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Supersymmetry

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Abstract: These are lecture notes for the BUSSTEPP Supersymmetry course, and are a snippet of my Part III 16 lecture course. You should already know some quantum field theory and group theory. You can watch videos of all of my Part III supersymmetry lectures on the web by following the link from

`http://users.hepforge.org/~allanach/teaching.html`

where the full lecture notes may also be found. I have a tendency to make trivial transcription errors on the board - please stop me if I make one.

Plan

- Review spinors and the Lorentz algebra
- Extend the Lorentz algebra to the global supersymmetry algebra
- Build the states of the supermultiplets that are used in the minimal supersymmetric standard model (MSSM)
- Introduce the MSSM.
- A quick computer tutorial on using `SOFTSUSY`, a computer program that calculates the MSSM particle spectrum and is usually the 'first step' in supersymmetry simulations etc.

In particular, this unfortunately does *not* include the supersymmetric interactions: you need lectures about superspace and superfields for that.

I welcome questions during lectures.

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1 Physical Motivation

Fundamental scalars (eg the Higgs) receive quantum corrections to their mass of order the heaviest mass scale in the theory (presumably $M_{Pl} \sim 10^{19}$ GeV), divided by a loop factor $16\pi^2$. The technical hierarchy problem is: how do we manage to end up with a ≈ 125 GeV Higgs boson in this case?

Supersymmetry solves the hierarchy problem because the large quantum corrections *cancel* between fermions and bosons in the loops. Combined with the GUT idea, it also unifies the three gauge couplings at one single point at larger energies. A supersymmetric version of the Standard Model also provides the most studied candidate for dark matter. Moreover, it provides well defined QFTs in which issues of strong coupling can be better studied than in the non-supersymmetric models, and is an important ingredient in string theory.

2 Supersymmetry algebra and representations

2.1 Poincaré symmetry and spinors

The Poincaré group corresponds to the basic symmetries of special relativity, it acts on space-time coordinates x^μ as follows:

$$x^\mu \;\mapsto\; x'^\mu \;=\; \underbrace{\Lambda^\mu{}_\nu}_{\text{Lorentz}}\, x^\nu \;+\; \underbrace{a^\mu}_{\text{translation}}$$

Lorentz transformations leave the metric tensor $\eta_{\mu\nu} = \text{diag}(1,\ -1,\ -1,\ -1)$ invariant:

$$\Lambda^T\,\eta\,\Lambda \;=\; \eta$$

They can be separated between those that are connected to the identity and this that are not (i.e. parity reversal $\Lambda_P = \text{diag}(1,\ -1,\ -1,\ -1)$ and time reversal $\Lambda_T = \text{diag}(-1,\ 1,\ 1,\ 1)$). We will mostly discuss those Λ continuously connected to identity, i.e. the *proper orthochronous group*[1] $SO(3,1)^\uparrow$. Generators for the Poincaré group are the hermitian $M^{\mu\nu}$ (rotations and Lorentz boosts) and P^σ (translations) with algebra

$$\begin{aligned}
\Big[P^\mu\ ,\ P^\nu\Big] \;&=\; 0\\
\Big[M^{\mu\nu}\ ,\ P^\sigma\Big] \;&=\; i\left(P^\mu\,\eta^{\nu\sigma} \;-\; P^\nu\,\eta^{\mu\sigma}\right)\\
\Big[M^{\mu\nu}\ ,\ M^{\rho\sigma}\Big] \;&=\; i\left(M^{\mu\sigma}\,\eta^{\nu\rho} \;+\; M^{\nu\rho}\,\eta^{\mu\sigma} \;-\; M^{\mu\rho}\,\eta^{\nu\sigma} \;-\; M^{\nu\sigma}\,\eta^{\mu\rho}\right)
\end{aligned}$$

A 4 dimensional matrix representation for the $M^{\mu\nu}$ is

$$(M^{\rho\sigma})^\mu{}_\nu \;=\; -i\left(\eta^{\mu\sigma}\,\delta^\rho{}_\nu \;-\; \eta^{\rho\mu}\,\delta^\sigma{}_\nu\right)\ .$$

2.1.1 Properties of the Lorentz group

- We now show that locally (i.e. in terms of the algebra), we have a correspondence

  $$SO(3,1) \;\cong\; SU(2)\times SU(2).$$

  The generators of $SO(3,1)$ (J_i of rotations and K_i of Lorentz boosts) can be expressed as

  $$J_i \;=\; \frac{1}{2}\,\epsilon_{ijk}\,M_{jk}\ , \qquad K_i \;=\; M_{0i}\ ,$$

  and the Lorentz algebra written in terms of J's and K's is

  $$[K_i,\ K_j] = -i\epsilon_{ijk}J_k, \qquad [J_i,\ K_j] = i\epsilon_{ijk}K_k, \qquad [J_i,\ J_j] = i\epsilon_{ijk}J_k.$$

  Their[2] linear combinations (which are neither hermitian nor anti hermitian)

  $$A_i \;=\; \frac{1}{2}\left(J_i + iK_i\right)\ , \qquad B_i \;=\; \frac{1}{2}\left(J_i - iK_i\right)$$

[1] These consist of the subgroup of transformations which have $\det\Lambda = +1$ and $\Lambda^0_0 \geq 1$.
[2] NB $\epsilon_{123} = +1$.

satisfy $SU(2) \times SU(2)$ commutation relations

$$\Big[A_i \ , \ A_j\Big] \ = \ i\epsilon_{ijk}\, A_k \ , \qquad \Big[B_i \ , \ B_j\Big] \ = \ i\epsilon_{ijk}\, B_k \ , \qquad \Big[A_i \ , \ B_j\Big] \ = \ 0$$

Under parity $\hat{P}$, ($x^0 \mapsto x^0$ and $\vec{x} \mapsto -\vec{x}$) we have

$$J_i \ \mapsto \ J_i \ , \qquad K_i \ \mapsto \ -K_i \ \Longrightarrow \ A_i \ \leftrightarrow \ B_i \ .$$

We can interpret $\vec{J} = \vec{A} + \vec{B}$ as the physical spin.

- On the other hand, there is a homeomorphism (not an isomorphism)

$$SO(3,1) \ \cong \ SL(2,\mathbb{C}) \ .$$

To see this, take a 4 vector X and a corresponding 2×2 - matrix $\tilde{x}$,

$$X \ = \ x_\mu\, e^\mu \ = \ (x_0 \ , \ x_1 \ , \ x_2 \ , \ x_3) \ , \qquad \tilde{x} \ = \ x_\mu\, \sigma^\mu \ = \ \begin{pmatrix} x_0 + x_3 & x_1 - ix_2 \\ x_1 + ix_2 & x_0 - x_3 \end{pmatrix} \ ,$$

where σ^μ is the 4 vector of *Pauli matrices*

$$\sigma^\mu \ = \ \left\{ \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \ , \ \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \ , \ \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \ , \ \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \right\} \ .$$

Transformations $X \mapsto \Lambda X$ under $SO(3,1)$ leaves the square

$$|X|^2 \ = \ x_0^2 \ - \ x_1^2 \ - \ x_2^2 \ - \ x_3^2$$

invariant, whereas the action of $SL(2,\mathbb{C})$ mapping $\tilde{x} \mapsto N\tilde{x}N^\dagger$ with $N \in SL(2,\mathbb{C})$ preserves the determinant

$$\det \tilde{x} \ = \ x_0^2 \ - \ x_1^2 \ - \ x_2^2 \ - \ x_3^2 \ .$$

The map between $SL(2,\mathbb{C})$ is 2-1, since $N = \pm 1$ both correspond to $\Lambda = 1$, but $SL(2,\mathbb{C})$ has the advantage of being simply connected, so $SL(2,\mathbb{C})$ is the universal covering group.

2.1.2 Representations and invariant tensors of $SL(2,\mathbb{C})$

The basic representations of $SL(2,\mathbb{C})$ are:

- The fundamental representation

$$\psi'_\alpha \ = \ N_\alpha{}^\beta\, \psi_\beta \ , \qquad \alpha, \beta = 1, 2 \tag{2.1}$$

The elements of this representation ψ_α are called *left-handed Weyl spinors.*

- The conjugate representation

$$\bar{\chi}'_{\dot{\alpha}} \ = \ N^{*}_{\dot{\alpha}}{}^{\dot{\beta}}\, \bar{\chi}_{\dot{\beta}} \ , \qquad \dot{\alpha}, \dot{\beta} = 1, 2$$

Here $\bar{\chi}_{\dot{\beta}}$ are called *right-handed Weyl spinors.*

- The contravariant representations are

$$\psi'^{\alpha} \;=\; \psi^{\beta}\,(N^{-1})_{\beta}{}^{\alpha}\;, \qquad \bar{\chi}'^{\dot{\alpha}} \;=\; \bar{\chi}^{\dot{\beta}}\,(N^{*-1})_{\dot{\beta}}{}^{\dot{\alpha}}.$$

The fundamental and conjugate representations are the basic representations of $SL(2,\mathbb{C})$ and the Lorentz group, giving then the importance to spinors as the basic objects of special relativity, a fact that could be missed by not realising the connection between the Lorentz group and $SL(2,\mathbb{C})$. We will see next that the contravariant representations are however not independent. Consider the different ways to raise and lower indices.

- The metric tensor $\eta^{\mu\nu} = (\eta_{\mu\nu})^{-1}$ is invariant under $SO(3,1)$ and is used to raise/lower indices.

- The analogy within $SL(2,\mathbb{C})$ is

$$\epsilon^{\alpha\beta} \;=\; \epsilon^{\dot{\alpha}\dot{\beta}} \;=\; -\epsilon_{\alpha\beta} \;=\; -\epsilon_{\dot{\alpha}\dot{\beta}}\;,\; \epsilon^{12} = +1, \epsilon^{21} = -1.$$

  since

$$\epsilon'^{\alpha\beta} \;=\; \epsilon^{\rho\sigma}\,N_{\rho}{}^{\alpha}\,N_{\sigma}{}^{\beta} \;=\; \epsilon^{\alpha\beta}\cdot\det N \;=\; \epsilon^{\alpha\beta}\;.$$

  That is why ϵ is used to raise and lower indices

$$\psi^{\alpha} \;=\; \epsilon^{\alpha\beta}\psi_{\beta}\;, \qquad \bar{\chi}^{\dot{\alpha}} \;=\; \epsilon^{\dot{\alpha}\dot{\beta}}\bar{\chi}_{\dot{\beta}} \;\Rightarrow\; \psi_{\alpha} \;=\; \epsilon_{\alpha\beta}\psi^{\beta}, \qquad \bar{\chi}_{\dot{\alpha}} \;=\; \epsilon_{\dot{\alpha}\dot{\beta}}\bar{\chi}^{\dot{\beta}}$$

  so contravariant representations are not independent from covariant ones.

- To handle mixed $SO(3,1)$- and $SL(2,\mathbb{C})$ indices, recall that the transformed components x_μ should look the same, whether we transform the vector X via $SO(3,1)$ or the matrix $\tilde{x} = x_\mu \sigma^\mu$ via $SL(2,\mathbb{C})$

$$(x_\mu\,\sigma^\mu)_{\alpha\dot{\alpha}} \;\mapsto\; N_{\alpha}{}^{\beta}\,(x_\nu\,\sigma^\nu)_{\beta\dot{\gamma}}\,N^{*}_{\dot{\alpha}}{}^{\dot{\gamma}} \;=\; \Lambda_{\mu}{}^{\nu}\,x_\nu\,\sigma^\mu\;,$$

  so the right transformation rule is

$$(\sigma^\mu)_{\alpha\dot{\alpha}} \;=\; N_{\alpha}{}^{\beta}\,(\sigma^\nu)_{\beta\dot{\gamma}}\,(\Lambda^{-1})^{\mu}{}_{\nu}\,N^{*}_{\dot{\alpha}}{}^{\dot{\gamma}}\;.$$

  Similar relations hold for

$$(\bar{\sigma}^\mu)^{\dot{\alpha}\alpha} \;:=\; \epsilon^{\alpha\beta}\,\epsilon^{\dot{\alpha}\dot{\beta}}\,(\sigma^\mu)_{\beta\dot{\beta}} \;=\; (1,\; -\vec{\sigma})\;.$$

  Question: 1. Check the following identities:

$$(\bar{\sigma}^\mu)^{\dot{\alpha}\alpha} \equiv \epsilon^{\alpha\beta}\epsilon^{\dot{\alpha}\dot{\beta}}\sigma^{\mu}_{\beta\dot{\beta}} = \left(1,\,-\sigma^1,\,-\sigma^2,\,-\sigma^3\right)$$

$$(\sigma^\mu)_{\alpha\dot{\beta}}\,(\bar{\sigma}_\mu)^{\dot{\gamma}\delta} \;=\; 2\,\delta_\alpha^\delta\,\delta_{\dot{\beta}}^{\dot{\gamma}}$$

$$(\sigma^\mu\bar{\sigma}^\nu \;+\; \sigma^\nu\bar{\sigma}^\mu)_\alpha^\beta \;=\; 2\,\eta^{\mu\nu}\delta_\alpha^\beta$$

$$\textbf{Trace}\left[\,\sigma^\mu\bar{\sigma}^\nu\,\right] \;=\; 2\,\eta^{\mu\nu}$$

2.1.3 Generators of $SL(2,\mathbb{C})$

Let us define tensors $\sigma^{\mu\nu}$, $\bar{\sigma}^{\mu\nu}$ as antisymmetrised products of σ matrices:

$$\begin{aligned}
(\sigma^{\mu\nu})_\alpha{}^\beta &:= \frac{i}{4}\left(\sigma^\mu\,\bar{\sigma}^\nu \;-\; \sigma^\nu\,\bar{\sigma}^\mu\right)_\alpha{}^\beta \\
(\bar{\sigma}^{\mu\nu})_{\dot{\alpha}}{}^{\dot{\beta}} &:= \frac{i}{4}\left(\bar{\sigma}^\mu\,\sigma^\nu \;-\; \bar{\sigma}^\nu\,\sigma^\mu\right)^{\dot{\alpha}}{}_{\dot{\beta}}
\end{aligned}$$

which satisfy the Lorentz algebra

$$\left[\sigma^{\mu\nu}\;,\;\sigma^{\lambda\rho}\right] \;=\; i\left(\eta^{\mu\rho}\,\sigma^{\nu\lambda} \;+\; \eta^{\nu\lambda}\,\sigma^{\mu\rho} \;-\; \eta^{\mu\lambda}\,\sigma^{\nu\rho} \;-\; \eta^{\nu\rho}\,\sigma^{\mu\lambda}\right) .$$

They thus form a representation of the Lorentz algebra (the spinor representation).

Under a finite Lorentz transformation with parameters $\omega_{\mu\nu}$, spinors transform as follows:

$$\begin{aligned}
\psi_\alpha &\mapsto \exp\left(-\frac{i}{2}\omega_{\mu\nu}\sigma^{\mu\nu}\right)_\alpha{}^\beta\,\psi_\beta && \text{(left-handed)} \\
\bar{\chi}^{\dot{\alpha}} &\mapsto \exp\left(-\frac{i}{2}\omega_{\mu\nu}\bar{\sigma}^{\mu\nu}\right)^{\dot{\alpha}}{}_{\dot{\beta}}\,\bar{\chi}^{\dot{\beta}} && \text{(right-handed)}
\end{aligned}$$

Now consider the spins with respect to the $SU(2)$s spanned by the A_i and B_i:

$$\begin{aligned}
\psi_\alpha : \qquad & (A,\;B) = \left(\frac{1}{2},\;0\right) \quad\Longrightarrow\quad J_i \;=\; \frac{1}{2}\,\sigma_i\;, \qquad K_i \;=\; -\frac{i}{2}\,\sigma_i \\
\bar{\chi}^{\dot{\alpha}} : \qquad & (A,\;B) = \left(0,\;\frac{1}{2}\right) \quad\Longrightarrow\quad J_i \;=\; \frac{1}{2}\,\sigma_i\;, \qquad K_i \;=\; +\frac{i}{2}\,\sigma_i
\end{aligned}$$

Let us just mention the identities[3]

$$\begin{aligned}
\sigma^{\mu\nu} &= \frac{1}{2i}\,\epsilon^{\mu\nu\rho\sigma}\,\sigma_{\rho\sigma} \\
\bar{\sigma}^{\mu\nu} &= -\frac{1}{2i}\,\epsilon^{\mu\nu\rho\sigma}\,\bar{\sigma}_{\rho\sigma}\;,
\end{aligned}$$

known as *self duality* and *anti self duality*. They are important because naively $\sigma^{\mu\nu}$ being antisymmetric seems to have $\frac{4\times 3}{2}$ components, but the self duality conditions reduces this by half. A reference book illustrating many of the calculations for two - component spinors is [2].

2.1.4 Products of Weyl spinors

Define the product of two Weyl spinors as

$$\begin{aligned}
\chi\psi &:= \chi^\alpha\,\psi_\alpha \;=\; -\chi_\alpha\,\psi^\alpha \\
\bar{\chi}\bar{\psi} &:= \bar{\chi}_{\dot{\alpha}}\,\bar{\psi}^{\dot{\alpha}} \;=\; -\bar{\chi}^{\dot{\alpha}}\,\bar{\psi}_{\dot{\alpha}}\;,
\end{aligned}$$

where in particular

$$\psi\psi \;=\; \psi^\alpha\,\psi_\alpha \;=\; \epsilon^{\alpha\beta}\,\psi_\beta\,\psi_\alpha \;=\; \psi_2\,\psi_1 \;-\; \psi_1\,\psi_2\;.$$

[3] $\epsilon_{0123} = 1 = -\epsilon^{0123}$

Choosing the ψ_α to be *anticommuting Grassmann numbers*, $\psi_1\psi_2 = -\psi_2\psi_1$, so $\psi\psi = 2\psi_2\psi_1$. Or, $\psi_\alpha\psi_\beta = \frac{1}{2}\epsilon_{\alpha\beta}(\psi\psi)$. Thus $\psi_\alpha\psi_\beta = \frac{1}{2}\epsilon_{\alpha\beta}(\psi\psi)$.
From the definitions

$$\psi^\dagger_\alpha \;:=\; \bar{\psi}_{\dot{\alpha}}\;, \qquad \bar{\psi}^{\dot{\alpha}} \;:=\; \psi^*_\beta\,(\sigma^0)^{\beta\dot{\alpha}}$$

it follows that

$$(\chi\psi)^\dagger \;=\; \bar{\chi}\bar{\psi}\;, \qquad (\psi\,\sigma^\mu\,\bar{\chi})^\dagger \;=\; \chi\,\sigma^\mu\,\bar{\psi}$$

which justifies the $\nearrow$ contraction of dotted indices in contrast to the $\searrow$ contraction of undotted ones.

Question: 2. Using the definition $\psi^\alpha \equiv \epsilon^{\alpha\beta}\psi_\beta$, and the SL(2, C) spinor transformation property

$$\psi'_\alpha = N_\alpha{}^\beta\psi_\beta \tag{2.2}$$

work through the following:

1. **Prove that under SL(2, C), ψ^α transforms as $\psi'^\alpha \;=\; \psi^\beta(N^{-1})_\beta{}^\alpha$.**
2. **Using the $SL(2,\,\mathbf{C})$ invariance of $\epsilon_{\alpha\beta}$, find an expression for N^{-1} in terms of N.**
3. **Prove that:**

$$\psi\chi \;\equiv\; \psi^\alpha\chi_\alpha \;=\; -\psi_\alpha\chi^\alpha \equiv \chi\psi$$

and check that $\psi\chi$ as well as $\bar{\psi}\bar{\chi} \;=\; \bar{\psi}_{\dot{\alpha}}\bar{\chi}^{\dot{\alpha}}$ are SL(2, C) scalars.

2.1.5 Dirac spinors

To connect the ideas of Weyl spinors with the more standard DIRAC theory, define

$$\gamma^\mu \;:=\; \begin{pmatrix} 0 & \sigma^\mu \\ \bar{\sigma}^\mu & 0 \end{pmatrix}\;,$$

then these γ^μ satisfy the *Clifford algebra*

$$\left\{\gamma^\mu\;,\;\gamma^\nu\right\} \;=\; 2\,\eta^{\mu\nu}\,1\;.$$

The matrix γ^5, defined as

$$\gamma^5 \;:=\; i\gamma^0\,\gamma^1\,\gamma^2\,\gamma^3 \;=\; \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}\;,$$

can have eigenvalues ± 1 (chirality). The generators of the Lorentz group are

$$\Sigma^{\mu\nu} \;=\; \frac{i}{4}\,\gamma^{\mu\nu} \;=\; \begin{pmatrix} \sigma^{\mu\nu} & 0 \\ 0 & \bar{\sigma}^{\mu\nu} \end{pmatrix}\;.$$

We define *Dirac spinors* to be the direct sum of two Weyl spinors of opposite chirality,

$$\Psi_D \;:=\; \begin{pmatrix} \psi_\alpha \\ \bar{\chi}^{\dot{\alpha}} \end{pmatrix}\;,$$

such that the action of γ^5 is given as

$$\gamma^5\,\Psi_D \;=\; \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \psi_\alpha \\ \bar{\chi}^{\dot{\alpha}} \end{pmatrix} \;=\; \begin{pmatrix} -\psi_\alpha \\ \bar{\chi}^{\dot{\alpha}} \end{pmatrix} .$$

We can define the following projection operators P_L, P_R,

$$P_L \;:=\; \frac{1}{2}\left(1 \;-\; \gamma^5\right) , \qquad P_R \;:=\; \frac{1}{2}\left(1 \;+\; \gamma^5\right) ,$$

eliminating one part of definite chirality, i.e.

$$P_L\,\Psi_D \;=\; \begin{pmatrix} \psi_\alpha \\ 0 \end{pmatrix} , \qquad P_R\,\Psi_D \;=\; \begin{pmatrix} 0 \\ \bar{\chi}^{\dot{\alpha}} \end{pmatrix} .$$

2.2 SUSY algebra

2.2.1 Graded algebra

We wish to *extend* the Poincaré algebra non-trivially. The *Coleman Mandula theorem* stated that in 3+1 dimensions, one cannot do this in a non-trivial way and still have non-zero scattering amplitudes. In other words, there is no non-trivial mix of Poincaré and internal symmetries with non-zero scattering except for the direct product

Poincaré $\times$ internal.

However (as usual with no-go theorems) there was a loop-hole because of an implicit axiom: the proof only considered "*bosonic* generators".
We wish to turn bosons into fermions, thus we need to introduce a fermionic generator Q. Heuristically:

$$Q|\text{boson}\rangle \propto |\text{fermion}\rangle, \qquad Q|\text{fermion}\rangle \propto |\text{boson}\rangle.$$

For this, we require a graded algebra - a generalisation of Lie algebra. If O_a is an operator of an algebra (such as a group generator), a graded algebra is

$$O_a O_b - (-1)^{\eta_a \eta_b} O_b O_a = i C^e_{ab} O_e, \tag{2.3}$$

where $\eta_a = 0$ if O_a is a *bosonic generator*, and $\eta_a = 1$ if O_a is a *fermionic generator*.
For supersymmetry, the generators are the Poincaré generators P^μ, $M^{\mu\nu}$ and the spinor generators Q^A_α, $\bar{Q}^A_{\dot{\alpha}}$, where $A = 1, ..., N$. In case $N = 1$ we speak of a simple SUSY, in case $N > 1$ of an extended SUSY. In this section, we will only discuss $N = 1$.

We know the commutation relations $[P^\mu, P^\nu]$, $[P^\mu, M^{\rho\sigma}]$ and $[M^{\mu\nu}, M^{\rho\sigma}]$ already from the Poincaré algebra, so we need to find

$$\text{(a) } \Big[Q_\alpha \,,\, M^{\mu\nu}\Big] \,,\ \text{(b) } \Big[Q_\alpha \,,\, P^\mu\Big] \,,$$
$$\text{(c) } \Big\{Q_\alpha \,,\, Q_\beta\Big\} \,,\ \text{(d) } \Big\{Q_\alpha \,,\, \bar{Q}_{\dot{\beta}}\Big\} \,,$$

also (for internal symmetry generators T_i)

$$\text{(e) } \Big[Q_\alpha \,,\, T_i\Big] \,.$$

We shall be using the fact that the right hand sides must be *linear* and that they must transform in the same way as the commutators under a Lorentz transformation, for instance. The relations for $Q \leftrightarrow \bar{Q}$ may then be obtained from these by taking hermitian conjugates.

- (a) $\Big[Q_\alpha \;,\; M^{\mu\nu}\Big]$: we can work this one out by knowing how Q_α transforms as a spinor and as an operator.

  Since Q_α is a spinor, it transforms under the exponential of the $SL(2,\mathbb{C})$ generators $\sigma^{\mu\nu}$:

$$Q'_\alpha \;=\; \exp\left(-\frac{i}{2}\omega_{\mu\nu}\sigma^{\mu\nu}\right)_\alpha^{\;\;\beta} Q_\beta \;\approx\; \left(1 \;-\; \frac{i}{2}\,\omega_{\mu\nu}\,\sigma^{\mu\nu}\right)_\alpha^{\;\;\beta} Q_\beta\,.$$

  Under an active transformation, as an operator. $|\psi\rangle \;\to\; U|\psi\rangle \;\Rightarrow\; \langle\psi|Q_\alpha|\psi\rangle \;\to\; \langle\psi|U^\dagger Q_\alpha U|\psi\rangle \to$, where $U = \exp\left(-\frac{i}{2}\omega_{\mu\nu}M^{\mu\nu}\right)$. Hence

$$Q'_\alpha \;=\; U^\dagger\, Q_\alpha\, U \;\approx\; \left(1 \;+\; \frac{i}{2}\,\omega_{\mu\nu}\,M^{\mu\nu}\right) Q_\alpha \left(1 \;-\; \frac{i}{2}\,\omega_{\mu\nu}\,M^{\mu\nu}\right)\,.$$

  Compare these two expressions for Q'_α up to first order in $\omega_{\mu\nu}$,

$$Q_\alpha \;-\; \frac{i}{2}\,\omega_{\mu\nu}\,(\sigma^{\mu\nu})_\alpha^{\;\;\beta}\, Q_\beta \;=\; Q_\alpha \;-\; \frac{i}{2}\,\omega_{\mu\nu}\left(Q_\alpha\, M^{\mu\nu} \;-\; M^{\mu\nu}\, Q_\alpha\right) \;+\; \mathcal{O}(\omega^2)$$

$$\Longrightarrow \quad \boxed{\Big[Q_\alpha \;,\; M^{\mu\nu}\Big] \;=\; (\sigma^{\mu\nu})_\alpha^{\;\;\beta}\, Q_\beta}$$

  Similarly,

$$\boxed{\Big[\bar{Q}^{\dot\alpha},\; M^{\mu\nu}\Big] \;=\; (\bar\sigma^{\mu\nu})^{\dot\alpha}_{\;\;\dot\beta}\, \bar{Q}^{\dot\beta}}$$

- (b) $\Big[Q_\alpha\,,\; P^\mu\Big]$: $c\cdot(\sigma^\mu)_{\alpha\dot\alpha}\bar{Q}^{\dot\alpha}$ is the only way of writing a sensible term with free indices μ, α which is linear in Q. To fix the constant c, consider $[\bar{Q}^{\dot\alpha}, P^\mu] = c^* \cdot (\bar\sigma^\mu)^{\dot\alpha\beta} Q_\beta$ (take adjoints using $(Q_\alpha)^\dagger = \bar{Q}_{\dot\alpha}$ and $(\sigma^\mu\bar{Q})^\dagger_\alpha = (Q\sigma^\mu)_{\dot\alpha}$). The Jacobi identity for P^μ, P^ν and Q_α

$$\begin{aligned}
0 \;&=\; \Big[P^\mu \;,\; \Big[P^\nu \;,\; Q_\alpha\Big]\Big] \;+\; \Big[P^\nu \;,\; \Big[Q_\alpha \;,\; P^\mu\Big]\Big] \;+\; \Big[Q_\alpha \;,\; \underbrace{\Big[P^\mu \;,\; P^\nu\Big]}_{0}\Big] \\
&=\; -c\,(\sigma^\nu)_{\alpha\dot\alpha}\Big[P^\mu \;,\; \bar{Q}^{\dot\alpha}\Big] \;+\; c\,(\sigma^\mu)_{\alpha\dot\alpha}\Big[P^\nu \;,\; \bar{Q}^{\dot\alpha}\Big] \\
&=\; |c|^2\,(\sigma^\nu)^{\alpha\dot\alpha}\,(\bar\sigma^\mu)^{\dot\alpha\beta}\, Q_\beta \;-\; |c|^2\,(\sigma^\mu)_{\alpha\dot\alpha}\,(\bar\sigma^\nu)^{\dot\alpha\beta}\, Q_\beta \\
&=\; |c|^2\, \underbrace{(\sigma^\nu\,\bar\sigma^\mu \;-\; \sigma^\mu\,\bar\sigma^\nu)_\alpha^{\;\;\beta}}_{\neq 0}\, Q_\beta
\end{aligned}$$

  can only hold for general Q_β, if $c = 0$, so

$$\boxed{\Big[Q_\alpha \;,\; P^\mu\Big] \;=\; \Big[\bar{Q}^{\dot\alpha} \;,\; P^\mu\Big] \;=\; 0}$$

- (c) $\Big\{Q_\alpha \ , \ Q_\beta\Big\}$

  Due to index structure, that commutator should look like

  $$\Big\{Q_\alpha \ , \ Q^\beta\Big\} \quad = \quad k\,(\sigma^{\mu\nu})_\alpha{}^\beta\, M_{\mu\nu} \ .$$

  Since the left hand side commutes with P^μ and the right hand side doesn't, the only consistent choice is $k=0$, i.e.

  $$\boxed{\Big\{Q_\alpha \ , \ Q_\beta\Big\} \quad = \quad 0, \qquad \Big\{\bar{Q}_{\dot{\alpha}} \ , \ \bar{Q}_{\dot{\beta}}\Big\} \quad = \quad 0}$$

- (d) $\Big\{Q_\alpha \ , \ \bar{Q}_{\dot{\beta}}\Big\}$

  This time, index structure implies an ansatz

  $$\Big\{Q_\alpha \ , \ \bar{Q}_{\dot{\beta}}\Big\} \quad = \quad t\,(\sigma^\mu)_{\alpha\dot{\beta}}\, P_\mu \ .$$

  There is no way of fixing t, so, by convention, set $t=2$, defining the normalisation of the operators:

  $$\boxed{\Big\{Q_\alpha \ , \ \bar{Q}_{\dot{\beta}}\Big\} \quad = \quad 2\,(\sigma^\mu)_{\alpha\dot{\beta}}\, P_\mu}$$

Notice that two symmetry transformations $Q_\alpha \bar{Q}_{\dot{\beta}}$ have the effect of a translation. Let $|B\rangle$ be a bosonic state and $|F\rangle$ a fermionic one, then

$$Q_\alpha\,|F\rangle \quad = \quad |B\rangle \ , \qquad \bar{Q}_{\dot{\beta}}\,|B\rangle \quad = \quad |F\rangle \quad \Longrightarrow \quad Q\bar{Q}: \ |B\rangle \quad \mapsto \quad |B \text{ (translated)}\rangle \ .$$

- (e) $\Big[Q_\alpha \ , \ T_i\Big]$

  Usually, this commutator vanishes due to the Coleman-Mandula theorem. Exceptions are $U(1)$ automorphisms of the supersymmetry algebra known as R *symmetry*. The algebra is invariant under the simultaneous change

  $$Q_\alpha \quad \mapsto \quad \exp(i\lambda)\,Q_\alpha \ , \qquad \bar{Q}_{\dot{\alpha}} \quad \mapsto \quad \exp(-i\lambda)\,\bar{Q}_{\dot{\alpha}} \ .$$

  Let R be a global $U(1)$ generator, then, since $Q_\alpha \mapsto e^{-iR\lambda}Q_\alpha e^{iR\lambda}$,

  $$\boxed{\Rightarrow \Big[Q_\alpha \ , \ R\Big] \quad = \quad Q_\alpha \ , \qquad \Big[\bar{Q}_{\dot{\alpha}} \ , \ R\Big] \quad = \quad -\bar{Q}_{\dot{\alpha}}.}$$

2.3 Representations of the Poincaré group

Since we are changing the Poincaré group, we must check to see if anything happens to the Casimirs of the changed group, since these are used to label irreducible representations (remember that one needs a complete commuting set of observables to label them). Recall the rotation group $\{J_i: \ i=1,2,3\}$ satisfying

$$\Big[J_i \ , \ J_j\Big] \quad = \quad i\epsilon_{ijk}\,J_k \ .$$

The Casimir operator

$$J^2 = \sum_{i=1}^{3} J_i^2$$

commutes with all the J_i and labels irreducible representations by eigenvalues $j(j+1)$ of J^2. Within these irreducible representations, the J_3 eigenvalues $j_3 = -j, -j+1, ..., j-1, j$ label each element. States are labelled like $|j, j_3\rangle$.
Also recall the two Casimirs in Poincaré group, one of which involves the *Pauli Ljubanski vector* W_μ describing generalised spin

$$W_\mu = \frac{1}{2}\, \epsilon_{\mu\nu\rho\sigma}\, P^\nu\, M^{\rho\sigma}$$

(where $\epsilon_{0123} = -\epsilon^{0123} = +1$).

The Poincaré Casimirs are then given by

$$C_1 = P^\mu\, P_\mu\ , \qquad C_2 = W^\mu\, W_\mu\ ,$$

since the C_i commute with all generators.

Poincaré multiplets are labelled $|m, \omega\rangle$, where m^2 is the eigenvalue of C_1 and ω is the eigenvalue of C_2. States within those irreducible representations carry the eigenvalue p^μ of the generator P^μ as a label. Notice that at this level the Pauli Ljubanski vector only provides a short way to express the second Casimir. Even though W_μ has standard commutation relations with the generators of the Poincaré group $M_{\mu\nu}$ (since it transforms as a vector under Lorentz transformations) and commutes with P_μ (it is invariant under translations), the commutator $[W_\mu, W_\nu] = i\epsilon_{\mu\nu\rho\sigma} W^\rho P^\sigma$ implies that the W_μ's by themselves are not generators of a closed algebra.
To find more labels we take P^μ as given and look for all elements of the Lorentz group that commute with P^μ. This defines little groups:

- Massless particles have $p^\mu = (|\mathbf{p}|,\ \mathbf{p})$ and W^μ eigenvalues λp^μ. Thus, $\lambda = \mathbf{j} \cdot \mathbf{p}/|\mathbf{p}|$ is the helicity.

  States are thus labelled $|0, 0;\ p^\mu, \lambda\rangle =: |p^\mu, \lambda\rangle$. Under CPT[4], those states transform to $|p^\mu, -\lambda\rangle$. λ must be integer or half integer $\lambda = 0, \frac{1}{2}, 1, ...$, e.g. $\lambda = 0$ (Higgs), $\lambda = \frac{1}{2}$ (quarks, leptons), $\lambda = 1$ (γ, $W^\pm$, Z^0, g) and $\lambda = 2$ (graviton). Note that massive representations are CPT self-conjugate.

2.4 $\mathcal{N} = 1$ supersymmetry representations

For $\mathcal{N} = 1$ supersymmetry, $C_1 = P^\mu P_\mu$ is still a good Casimir, $C_2 = W^\mu W_\mu$, however, is not because it doesn't commute with Q_α. One can have particles of different spin within one multiplet. To get a new Casimir $\tilde{C}_2$ (corresponding to superspin), we define

$$B_\mu := W_\mu - \frac{1}{4}\, \bar{Q}_{\dot{\alpha}}\, (\bar{\sigma}_\mu)^{\dot{\alpha}\beta}\, Q_\beta\ , \qquad C_{\mu\nu} := B_\mu\, P_\nu - B_\nu\, P_\mu$$

$$\tilde{C}_2 := C_{\mu\nu}\, C^{\mu\nu}\ .$$

[4]The CPT theorem states that *any local Lorentz invariant quantum field theory is CPT invariant [3].*

2.4.1 Massless supermultiplet

States of massless particles have P^μ - eigenvalues $p^\mu = (E,\ 0,\ 0,\ E)$. The Casimirs $C_1 = P^\mu P_\mu$ and $\tilde{C}_2 = C_{\mu\nu}C^{\mu\nu}$ are zero. Consider the algebra (implicitly acting on our massless state $|p^\mu,\ \lambda\rangle$ on the right hand side)

$$\left\{Q_\alpha\ ,\ \bar{Q}_{\dot\beta}\right\} \ = \ 2\,(\sigma^\mu)_{\alpha\dot\beta}\,P_\mu \ = \ 2\,E\,(\sigma^0 - \sigma^3)_{\alpha\dot\beta} \ = \ 4\,E\begin{pmatrix}0 & 0\\ 0 & 1\end{pmatrix}_{\alpha\dot\beta}\ ,$$

which implies that Q_1 is zero in the representation:

$$\langle p^\mu,\lambda|\left\{Q_1\ ,\ \bar{Q}_{\dot 1}\right\}|p^\mu,\ \lambda\rangle \ = \ 0 \Leftrightarrow \bar{Q}_{\dot 1}|p^\mu,\ \lambda\rangle \ = \ Q_1|p^\mu,\ \lambda\rangle \ = \ 0.$$

We may also find one element $|p^\mu,\lambda\rangle$ such that $Q_2|p^\mu,\lambda\rangle = 0$.
From our previous commutation relation, and the definition of W_μ, in this representation

$$[W_\mu,\ \bar{Q}^{\dot\alpha}] = \frac{1}{2}\epsilon_{\mu\nu\rho\sigma}P^\nu[M^{\rho\sigma},\bar{Q}^{\dot\alpha}] = -\frac{1}{2}\epsilon_{\mu\nu\rho\sigma}P^\nu(\bar\sigma^{\rho\sigma})^{\dot\alpha}_{\ \dot\beta}\bar{Q}^{\dot\beta}$$

$$\Rightarrow [W_0,\ \bar{Q}^{\dot\alpha}]|p^\mu,\ \lambda\rangle = -\frac{i}{8}\epsilon_{03jk}p^3\left([\bar\sigma^j,\ \sigma^k]\bar{Q}\right)^{\dot\alpha}|p^\mu,\ \lambda\rangle = -\frac{1}{2}p^3(\sigma^3\bar{Q})^{\dot\alpha}|p^\mu,\ \lambda\rangle. \quad (2.4)$$

So, remembering that for massless representations, W_0 measures the *helicity* λ,

$$W_0\bar{Q}^{\dot 2}|p^\mu,\ \lambda\rangle = \left([W_0,\bar{Q}^{\dot 2}] + \bar{Q}^{\dot 2}\lambda p_0\right)|p^\mu,\ \lambda\rangle = (\lambda + \frac{1}{2})p_0\bar{Q}^{\dot 2}|p^\mu,\ \lambda\rangle.$$

Thus, $\bar{Q}^{\dot 2} = -\bar{Q}_{\dot 1}$ increases the helicity by 1/2 a unit[5]. The normalised state is then

$$|p^\mu,\ \lambda + \frac{1}{2}\rangle = \frac{\bar{Q}_{\dot 1}}{\sqrt{4E}}|p^\mu,\ \lambda\rangle \quad (2.5)$$

and there are no other states, since Eq. 2.5 $\Rightarrow \bar{Q}_{\dot 1}|p^\mu,\ \lambda + \frac{1}{2}\rangle = 0$ and

$$Q_2|p^\mu,\ \lambda + \frac{1}{2}\rangle = \frac{1}{\sqrt{4E}}Q_2\bar{Q}_{\dot 2}|p^\mu,\ \lambda\rangle = \frac{1}{\sqrt{4E}}\left(\{Q_2,\ \bar{Q}_{\dot 2}\} - \bar{Q}_{\dot 2}Q_2\right)|p^\mu,\ \lambda\rangle = \sqrt{4E}|p^\mu,\ \lambda\rangle,$$

Thus, we have two states in the supermultiplet: a boson and a fermion, plus CPT conjugates:

$$|p^\mu, \pm\lambda\rangle\ , \qquad |p^\mu, \pm\left(\lambda + \tfrac{1}{2}\right)\rangle\ .$$

There are, for example, chiral multiplets with $\lambda = 0, \frac{1}{2}$, vector- or gauge multiplets ($\lambda = \frac{1}{2}, 1$ gauge and gaugino)

$\lambda = 0$ scalar	$\lambda = \frac{1}{2}$ fermion
squark	quark
slepton	lepton
Higgs	Higgsino

$\lambda = \frac{1}{2}$ fermion	$\lambda = 1$ boson
photino	photon
gluino	gluon
Wino, Zino	$W,\ Z$

,

as well as the graviton with its partner:

$\lambda = \frac{3}{2}$ fermion	$\lambda = 2$ boson
gravitino	graviton

Question: 3. Why do we put matter fields in the $\lambda = \{0, \frac{1}{2}\}$ supermultiplets rather than in the $\lambda = \{\frac{1}{2},\ 1\}$ ones?

[5] Note that we have used natural units, therefore $\hbar = 1$.

3 Introducing the minimal supersymmetric standard model (MSSM)

The MSSM is based on $SU(3)_C \times SU(2)_L \times U(1)_Y \times N = 1$ SUSY. We must fit all of the experimentally discovered field states into $N = 1$ supermultiplets.

3.1 Particles

First of all, we have vector superfields containing the Standard Model gauge bosons. We write their representations under $(SU(3)_C,\ SU(2)_L\ U(1)_Y)$ as (pre-Higgs mechanism):

- gluons/gluinos

$$G = (8, 1, 0)$$

- W bosons/winos

$$W = (1, 3, 0)$$

- B bosons/gauginos

$$B = (1, 1, 0),$$

  which contains the gauge boson of $U(1)_Y$.

Secondly, there are supermultiplets containing Standard Model matter and Higgs fields. Since $\lambda = 0$ superfields only contain left-handed fermions, we use charge conjugated, i.e. *anti* right handed fermionic fields (which are actually left-handed), denoted by ^{c}

- (s)quarks: lepton number $L = 0$, whereas baryon number $B = 1/3$ for a (s)quark, $B = -1/3$ for an anti-quark.

$$\underbrace{Q_i = \left(3,\ 2, \tfrac{1}{6}\right)}_{\text{left-handed}}, \quad \underbrace{u_i^c = \left(\bar{3},\ 1,\ -\tfrac{2}{3}\right), \quad d_i^c = \left(\bar{3},\ 1,\ \tfrac{1}{3}\right)}_{\text{anti (right-handed)}}$$

- (s)leptons $L = 1$ for a lepton, $L = -1$ for an anti-lepton. $B = 0$.

$$\underbrace{L_i = \left(1,\ 2,\ -\tfrac{1}{2}\right)}_{\text{left-handed}}, \quad \underbrace{e_i^c = (1,\ 1,\ +1)}_{\text{anti (right-handed)}}$$

- higgs bosons/higgsinos: $B = L = 0$.

$$H_2 = \left(1,\ 2,\ \tfrac{1}{2}\right), \quad H_1 = \left(1,\ 2,\ -\tfrac{1}{2}\right)$$

  the second of which is a new Higgs doublet not present in the Standard Model. Thus, the MSSM is a *two Higgs doublet model.* The extra Higgs doublet is needed in order to avoid a gauge anomaly, and to give masses to down-type quarks and leptons.

Note that after the breaking of electroweak symmetry (see the Standard Model course), the electric charge generator is $Q = T_3^{SU(2)_L} + Y/2$. Baryon and lepton number correspond to multiplicative discrete perturbative symmetries in the SM, and are thus conserved, perturbatively.

Chiral fermions may generate an *anomaly* in the theory, as shown by Fig. 1. This is where a symmetry that is present in the tree-level Lagrangian is broken by quantum corrections. Here, the symmetry is $U(1)_Y$: all chiral fermions in the theory travel in the loop, and yield a logarithmic divergence proportional to

$$A := \sum_{LH\ f_i} Y_i^3 - \sum_{RH\ f_i} Y_i^3$$

multiplied by some kinematic factor which is the same for each fermion. If A is non-zero, one must renormalise the diagram away by adding a $B_\mu B_\nu B_\rho$ counter term in the Lagrangian. But this breaks $U(1)_Y$, meaning that $U(1)_Y$ would not be a consistent symmetry at the quantum level. Fortunately, $A = 0$ for each fermion family in the Standard Model.

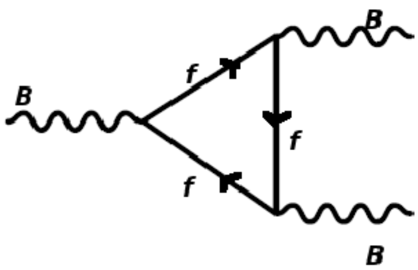

Figure 1. Anomalous graph proportional to $\mathrm{Tr}\{Y^3\}$ which must vanish for $U(1)_Y$ to be a valid symmetry at the quantum level. Hyper-charged chiral fermions f travel in the loop contributing to a three-hypercharge gauge boson B vertex.

Question: 4. Can you show that $A = 0$ in a Standard Model family?

In SUSY, we add the Higgsino doublet $\tilde{H}_1$, which yields a non-zero contribution to A. This must be cancelled by another Higgsino doublet with opposite Y: H_2.

3.2 Supersymmetry breaking in the MSSM

Supersymmetry predicts that, say $m_e = m_{\tilde{e}}$, but an electron mass charge -1 scalar has not been observed. Somehow, we must break supersymmetry to make $m_{\tilde{e}}$ heavier, but in a way that still solves the technical hierarchy problem.

We cannot have supersymmetry breaking directly in the MSSM, since it preserves $\mathrm{STr}\{M^2\} = 0$. Applying this to the photon, say: $-3m_\gamma^2 + 2m_{\tilde{\gamma}}^2 = 0$, which would predict a massless photino that hasn't been observed. Applying it to up quarks: $2m_u^2 - m_{\tilde{u}_L}^2 - m_{\tilde{u}_R}^2 = 0$, thus one up squark must be *lighter* than the up quark, again this hasn't been observed. We introduce a *hidden* sector, which breaks SUSY and has its own fields (which do not directly interact with MSSM fields) and interactions, and an additional *messenger sector*

$$\begin{pmatrix}\text{observable}\\ \text{sector, MSSM}\end{pmatrix} \longleftrightarrow \begin{pmatrix}\text{messenger -}\\ \text{sector}\end{pmatrix} \longleftrightarrow \begin{pmatrix}\text{hidden}\\ \text{sector}\end{pmatrix}.$$

This gets around the supertrace rule. There is typically an overall gauge group

$$\big(SU(3)\ \times\ SU(2)\ \times\ U(1)\big)\ \times\ G_{\not{SUSY}}\ =:\ G_{SM}\ \times\ G_{\not{SUSY}},$$

where the MSSM fields are singlets of $G_{\not{SUSY}}$ and the hidden sector fields are singlets of G_{SM}.

One popular SUSY-breaking sector in the MSSM context is that of *gaugino condensation*: here, some asymptotically free gauge coupling g becomes large at some energy scale Λ. One obtains $\Lambda = M \exp[1/\beta g^2(M)]$, with β some order 1 number. M could be some large scale such as the string scale, $\sim 5 \times 10^{17}$ GeV. It is easy to arrange for $\Lambda \ll M$. When the gauge coupling becomes large, and the theory becomes non-perturbative, one can obtain $\langle \tilde{g}\tilde{g} \rangle \neq 0$, breaking SUSY dynamically[6].

The SUSY breaking fields have couplings with the messenger sector, which in turn have couplings with the MSSM fields, and carry the SUSY breaking over to them. There are several possibilities for the messenger sector fields, which may determine the explicit form of SUSY breaking terms in the MSSM, including:

- gravity mediated $\not{SUSY}$
- gauge mediated $\not{SUSY}$
- anomaly mediated $\not{SUSY}$

Each if these scenarios has phenomenological advantages and disadvantages and solving their problems is an active field of research. In all scenarios, the Lagrangian for the observable sector has contributions

$$\mathcal{L} \;=\; \mathcal{L}_{\rm SUSY} \;+\; \mathcal{L}_{\not{\rm SUSY}}.$$

In the second term, we write down all renormalisable symmetry invariant terms which do not reintroduce the hierarchy problem. They are of the form (where i and j label different fields):

$$\boxed{\mathcal{L}_{\not{\rm SUSY}} \;=\; \underbrace{m^2_{ij}\,\varphi^*_i\,\varphi_j + m'^2_{ij}(\varphi_i\varphi_j + h.c.)}_{\text{scalar masses}} + \left(\underbrace{\frac{1}{2}M_\lambda\,\lambda\lambda}_{\text{gaugino masses}} \;+\; \underbrace{A_{ijk}\,\varphi_i\varphi_j\varphi_k}_{\text{trilinear couplings}} \;+ h.c.\right).}$$

$M_\lambda, m'^2_{ij}, m^2_{ij}, A_{ijk}$ are called *soft SUSY breaking terms*: they do not reintroduce large sensitivity to the heavy mass scales in the quantum corrections. Particular forms of SUSY breaking mediation can give relations between the different soft SUSY breaking terms. They determine the amount by which supersymmetry is expected to be broken in the observable sector, and the masses of the superparticles for which the LHC is searching.

Explicitly, we parametrise all of the terms that softly break SUSY in the R_p preserving MSSM, suppressing gauge indices:

$$\begin{aligned}\mathcal{L}^{\not{\rm SUSY}}_{R_p} = {} & (A_U)_{ij}\tilde{Q}_{Li}H_2\tilde{u}^*_{Rj} + (A_D)_{ij}\tilde{Q}_{Li}H_1\tilde{d}^*_{Rj} + (A_E)_{ij}\tilde{L}_{Li}H_1\tilde{e}^*_{Rj} + \\ & \tilde{Q}^*_{Li}(m^2_{\tilde{Q}})_{ij}\tilde{Q}_{Lj} + \tilde{L}^*_i(m^2_{\tilde{L}})_{ij}\tilde{L}_j + \tilde{u}_{Ri}(m^2_{\tilde{U}})_{ij}\tilde{u}^*_{Rj} + \tilde{d}_{Ri}(m^2_{\tilde{D}})_{ij}\tilde{d}^*_{Rj} + \tilde{e}_{Ri}(m^2_{\tilde{E}})_{ij}\tilde{e}^*_{Rj} + \\ & (m^2_3 H_1H_2 + h.c.) + m^2_1|H^2_1| + m^2_2|H_2|^2 + \frac{1}{2}M_3\tilde{g}\tilde{g} + \frac{1}{2}M_2\tilde{W}\tilde{W} + \frac{1}{2}M_1\tilde{B}\tilde{B}.\end{aligned}$$

[6] Here, $\tilde{g}$ is the gaugino of the hidden sector gauge group.

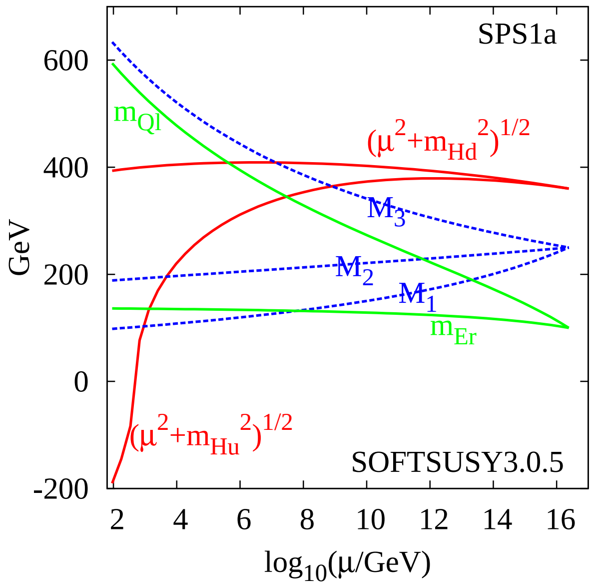

Figure 2. An example of renormalisation in the MSSM. A particular high energy theory is assumed, which has GUT symmetry and implies that the gauginos are all mass degenerate at the GUT scale. The scalars (e.g the right-handed electron Er and the left-handed squarks Ql) are also mass-degenerate at the GUT scale. Below the GUT scale though, the masses split and renormalise separately. When we are scattering at energies $\sim O(1)$ GeV, it is a good approximation to use the masses evaluated at that renormalisation scale $\mu \approx E$. We see that one of the Higgs mass squared parameters, $\mu^2 + M_{Hu}^2$, becomes negative at the electroweak scale, triggering electroweak symmetry breaking.

Question: Which symmetry bans say $m e_R e_R$?

Sometimes, m_3^2 is written as μB. Often, specific high scale models provide relations between these many parameters. For instance, the Constrained MSSM (which may come from some string theory or other field theory) gives the constraints

$$\begin{aligned} M_1 &= M_2 = M_3 =: M_{1/2} \\ m_{\tilde{Q}}^2 &= m_{\tilde{L}}^2 = m_{\tilde{U}}^2 = m_{\tilde{D}}^2 = m_{\tilde{E}}^2 := m_0^2 I_3 \\ m_1^2 &= m_2^2 = m_0^2 \\ A_U &= A_0 Y_U, \; A_D = A_0 Y_D, \; A_E = A_0 Y_E \end{aligned}$$

where I_3 is the 3 by 3 identity matrix. Thus in the 'CMSSM', we reduce the large number of free SUSY breaking parameters down to[7] 3: $M_{1/2}$, m_0 and A_0. These relations hold at the GUT scale, and receive large radiative corrections, as Fig. 2 shows.

3.3 Pros and Cons of the MSSM

We start with a list of unattractive features of the MSSM:

[7]One should really include $\tan\beta = v_2/v_1$ as well, the ratio of the two Higgs vacuum expectation values.

- There are ~ 100 extra free parameters in the SUSY breaking sector, making for a complicated parameter space.
- Nearly all of this parameter space is ruled out from flavour physics constraints: SUSY particles could heavily mix in general, then this mixing could appear in loops and make the quarks mix in a flavour changing neutral current, upon which there are very strong experimental bounds. It could be that this clue is merely telling us that there is more structure to the MSSM parameter space, though (like in the CMSSM).
- The μ problem. μ is a parameter in the supersymmetric part of the Lagrangian, and must be $< \mathcal{O}(1)$ TeV, since it contributes at tree-level to m_H. Why should this be, when in principle we could put it to be $\sim \mathcal{O}(M_{Pl})$, because it does not break any SM symmetries?

These SUSY problems can be solved with further model building.

We close with an ordered list of weak-scale SUSY's successes:

- SUSY solves the technical hierarchy problem.
- Gauge unification works.
- The MSSM contains a viable dark matter candidate, if R_p is conserved.
- Electroweak symmetry breaks radiatively.

4 Using SOFTSUSY

`SOFTSUSY` [12] is a computer program that calculates the masses of supersymmetric particles (and couplings of the particles etc) in the Minimal Supersymmetric Standard Model, consistent with some theoretical boundary conditions on supersymmetry breaking terms, and consistent with measurements of Standard Model fermion masses and gauge boson masses and couplings and electroweak symmetry breaking. We will download the SOFTSUSY computer program, compile it and run it, preparing computer input with the SUSY Les Houches Accord. The output can be fed into collider (eg LHC) simulations, programs that calculate dark matter properties, decay calculators etc.

4.1 SOFTSUSY Tutorial

First, we need to download and compile the program.

1. Google `SOFTSUSY`
2. Click on the `SOFTSUSY` homepage: it should be at `http://softsusy.hepforge.org`.
3. Scroll down to "latest release", then click on the most recent "source" link (at the time of writing, this is `source 3.3.3`).
4. Your browser may save it, or ask whether you want to do so (you should).

5. Now, do

```
> tar -xvzf softsusy-3.3.3.tar.gz
> cd softsusy-3.3.3
```

6. Now we compile the code:

```
> ./configure F77=gfortran
> make
```

7. We are going to use the main program `softpoint.x`. If it's run without any arguments, you'll see some options come up:

```
> ./softpoint.x
```

8. Now let's run the program. We're going to pick an mSUGRA/CMSSM parameter point that is called CMSSM10.1.1 [13]. We have a universal scalar SUSY breaking mass of $m_0 = 125$ GeV, a universal gaugino mass of $m_{1/2} = 500$ GeV, ratio of the two Higgs vacuum expectation values $\tan\beta = v_u/v_d = 10$ and SUSY breaking trilinear scalar couplings all $A_0 = 0$. We also pick the μ parameter from the superpotential to be positive.

```
> more lesHouchesInput
```

You can find out more about how to change this input file, and understand the output at `http://arxiv.org/pdf/hep-ph/0311123v4.pdf` [14].

```
> ./softpoint.x leshouches < lesHouchesInput > lesHouchesOutput
```

9. Now have a look at the output file

```
> more lesHouchesOutput
```

In `Block MASS`, you can see the masses, in units of GeV, of the various particles. The '`#`' denotes the start of a comment, and the particles are all handily labelled after this. So, for example, you can see that at this point, the gluino mass is predicted to be $m_{\tilde{g}} = 1147$ GeV, whereas the lightest higgs is at 115.4 GeV (note that this is *inconsistent* with the recent Higgs measurements, so this point doesn't agree with data).

10. Try changing

m_0 to 10^4 GeV. You should see a

```
      4   Point  invalid: [ MuSqWrongsign ]
```

message under `Block SPINFO`.

11. Next reduce m_0 to 5×10^3 GeV. What is the Higgs mass now?

12. Competition: play with the numerical values of the inputs in `Block MINPAR`: can you get the Higgs mass higher? The first one to hit 125±1 GeV without any warning message wins.

13. If you're mega fast at this, try a harder problem: by referring to the SUSY Les Houches accord `hep-ph/0311123`, produce some output at CMSSM10.1.1 (that was originally used above) but with a separate mass parameter for the stop squarks at the GUT scale: $m_{\tilde{t}_R} = m_{\tilde{Q}_{3,L}}$ =5000 GeV.

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