BUSSTEPP 2012
Standard Model and Beyond
Exercises

Part I
The Standard Model

1 The Weyl formalism

Consider two complex doublets ψ_α and χ_α transforming as $\psi \to U\psi$, $\chi \to U\chi$ under $U \in$ SU(2) transformations. Determine the most general SU(2)-invariant bilinear involving one out of ψ and ψ^* and one out of χ and χ^*.

Assume now that ψ and χ are left-handed Lorentz spinors. The Lorentz transformations act on them through SL(2,C), the set of 2×2 complex matrices with unit determinant: $\psi \to L\psi$, $\chi \to L\chi$, $L \in$ SL(2,C). Determine the most general SL(2,C)-invariant bilinear involving one out of ψ and ψ^* and one out of χ and χ^*. Use the relation between $L \in$ SL(2,C) and the Lorentz transformation Λ given by $L(x_\mu \sigma^\mu)L^\dagger = (\Lambda^\nu_\mu x_\nu)\sigma^\mu$.

Consider now a system of 4 left-handed Lorentz spinors $\psi_1, \ldots \psi_4$. Determine the most general Lorentz invariant involving all the four fields (or their conjugated). You may want to use the following relation involving the Pauli matrices σ_i, $i = 1, 2, 3$: $\sum_i (\sigma_i)_{ab}(\sigma_i)_{cd} = 2\delta_{ad}\delta_{bc} - \delta_{ab}\delta_{cd}$.

2 The Standard Model is chiral

Consider the left-handed fermion content of the Standard Model shown in the lectures:

	$\mathrm{SU}(3)_c$	$\mathrm{SU}(2)_L$	Y
q_i	3	2	1/6
u_i^c	$\overline{3}$	1	-2/3
d_i^c	$\overline{3}$	1	1/3
l_i	1	2	-1/2
e_i^c	1	1	1

Remembering that the most generic mass term one can build out of the Weyl fermions $\psi_1, \ldots, \psi_n$ is $m_{ij}\psi_i\psi_j/2 + \text{h.c.}$, show that no gauge invariant mass term is allowed in the Standard Model.

3 The effective four-fermion Fermi operator

Consider the neutrino scattering process $\nu_\mu e \to \mu \nu_e$. Derive the four-fermion effective lagrangian density describing that process at energies much smaller than the W mass (in the Dirac formalism).

What is the parametric dependence of the cross section σ on the center of mass energy of the process E for $E \ll M_W$ and $E \gg M_W$?

4 Higgs decay into W^+W^-

The Higgs doublet can be written in the unitary gauge as

$$H = \begin{pmatrix} 0 \\ v + \dfrac{h}{\sqrt{2}} \end{pmatrix},$$

where h is the physical Higgs field and $v = \sqrt{2} M_W / g$ (g is the $\mathrm{SU}(2)_L$ coupling).

- Write the hW^+W^- Lagrangian interaction term responsible of the Higgs decay into W^+W^-.
- Calculate the total Higgs decay width into W^+W^- at the tree level.
- In the limit in which the hypercharge $\mathrm{U}(1)_Y$ gauge coupling is negligible, how are the decay widths into W^+W^- and ZZ bosons related?

5 The custodial symmetry

Suppose the SM Higgs was an SU(2) triplet with hypercharge Y_H and the Higgs potential generates a vacuum expectation value for the Higgs. What would be the values of Y_H allowing $Q = T_3 + Y$ to be conserved? What would be the values of Y_H in which all generators of SU(2)×U(1) are broken except those proportional to Q? Suppose that $Y_H = 1$ and that the Higgs obtains a vev that indeed does not break Q. Calculate the gauge boson masses generated by the Higgs gauge couplings. What would be the tree-level value of the ρ parameter?

6 The Higgs potential

Starting from the Higgs potential shown in the lectures, $V(H) = -\mu^2 H^\dagger H + \lambda_H (H^\dagger H)^2/2$ and using the unitary gauge shown in exercise 4, recover the Higgs potential for the physical degree of freedom h and in particular its mass.