BUSSTEPP 2012: Questions - LHC physics

Kinematic Invariants of $2 \to 2$ Scattering

Consider a $2 \to 2$ scattering process

$$a + b \to c + d \tag{1}$$

with four momenta p_a, p_b, p_c, and p_d.

- How many independent Lorentz-invariant quantities can one construct from the incoming and outgoing four momenta? Take into account energy-momentum conservation of the scattering process.
- Show that the Mandelstam variables

$$s = (p_a + p_b)^2 \tag{2}$$
$$t = (p_a - p_c)^2 \tag{3}$$
$$u = (p_a - p_d)^2 \tag{4}$$

  suffice to fully describe the $2 \to 2$ scattering's kinematics.
- Show that

$$s + t + u = \sum_{i=1}^{4} m_i^2. \tag{5}$$

- Now assume that $m_a = m_c$ and $m_b = m_d$. Show that

$$t = -2p_{cm}^2(1 - \cos\theta_{ac}^*), \tag{6}$$
$$u = -2p_{cm}^2(1 + \cos\theta_{ad}^*) + \frac{(m_c^2 - m_d^2)^2}{s} \tag{7}$$

  $p_{cm} = \lambda^{1/2}(s, m_1^2, m_2^2)/(2\sqrt{s})$ is the momentum magnitude in the c.m. frame and $\lambda(x, y, z) = x^2 + y^2 + z^2 - 2xy - 2xz - 2yz$. θ^* is the polar angle in the c.m. frame.

Splitting function

In collinear approximation the polarization and color summed/averaged matrix element for $n + 1$ partons can be expressed by the matrix element for n partons using

$$\overline{|\mathcal{M}_{n+1}|^2} = \frac{2g_s^2}{p_a^2} P(z) \overline{|\mathcal{M}_n|^2}. \tag{8}$$

Show that the splitting kernel, $P(z)$, for a gluon splitting $g \to gg$ is given by

$$P_{g \leftarrow g}(z) = C_A \left[\frac{z}{1-z} + \frac{1-z}{z} + z(1-z) \right] \tag{9}$$

using $C_A = N_c$ and averaging over the color and polarization of the mother particle a.

Top quark phenomenology

At the LHC top quarks will be mainly produced in pairs from $gg \to t\bar{t}$. The best sensitivity in measurments of top properties is expected from the channel where one top decays leptonically and the other hadronically, i.e. $t\bar{t} \to b\bar{b}l\nu\bar{q}q'$.

- Show that the measured charged lepton momentum, $E_{T,mis}$ and the known W boson mass suffice to reconstruct the longitudinal component of the neutrino momentum (except for a twofold ambiguity).

- Consider the decay of a top quark. Assuming the top is at rest, show that $p_{T,b}$ has a Jacobian peak. What is the position of this peak?
- Express the mass of the leptonically dacaying top quark using the b quark's four-momentum and the neutrino momentum calculated before. Based on what you know about the reconstruction of physical objects, from where come the largest uncertainties in measuring the top mass this way?