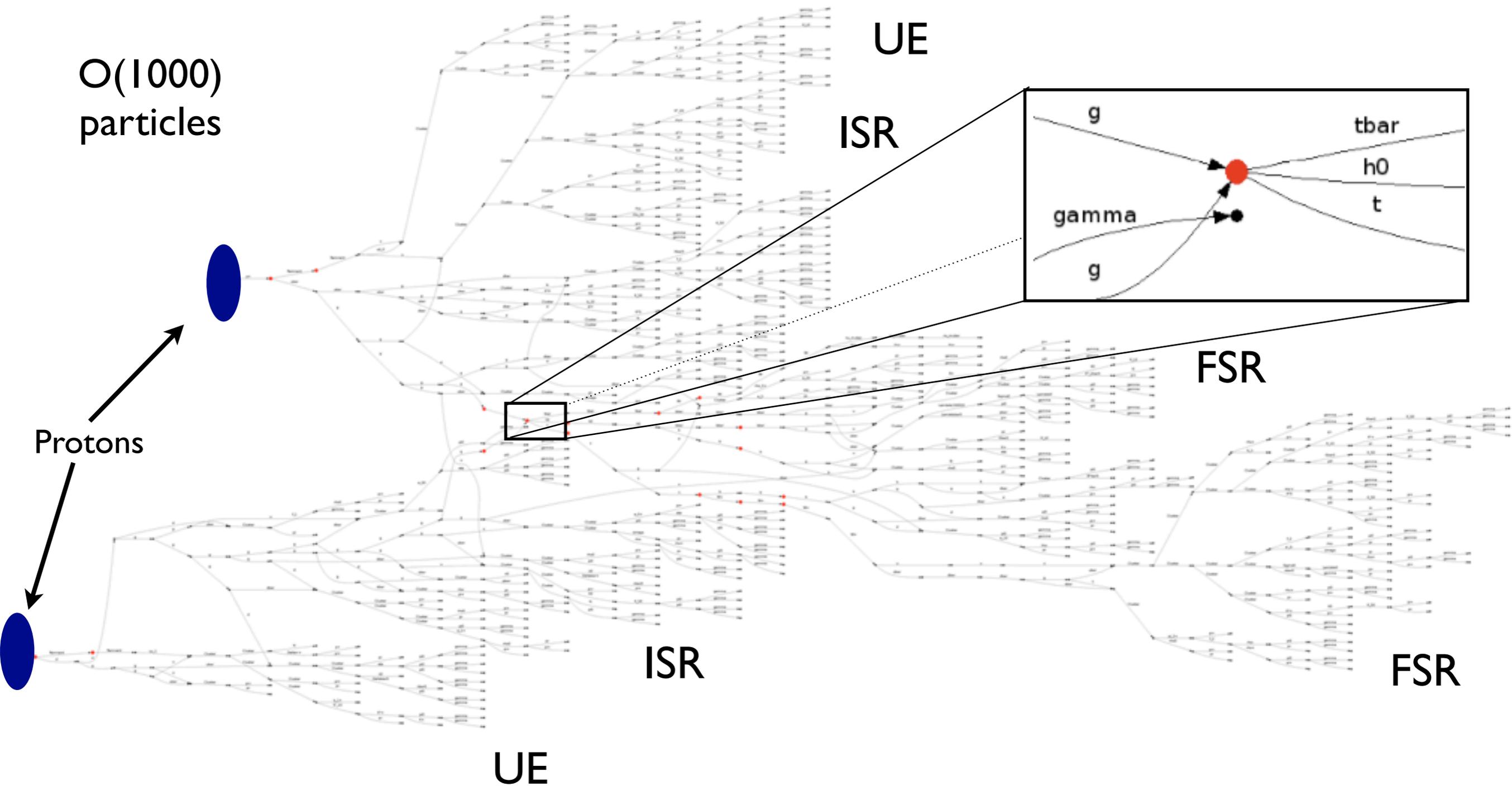


# LHC hosts complex environment!

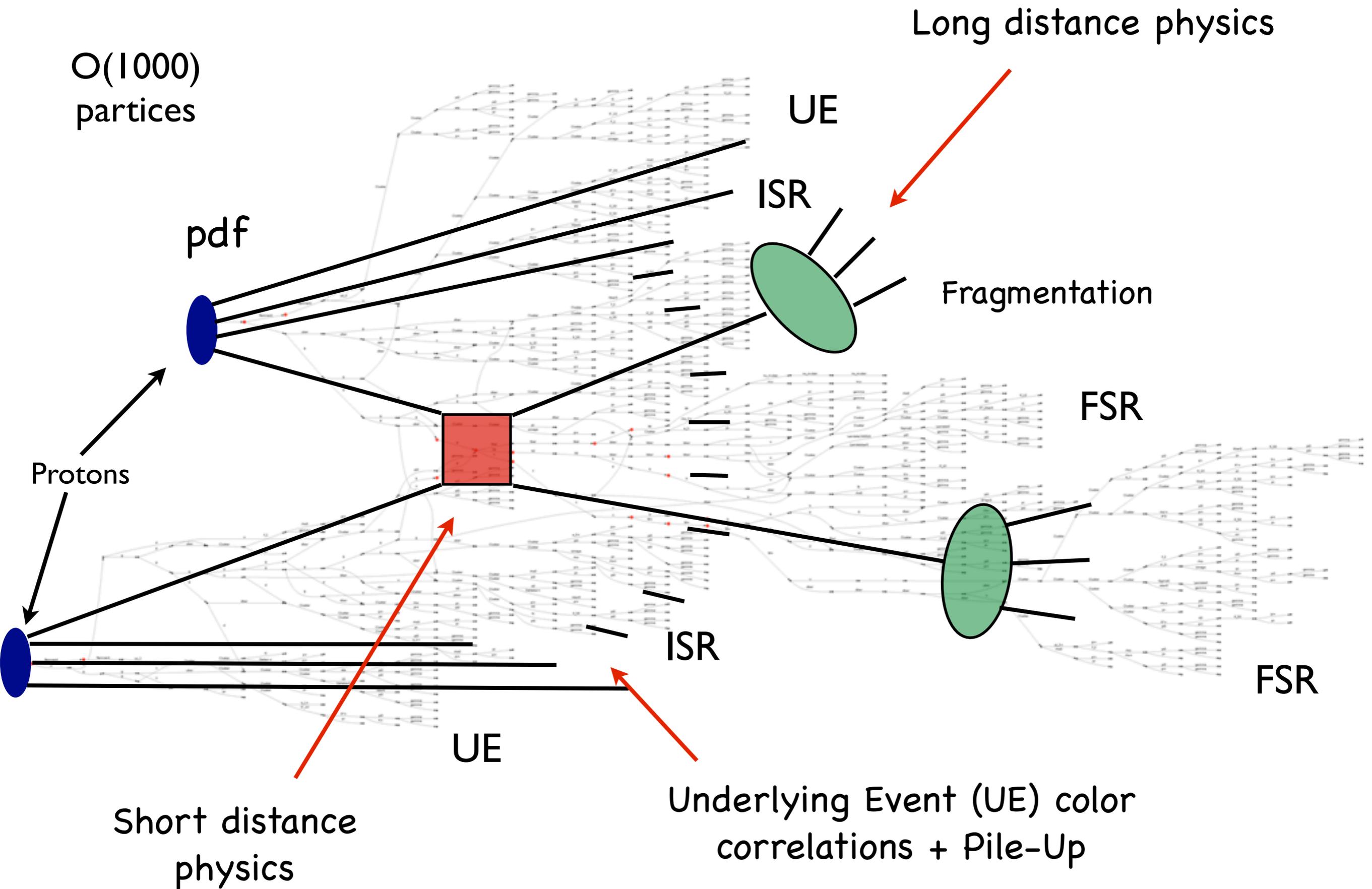


Tedious for theorists and experimentalists

# LHC hosts complex environment!



Tedious for theorists and experimentalists



## More detailed discussion of master formula

Partonic cross section  $\hat{\sigma}$ :

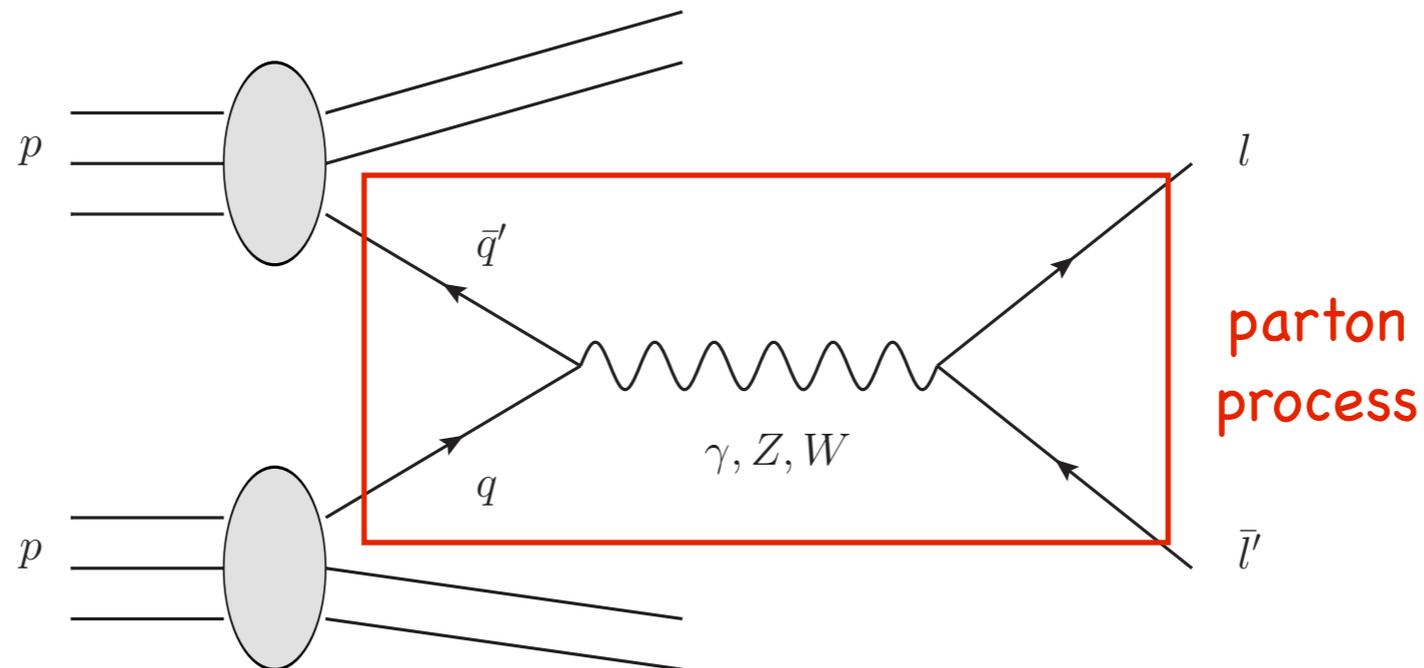
Partons (e.g. quarks) of incoming hadrons (e.g. protons) interact at short distance (large momentum transfer). Example Drell-Yan process:  $\hat{\sigma}(q\bar{q} \rightarrow l^+l^-)$

calculable with perturbation theory in powers of

$$\hat{\sigma}_{ab \rightarrow k} = \left[ \hat{\sigma}_0 + \alpha_s(\mu_R^2) \hat{\sigma}_1 + \alpha_s^2(\mu_R^2) \hat{\sigma}_2 + \dots \right]$$

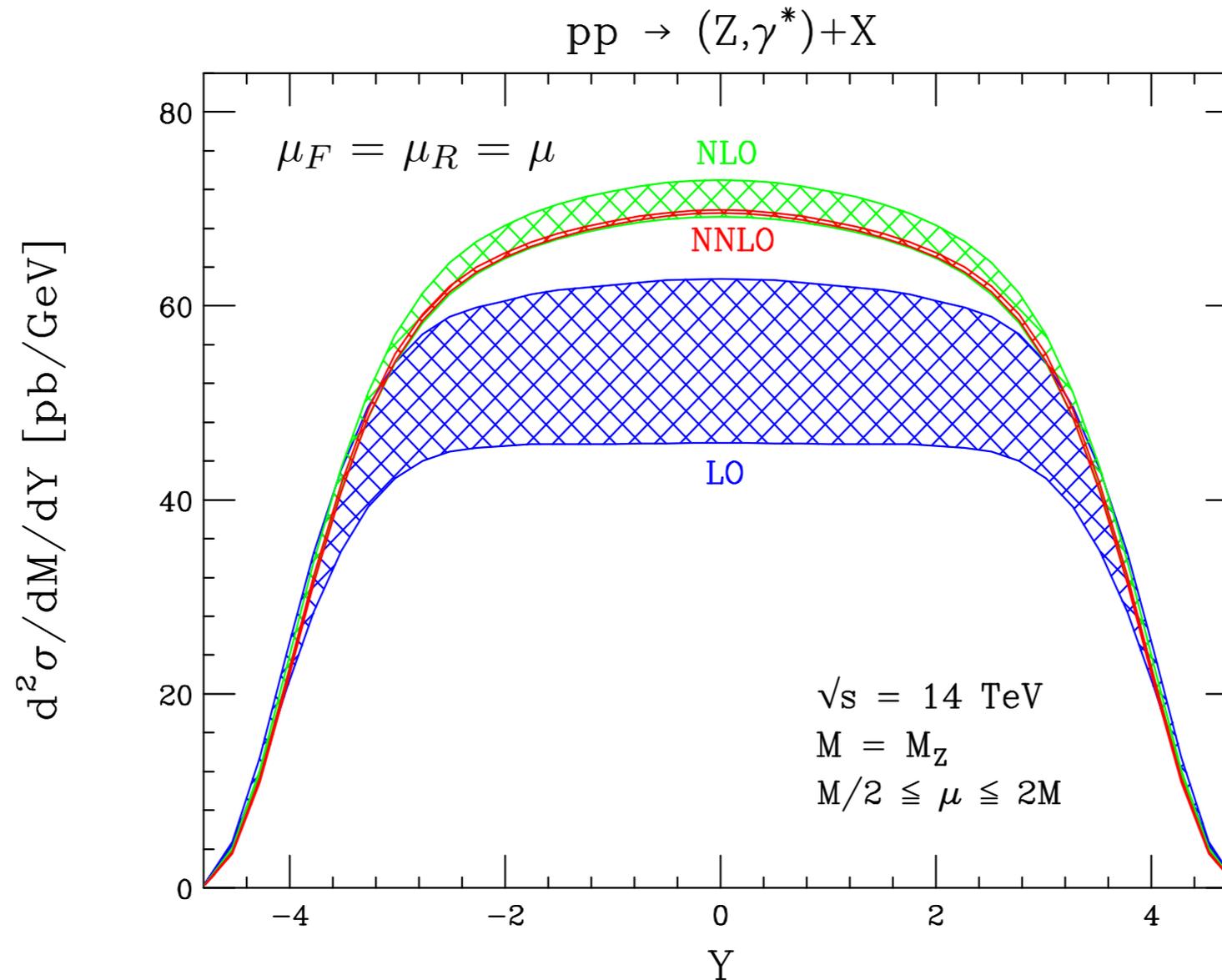
Example: Drell-Yan Process

Leading-order  
diagram



# More detailed discussion of master formula

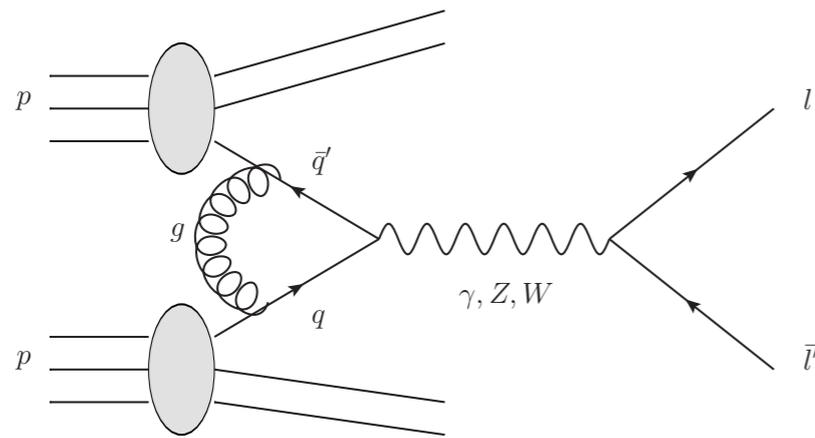
NLO contributions are important for precision



- NLO-K-factor is ratio between LO and NLO
- Higher-Order corrections important for phenomenology at colliders

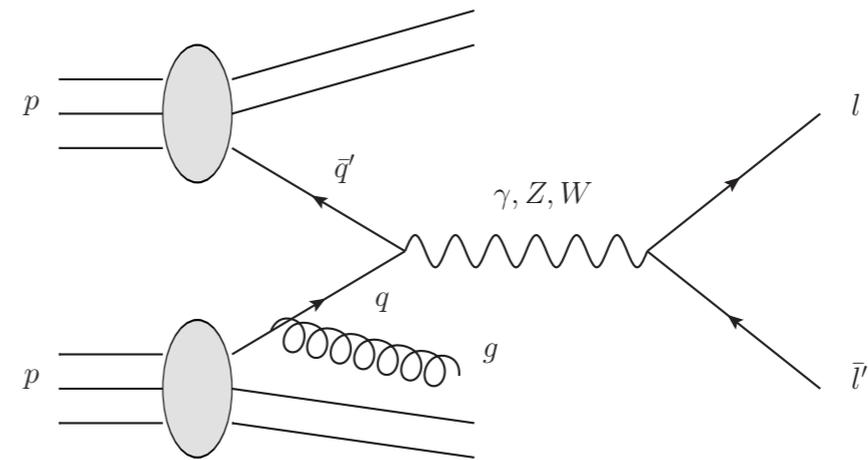
# More detailed discussion of master formula

virtual correction:



UV divergences  
soft/coll. divergences

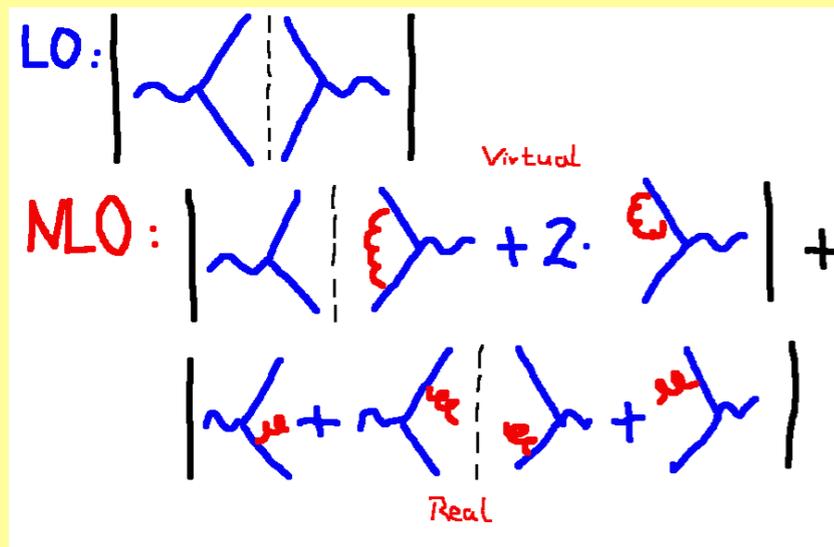
real correction:



soft divergences  
collinear divergences

$$|\mathcal{M}_B + \mathcal{M}_V + \mathcal{M}_R|^2 = \mathcal{M}_B^2 + 2\text{Re}(\mathcal{M}_B \mathcal{M}_V^*) + \mathcal{M}_R^2 + \mathcal{O}(\alpha_s^2)$$

The NLO matrix element is formally  $\alpha_s^{n+1}$  in pert. expansion:



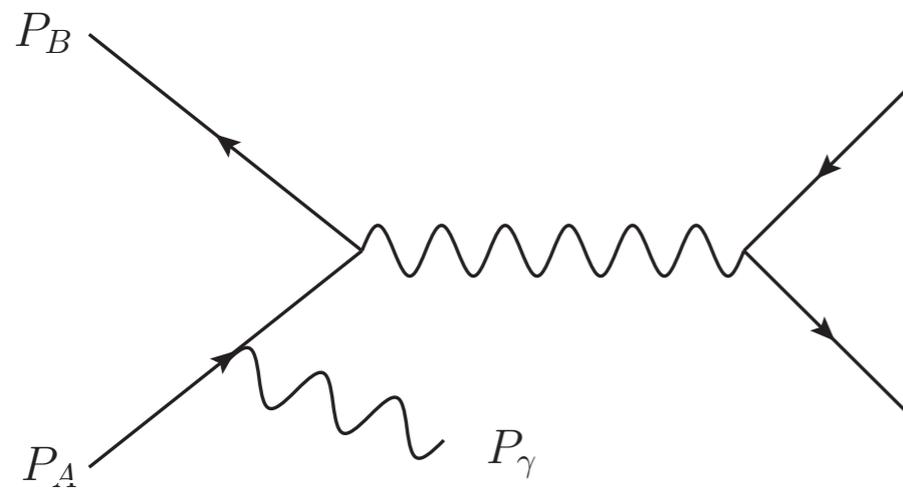
UV divergences: **renormalization**

all soft and coll. final state divergences:  
**(KLN theorem) cancel between virtual and real corrections**

coll. initial state divergences:  
**absorbed in pdfs**

## Closer look at structure of radiative corrections:

- Use QED as first approximation:



$$\sim \bar{u}_B \gamma^\mu \frac{\not{p}_A - \not{p}_\gamma}{(p_A - p_\gamma)^2} \gamma^\nu u_A \epsilon_\nu(p_\gamma) J_\mu$$

$$p_A = (E, 0, 0, E) \quad p_\gamma = \left( zE, \vec{p}_\perp, \sqrt{(zE)^2 - \vec{p}_\perp^2} \right)$$

$$(p_A - p_\gamma)^2 = -2p_A p_\gamma = -2zE^2 \left( 1 - \sqrt{1 - \frac{\vec{p}_\perp^2}{(zE)^2}} \right)$$

➔  $\mathcal{M} \rightarrow \infty$  when  $z \rightarrow 0$  “soft singularity”

or  $|\vec{p}_\perp| \rightarrow 0$  “collinear singularity”

- photons/gluons emitted at small angle unresolvable
- photons/gluons emitted with small energy unobservable

Define energy scale Q:  $p_{\perp} > Q$  - observe  $2 \rightarrow 3$

$p_{\perp} < Q$  - observe  $2 \rightarrow 2$

$$(p_A - p_{\gamma})^2 \rightarrow 0 \implies \not{p}_A - \not{p}_{\gamma} \simeq \sum_s u_s (p_A - p_{\gamma}) \bar{u}_s (p_A - p_{\gamma})$$

$$(p_A - p_{\gamma})^2 \simeq -\frac{p_{\perp}^2}{z}$$

[Peskin, Schroeder]

$$\mathcal{M} \sim -\frac{z}{p_{\perp}^2} \sum_s [\bar{u}_s \gamma^{\nu} u_A \epsilon_{\nu}(p_{\gamma})] \cdot \mathcal{M}(p_A - p_{\gamma}, p_B \rightarrow p_1, p_2)$$

→ factorization:  $e \rightarrow e\gamma$  ×  $2 \rightarrow 2$  process

$$d\sigma_{2 \rightarrow 3} \sim d\Pi_{\gamma} \left(\frac{z}{p_{\perp}^2}\right)^2 \sum_s |\mathcal{M}(e \rightarrow e\gamma)|^2 \cdot |\mathcal{M}_{2 \rightarrow 2}(\hat{s} = (1-z)s)|^2 d\Pi_1 d\Pi_2 \frac{1}{2\hat{s}}$$

$$d\Pi_{\gamma} = \frac{d^3 p_{\gamma}}{(2\pi)^3} \frac{1}{2E_{\gamma}} = \frac{p_{\perp} dp_{\perp} dz}{8\pi^2 z}$$

$$\frac{4e^2 p_{\perp}^2}{z(1-z)} \left[ \frac{1 + (1-z)^2}{z} \right]$$

$$d\sigma_{2 \rightarrow 2}^{LO}(\hat{s})$$

$$\sigma_{2 \rightarrow 3} \sim \int_{z_{min}}^1 \frac{dz}{z} \int_Q^{\sim E} \frac{p_{\perp} dp_{\perp}}{8\pi^2} (1-z) \left(\frac{z}{p_{\perp}^2}\right)^2 \frac{4e^2 p_{\perp}^2}{z(1-z)} \left[\frac{1+(1-z)^2}{z}\right] \sigma_{2 \rightarrow 2}^{LO}(\hat{s})$$

$$z_{min} = \frac{Q}{E}$$

“Sudakov double log”

$$\sim \frac{2}{\pi} \log^2 \frac{\sqrt{s}}{Q} \sigma_{2 \rightarrow 2}^{LO}(s) + \dots$$

In QCD:

For final state with invariant mass  $M(X)$

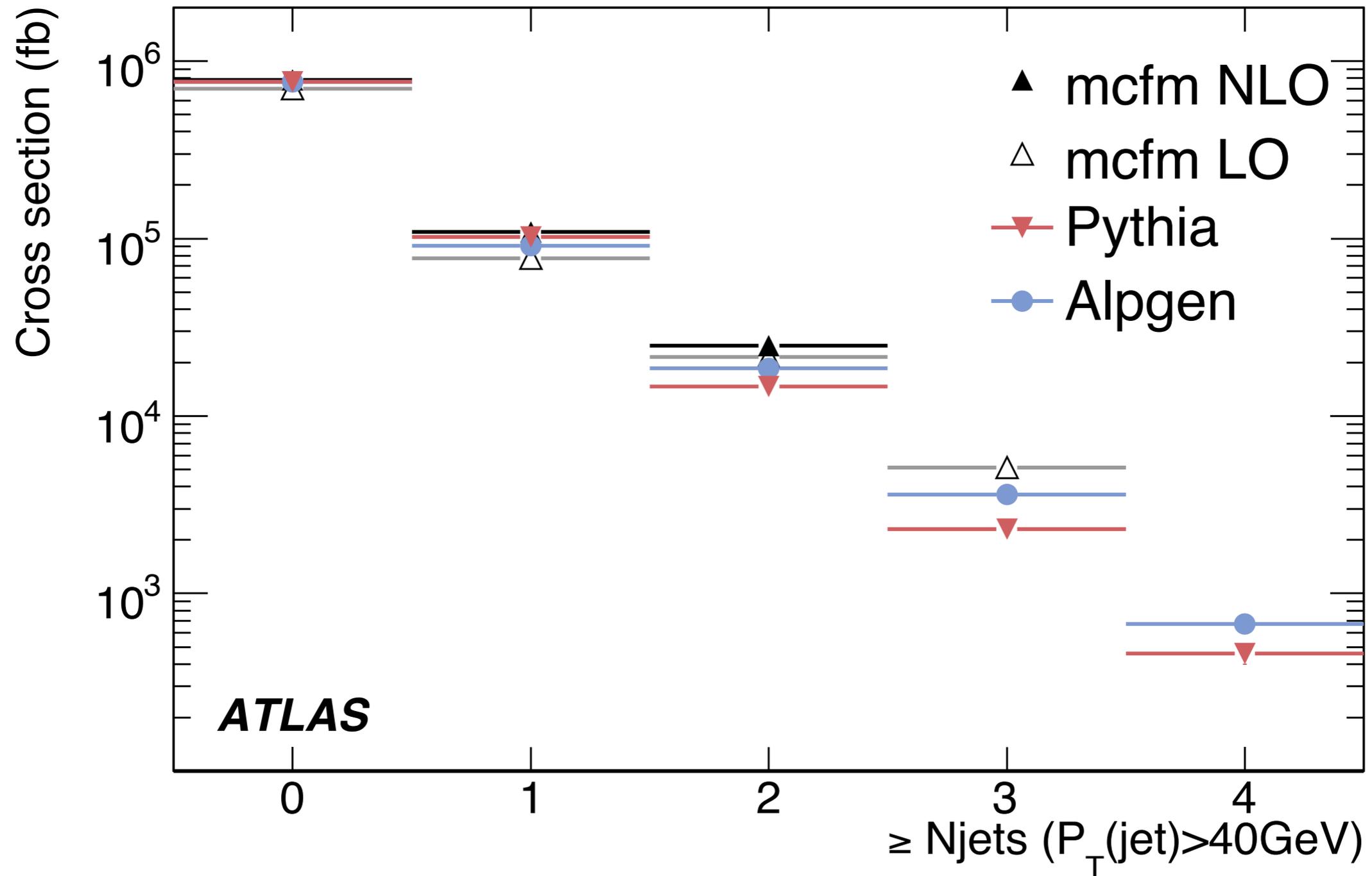
**exclusive:**  $\sigma(X + \text{jet}, p_{T,j} > Q) \simeq \frac{\alpha}{\pi} \log^2 \frac{M(X)}{Q} \sigma_{LO}(X \text{ without jet})$

**inclusive:**  $\sigma(X) + \sigma(X + \text{jet}) = \sigma_{LO}(X) \left(1 + c \frac{\alpha}{\pi}\right)$  (no uncanceled logs)

**Example**  $pp \rightarrow Z + j$  :  $\frac{\alpha_s}{\pi} \log^2 \frac{M_Z^2}{Q^2} \sim 1$  for  $Q \sim 10 \text{ GeV}$

→ Z prod. typically accompanied with 10 GeV jets

# # of Jets in $pp \rightarrow Z \rightarrow e^+e^-/\mu^+\mu^-$



## More detailed discussion of master formula

Parton distribution functions (pdf)  $f_{a/A}(x_1, \mu_F^2)$   $f_{b/B}(x_2, \mu_F^2)$

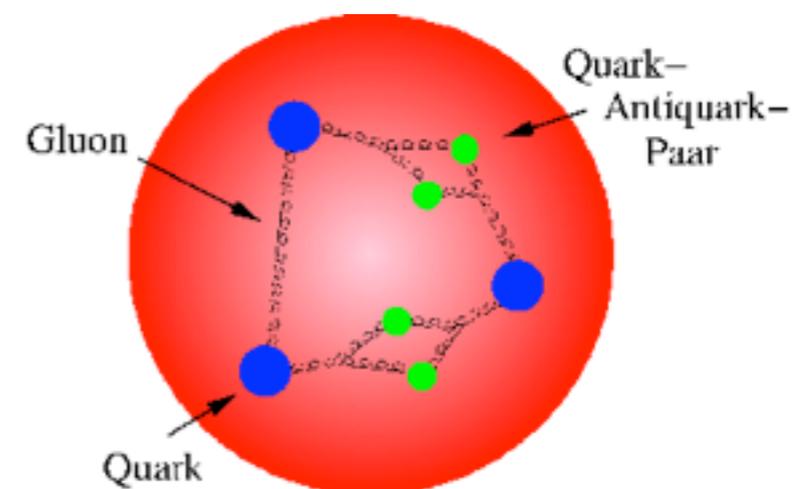
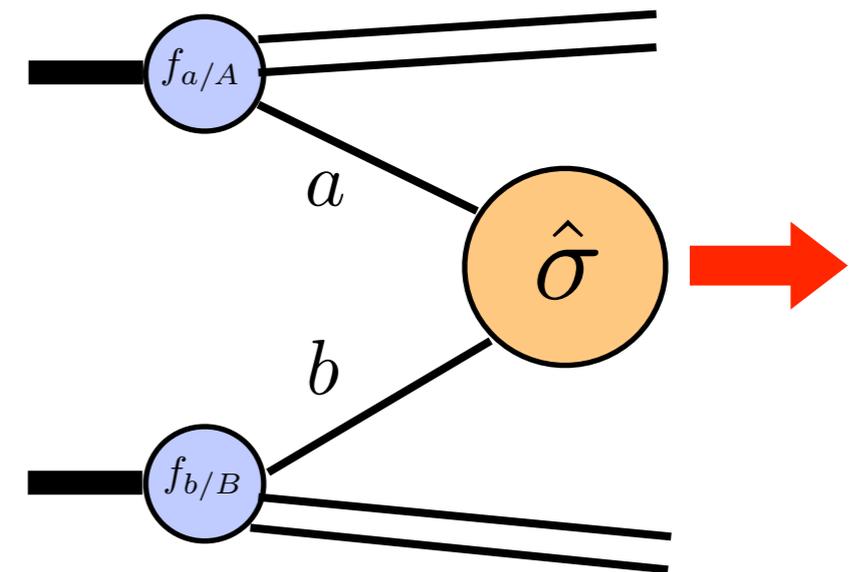
$x_1 = 2E_a/\sqrt{S}$  and  $x_2 = 2E_b/\sqrt{S}$  momentum fraction carried by incoming partons

$\mu_F^2$  factorization scale (separates short- and long-distance physics)

pdfs experimentally extracted from deep-inelastic scattering

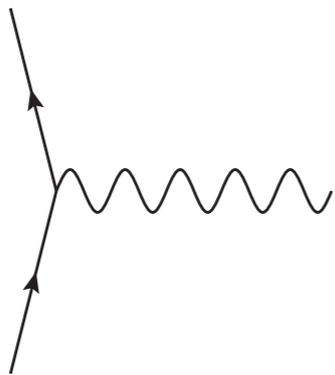
### Factorization theorem of QCD

Partonic cross sections have collinear divergences associated with the hadronic initial state, which factorize universally (i.e. independent of the process) from the hard scattering process and can be absorbed in the renormalized parton densities of the initial state. These renormalized parton densities are solutions of the DGLAP equations.



Drell-Yan production at LO:

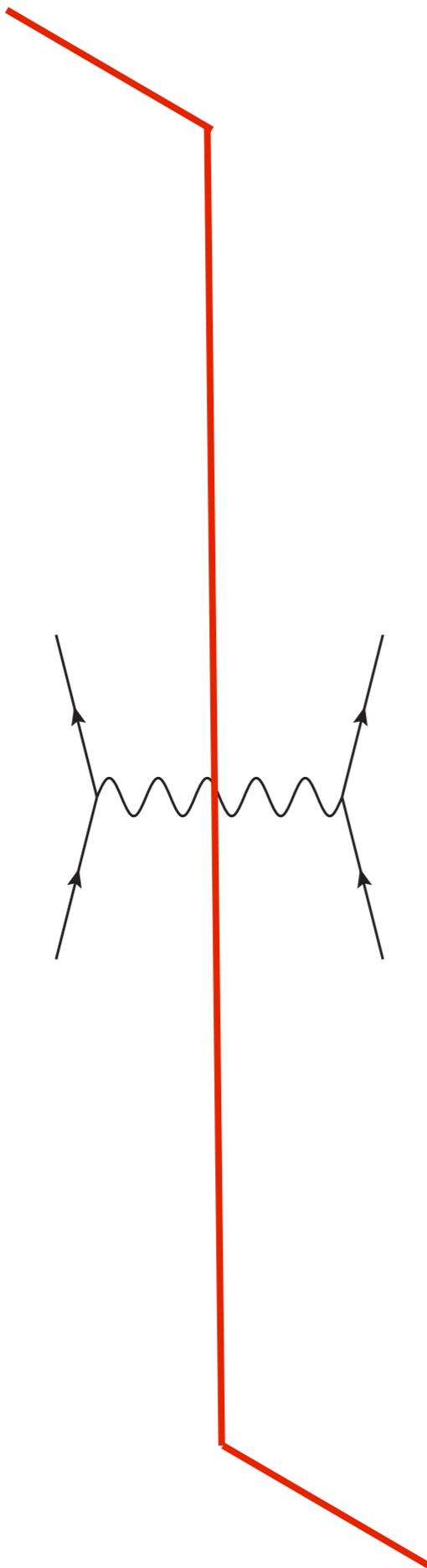
$$q\bar{q} \rightarrow V \quad \text{with} \quad V = \{W, Z, \gamma\}$$



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$$\text{Cut diagram:} \quad |\mathcal{M}|^2 = \mathcal{M}\mathcal{M}^*$$



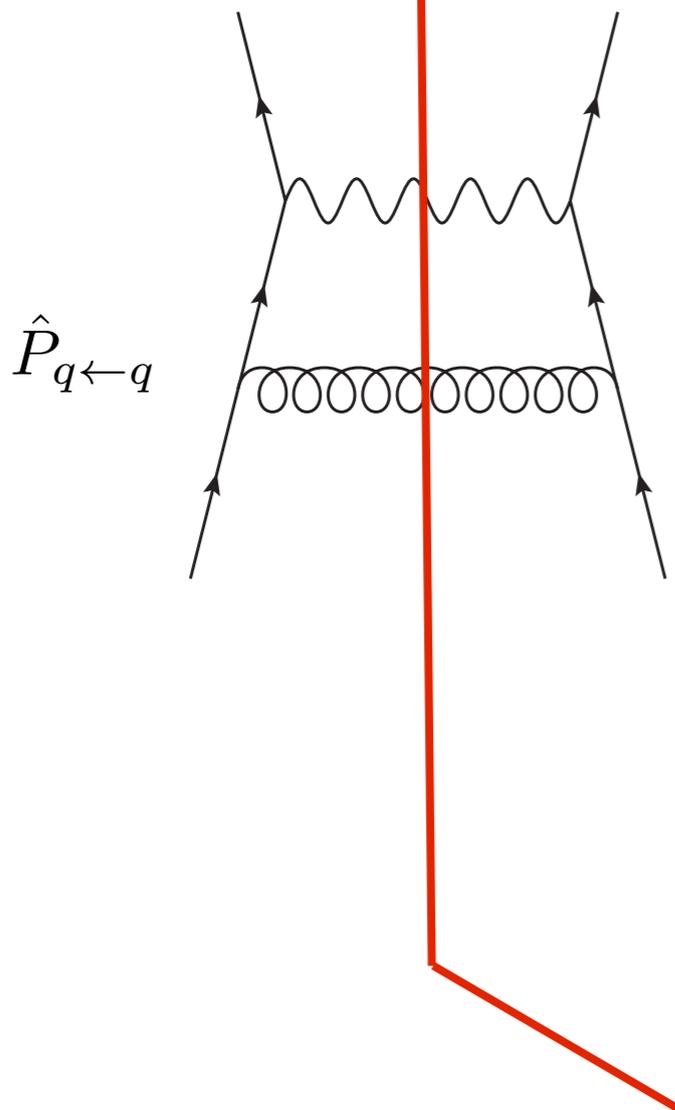
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Large logarithm from collinear gluon emission:

$$\int_{k_0^2}^{Q^2} (dk_T^2/k_T^2) \frac{\alpha_s}{2\pi} \hat{P}_{q\leftarrow q}(z)$$



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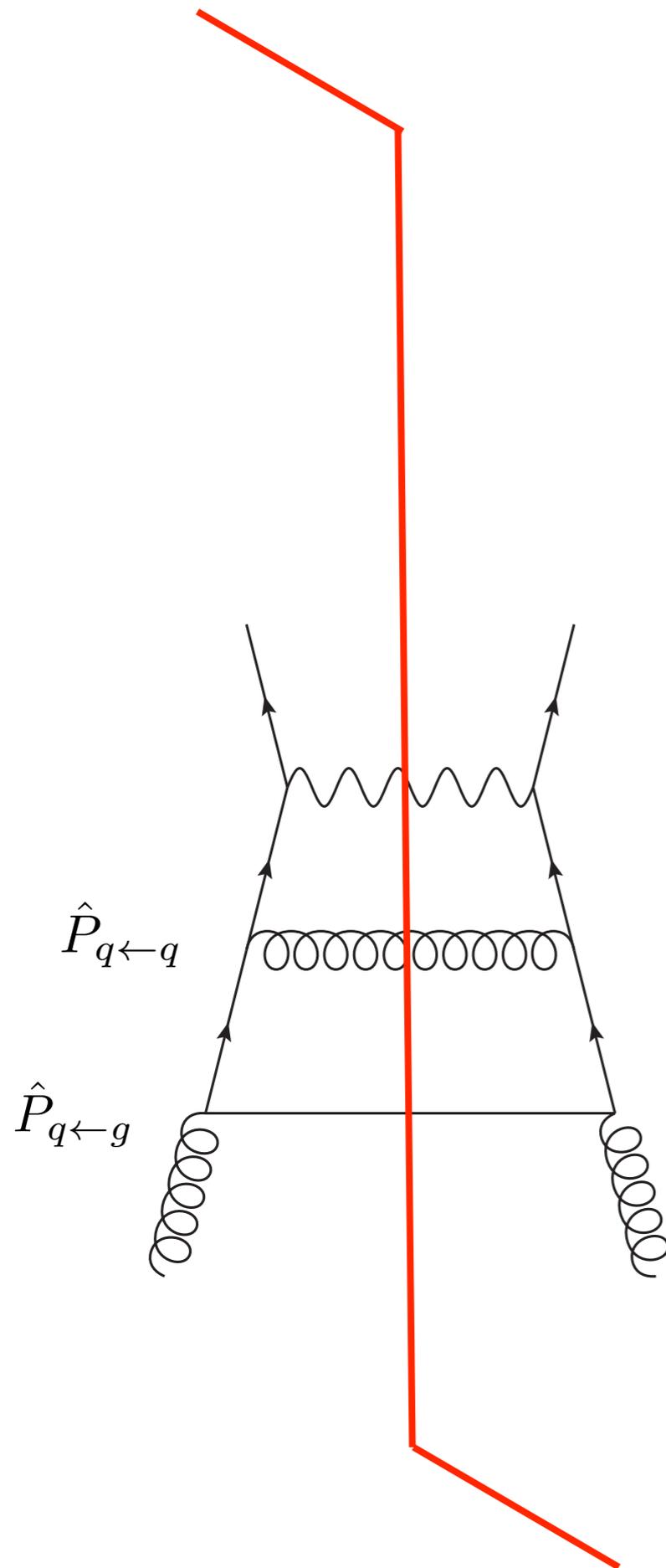
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Similar contributions from other parton splittings



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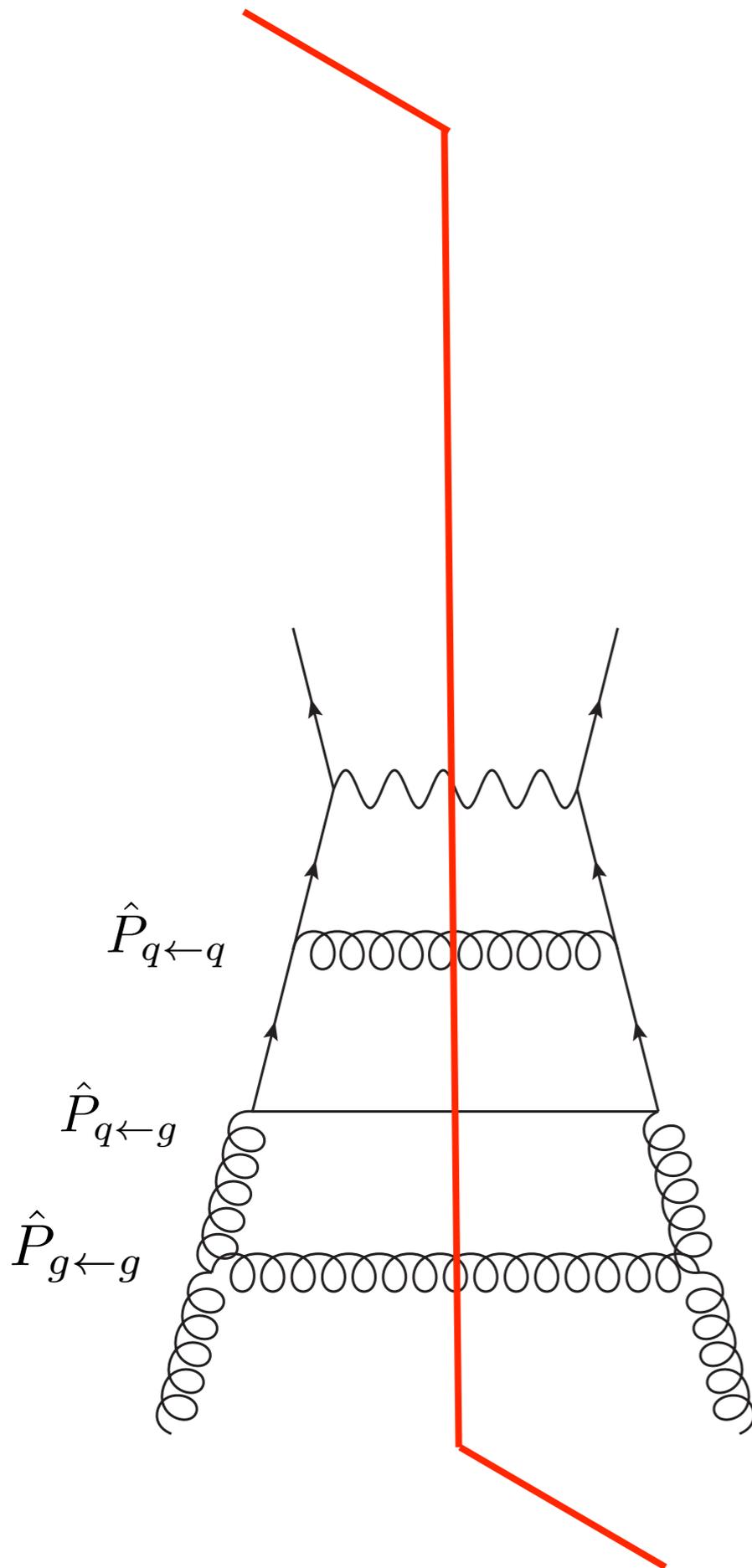
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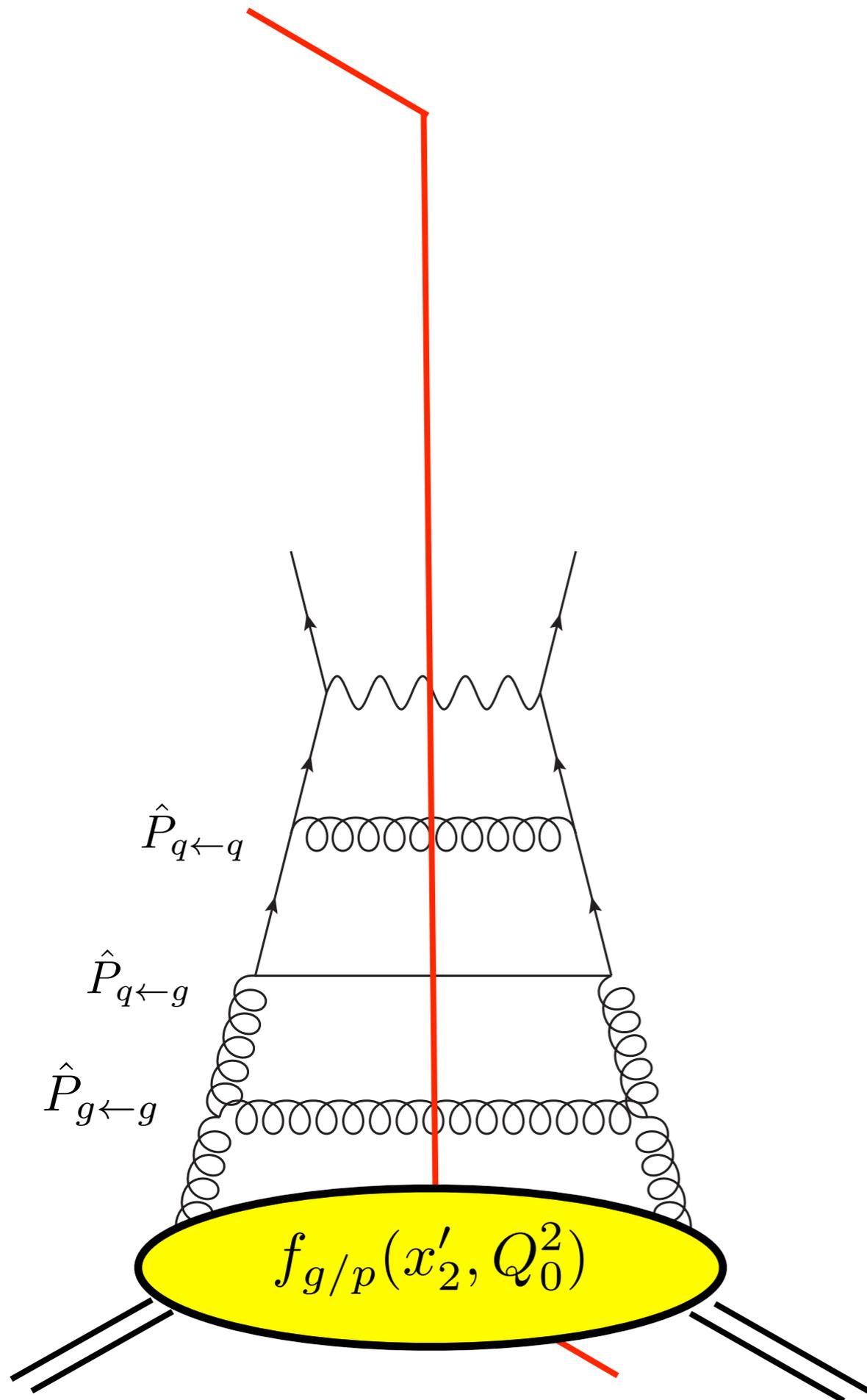
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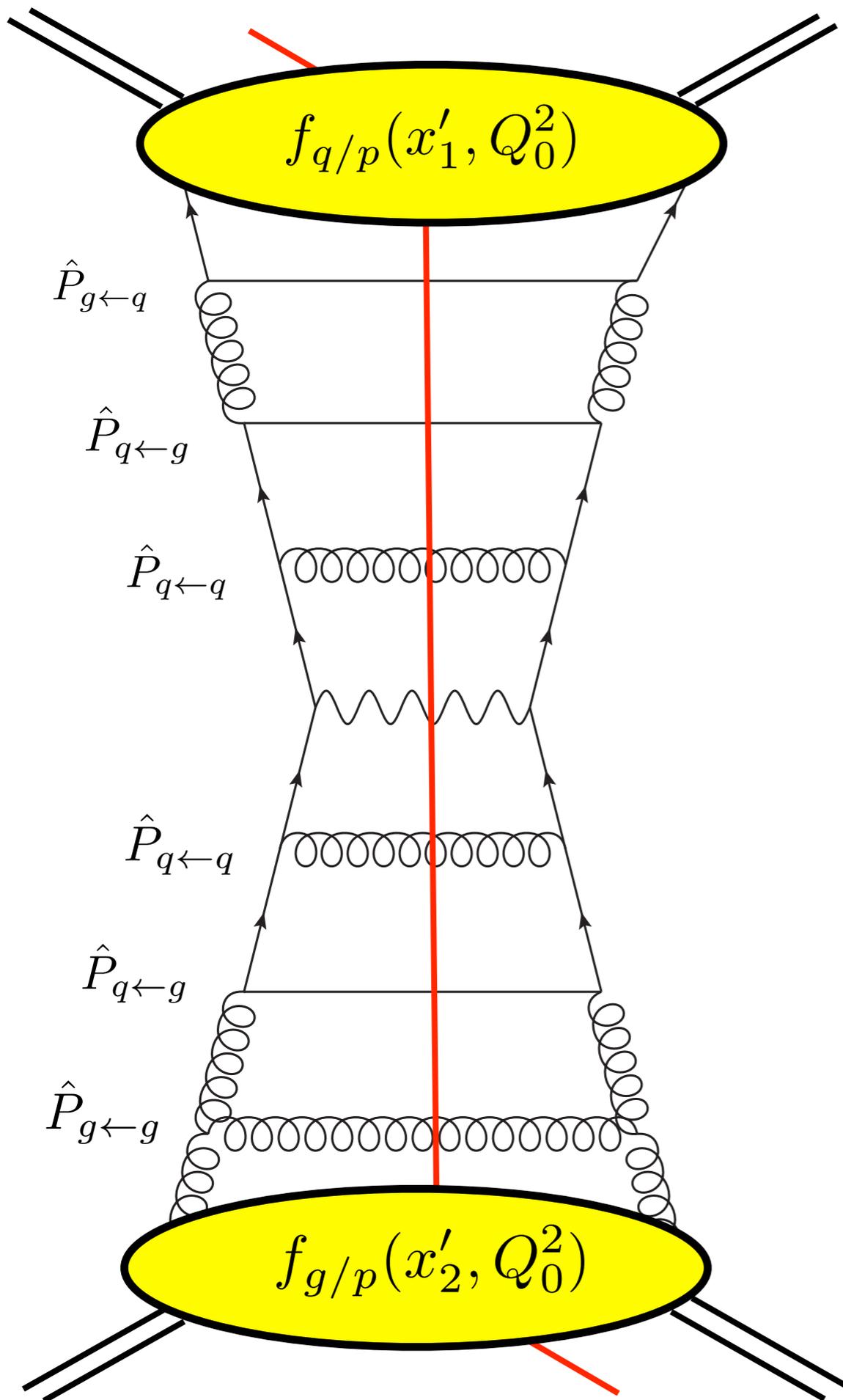
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DGLAP evolution equation:

$$\frac{\partial f_{a/p}}{\partial \ln Q^2} = \frac{\alpha_s}{2\pi} \sum_{a'=q,g} P_{a \leftarrow a'} \otimes f_{a'/p}$$





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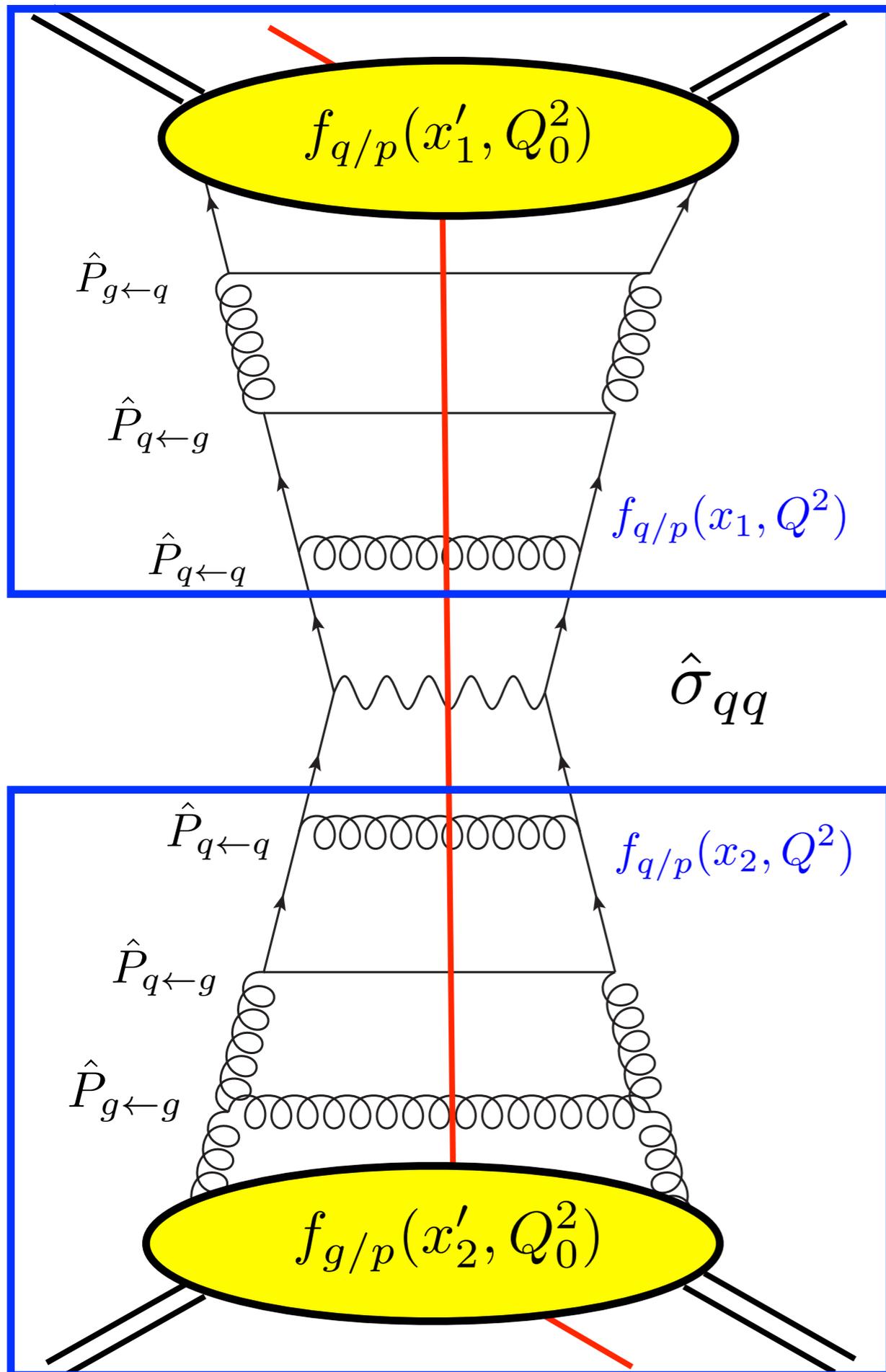
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Connect PDF at different scales

$$f_{a/p}(x, Q_0^2) \rightarrow f_{a/p}(x, Q^2)$$

## Leading-order splitting functions

$$\hat{P}_{q \leftarrow q}(z) = C_F \left[ \frac{1+z^2}{(1-z)_+} + \frac{3}{2} \delta(1-z) \right] \quad \int_0^1 dz \frac{z}{(1-z)_+} = \int_0^1 dz \frac{f(z) - f(1)}{1-z}$$

$$\hat{P}_{g \leftarrow q}(z) = C_F \left[ \frac{1+(1-z)^2}{z} \right] \quad \hat{P}_{q \leftarrow g}(z) = T_R [z^2 + (1-z)^2]$$

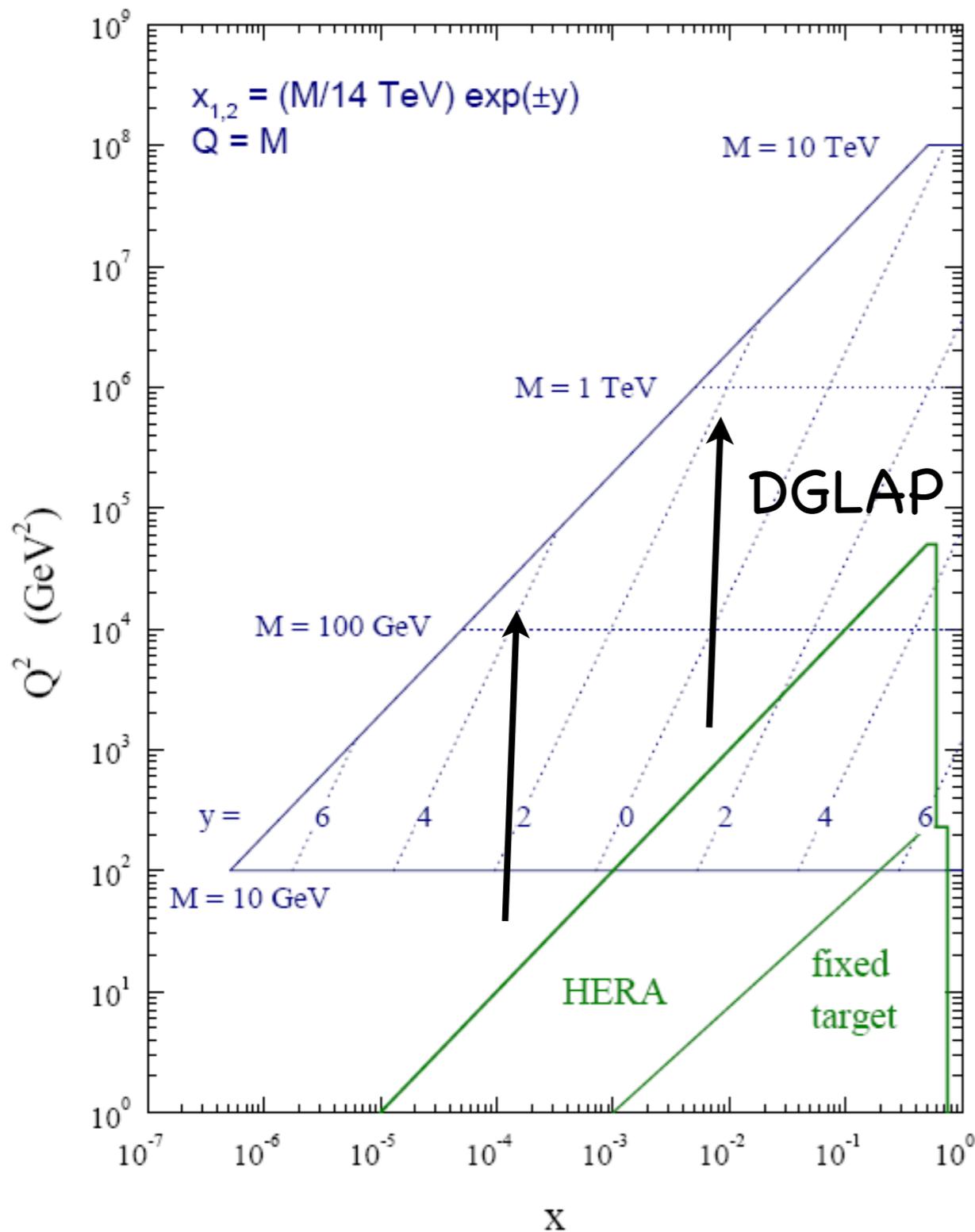
$$\hat{P}_{g \leftarrow g}(z) = 2C_A \left[ \frac{z}{(1-z)_+} + \frac{1-z}{z} + z(1-z) + \frac{1}{6}(11C_A - 4n_f T_R) \delta(1-z) \right]$$

Number- and momentum-sum rules (preserved by evolution)

$$\int_0^1 dx [f_{q/p}(x, Q_0^2) - f_{\bar{q}/p}(x, Q_0^2)] = n_q \quad (n_u = 2, n_d = 1, n_s = 0)$$

$$\sum_{a=q, \bar{q}, g} \int_0^1 dx x f_{a/p}(x, Q_0^2) = 1$$

## LHC parton kinematics

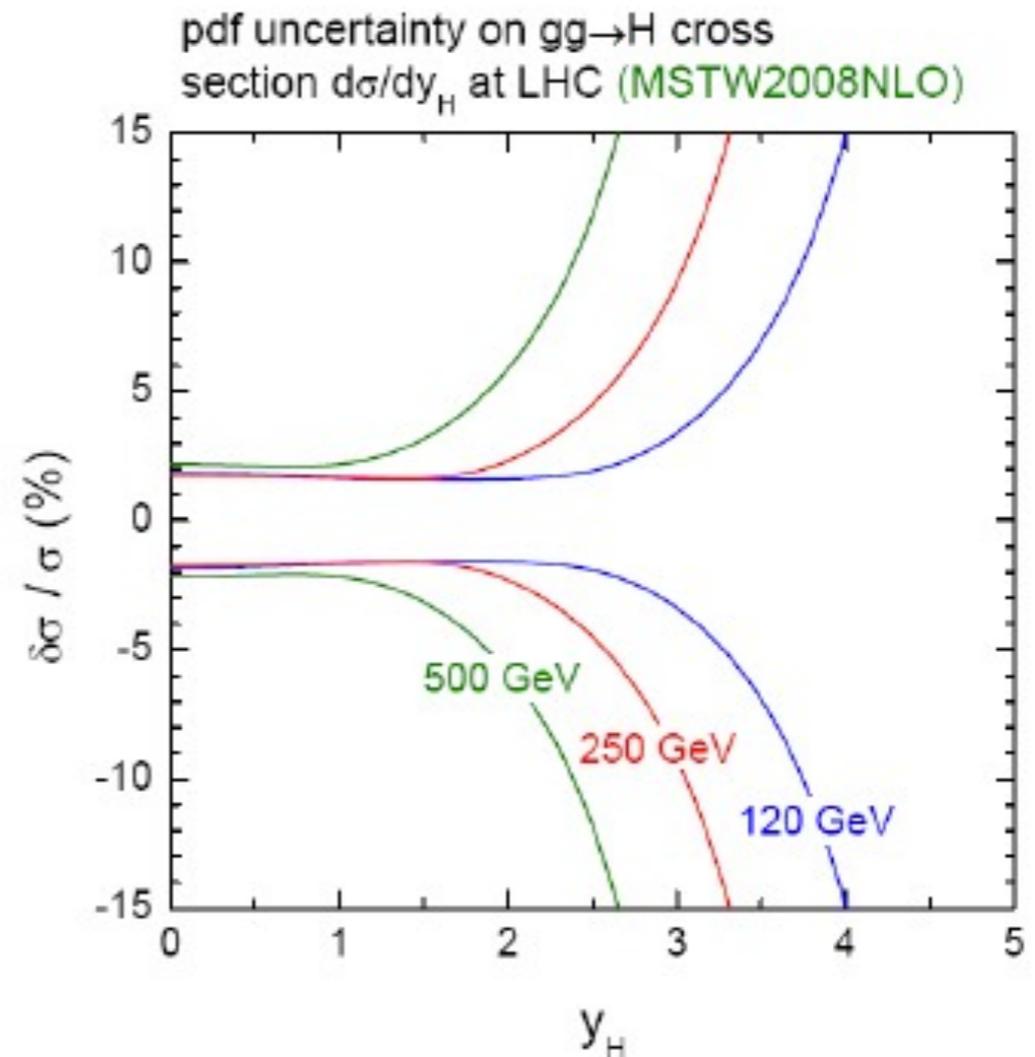


PDFs are universal

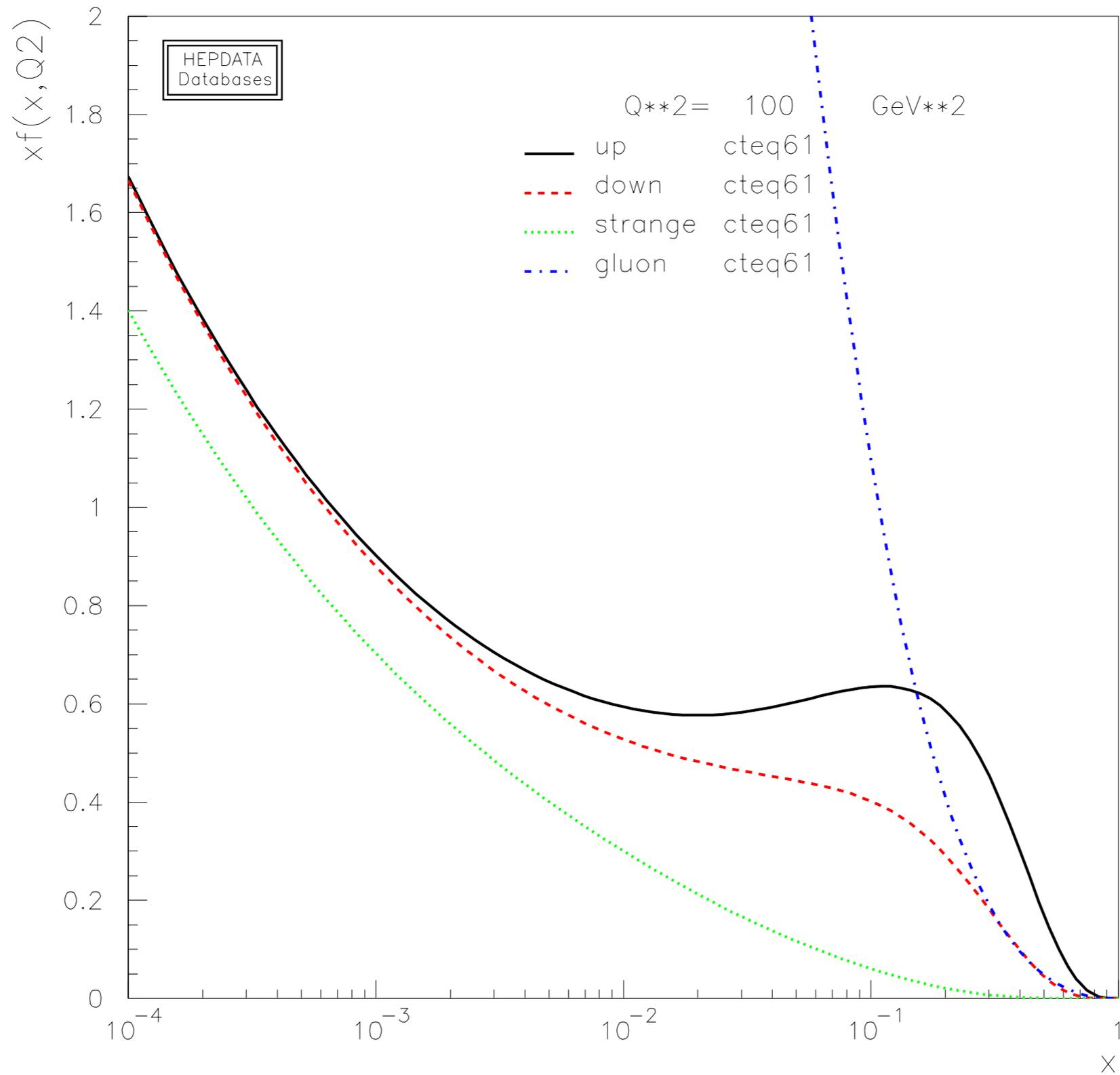
Fit from HERA, fixed-target experiments and Tevatron

DGLAP evolution gives PDFs at higher  $Q$ , e.g. for LHC

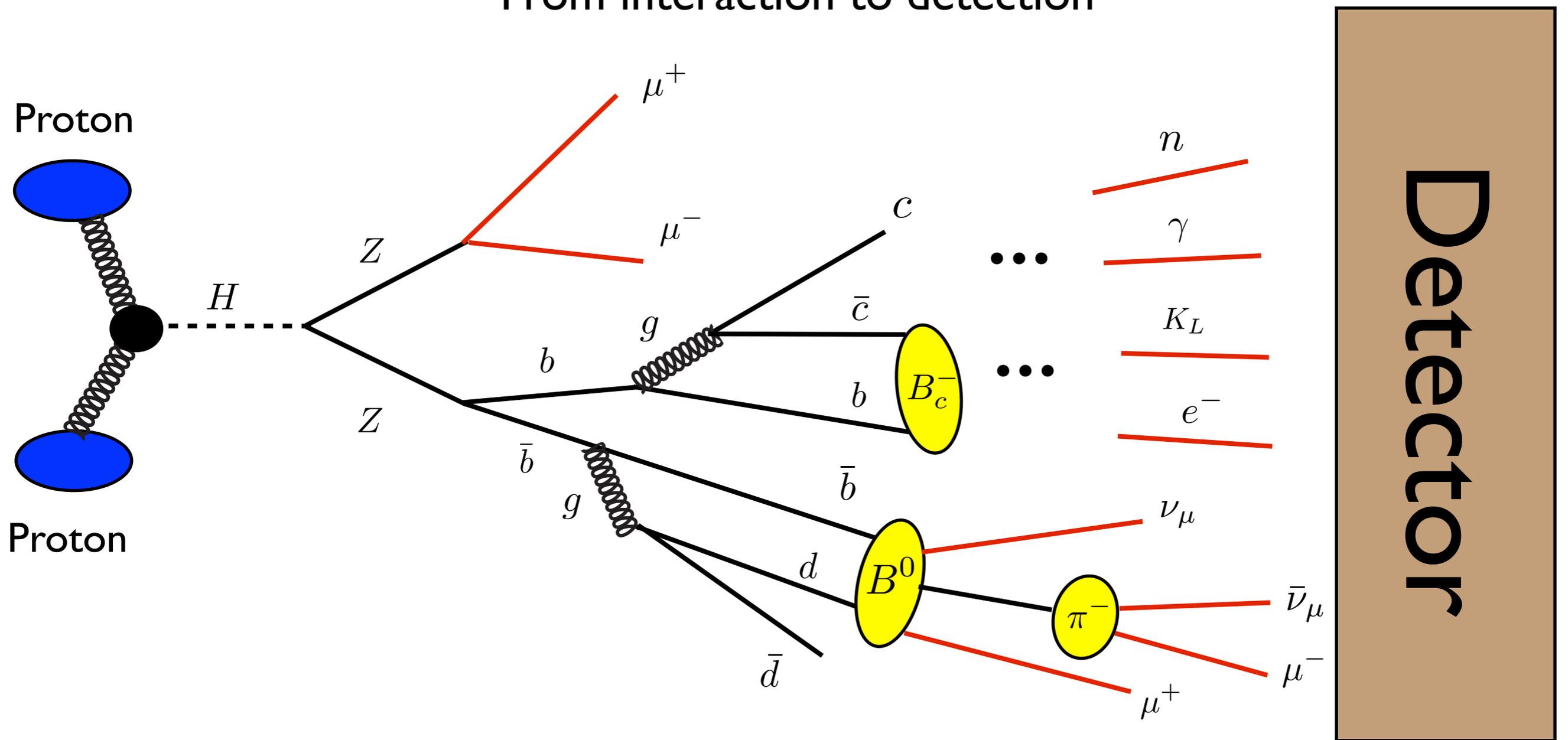
Potentially important source of theory uncertainty



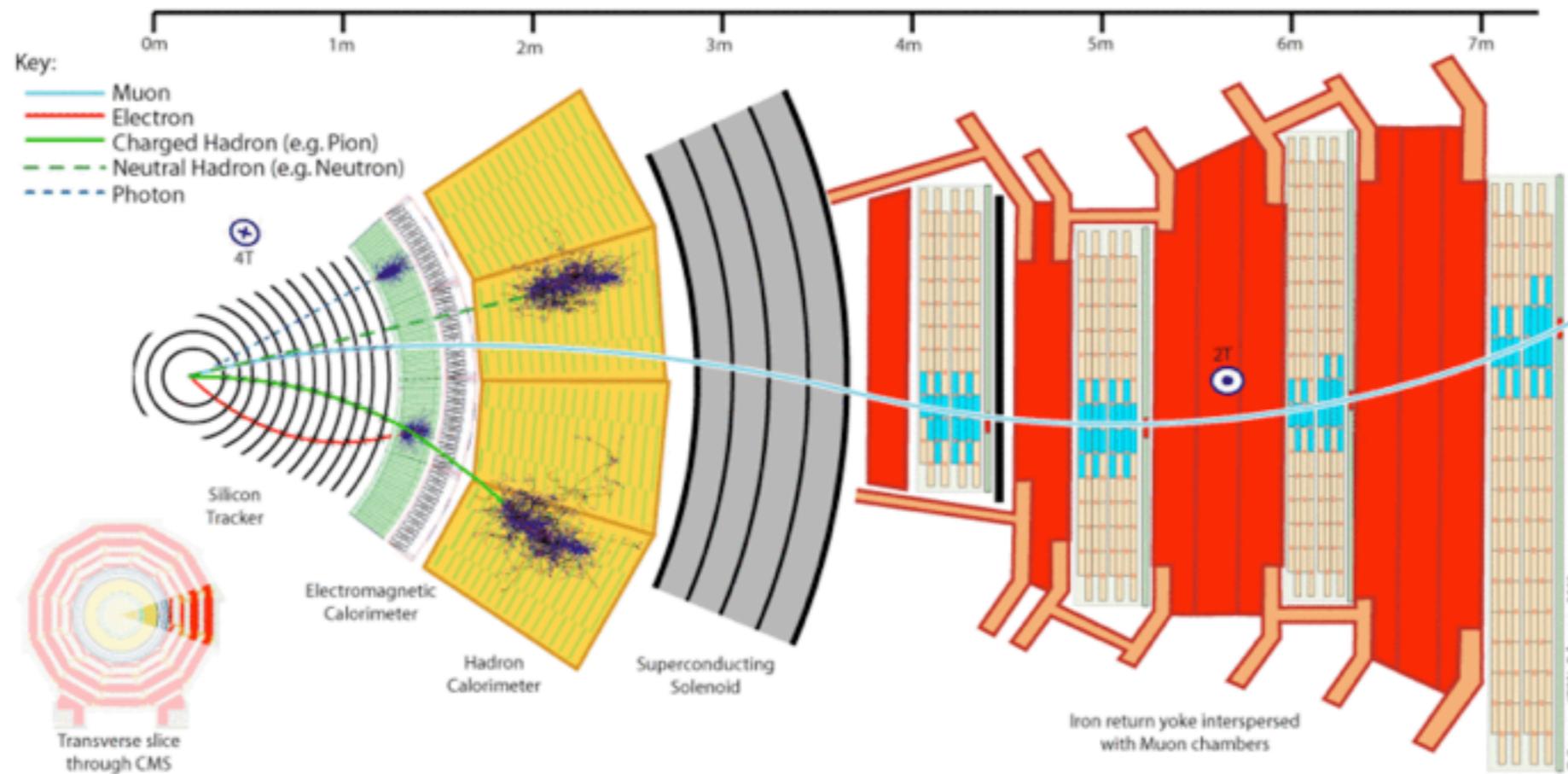
# parton distribution functions for gluon, up, down and strange



## From interaction to detection

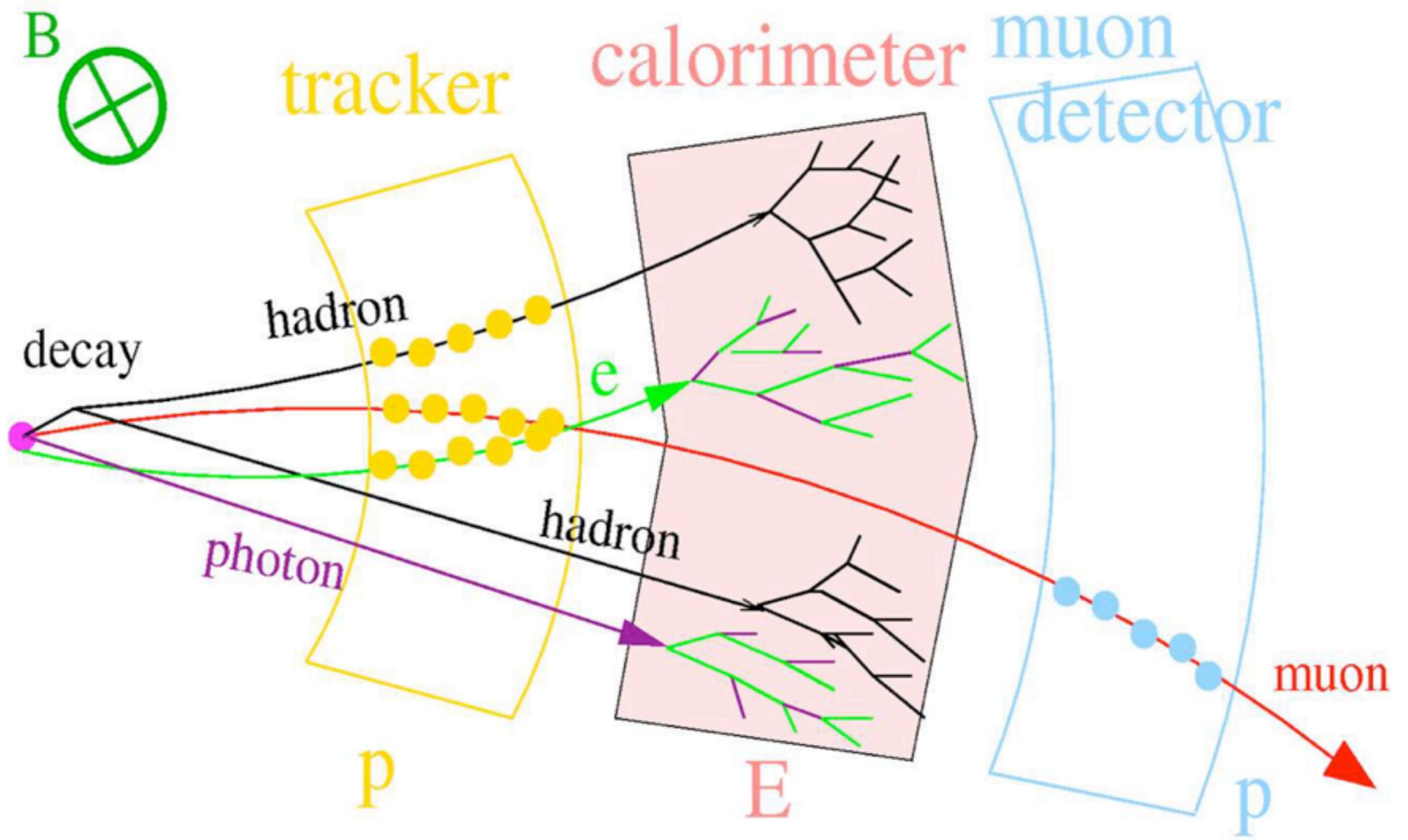


- Higgs boson generated in proton-proton collision with subsequent decay into Z bosons
- Z bosons decay into quarks (e.g. bottom quarks) or leptons (e.g. long-lived muons)
- After bottom quarks radiate gluons a process called hadronization starts. During hadronization the coloured partons are regrouped in colour-singlet mesons (2 quarks) or baryons (3 quarks)
- Most mesons/baryons are not stable and decay into long-lived hadrons or leptons before reaching the detector



- Tracker: Immediately around the interaction point the inner tracker serves to identify the tracks of individual particles and match them to the vertices from which they originated. The curvature of charged particle tracks in the magnetic field allows their charge and momentum to be measured.
- Electromagnetic Calorimeter: The Electromagnetic Calorimeter (ECAL) is designed to measure with high accuracy the energies of photons, charged leptons and hadrons.
- Hadron Calorimeter: The purpose of the Hadronic Calorimeter (HCAL) is both to measure the energy of individual hadrons produced in each event, and to be as near to hermetic around the interaction region as possible to allow events with missing energy to be identified.
- Return yoke with muon chambers: Its purpose is to identify muons and measure their momenta.

Watch also: <http://www.atlas.ch/multimedia/html-nc/feature-episode2.html>



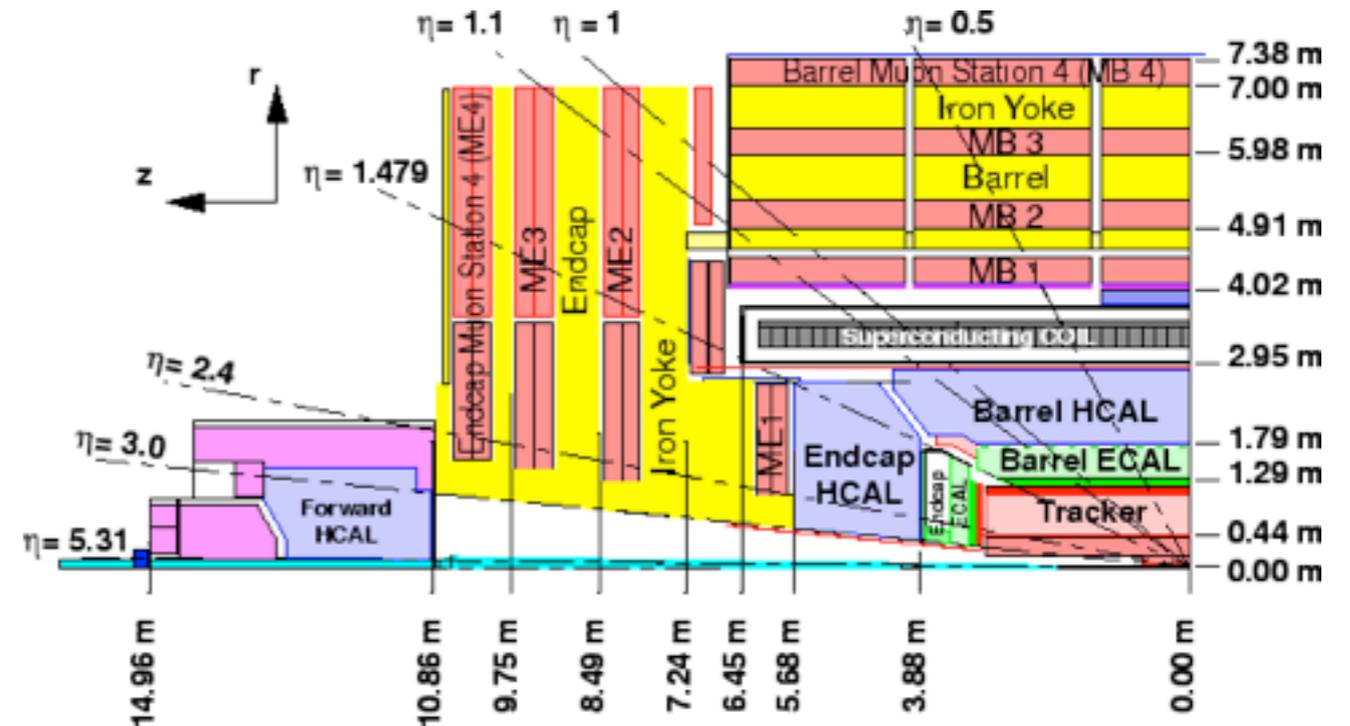
## Important phenomenological quantities

Rapidity:

$$y = \frac{1}{2} \ln \left( \frac{E + p_z}{E - p_z} \right)$$

Pseudo-Rapidity:

$$\eta = \frac{1}{2} \ln \left( \frac{|\vec{p}| + p_z}{|\vec{p}| - p_z} \right)$$



In an detector experiment, the direction along the beam axis is usually parametrized using the Rapidity or Pseudo-Rapidity

The detectors are shaped like a cylinder.  $\phi$  symbolizes the azimuthal angle in this configuration.

The transverse momentum of a particle, if we align the beam axis with the z component, is simply:  $p_T = \sqrt{p_x^2 + p_y^2}$

The 4-vector of a massless particles can be written using  $p_T$ ,  $\phi$  and  $\eta$  :

$$p = (p_T \cosh y, p_T \cos \phi, p_T \sin \phi, p_T \sinh y)$$