Busstepp 2012 – Cosmology
Problems 1

1. Given the (isotropic) metric:

$$ds^2 = dt^2 - a(t)^2(f(r)dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2)),$$

with r, θ and ϕ comoving coordinates, and $f(r)$ an arbitrary function of r, compute the scalar curvature ${}^3R$ for the 3-dimensional space. Show then that the homogeneity of space implies

$$f(r) = \frac{1}{1-kr^2},$$

where $k = +1, 0, -1$ (Robertson-Walker-metric).
Hint: the curvature can not depend on the spatial position if the universe is homogeneous)

2. Suppose that, in addition to the Hubble expansion, the typical rms peculiar velocity of galaxies is $600\,\mathrm{km\,sec}^{-1}$. Estimate the minimum distance at which a galaxy must be in order for the redshift to give an estimate of the true distance which is accurate to 10% or better.

3. Starting with the definitions of the number density n, energy density ρ and pressure p of a gas of particles of energy $E^2(\mathbf{p}) = \mathbf{p}^2 + m^2$

$$\begin{aligned} n &= \frac{g}{(2\pi)^3}\int d^3p f(\mathbf{p}) \\ \rho &= \frac{g}{(2\pi)^3}\int d^3p f(\mathbf{p})E(\mathbf{p}) \\ p &= \frac{g}{(2\pi)^3}\int d^3p f(\mathbf{p})\frac{p^2}{3E(\mathbf{p})} \end{aligned}$$

where $f(E,T) = \frac{1}{e^{E/T}\pm 1}$, $(\mu = 0)$, show that

$$\frac{\partial p}{\partial T} = \frac{\rho + p}{T}.$$

4. Estimates in cosmology are really useful as the numbers you are dealing with can be so extreme that sometimes after lots of calculations you are still not sure about the answer – for example can you really get your head around the order of magnitude that is the critical density of the Universe? Enrico Fermi one of the most brilliant physicists of the 20th century was a great advocate of the power of estimates. He has a series of questions which came to be known as Fermi questions and require nothing more than back of the envelope estimates to get a ball park figure for things which would typically be accurate to around 10%. Try a few with your mates (perhaps with a drink for company – but not if you are driving of course!):
a). How many piano tuners are there in Nottingham? (Estimate it and check against the phone directory)
b). How many jelly beans fill a one-liter bottle? Then eat them.
c). How many passengers are in the departure lounge at the South terminal of Gatwick airport at any given time?
d). How many petrol stations are there in the UK?