BUSSTEPP 2012
Standard Model and Beyond
Exercises

Part III
A non standard Higgs

1 The SM Higgs as a specific limit

Consider the lagrangian

$$\mathcal{L}_H = \frac{1}{2}(\partial_\mu h)^2 + V(h) + \frac{v^2}{4}\mathrm{Tr}\left[(D_\mu\Sigma)^\dagger(D_\mu\Sigma)\right]\left(1 + 2a\,\frac{h}{v} + b\,\frac{h^2}{v^2}\right)$$
$$- \frac{v}{\sqrt{2}}\sum_{i,j}\left(\bar{u}_L^{(i)} d_L^{(i)}\right)\Sigma\left(1 + c\,\frac{h}{v}\right)\begin{pmatrix}\lambda_{ij}^u\, u_R^{(j)} \\ \lambda_{ij}^d\, d_R^{(j)}\end{pmatrix} + h.c.$$

where h is a real scalar field, $\Sigma = e^{i\sigma^a\chi^a(x)/v}$ is a 2× 2 unitary matrix accounting for the Goldstone bosons χ_a transforming as $\Sigma \to U_L(x)\,\Sigma\,U_Y^\dagger(x)$ under a Standard Model SU(2)×U(1) gauge transformation (with $U_Y = e^{i\alpha_Y\sigma_3/2}$), v is the electroweak scale, and a Dirac notation is used for the SM quarks. Consider the limit $a = b = c = 1$ and define the complex doublet field

$$H(x) = \frac{1}{\sqrt{2}}\,e^{i\sigma^a\chi^a(x)/v}\begin{pmatrix}0 \\ v + h(x)\end{pmatrix}.$$

Show that in terms of the above field, the lagrangian above can be written as

$$\mathcal{L}_H = (D_\mu H)^\dagger(D^\mu H) - \hat{V}(H^\dagger H) - \left(\lambda_{ij}^u\bar{q}_L^{(i)}Hu_R^{(j)} + \lambda_{ij}^d\bar{q}_L^{(i)}H^*d_R^{(j)} + \text{h.c.}\right),$$

provided that $V(h) = (m^2/2)h^2 + (m^2/(2v))h^3 + (m^2/v^2)h^4$.

2 An Higgs doublet as a pseudo-Goldstone boson

Consider a toy extension of the SM in which the gauge group reduces to $\mathrm{SU}(2)_L$. Assume that the scalar sector of the theory has a SU(3) global symmetry spontaneously broken by the vev of a real scalar A in the adjoint representation of SU(3), $\langle A\rangle = f\,\mathrm{diag}(1,1,-2)$, where f is a scale larger than the electroweak scale v. The $\mathrm{SU}(2)_L$ gauge group corresponds to the upper 2×2 block of SU(3). Show that the Goldstone bosons contain an $\mathrm{SU}(2)_L$ doublet. Show that the SU(3) global symmetry is explicitly broken by the gauge interactions.