BUSSTEPP

Neutrino physics II: Neutrino Oscillation Phenomenology

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Outline

Lepton mixing

Neutrino oscillations
- Oscillations in vacuum
- Oscillations in matter
- Varying matter density and MSW

Global data and 3-flavour oscillations
- The θ_{13} revolution

Outlook
- CPV, mass hierarchy

Outline

Flavour neutrinos

A neutrino of flavour α is defined by the charged current interaction with the corresponding charged lepton:

$$\mathcal{L}_{\rm CC} = -\frac{g}{\sqrt{2}} W^\rho \sum_{\alpha=e,\mu,\tau} \bar{\nu}_{\alpha L} \gamma_\rho \ell_{\alpha L} + \text{h.c.}$$

for example

$$\pi^+ \to \mu^+ \nu_\mu$$

the muon neutrino ν_μ comes together with the charged muon μ^+

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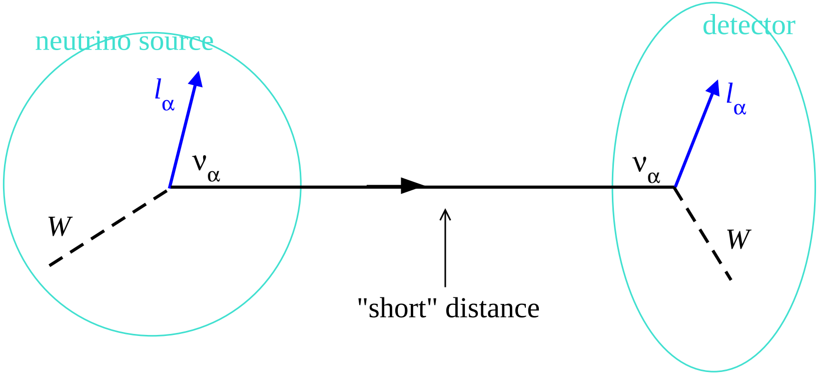
Flavour neutrinos



Let's give mass to the neutrinos

Majorana mass term:

$$\mathcal{L}_{\mathrm{M}} = -\frac{1}{2} \sum_{\alpha,\beta=e,\mu,\tau} \nu_{\alpha L}^{T} C^{-1} \mathcal{M}_{\alpha\beta} \nu_{\beta L} + \mathrm{h.c.}$$

$\mathcal{M}$: symmetric mass matrix

In the basis where the CC interaction is diagonal the mass matrix is in general not a diagonal matrix

any complex symmetric matrix $\mathcal{M}$ can be diagonalised by a unitary matrix

$$U_\nu^T \mathcal{M} U_\nu = m \,, \qquad m : \text{ diagonal, } m_i \geq 0$$

Lepton mixing

$$\mathcal{L}_{\rm CC} = -\frac{g}{\sqrt{2}} W^\rho \sum_{\alpha=e,\mu,\tau} \sum_{i=1}^{3} \bar{\nu}_{iL} U^*_{\alpha i} \gamma_\rho \ell_{\alpha L} + \text{h.c.}$$

$$\mathcal{L}_{\rm M} = -\frac{1}{2} \sum_{i=1}^{3} \nu_{iL}^T C^{-1} \nu_{iL} m_i^\nu - \sum_{\alpha=e,\mu,\tau} \bar{\ell}_{\alpha R} \ell_{\alpha L} m_\alpha^\ell + \text{h.c.}$$

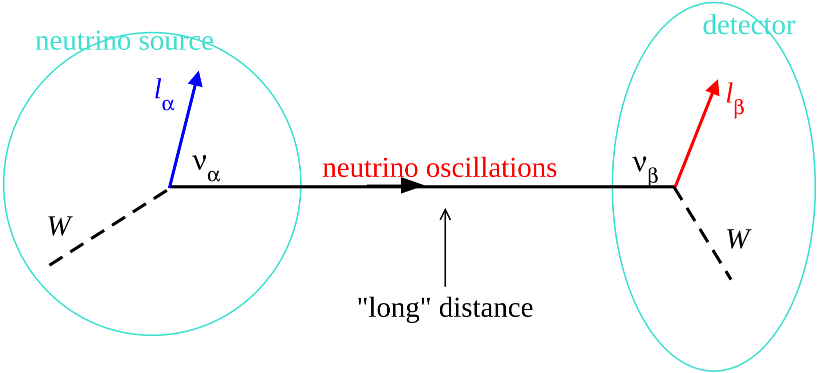
The unitary lepton mixing matrix:

$$(U_{\alpha i}) \equiv U_{\rm PMNS} = V^{\rm Dirac}\ D^{\rm Maj}$$

$$D^{\rm Maj} = \text{diag}(e^{i\alpha_i/2})$$

Outline

Neutrino oscillations



$|\nu_\alpha\rangle = U^*_{\alpha i}|\nu_i\rangle$ $\quad e^{-i(E_i t - p_i x)}$ $\quad |\nu_\beta\rangle = U^*_{\beta i}|\nu_i\rangle$

Neutrino oscillations in vacuum

oscillation amplitude:

$$\mathcal{A}_{\nu_\alpha \to \nu_\beta} = \langle \nu_\beta | \quad \text{propagation} \quad | \nu_\alpha \rangle$$

$$= \sum_{i,j} U_{\beta j} \langle \nu_j | \quad e^{-i(E_i t - p_i x)} \quad | \nu_i \rangle U^*_{\alpha i} = \sum_i U_{\beta i} U^*_{\alpha i} e^{-i(E_i t - p_i x)}$$

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oscillation probability:

$$P_{\nu_\alpha \to \nu_\beta} = \left| \mathcal{A}_{\nu_\alpha \to \nu_\beta} \right|^2$$

Neutrino oscillations in vacuum

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oscillation probability:

$$P_{\nu_\alpha \to \nu_\beta} = \left| \mathcal{A}_{\nu_\alpha \to \nu_\beta} \right|^2$$

To derive the oscillation probability rigorously one needs either a wave-packet treatment or field theory Akhmedov, Kopp, JHEP 1004:008 (2010) [1001.4815]

The oscillation probability in vacuum

$$\mathcal{A}_{\nu_\alpha\to\nu_\beta} = \sum_i U_{\beta i} U^*_{\alpha i} e^{-i(E_i t - p_i x)}$$

$$P_{\nu_\alpha\to\nu_\beta}(L) = \sum_{jk} U_{\alpha j} U^*_{\beta j} U^*_{\alpha k} U_{\beta k} \exp\left[-i\frac{\Delta m^2_{kj} L}{2\,E_\nu}\right]$$

$\Delta m^2_{kj} \equiv m^2_k - m^2_j$: oscillations are sensitive only to mass-squared differences (not to absolute mass!)

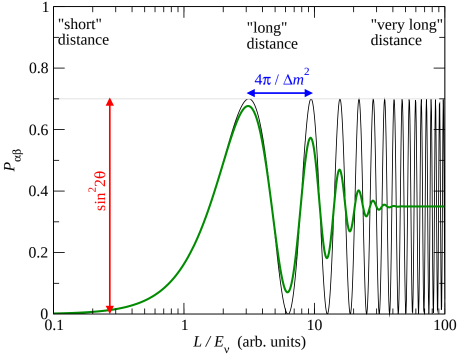
to observe oscillations one needs

- non-trivial mixing $U_{\alpha i}$
- non-zero mass-squared differences Δm^2_{kj}
- a suitable value for L/E_ν

2-neutrino oscillations

Two-flavour limit:

$$U = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}, \qquad P = \sin^2 2\theta \, \sin^2 \frac{\Delta m^2 L}{4E_\nu}$$



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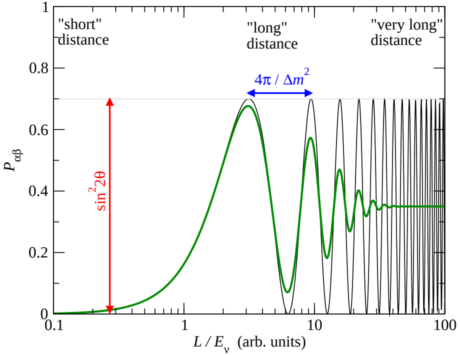


$$\frac{\Delta m^2 L}{4E_\nu} = 1.27 \frac{\Delta m^2[\mathrm{eV}^2]\, L[\mathrm{km}]}{E_\nu[\mathrm{GeV}]}$$

Appearance vs. disappearance

- appearance experiments:

$$P_{\nu_\alpha \to \nu_\beta} \,, \qquad \alpha \neq \beta$$

"appearance" of a neutrino of a new flavour $\beta \neq \alpha$ in a beam of ν_α

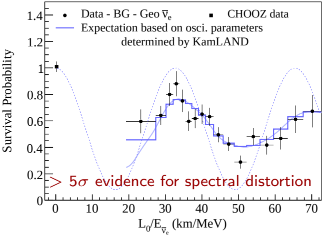
- disappearance experiments:

$$P_{\nu_\alpha \to \nu_\alpha}$$

measurement of the "survival" probability of a neutrino of given flavour

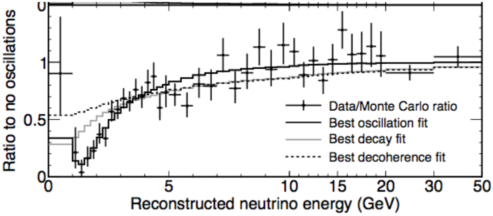
Neutrinos oscillate!

KamLAND ($\bar{\nu}_e \to \bar{\nu}_e$)



$> 5\sigma$ evidence for spectral distortion

MINOS ($\nu_\mu \to \nu_\mu$)



$$P_{\text{survival}} \approx 1 - \sin^2 2\theta \sin^2\left(\frac{\Delta m^2}{4}\frac{L}{E}\right)$$

General properties of vacuum oscillations

$$P_{\nu_\alpha \to \nu_\beta} = \sum_{jk} U_{\alpha j} U^*_{\beta j} U^*_{\alpha k} U_{\beta k} \exp\left[-i \frac{\Delta m^2_{kj} L}{2 E_\nu}\right]$$

- Unitarity: $\sum_\beta P_{\nu_\alpha \to \nu_\beta} = 1$
- For anti-neutrinos replace $U_{\alpha i} \to U^*_{\alpha i}$
- $P_{\nu_\alpha \to \nu_\beta} = P_{\bar\nu_\beta \to \bar\nu_\alpha}$ (CPT invariance)
- Phases in U induce CP violation: $P_{\nu_\alpha \to \nu_\beta} \neq P_{\bar\nu_\alpha \to \bar\nu_\beta}$
- there is no CP violation in disappearance experiments:

$$P_{\nu_\alpha \to \nu_\alpha} = P_{\bar\nu_\alpha \to \bar\nu_\alpha} = \sum_{k,j} |U_{\alpha k}|^2 |U_{\alpha j}|^2 e^{-i \Delta m^2_{kj} L/2E}$$

Eff. Schrödinger equation

The evolution of the flavour state can be described by an effective Schrödinger equation:

$$i\frac{d}{dt}\begin{pmatrix} a_e \\ a_\mu \\ a_\tau \end{pmatrix} = H_{\rm vac}\begin{pmatrix} a_e \\ a_\mu \\ a_\tau \end{pmatrix}$$

where

$$\begin{aligned} H^{\nu}_{\rm vac} &= U \mathrm{diag}\left(0, \frac{\Delta m^2_{21}}{2E_\nu}, \frac{\Delta m^2_{31}}{2E_\nu}\right) U^\dagger \\ H^{\bar\nu}_{\rm vac} &= U^* \mathrm{diag}\left(0, \frac{\Delta m^2_{21}}{2E_\nu}, \frac{\Delta m^2_{31}}{2E_\nu}\right) U^T \end{aligned}$$

The matter effect

When neutrinos pass through matter the interactions with the particles in the background induce an effective potential for the neutrinos

The coherent forward scattering amplitude leads to an index of refraction for neutrinos

L. Wolfenstein, Phys. Rev. D **17**, 2369 (1978); *ibid.* D **20**, 2634 (1979)

Effective Hamiltonian in matter

$$H^{\nu}_{\rm mat} = U \text{diag}\left(0, \frac{\Delta m^2_{21}}{2E_\nu}, \frac{\Delta m^2_{31}}{2E_\nu}\right) U^\dagger + \text{diag}(\sqrt{2} G_F N_e, 0, 0)$$

$$H^{\bar\nu}_{\rm mat} = \underbrace{U^* \text{diag}\left(0, \frac{\Delta m^2_{21}}{2E_\nu}, \frac{\Delta m^2_{31}}{2E_\nu}\right) U^T}_{H_{\rm vac}} - \underbrace{\text{diag}(\sqrt{2} G_F N_e, 0, 0)}_{V_{\rm mat}}$$

$N_e(x)$: electron density along the neutrino path

matter effect introduces "enviromental" CP violation
(even for real U and in disappearance experiments)

for non-constant matter the Hamiltonian depends on time:

$$i \frac{d}{dt} a = H_{\rm mat}(t) a$$

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Effective matter potential - 1

Effective 4-point interaction Hamiltonian in the SM

$$H_{\rm int}^{\nu_\alpha} = \frac{G_F}{\sqrt{2}}\,\bar\nu_\alpha\gamma_\mu(1-\gamma_5)\nu_\alpha \underbrace{\sum_f \bar f\gamma^\mu(g_V^{\alpha,f} - g_A^{\alpha,f}\gamma_5)f}_{J^\mu_{\rm mat}}$$

ordinary matter: e^-, p, n

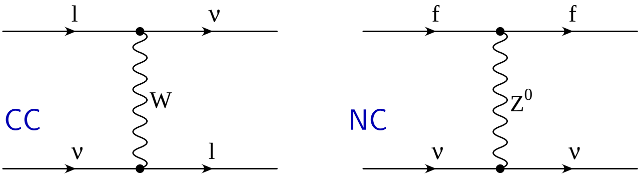
non-relativistic: $\langle \bar f\gamma^\mu f\rangle = \frac{1}{2}N_f\delta_{\mu 0}$

unpolarised: $\langle \bar f\gamma_5\gamma^\mu f\rangle = 0$

neutral: $N_e = N_p$

Effective matter potential - 2

$$V_{\mathrm{mat}} = \sqrt{2} G_F \mathrm{diag}\left(N_e - N_n/2, -N_n/2, -N_n/2\right)$$



- only ν_e feel CC (there are no μ, τ in normal matter)
- NC is the same for all flavours $\Rightarrow$ potential proportional to identiy has no effect on the evolution
- NC has no effect for 3-flavour active neutrinos, but is important in the presence of sterile neutrinos

Neutrino oscillations in constant matter

diagonalize the Hamiltonian in matter:

$$\begin{aligned} H^{\nu}_{\mathrm{mat}} &= U \mathrm{diag}\left(0, \frac{\Delta m^2_{21}}{2E_\nu}, \frac{\Delta m^2_{31}}{2E_\nu}\right) U^\dagger + \mathrm{diag}(\sqrt{2} G_F N_e, 0, 0) \\ &= U_m \mathrm{diag}\left(\lambda_1, \lambda_2, \lambda_3\right) U_m^\dagger \end{aligned}$$

Same expression for oscillation probability, but replace "vacuum" parameters by "matter" parameters

2-neutrino oscillations in constant matter

Two-flavour case:

$$P_{\rm mat} = \sin^2 2\theta_{\rm mat} \, \sin^2 \frac{\Delta m^2_{\rm mat} L}{4E}$$

with

$$\sin^2 2\theta_{\rm mat} = \frac{\sin^2 2\theta}{\sin^2 2\theta + (\cos 2\theta - A)^2}$$

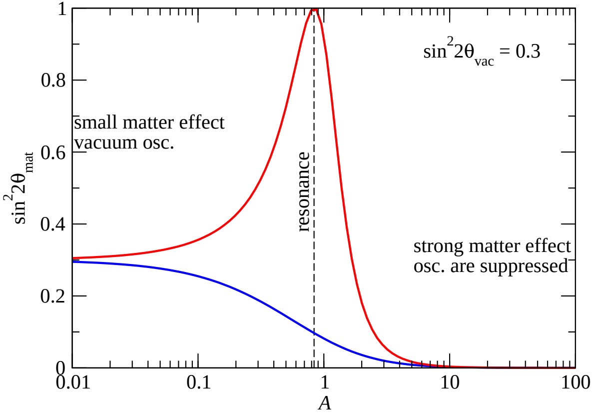
$$\Delta m^2_{\rm mat} = \Delta m^2 \sqrt{\sin^2 2\theta + (\cos 2\theta - A)^2}$$

$$A \equiv \frac{2EV}{\Delta m^2}$$

2-neutrino oscillations in constant matter

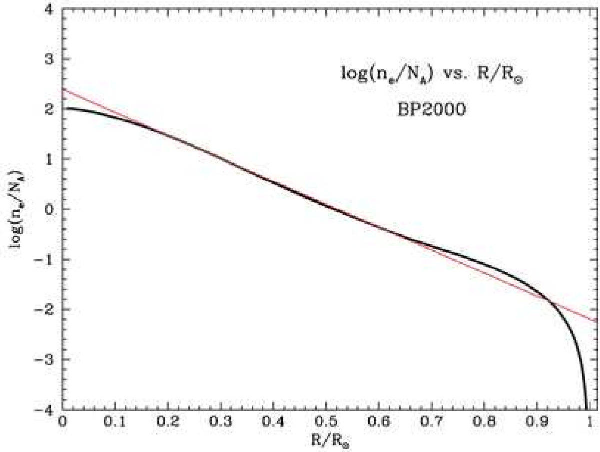
$$\sin^2 2\theta_{\rm mat} = \frac{\sin^2 2\theta}{\sin^2 2\theta + (\cos 2\theta - A)^2} \qquad A \equiv \frac{2EV}{\Delta m^2}$$

resonance for $\cos 2\theta = A$: "MSW resonance" Mikheev, Smirnov, Sov. J. Nucl. Phys. 42, 913 (1985)



Varying matter density: example solar neutrinos

The electron density in the sun:



The LMA-MSW mechanism

evolution is adiabatic if $\left(\frac{1}{\theta_m}\frac{d\theta_m}{dx}\right)^{-1} \gg L_\mathrm{osc}$

using $\Delta m^2 = 8 \times 10^{-5}$ eV^2 the oscillation length is

$$L_\mathrm{osc} = \frac{4\pi E}{\Delta m^2} \simeq 30\,\mathrm{km}\left(\frac{E}{\mathrm{MeV}}\right)$$

for large mixing angles ($\sin^2\theta_{12} \simeq 0.3$):

$$\left(\frac{1}{\theta_m}\frac{d\theta_m}{dx}\right)^{-1} \sim \left(\frac{1}{V}\frac{dV}{dx}\right)^{-1} \sim \text{size of sun} \gg 30\,\mathrm{km}$$

$\Rightarrow$ **adiabatic evolution**

The LMA-MSW mechanism

the electron neutrino is born at the center of the sun as

$$|\nu_e\rangle = \cos\theta_m |\nu_1\rangle + \sin\theta_m |\nu_2\rangle$$

then $|\nu_1\rangle$ and $|\nu_2\rangle$ evolve adiabatically to the Earth

$$P_{ee} = P_{e1}^{\rm prod} P_{1e}^{\rm det} + P_{e2}^{\rm prod} P_{2e}^{\rm det}$$

$P_{e3}^{\rm prod} \approx \sin^2\theta_{13} \approx 0$, interference term averages out

$$P_{e1}^{\rm prod} = \cos^2\theta_m\,, \quad P_{1e}^{\rm det} = \cos^2\theta$$
$$P_{e2}^{\rm prod} = \sin^2\theta_m\,, \quad P_{2e}^{\rm det} = \sin^2\theta$$

$$\Rightarrow \qquad P_{ee} = \cos^2\theta_m \cos^2\theta + \sin^2\theta_m \sin^2\theta$$

The LMA-MSW mechanism

in the center of the sun we have

$$A \equiv \frac{2EV}{\Delta m^2} \simeq 0.2 \left(\frac{E_\nu}{\mathrm{MeV}}\right) \left(\frac{8 \times 10^{-5}\,\mathrm{eV}^2}{\Delta m^2}\right)$$

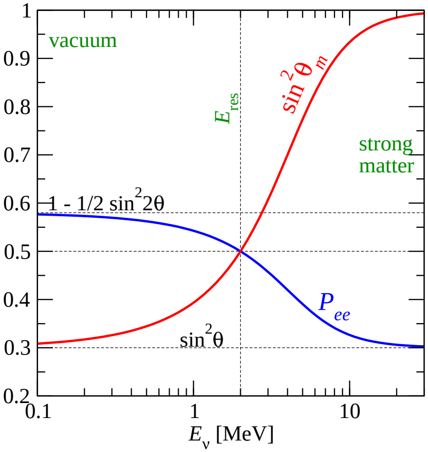
resonance occurs for

$$A = \cos 2\theta = 0.4$$

$$\Rightarrow \qquad E_{\mathrm{res}} \simeq 2\,\mathrm{MeV}$$

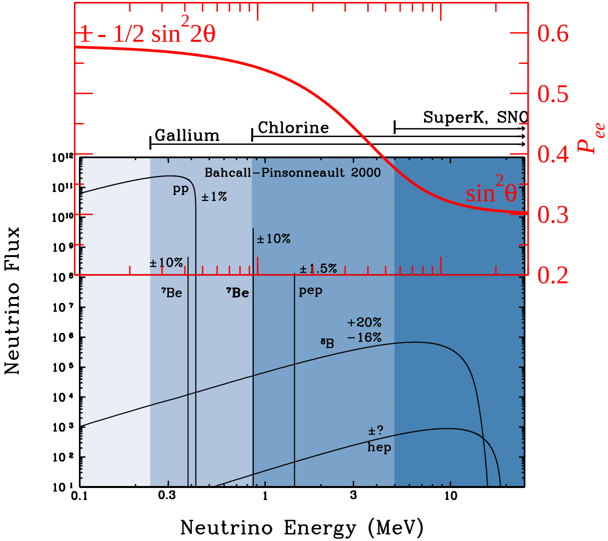
The LMA-MSW mechanism

$$P_{ee} = \cos^2\theta_m \cos^2\theta + \sin^2\theta_m \sin^2\theta$$



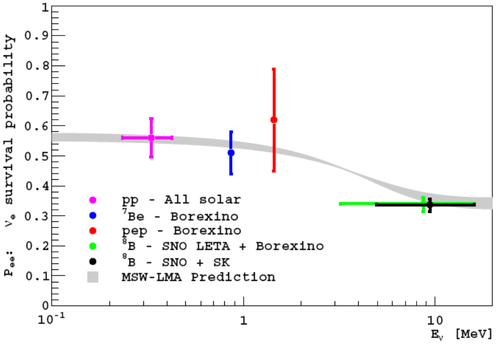
$$P_{ee} = \begin{cases} c^4 + s^4 = 1 - \frac{1}{2}\sin^2 2\theta & \text{vacuum} \quad (\theta_m = \theta) \\ \sin^2\theta & \text{strong matter} \quad (\theta_m = \pi/2) \end{cases}$$

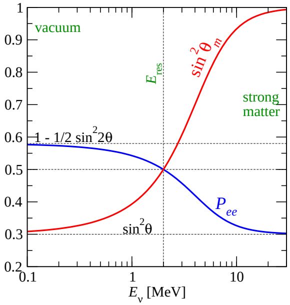
LMA-MSW and the solar neutrino spectrum



Evidence for LMA-MSW

Measurements of the solar neutrino rate at SNO and Borexino





Borexino, 1110.3230

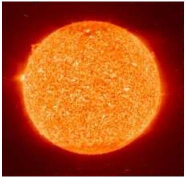
$\sin^2\theta < 0.5$ is strong evidence for MSW conversion

Outline

Global data on neutrino oscillations

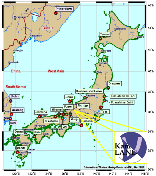
from various neutrino sources and
vastly different energy and distance scales:

sun



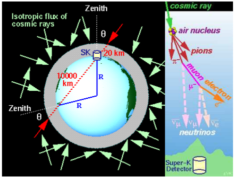
Homestake,SAGE,GALLEX
SuperK, SNO, Borexino

reactors



KamLAND, CHOOZ

atmosphere



SuperKamiokande

accelerators

K2K, MINOS, T2K

- global data fits nicely with the 3 neutrinos from the SM
- a few "anomalies" at 2-3 σ: LSND, MiniBooNE, reactor anomaly, no LMA MSW up-turn of solar neutrino spectrum

3-flavour oscillation parameters

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

$$\mathrm{U} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & e^{-i\delta} s_{13} \\ 0 & 1 & 0 \\ -e^{i\delta} s_{13} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

3-flavour oscillation parameters

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Δm^2_{31} Δm^2_{21}

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & e^{-i\delta} s_{13} \\ 0 & 1 & 0 \\ -e^{i\delta} s_{13} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

atm+LBL(dis) react+LBL(app) solar+KamLAND

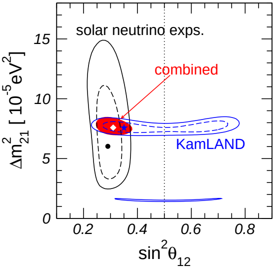
3-flavour effects are suppressed: $\Delta m^2_{21} \ll \Delta m^2_{31}$ and $\theta_{13} \ll 1$ ($U_{e3} = s_{13} e^{-i\delta}$)

$\Rightarrow$ dominant oscillations are well described by effective two-flavour oscillations

$\Rightarrow$ CP-violation is suppressed by θ_{13}

The dominating oscillation modes

- solar neutrinos Homestake, SAGE+GNO, Super-K, SNO, Borexino
  $\nu_e \to \nu_{\mu,\tau}$ LMA-MSW, $\Delta m^2 \sim 7 \times 10^{-5} \mathrm{eV}^2$
- Kamland reactor experiment (180 km)
  $\bar{\nu}_e$ disappearance with $E_\nu / L \sim 7 \times 10^{-5} \mathrm{eV}^2$



$$\sin^2\theta_{12} = 0.3^{+0.13}_{-0.12}$$

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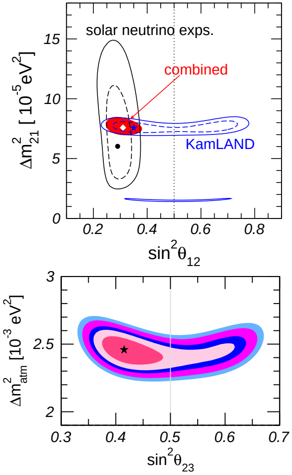
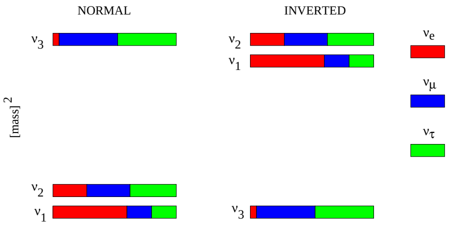
- Kamland reactor experiment (180 km)
  $\bar{\nu}_e$ disappearance with $E_\nu/L \sim 7 \times 10^{-5}\mathrm{eV}^2$
- atmospheric neutrinos Super-Kamiokande
  long-baseline accelerator experiments
  K2K (250 km), MINOS (735 km)
  $\nu_\mu \to \nu_\mu$ disapp. with $E_\nu/L \sim 2 \times 10^{-3}\mathrm{eV}^2$



$$\sin^2\theta_{12} = 0.3^{+0.13}_{-0.12} \qquad \sin^2\theta_{23} = 0.42^{+0.037}_{-0.031} \qquad \frac{\Delta m^2_{21}}{\Delta m^2_{31}} \approx \frac{1}{30}$$

Two possibilities for the neutrino mass spectrum



$\Delta m^2_{31} > 0$ $\Delta m^2_{31} < 0$

- We know that the mass state containing most of ν_e is the lighter of the two "solar mass" states: $\Delta m^2_{21} \equiv m^2_2 - m^2_1 > 0, \quad \theta_{12} < 45^o$
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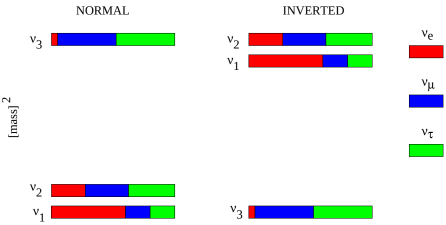
  Has to look for matter effect in $\nu_e \leftrightarrow \nu_\mu$ due to $\Delta m^2_{31}, \theta_{13}$

Two possibilities for the neutrino mass spectrum



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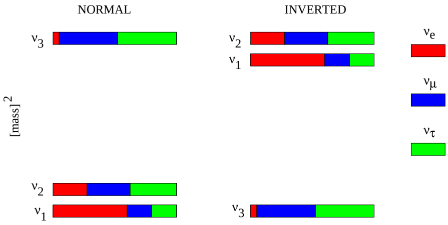
Effects of θ_{13}



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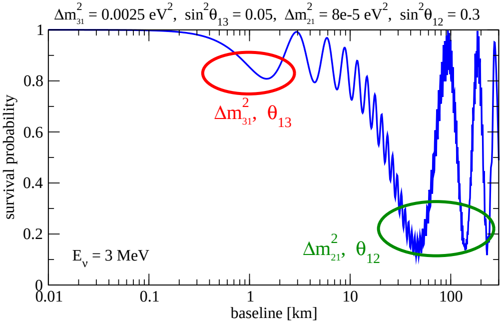
transitions of ν_e involving Δm^2_{31}:

- $\bar{\nu}_e \to \bar{\nu}_e$ disappearance reactor experiments
- long-baseline accelerator experiments looking for $\nu_\mu \to \nu_e$ appearance

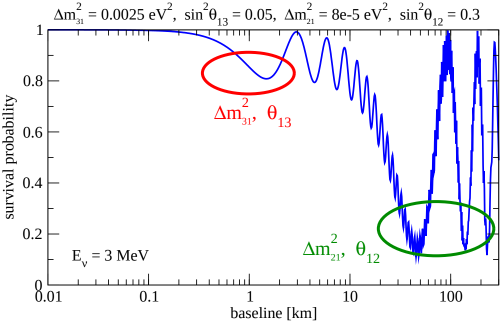
Reactor experiments

$$P_{\bar\nu_e\to\bar\nu_e} \approx 1 - \sin^2 2\theta_{13}\ \sin^2\frac{\Delta m^2_{31} L}{4E_\nu} - \cos^4\theta_{13}\sin^2 2\theta_{12}\ \sin^2\frac{\Delta m^2_{21} L}{4E_\nu}$$



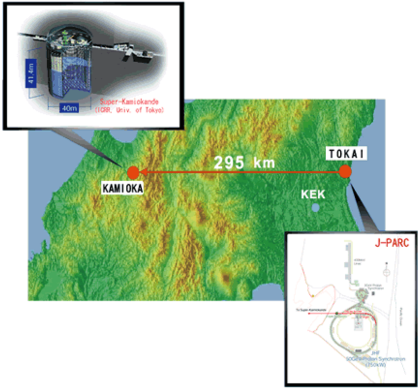
Reactor experiments

$$P_{\bar\nu_e\to\bar\nu_e} \approx 1 - \sin^2 2\theta_{13}\,\sin^2\frac{\Delta m^2_{31}L}{4E_\nu} - \cos^4\theta_{13}\sin^2 2\theta_{12}\,\sin^2\frac{\Delta m^2_{21}L}{4E_\nu}$$



- experiments at $\sim$1 km provide "clean" measurment of $\sin^2 2\theta_{13}$
- up to last year: $\sin^2 2\theta_{13} \lesssim 0.1$ dominated by the CHOOZ reactor exp.

June 2011: T2K data on $\nu_\mu \to \nu_e$ appearance



search for $\nu_\mu \to \nu_e$ oscillations
with $L = 295$ km and
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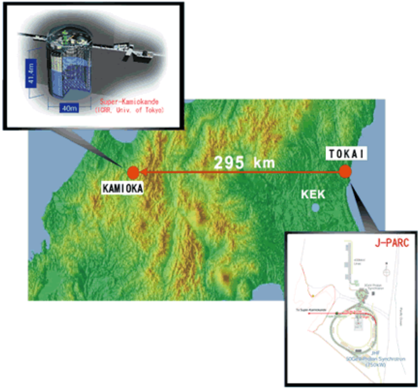
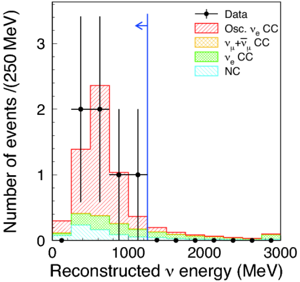




search for $\nu_\mu \to \nu_e$ oscillations
with $L = 295$ km and
$E_\nu \simeq 0.7$ GeV

K. Abe et al., 1106.2822

6 events obs, 1.5 ± 0.3 exp for $\theta_{13} = 0$
2.5σ indication for $\theta_{13} > 0$

Kobayashi @ ICHEP 2012

11 obs, 3.2 ± 0.4 exp for $\theta_{13} = 0$
3.2σ indication for $\theta_{13} > 0$

The LBL appearance oscillation probability

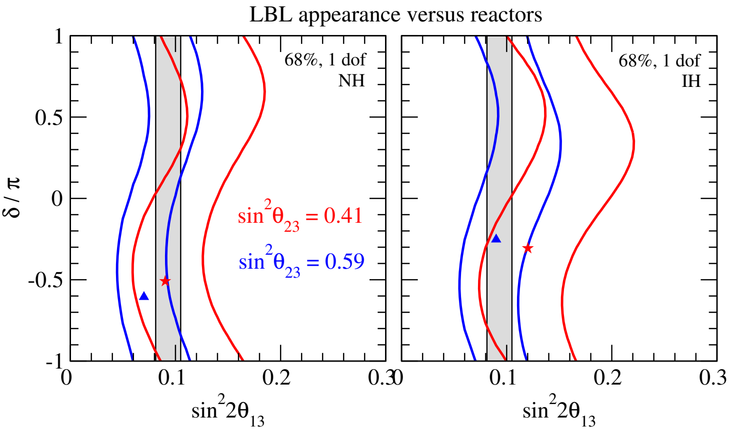
θ_{13} correlated with other parameters, especially δ, $\mathrm{sgn}(\Delta m^2_{31})$, θ_{23}

$$
\begin{aligned}
P_{\mu e} \simeq\ & \sin^2 2\theta_{13} \sin^2\theta_{23} \frac{\sin^2(1-A)\Delta}{(1-A)^2} \\
+\ & \sin 2\theta_{13}\, \hat{\alpha} \sin 2\theta_{23} \frac{\sin(1-A)\Delta}{1-A} \frac{\sin A\Delta}{A} \cos(\Delta + \delta_{\mathrm{CP}}) \\
+\ & \hat{\alpha}^2 \cos^2\theta_{23} \frac{\sin^2 A\Delta}{A^2}
\end{aligned}
$$

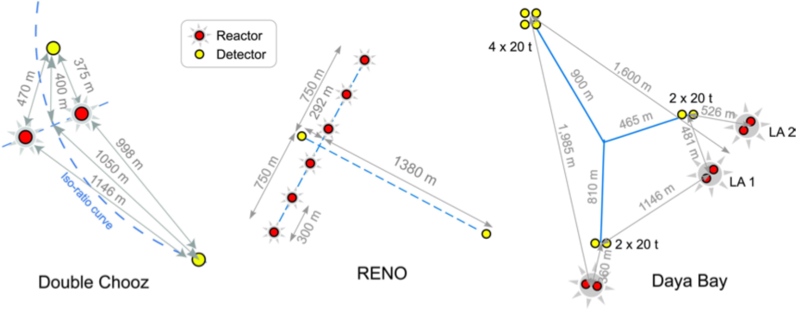
with $$\Delta \equiv \frac{\Delta m^2_{31} L}{4E_\nu}\,, \quad \hat{\alpha} \equiv \frac{\Delta m^2_{21}}{\Delta m^2_{31}} \sin 2\theta_{12}\,, \quad A \equiv \frac{2E_\nu V}{\Delta m^2_{31}}$$

anti-ν: $\delta_{\mathrm{CP}} \to -\delta_{\mathrm{CP}}$, $A \to -A$, $P_{e\mu}$: $\delta_{\mathrm{CP}} \to -\delta_{\mathrm{CP}}$
other hierarchy: $\Delta \to -\Delta$, $A \to -A$, $\hat{\alpha} \to -\hat{\alpha}$

T2K and MINOS data on $\nu_\mu \rightarrow \nu_e$ appearance



New generation of θ_{13} reactor experiments



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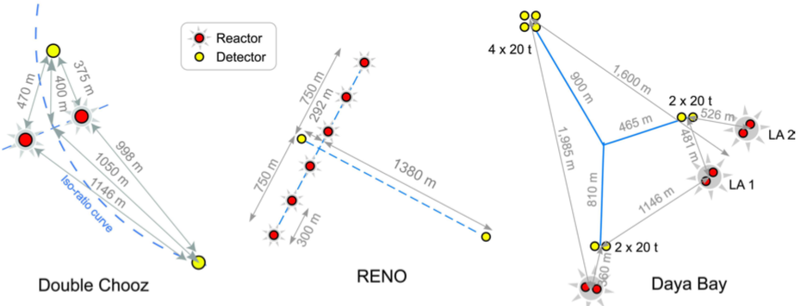

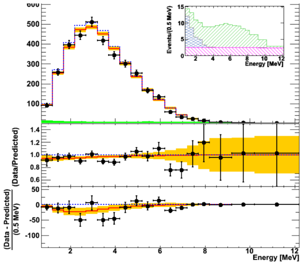

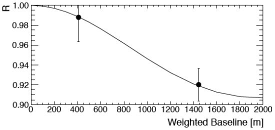

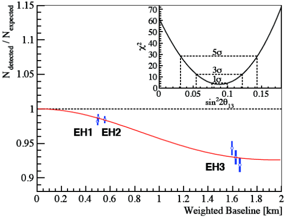

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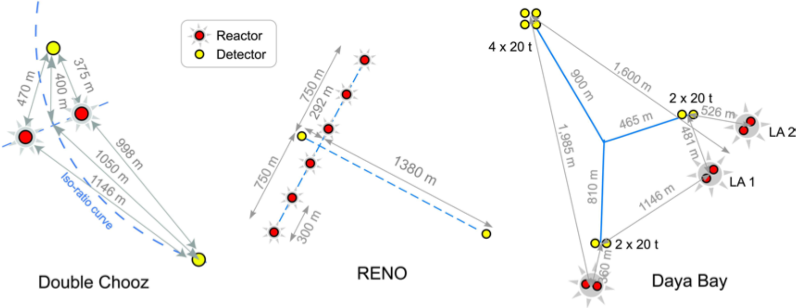
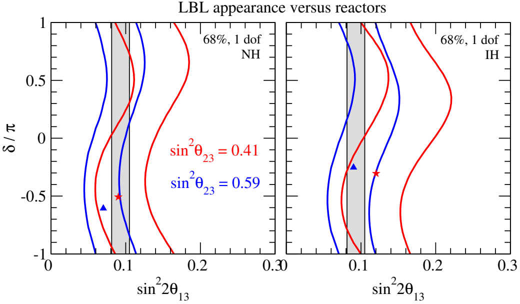
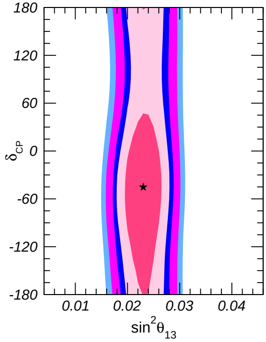


DC:	1112.6353	101d	4121ev	$\sin^2 2\theta_{13} = 0.086 \pm 0.041 \pm 0.030$	(1.9σ)
DB:	1203.1669	55d	10416ev	$\sin^2 2\theta_{13} = 0.092 \pm 0.016 \pm 0.005$	(5.2σ)
RE:	1204.0626	229d	17102ev	$\sin^2 2\theta_{13} = 0.113 \pm 0.013 \pm 0.019$	(4.9σ)

naive comb.: $\sin^2 2\theta_{13} = 0.098 \pm 0.013$ ($\chi^2 = 0.6/2$ dof)

θ_{13} global fit





$$\theta_{13} = (8.7 \pm 0.04)^o \qquad \Delta\chi^2(\theta_{13} = 0) \approx 100$$

Gonzalez-Garcia, Maltoni, Salvado, TS, in prep.

Outline

Lepton mixing

Neutrino oscillations
- Oscillations in vacuum
- Oscillations in matter
- Varying matter density and MSW

Global data and 3-flavour oscillations
- The θ_{13} revolution

Outlook
- CPV, mass hierarchy

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- test the three-flavour picture and search for deviations
  light sterile neutrinos, non-unitarity, non-standard neutrino interactions,. . .

Determination of the mass hierarchy

the vacuum $\nu_\mu \to \nu_e$ probability is invariant under

$$\Delta m^2_{31} \to -\Delta m^2_{31} \qquad \delta_{\rm CP} \to \pi - \delta_{\rm CP}$$

$\to$ the key to resolve the hierarchy degeneracy is the matter effect
resonance condition for $\nu_\mu \to \nu_e$ oscillations:

$$\pm \frac{2EV}{\Delta m^2_{31}} = \cos 2\theta_{13} \approx 1$$

can be fulfilled for
neutrinos if $\Delta m^2_{31} > 0$ (normal hierarchy)
anti-neutrinos if $\Delta m^2_{31} < 0$ (inverted hierarchy)

The size of the matter effect

$$A \equiv \left|\frac{2EV}{\Delta m_{31}^2}\right| \simeq 0.09 \left(\frac{E}{\mathrm{GeV}}\right)\left(\frac{|\Delta m_{31}^2|}{2.5\times 10^{-3}\,\mathrm{eV}^2}\right)^{-1}$$

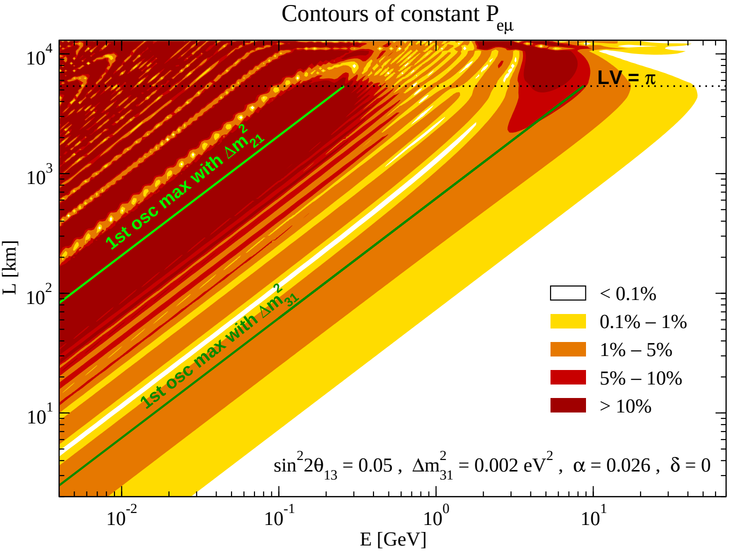
for experiments at the 1st osc. max, $|\Delta m_{31}^2|L/2E \simeq \pi$, and

$$A \simeq 0.02 \left(\frac{L}{100\,\mathrm{km}}\right)$$

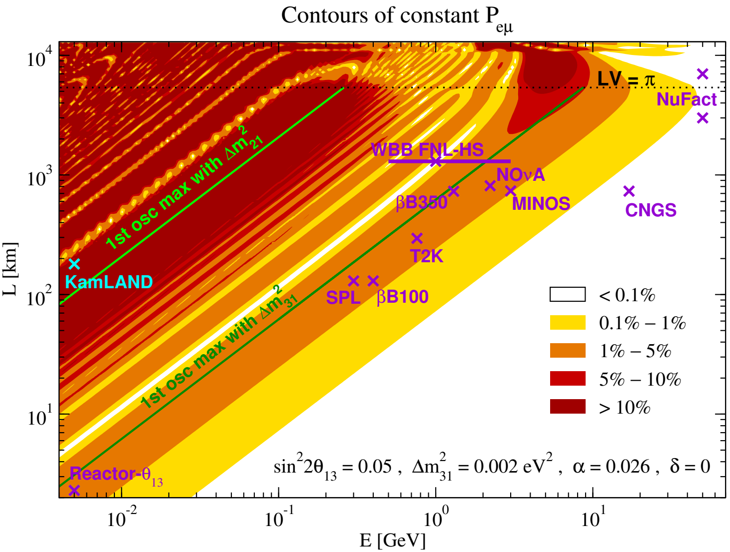
need $L \gtrsim 1000$ km and $E_\nu \gtrsim 3$ GeV in order to reach the regime of strong matter effect $A \gtrsim 0.2$.

terms linear in A do not break the degeneracy $\rightarrow$
have to be sensitive to higher order terms in A TS, hep-ph/0703279

$P_{\nu_e \to \nu_\mu}$


Contours of constant P_eμ
L [km]
E [GeV]
LV = π
1st osc max with Δm²21
1st osc max with Δm²31
< 0.1%
0.1% – 1%
1% – 5%
5% – 10%
> 10%
sin²2θ13 = 0.05 , Δm²31 = 0.002 eV² , α = 0.026 , δ = 0

$P_{\nu_e \to \nu_\mu}$


Contours of constant Peμ
L [km]
E [GeV]
1st osc max with Δm²21
1st osc max with Δm²31
LV = π
NuFact
WBB FNL-HS
NOνA
βB350
MINOS
CNGS
T2K
KamLAND
SPL
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Reactor-θ13
< 0.1%
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sin²2θ13 = 0.05 , Δm²31 = 0.002 eV² , α = 0.026 , δ = 0

Subsequent generation of LBL experiments

- superbeam upgardes
- beta beams
- neutrino factory

under intense study e.g.

EUROν http://www.euronu.org
NF-IDS http://www.ids-nf.org