

Topics related to MSTW PDFs

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Together with Alan Martin, James Stirling, Graeme Watt
and Arnold Mathijssen and Ben Watt

Variety of topics - related in various ways.

- Brief reminder of results from Monte Carlo approach using **MSTW** PDFs from **JHEP 1208 (2012) 052 (G. Watt and RT)**.
- Some investigations using a **3-flavour FFNS** fit - (**RT to be in PRD**).
- Comparison of **MSTW** PDFs with LHC data and implications.
- Investigation of parameterisation extension dependence. Related to deuterium corrections. Implication for **LHC** data.

Study supported correctness of “dynamic tolerance” approach. Easiest in Hessian study with eigenvectors.

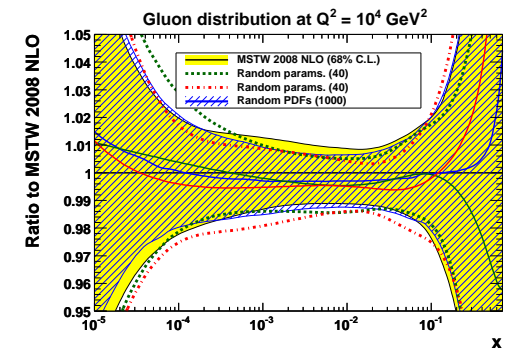
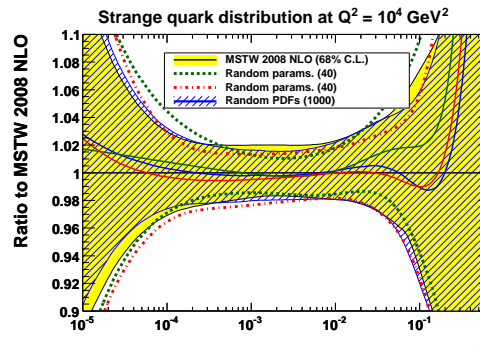
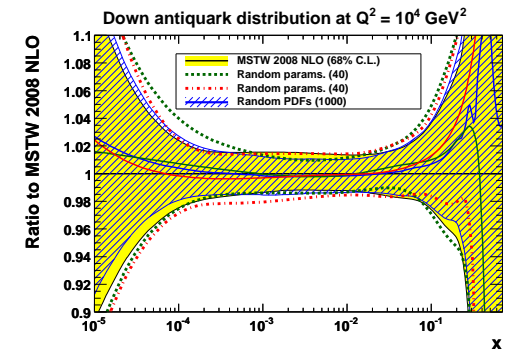
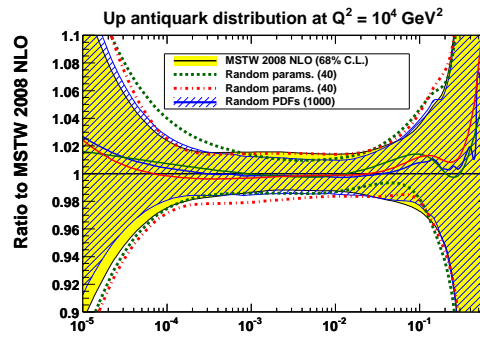
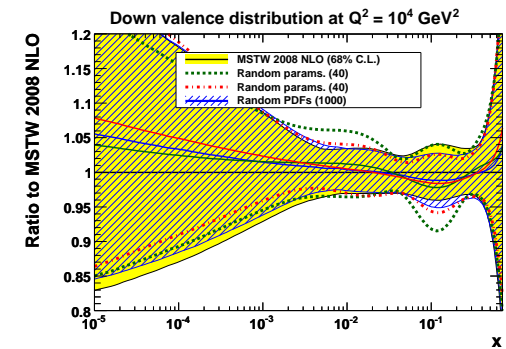
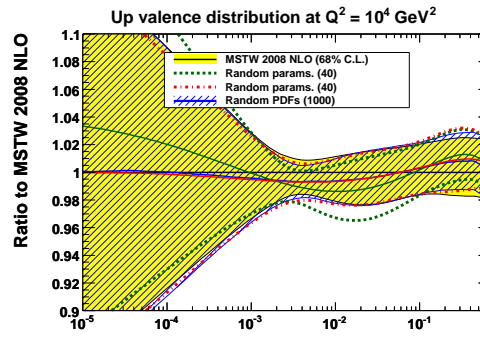
However, can generate “random” PDF sets directly from parameters and variation from eigenvectors.

$$a_i(\mathcal{S}_k) = a_i^0 + \sum_{j=1}^n e_{ij} (\pm t_j^\pm) |R_{jk}|$$

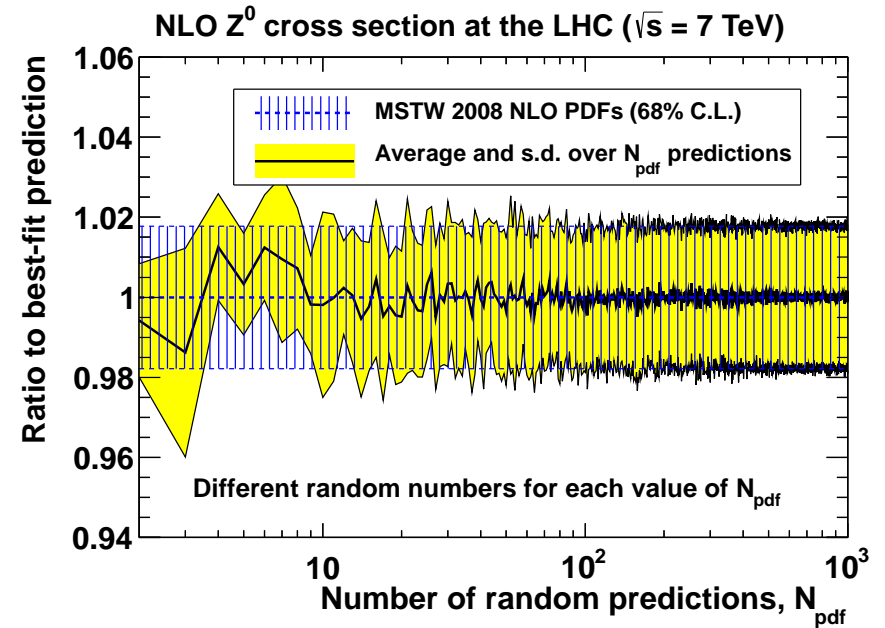
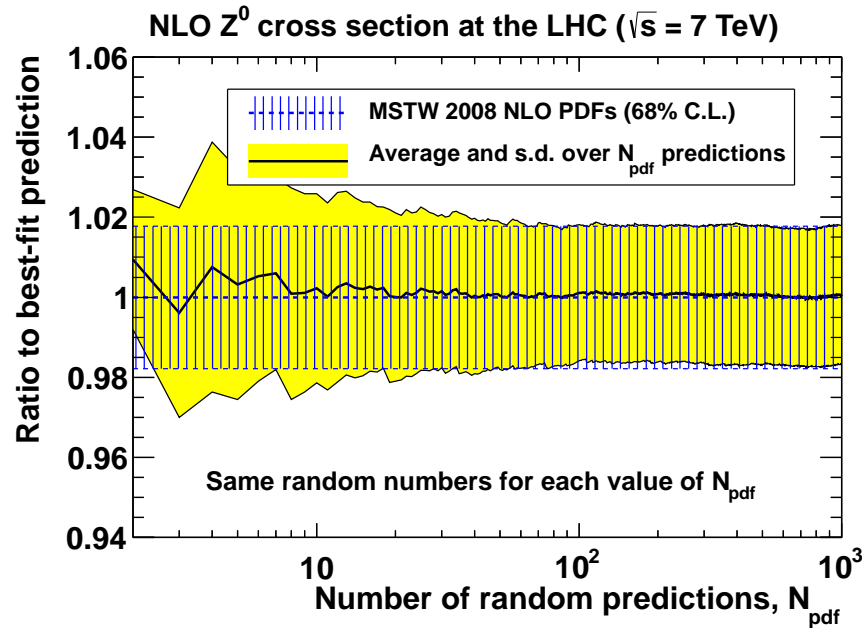
($k = 1, \dots, N_{\text{pdf}}$). Or from eigenvectors directly (see LHCb study and De Lorenzi thesis). Far quicker.

$$F(\mathcal{S}_k) = F(S_0) + \sum_j [F(S_j^\pm) - F(S_0)] |R_{jk}|$$

Use in reweighting studies as NNPDF.



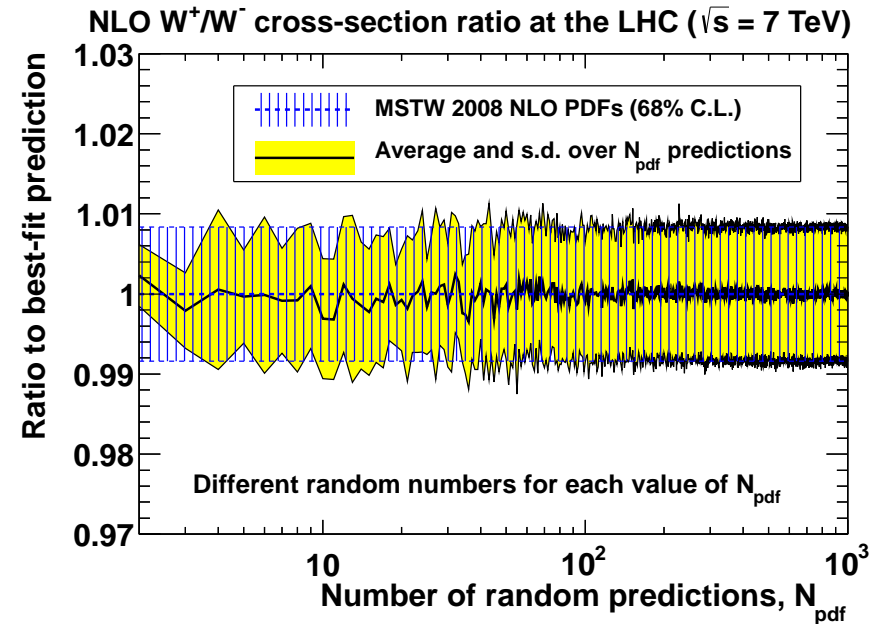
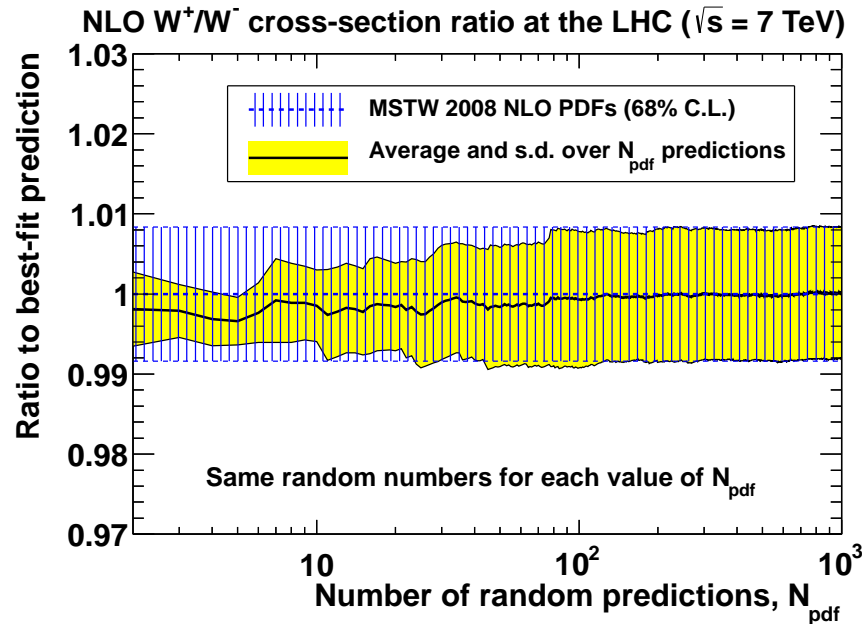
Speed of convergence of prediction for Z cross section.



Left, add a new random set to existing ones sequentially. Right, increasing numbers of independent random sets.

Very good with 40 sets. Excellent with 100 sets.

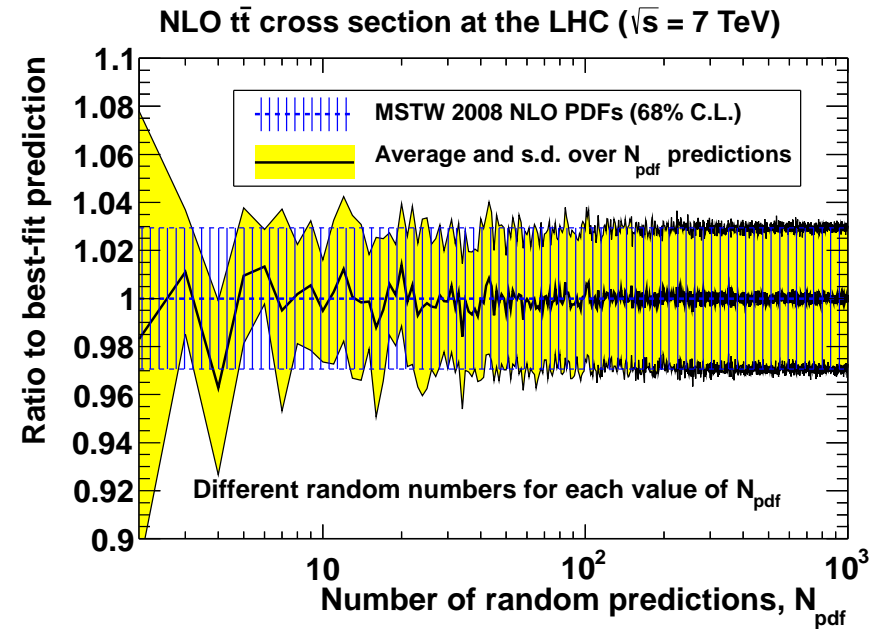
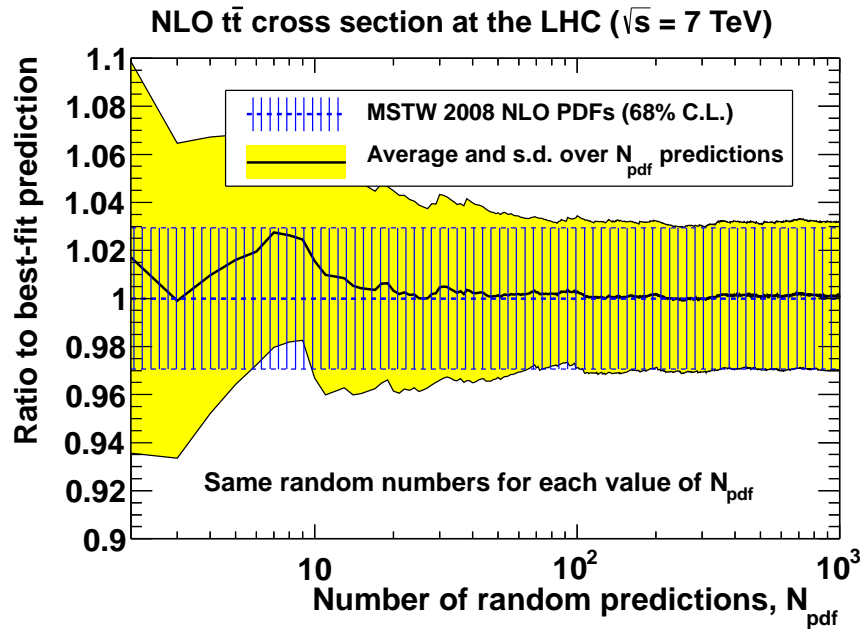
Speed of convergence of prediction for W^+/W^- cross section ratio.



Left, add a new random set to existing ones sequentially. Right, increasing numbers of independent random sets.

Very good with 40 sets. Excellent with 100 sets.

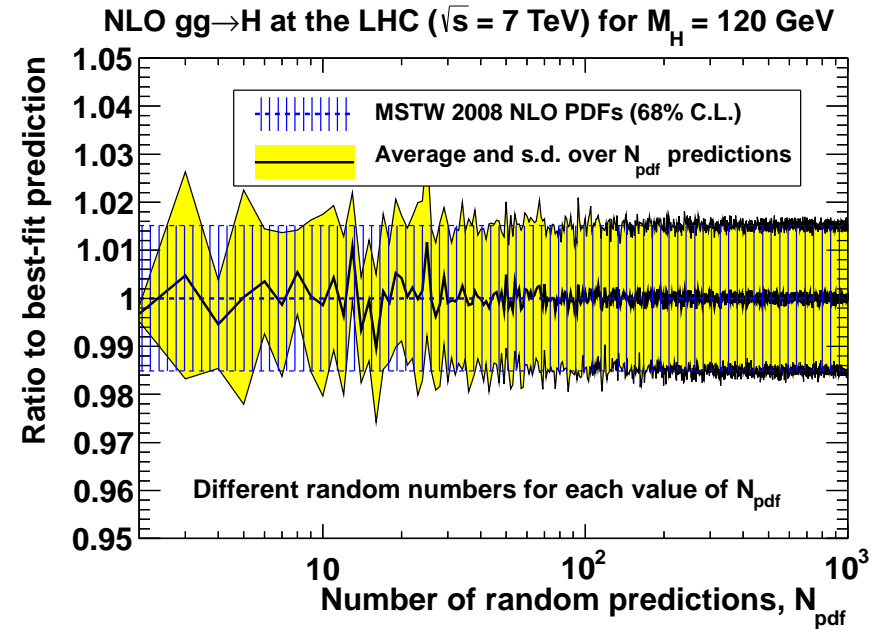
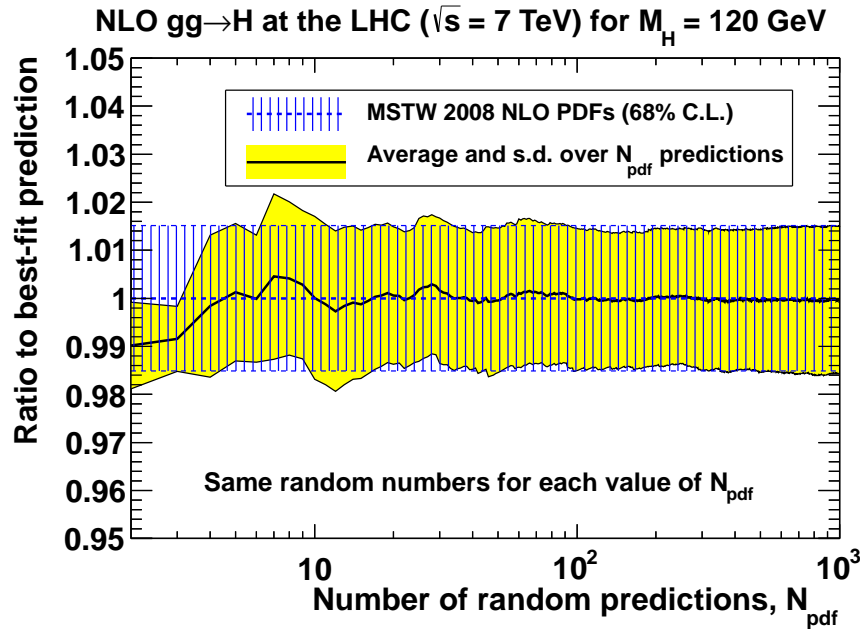
Speed of convergence of prediction for $t\bar{t}$ cross section.



Left, add a new random set to existing ones sequentially. Right, increasing numbers of independent random sets.

Very good with 40 sets. Excellent with 100 sets.

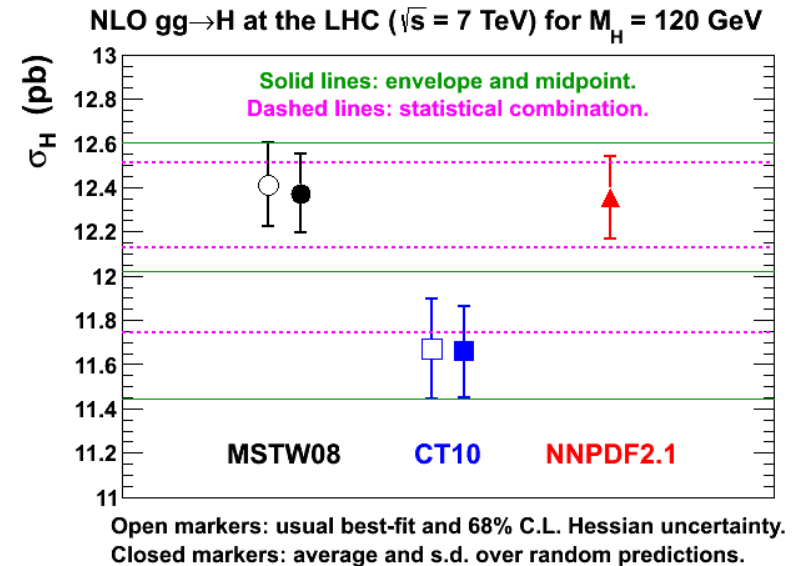
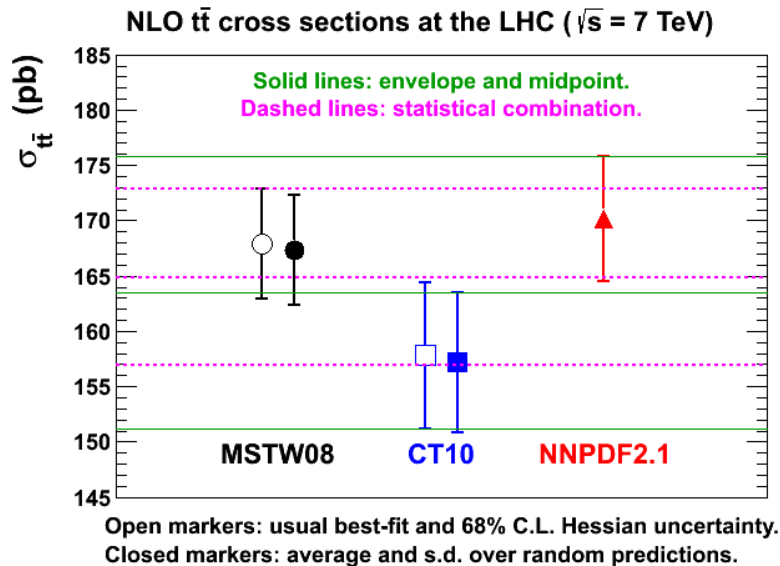
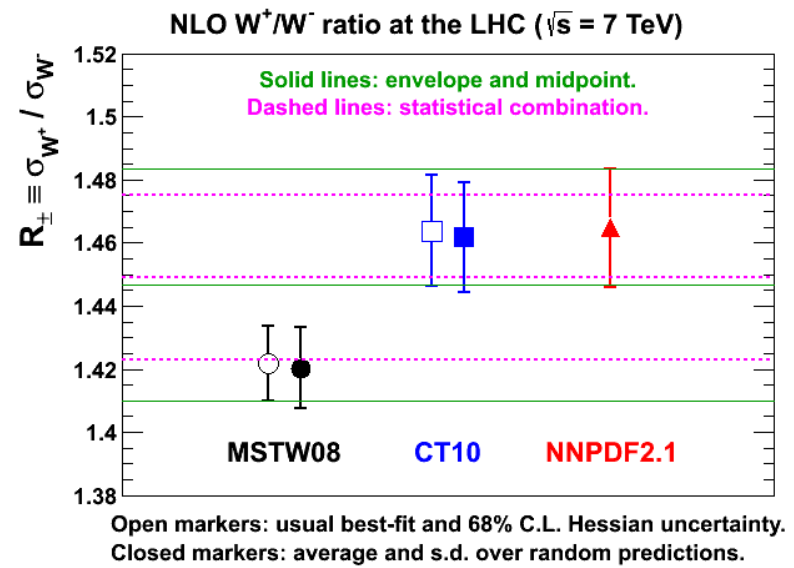
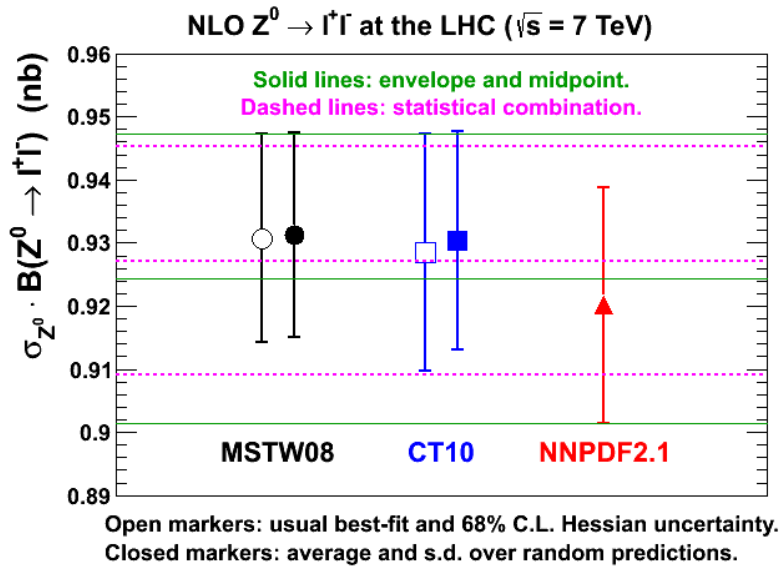
Speed of convergence of prediction for H cross section.



Left, add a new random set to existing ones sequentially. Right, increasing numbers of independent random sets.

Very good with 40 sets. Excellent with 100 sets.

Can combine different PDF sets, e.g. comparison to PDF4LHC prescription.



Smaller uncertainty and shifted central value if disagreement between individual predictions. (Plots by G. Watt at <http://mstwpdf.hepforge.org/random/>).

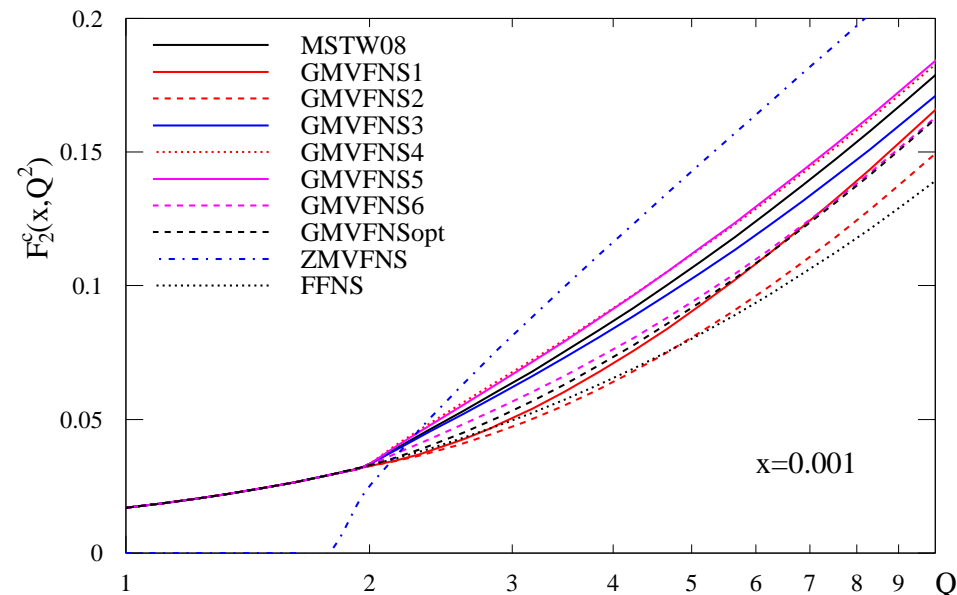
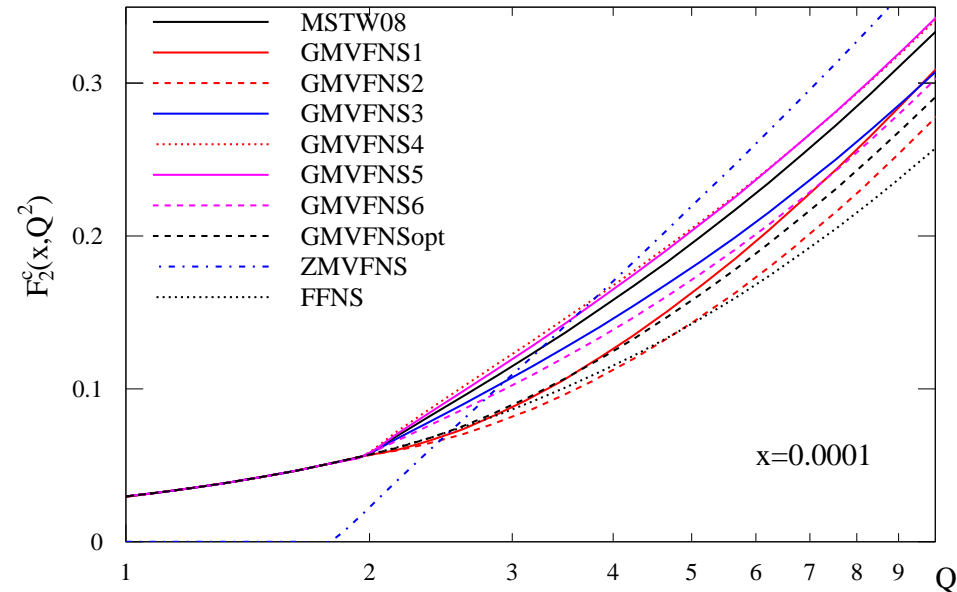
Results using a FFNS

Performed a fit to DIS only data using the FFNS scheme. (At NLO since NNLO still requires potentially significant approximations).

Do not include Drell-Yan or Tevatron jet data as FFNS calculations do not exist.

As seen at higher Q^2 charm structure function for FFNS always lower than any GM-VFNS.

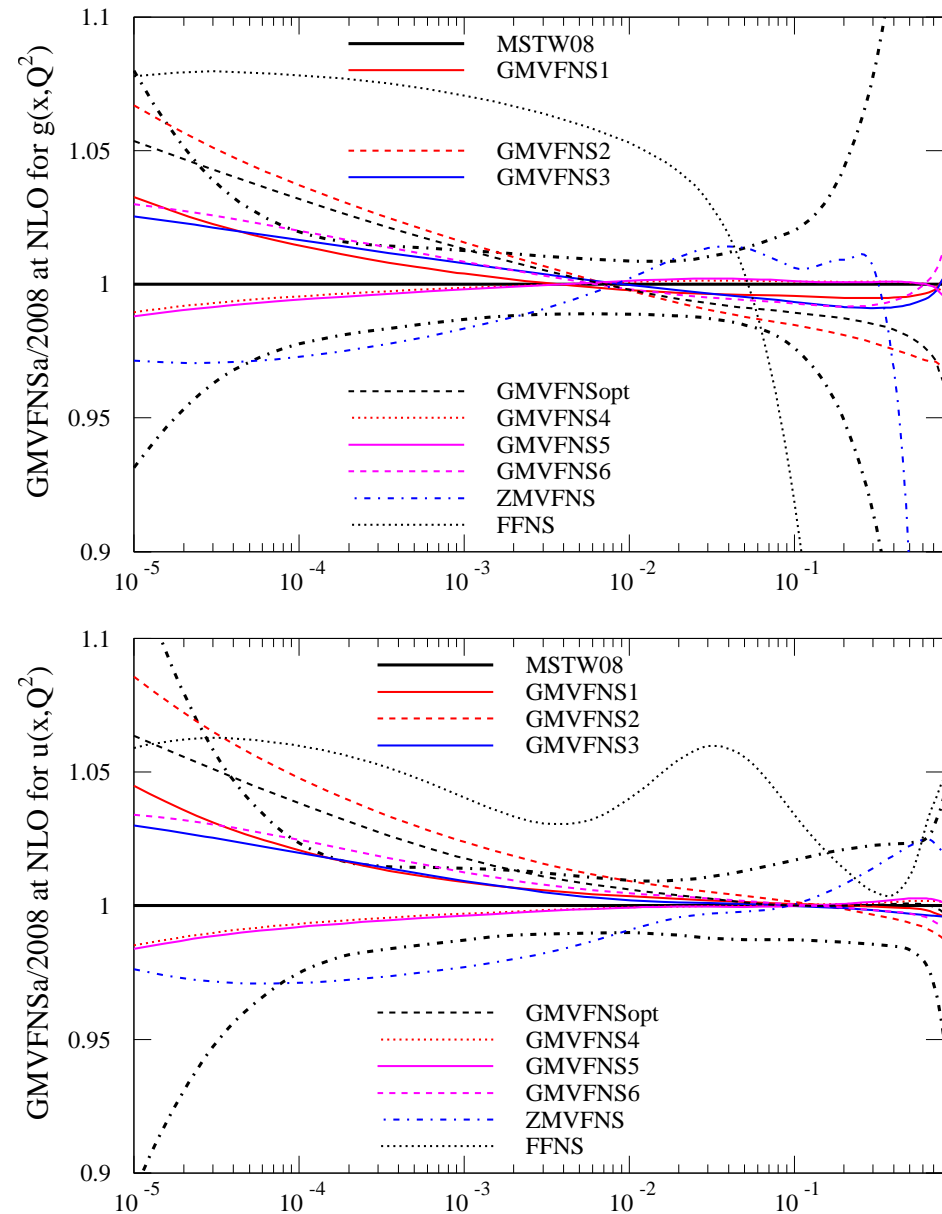
Fit a few tens of units worse than MSTW08 to same data (even without refitting). Slightly better for $F_2^c(x, Q^2)$, but flatter in Q^2 for $x \sim 0.01$ for inclusive structure function.

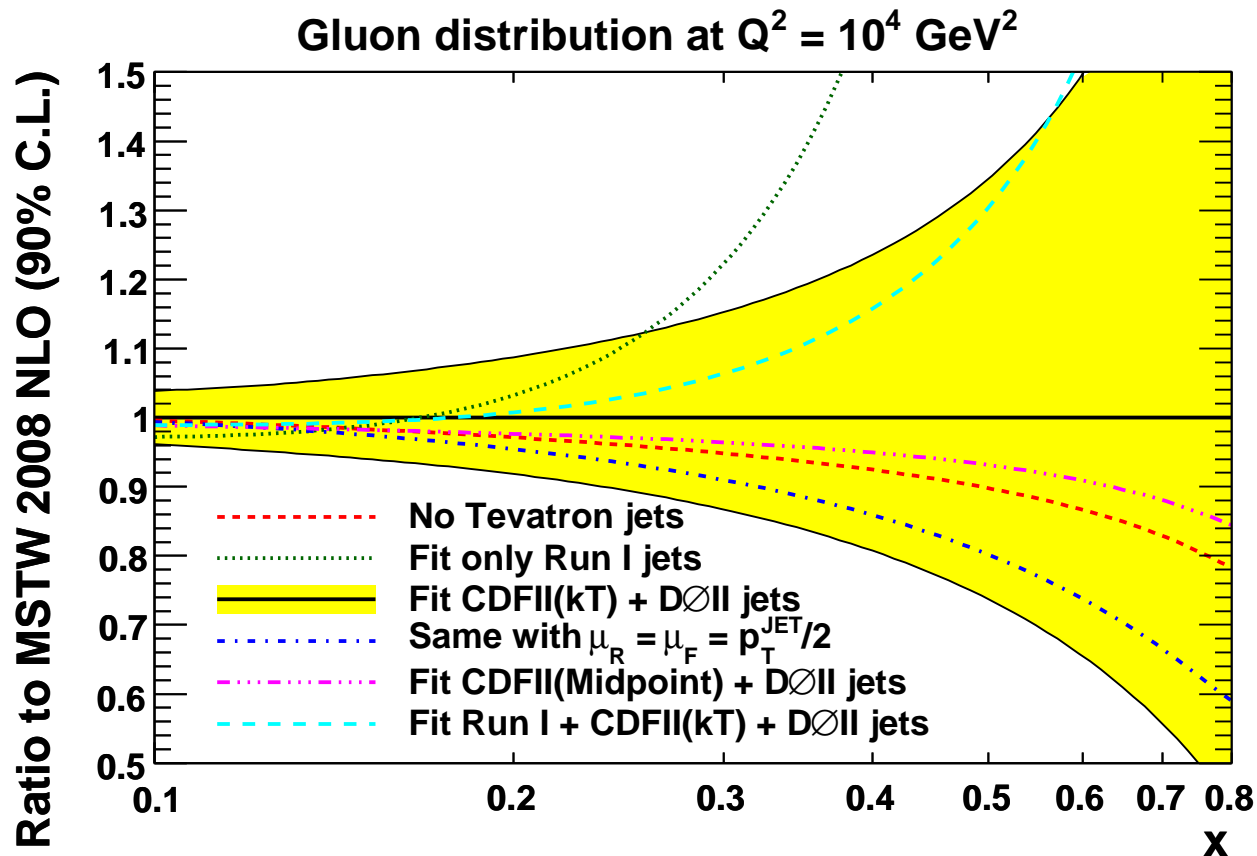


PDFs evolved up to $Q^2 = 10,000\text{GeV}^2$ (using variable flavour evolution for consistent comparison) different in form to **MSTW08** and **GM-VFNS** variants.

$\alpha_S(M_Z^2) = 0.1187$, a bit lower than **MSTW NLO** value of 0.1202 .

PDFs do not automatically fit **Tevatron** jet data well at all, and are not good for **CDF Z** rapidity data.





In contrast in [MSTW2008](#) fit central gluon hardly changed if Tevatron jet data left out, and only slight further rearrangement of quark flavours if [Drell-Yan](#) data left out (actually improves [CDF](#) rapidity data).

Main effect loss of tight constraint on $\alpha_S(M_Z^2)$. Similar results from various other groups.

At [NLO](#) see qualitative effect from using [FFNS](#) as opposed to any [GM-VFNS](#).

Comparison to LHC data.

Start with ATLAS jets. Use APPLGrid or FastNLO at NLO (Ben Watt) and correlated errors treated as in the formula,

$$\chi^2 = \sum_{i=1}^{N_{\text{pts.}}} \left(\frac{\hat{D}_i - T_i}{\sigma_i^{\text{uncorr.}}} \right)^2 + \sum_{k=1}^{N_{\text{corr.}}} r_k^2,$$

where $\hat{D}_i \equiv D_i - \sum_{k=1}^{N_{\text{corr.}}} r_k \sigma_{k,i}^{\text{corr.}} \cdot D_i$ are the data points allowed to shift by the systematic errors in order to give the best fit, and $\sigma_{k,i}^{\text{corr.}}$ is a fractional uncertainty. Normalisation is treated as the other correlated uncertainties.

MSTW fit very good (χ^2 per point below left), though numbers lower for inclusive data. Always close to, if not best, particularly for $R = 0.6$. Not huge variation in PDFs though.

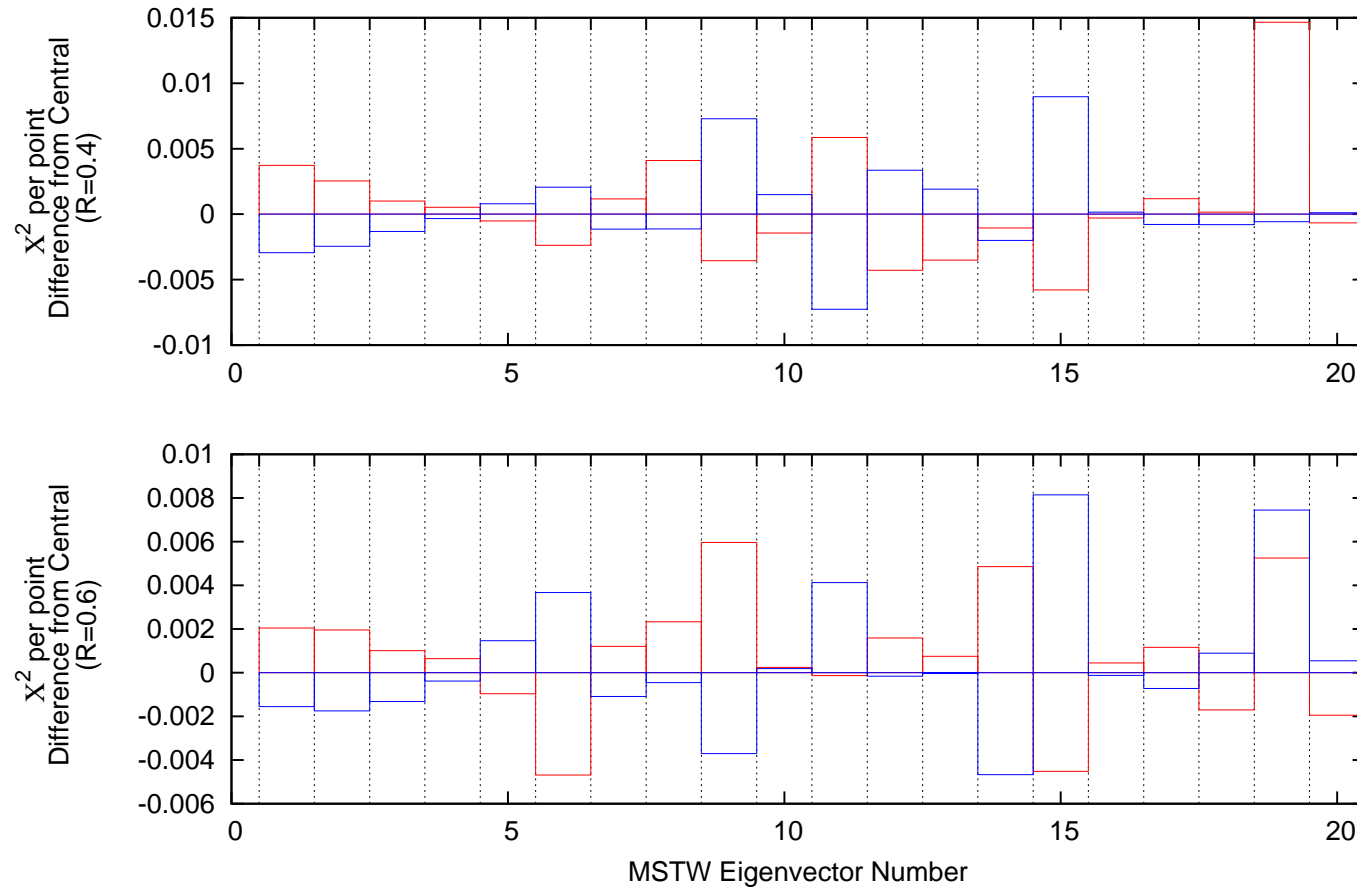
Scale	pT/2	pT	2pT
Inclusive (R=0.4)	0.752	0.773	0.703
Inclusive (R=0.6)	0.845	0.790	0.721
Dijet (R=0.4)	2.53	2.24	2.20
Dijet (R=0.6)	2.44	2.04	1.74

	$ r_k < 1$	$1 < r_k < 2$	$2 < r_k < 3$	$3 < r_k < 4$
Inclusive (R=0.4)	85	2	1	0
Inclusive (R=0.6)	87	1	0	0
Dijet (R=0.4)	82	6	0	0
Dijet (R=0.6)	74	12	2	0

Can see how fit varies across eigenvectors.

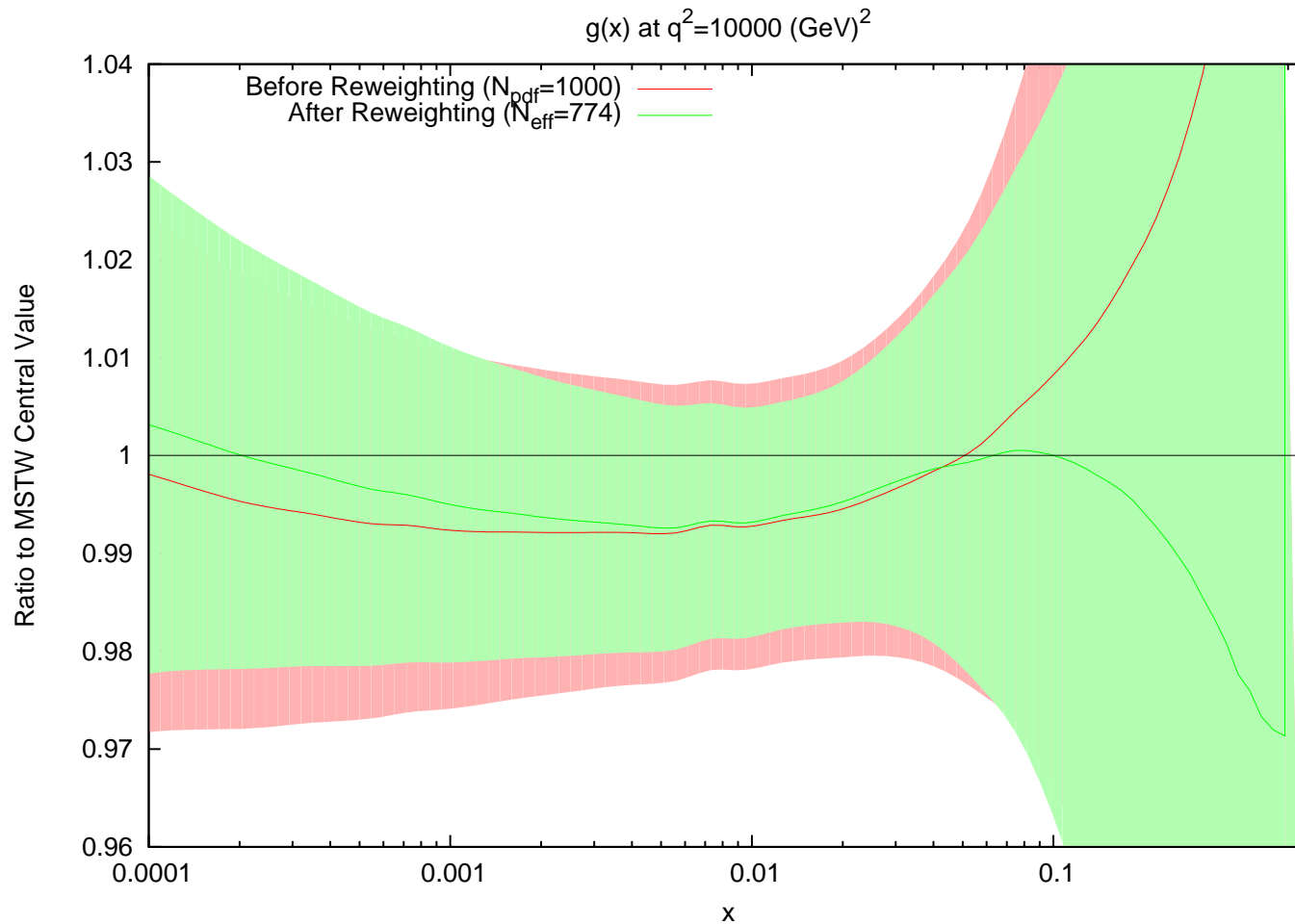
Clearly no pull with present data. (Eigenvector χ^2 variation lower than PDF variation.)

χ^2 per point for ATLAS Inclusive Jets

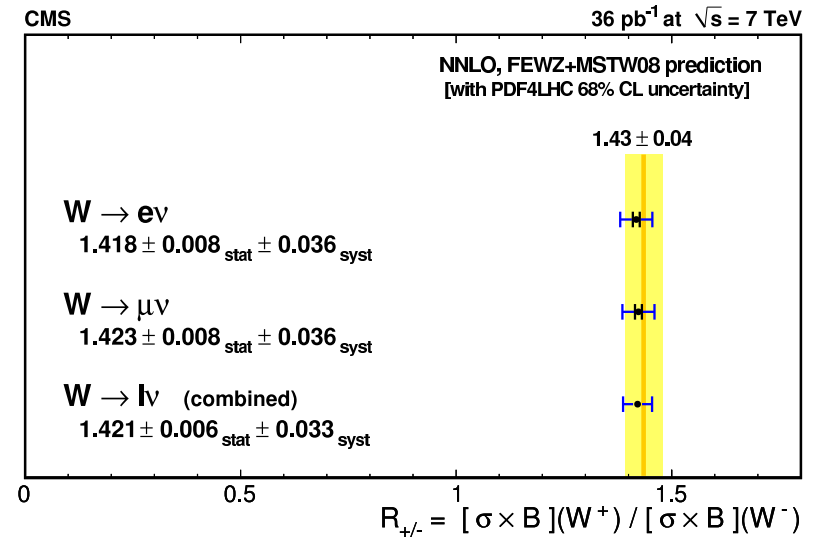
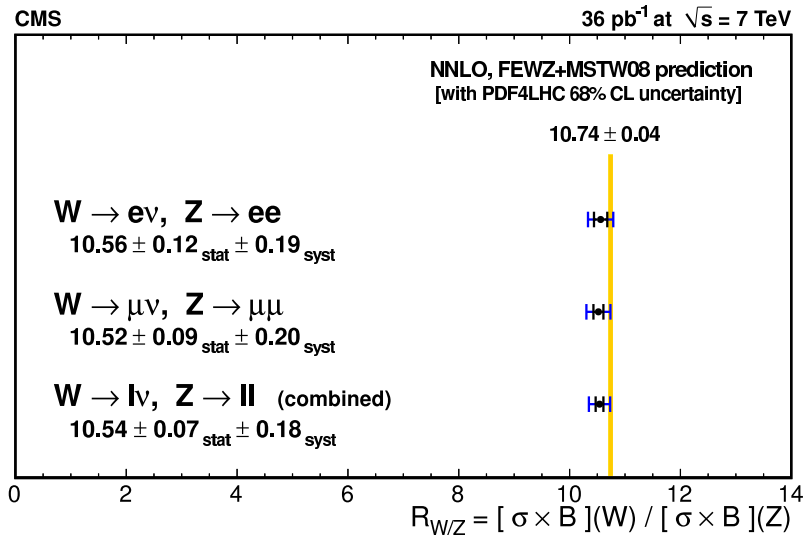
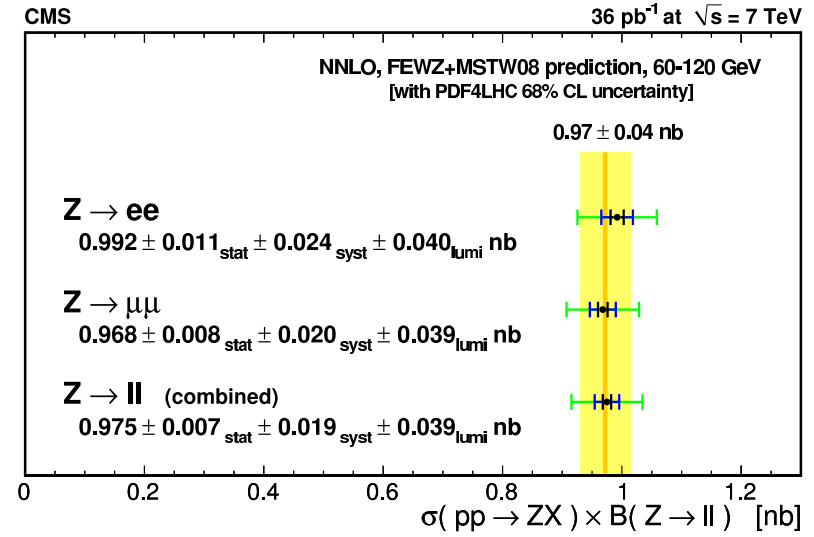
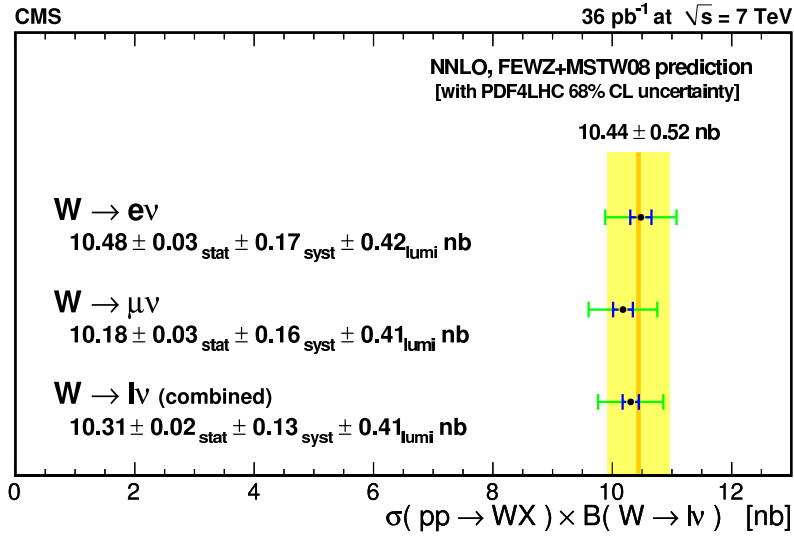


Can see effect of data on the gluon using reweighting technique, $R = 0.6$ ($R = 0.4$ similar).

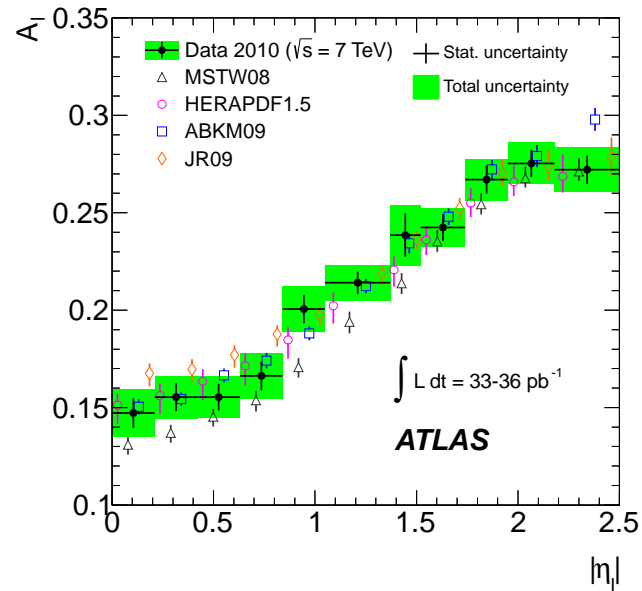
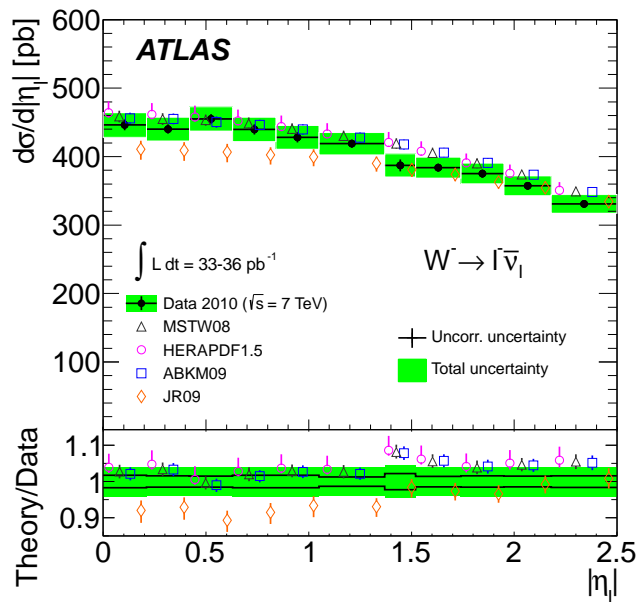
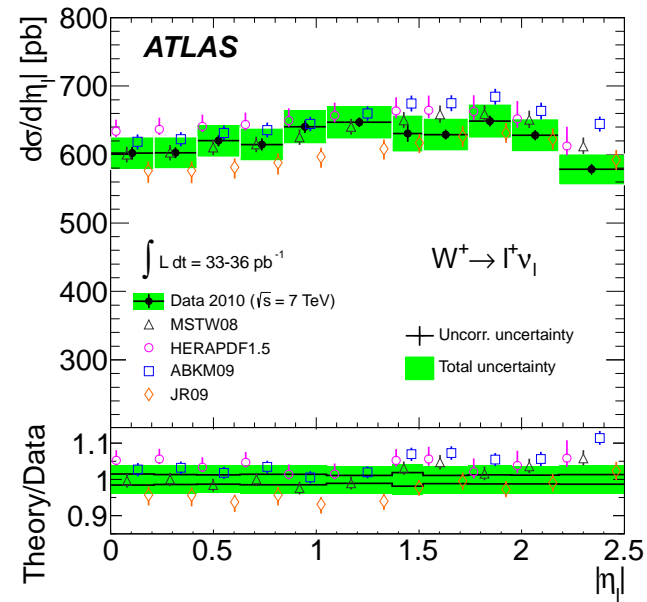
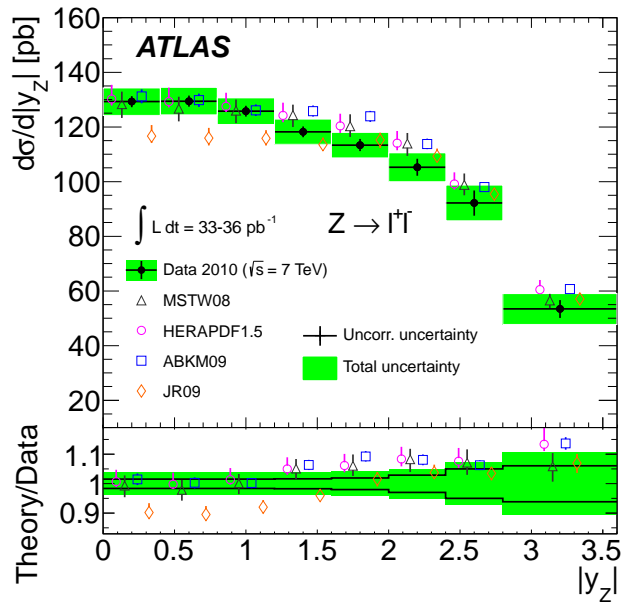
Clearly little pull with present data.

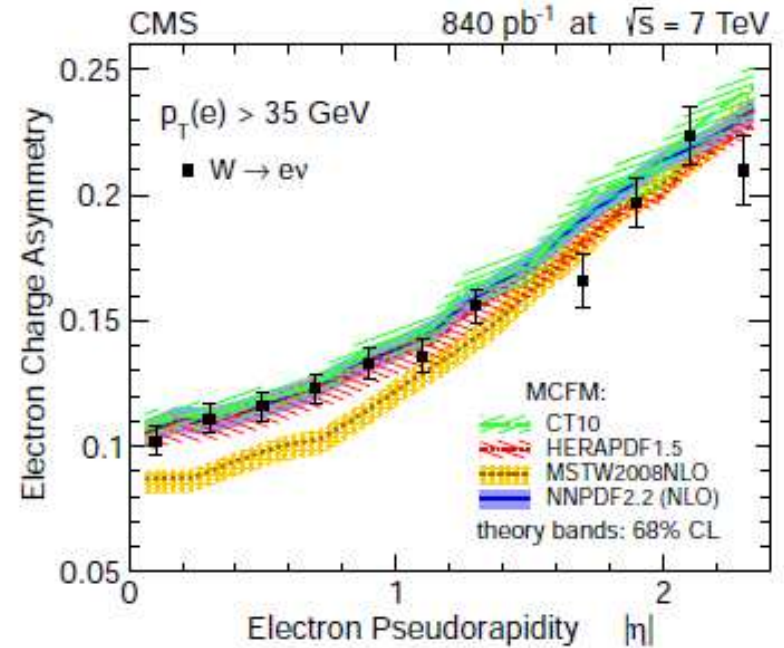
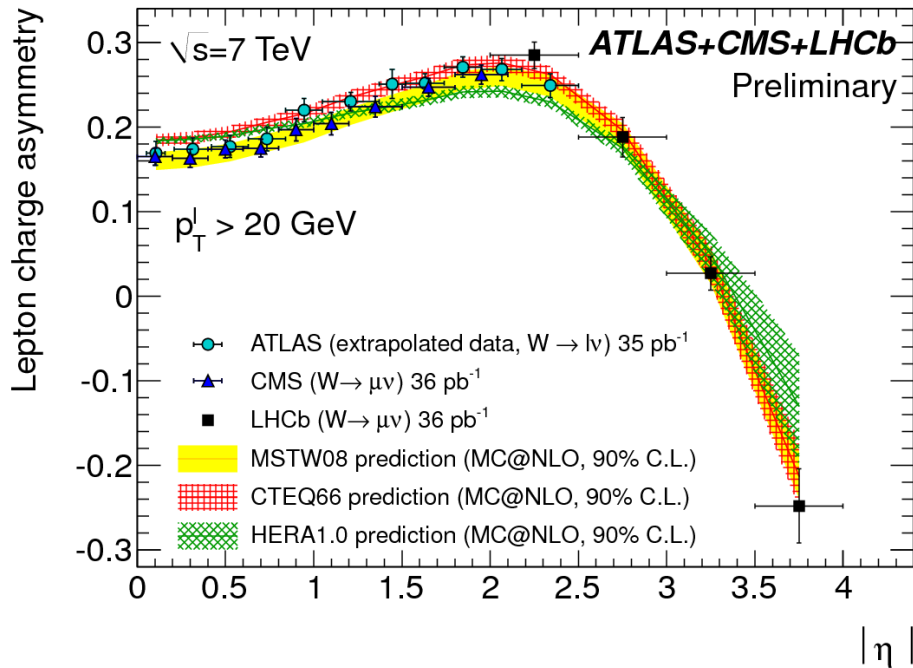


Comparison of MSTW2008 to total W, Z excellent.

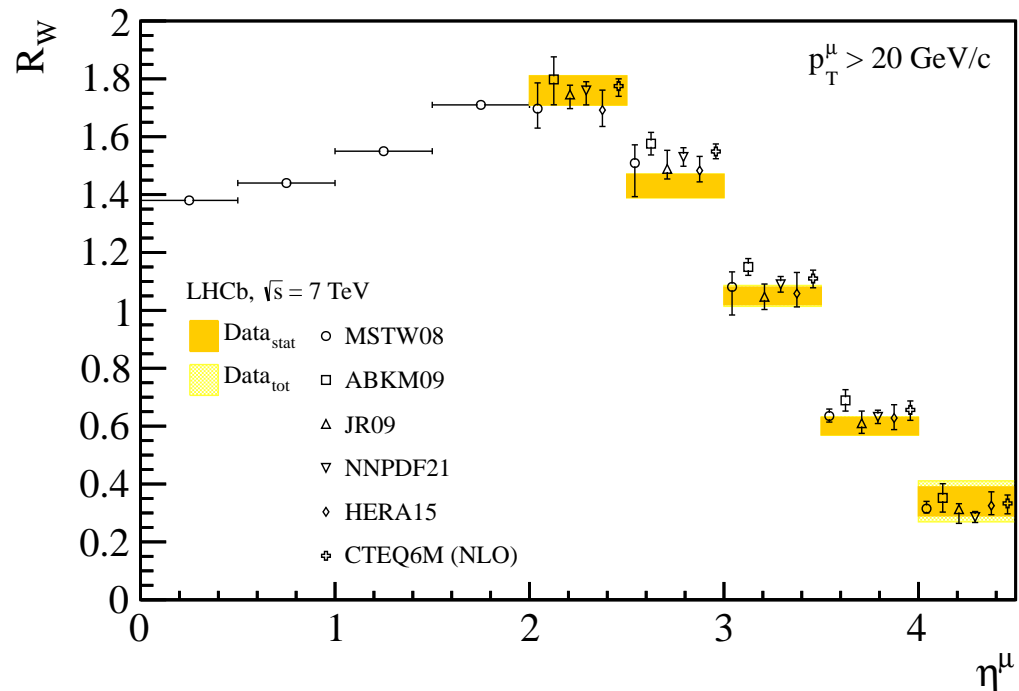


Also pretty good for inclusive distributions. Except some problems with asymmetry.

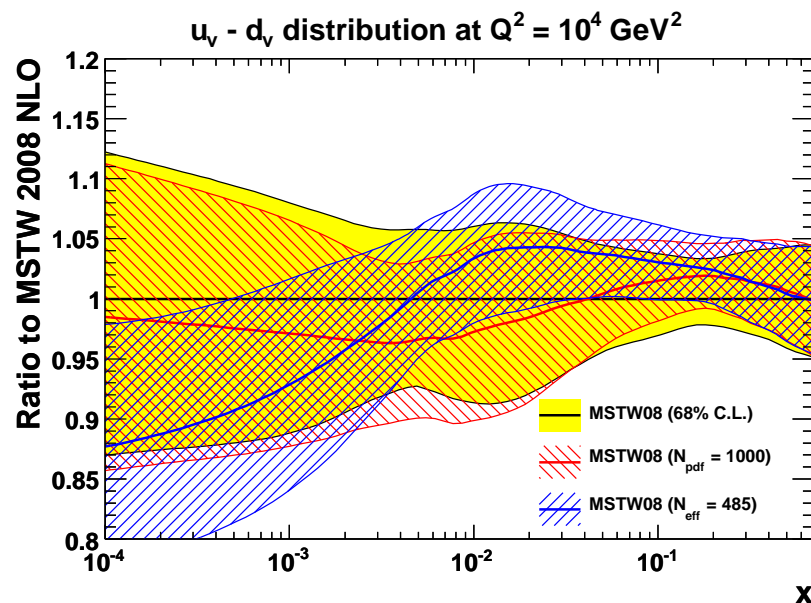
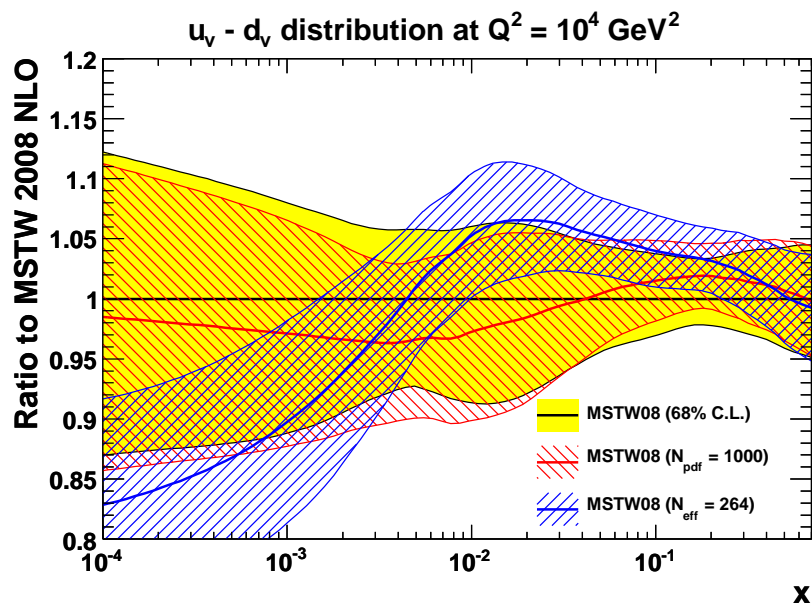




Similar for CMS data (will return to this later), though depends on p_T cut. Generally very good for LHCb.



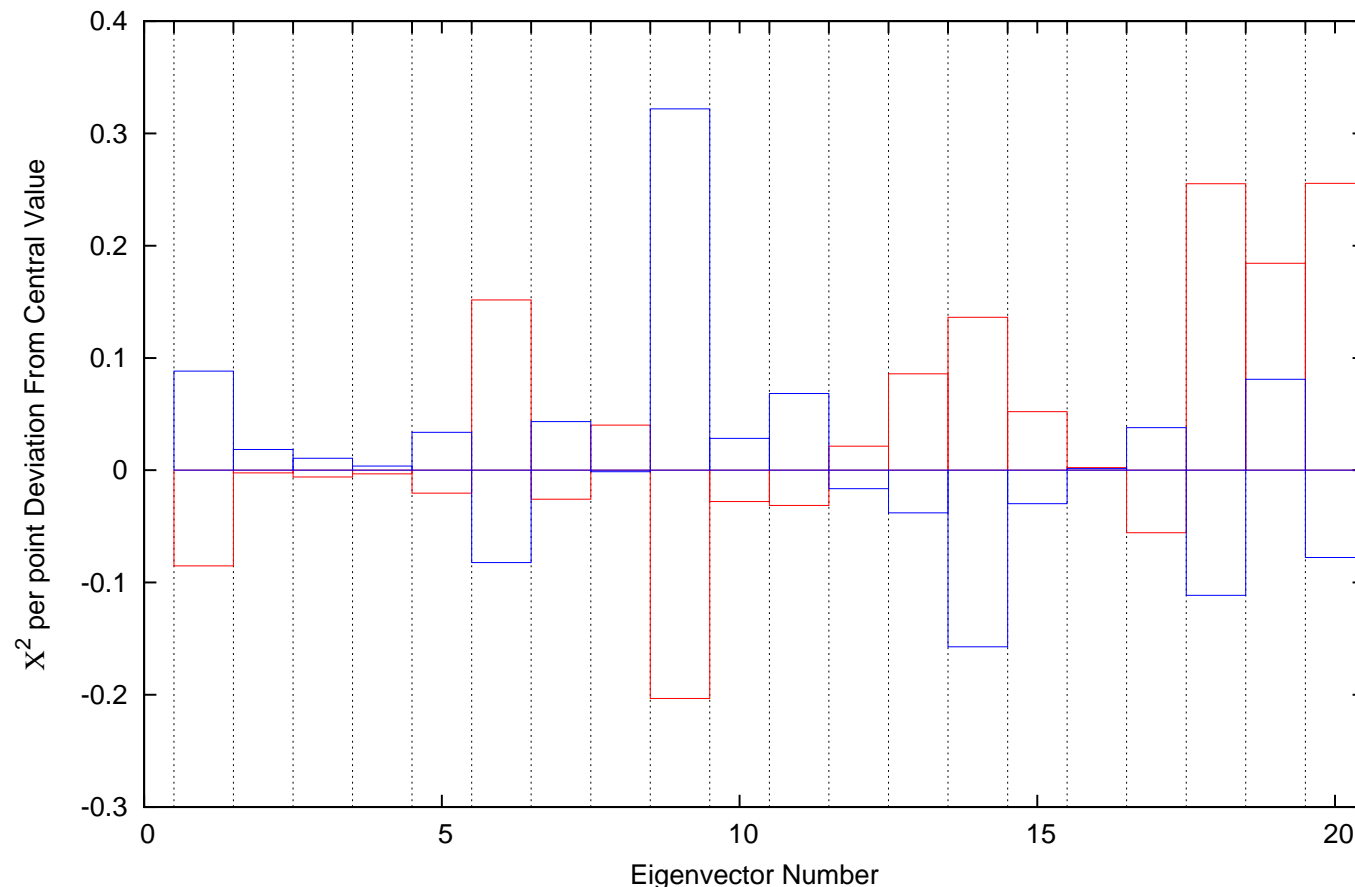
Asymmetry used by Graeme Watt in reweighting, and moves $u_V - d_V$ up around $x = 0.01$ - where parameterisation perhaps underestimates uncertainty. (ATLAS left, CMS $p_T > 25\text{GeV}$ right).



Calculate $\chi^2/N = 60/30$ for ATLAS W, Z data again at NLO using APPLGrid. Not best, but fairly close to any other set except CT10 which is best. Again look at eigenvectors (Ben Watt).

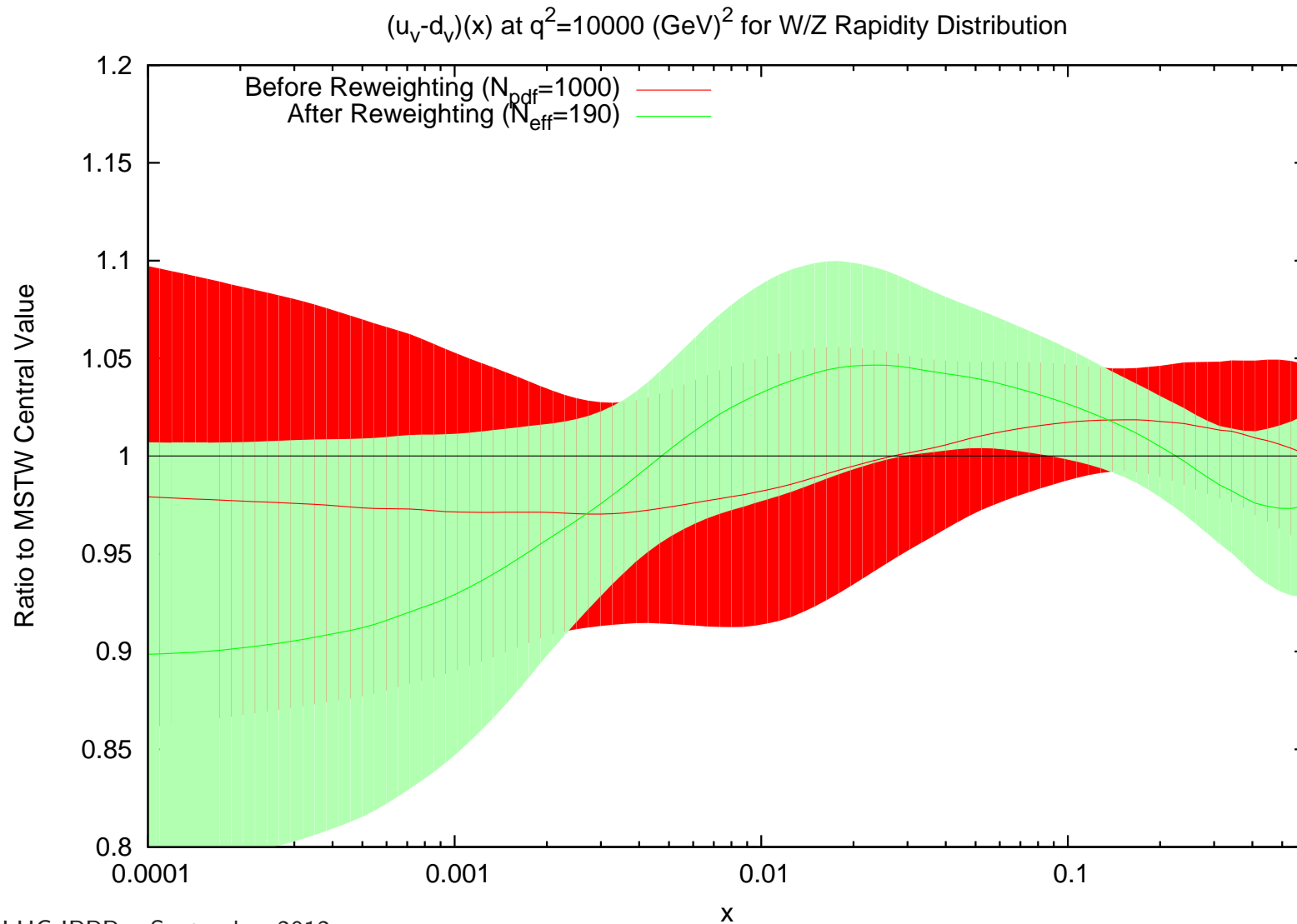
Fit improves markedly in one direction with eigenvector 9, gluon, which alters common shape and normalisation, and 14 and 18 which alter d_V and u_V , i.e. affect asymmetry. Not much variation in strange normalisation.

MSTW Eigenvectors for WZ Fit

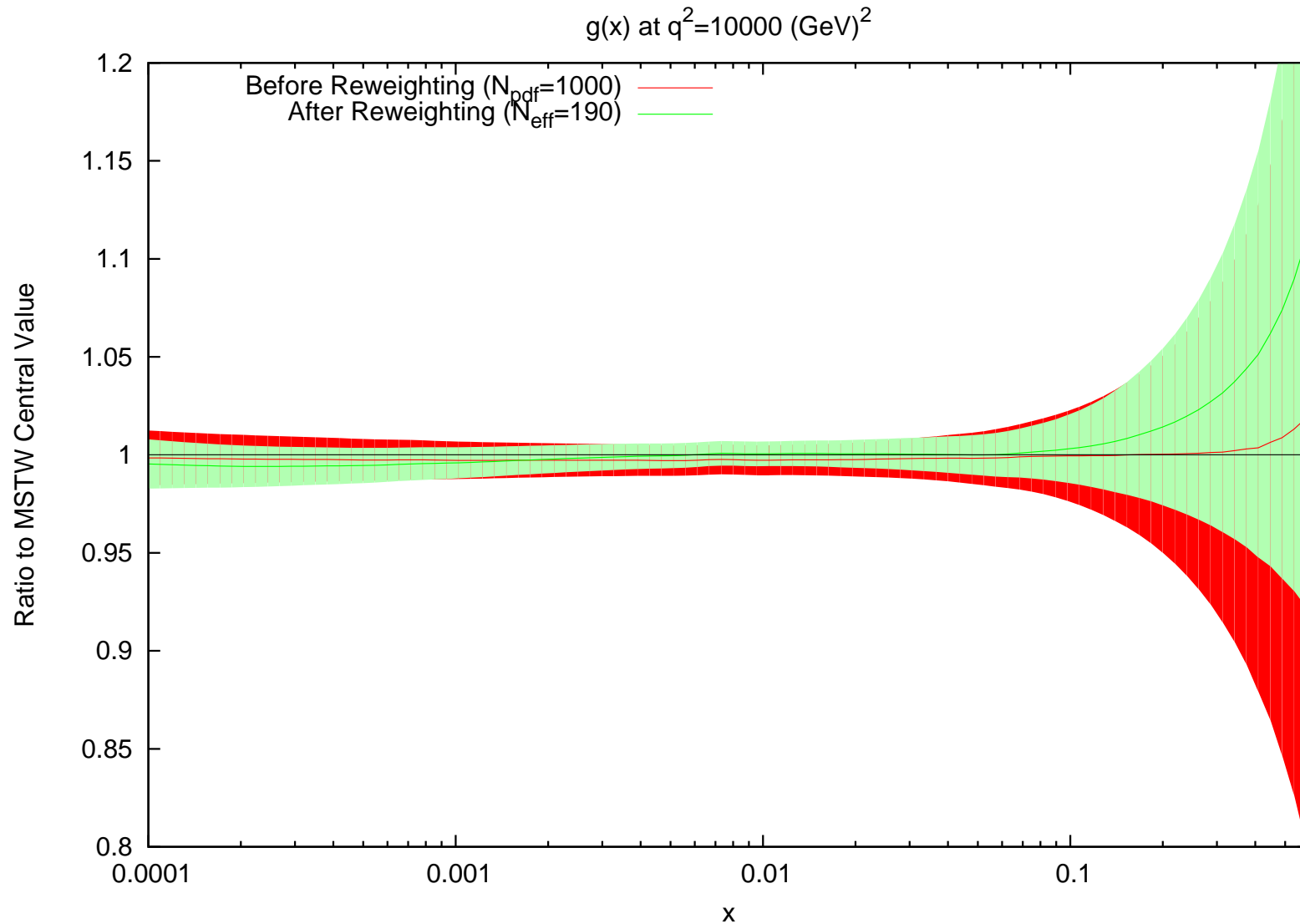


Can see effect of total rapidity data using reweighting. Fairly small effective number of PDFs = 190.

Slightly smaller effect on $u_V - d_V$ than asymmetry alone.



Can also see the effect on the gluon. Slight raise near $x = 0.01$ preferred. Improves overall shape of rapidity distribution. After reweighting $\chi^2 = 48/60$.



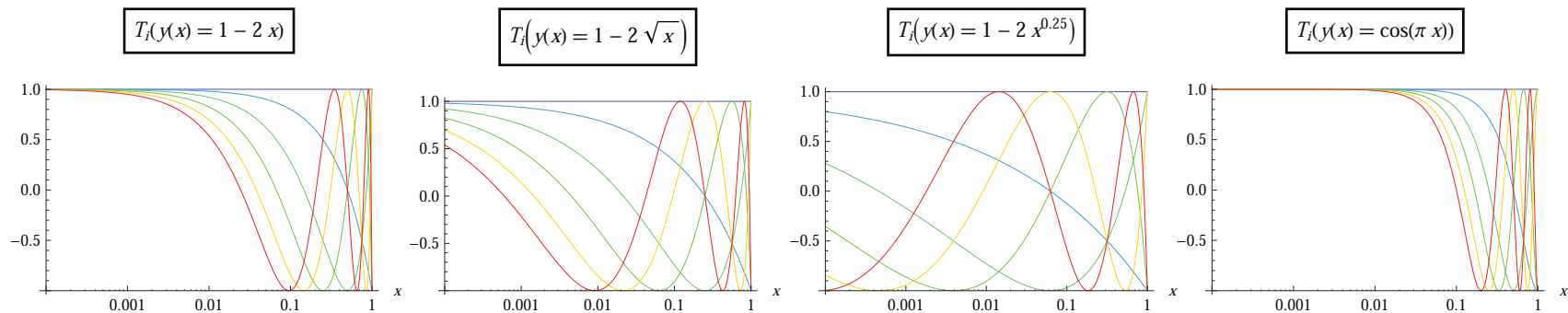
Investigation of Parameterisation Issues - with A. Mathijssen.

In the light of Monte Carlo studies investigate parameterisation dependence, initially concentrating on valence quarks.

Decide to use Chebyshev polynomials (looked at other possibilities)

$$xf(x, Q_0^2) = A(1-x)^\eta x^\delta \left(1 + \sum_n a_n T_n(y)\right)$$

i.e. keep high and low x limits. Choose $y = 1 - 2\sqrt{x}$.

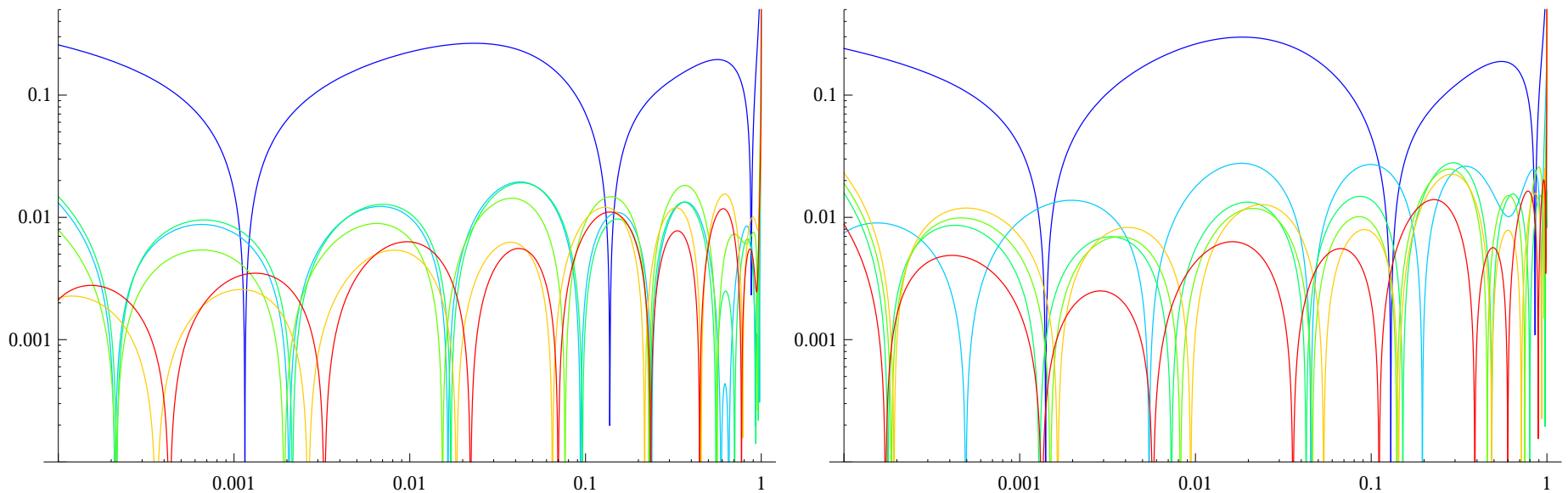


Same choice as in [Pumplin](#) study. Slightly different to [Glazov, Moch and Radescu](#).

Fit to 1000 pseudodata points for valence quark generated from very large order polynomial with smoothness constraints applied. Distributed evenly in $\ln(1/x)$ with percentage error constant down to $x = 0.00001$.

Percent deviation for full function. Order increase across the visible spectrum (i.e. dark blue to red).

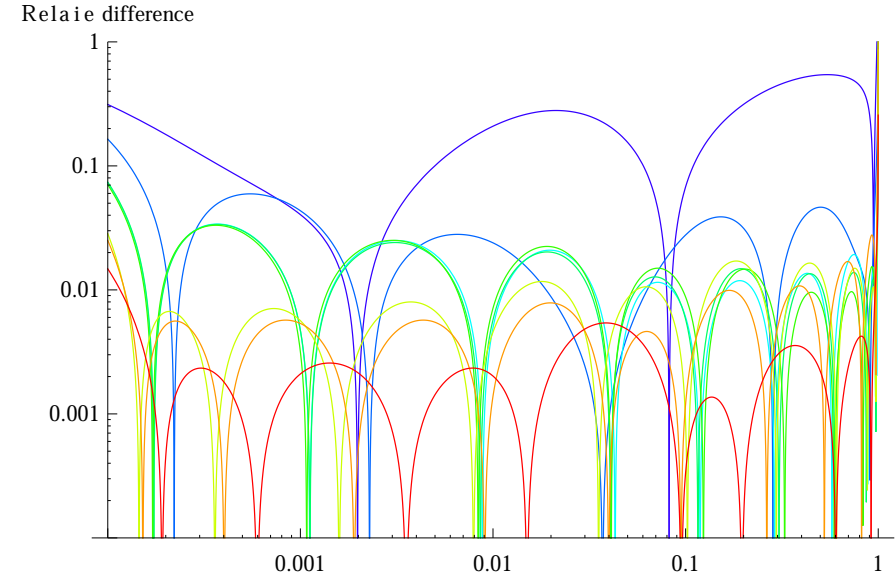
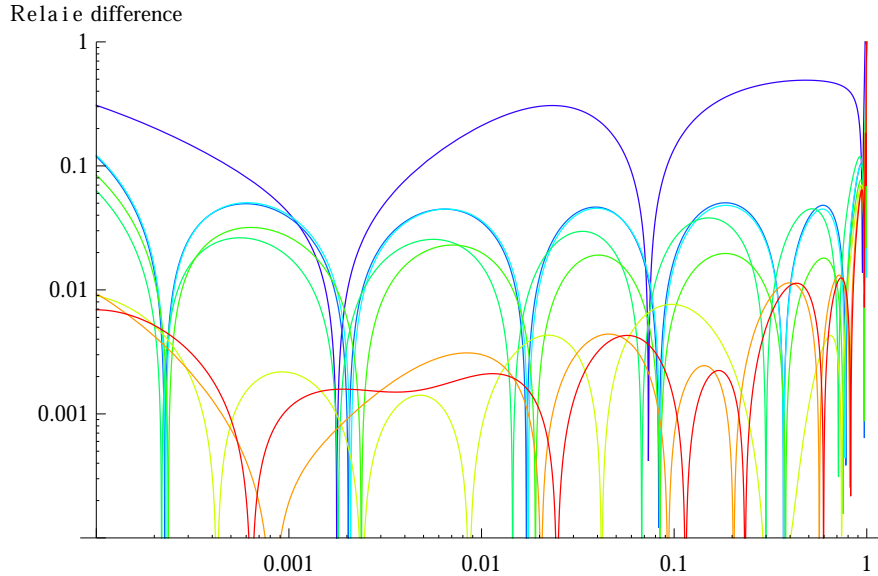
2 terms in polynomial mainly $\leq 2\%$ deviation. 4 terms in polynomial $\leq 0.5 - 1\%$ deviation except high x .



After on average ~ 6 polynomials start fitting noise, i.e. χ^2 lower than real function.

Conclude 4 parameters fine.

Deviations from true function for sea-like distribution.

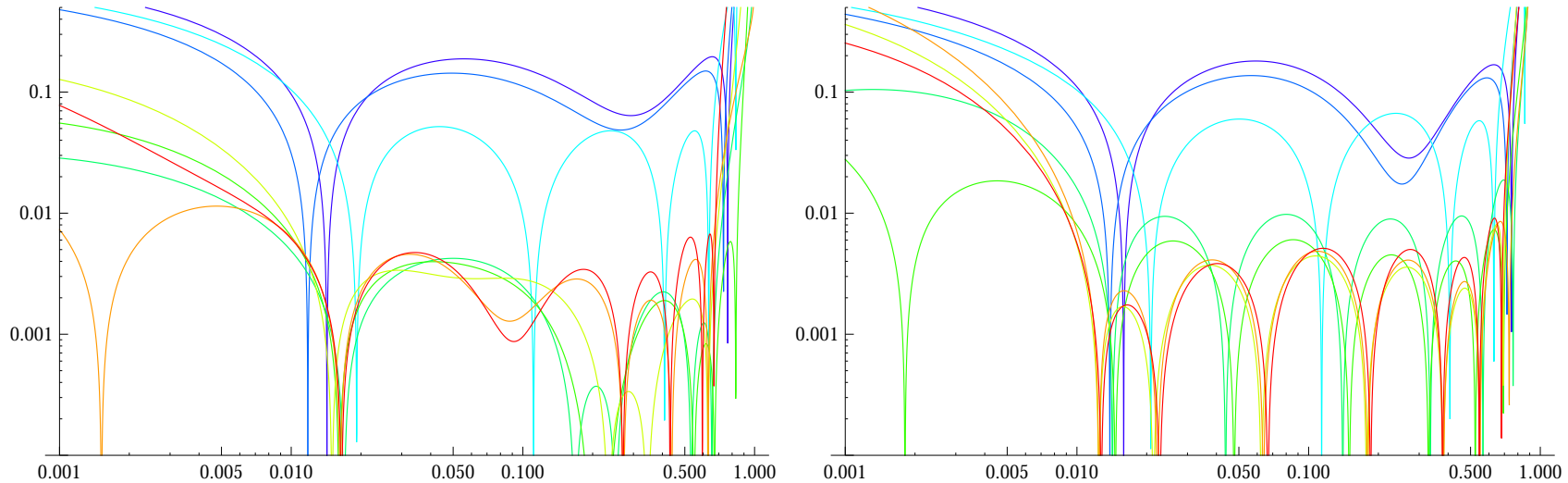


A bit more difficult in this case.

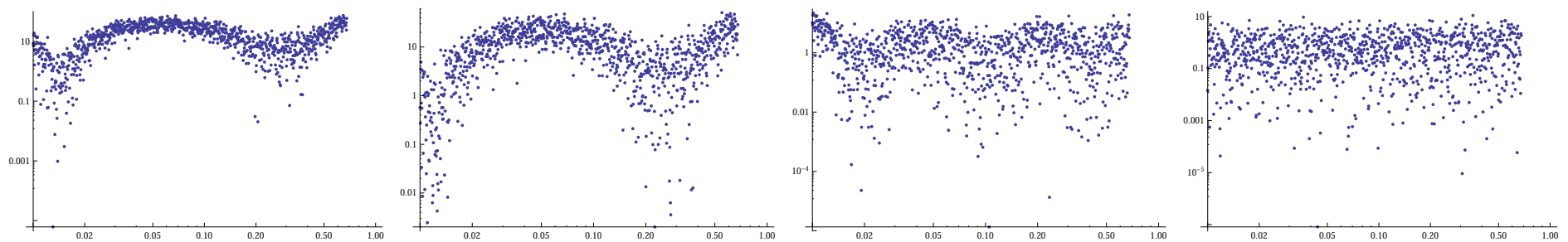
4 terms in polynomial $\leq 2\%$ deviation except high x .

Note uncertainty in input MSTW2008 sea is $\sim 5 - 6\%$ at best.

Also to pseudodata for valence quark generated only between $x = 0.01$ and $x = 0.7$. Typically slightly more deviation again, especially with only two terms.

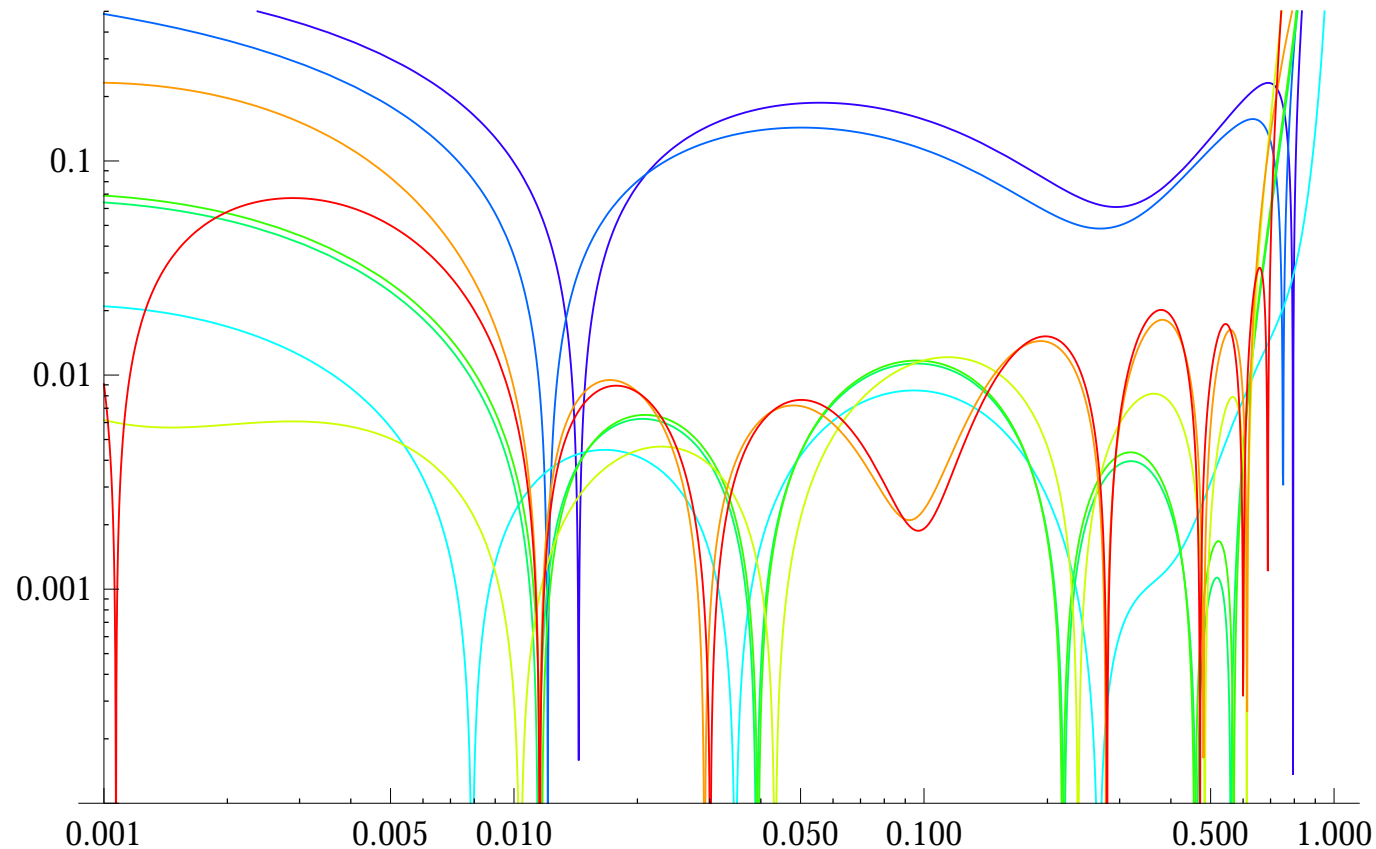


Look at χ^2 distribution with increasing terms in polynomial.



Very good fits with 4 parameters. More tends to give over-fitting and peculiarities outside of range of x fit.

In this case see some variation with number of pseudodata points fit (not so clear in other cases), e.g. result for 100 points.



Lower no. points allows better fit with fewer parameters, but best possible fit less good match to true function, $\sim 1\%$ deviations.

General result that ~ 4 terms is optimal unchanged.

Fits to same data as MSTW2008

Just applying to valence quarks, 4 new parameters, $\Delta\chi^2 = -4$.

Significant change in $u_V(x), x \leq 0.03$

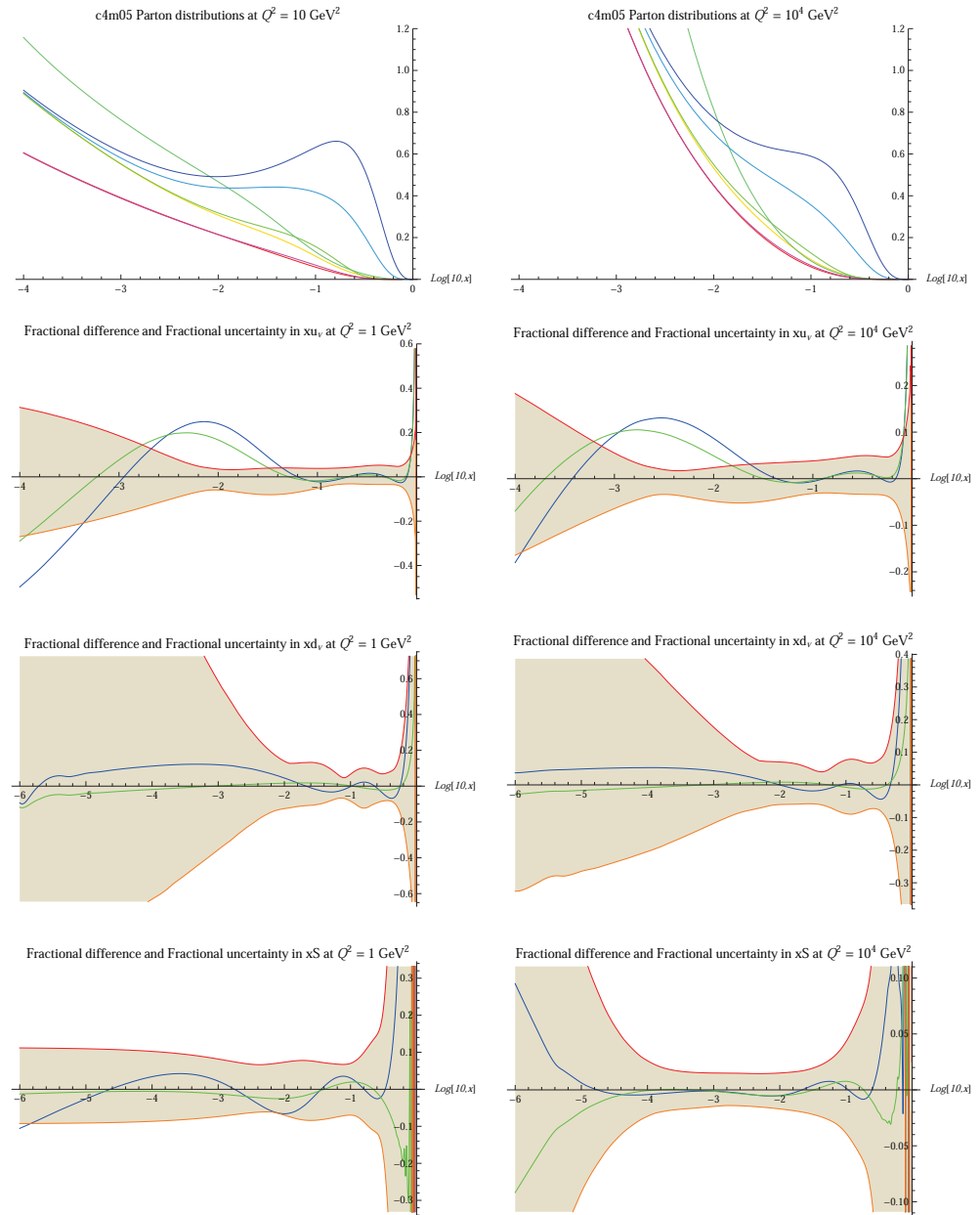
similar to earlier conclusion adding x^2 term to parameterisation.

Applying also to sea and gluon, 8 new parameters, $\Delta\chi^2 = -29$ (mainly BCDMS and Drell Yan data).

Still change significant only for $u_V(x), x \leq 0.03$.

Fits with requirement for fitting lepton asymmetry at LHC.

— valence, — valence + sea

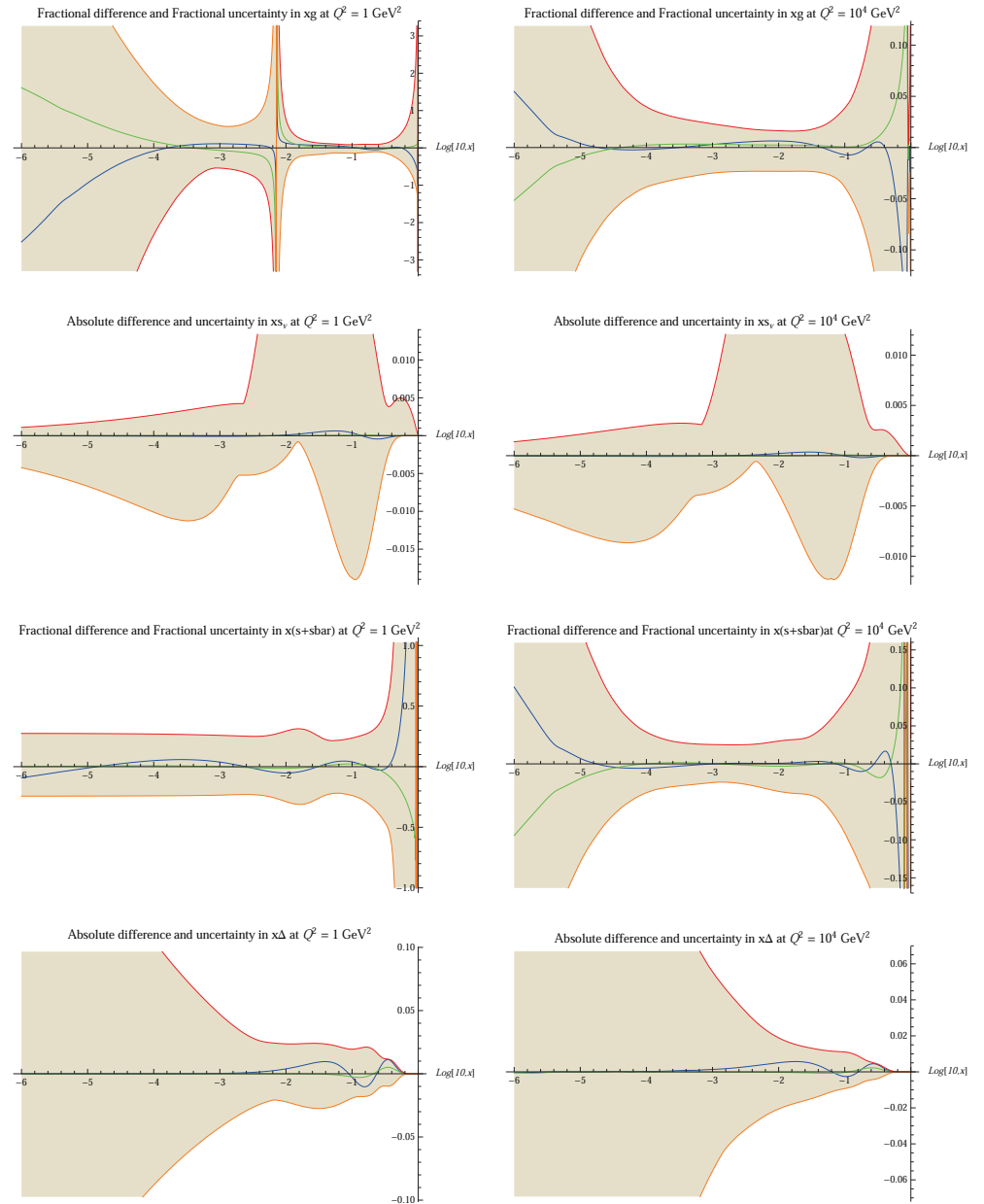


Little change in other PDFs.

Already 7 free parameters in the gluon. Sticking with two terms in Chebyshev polynomial leads to no change.

Take this a default - MSTW2008Cp (preliminary), 6 new parameters - 34 in total.

Prelim. study of uncertainties with 23 eigenvectors (one extra for valence quarks and sea). Little change except valence for $x \leq 0.03$, where significant increase.

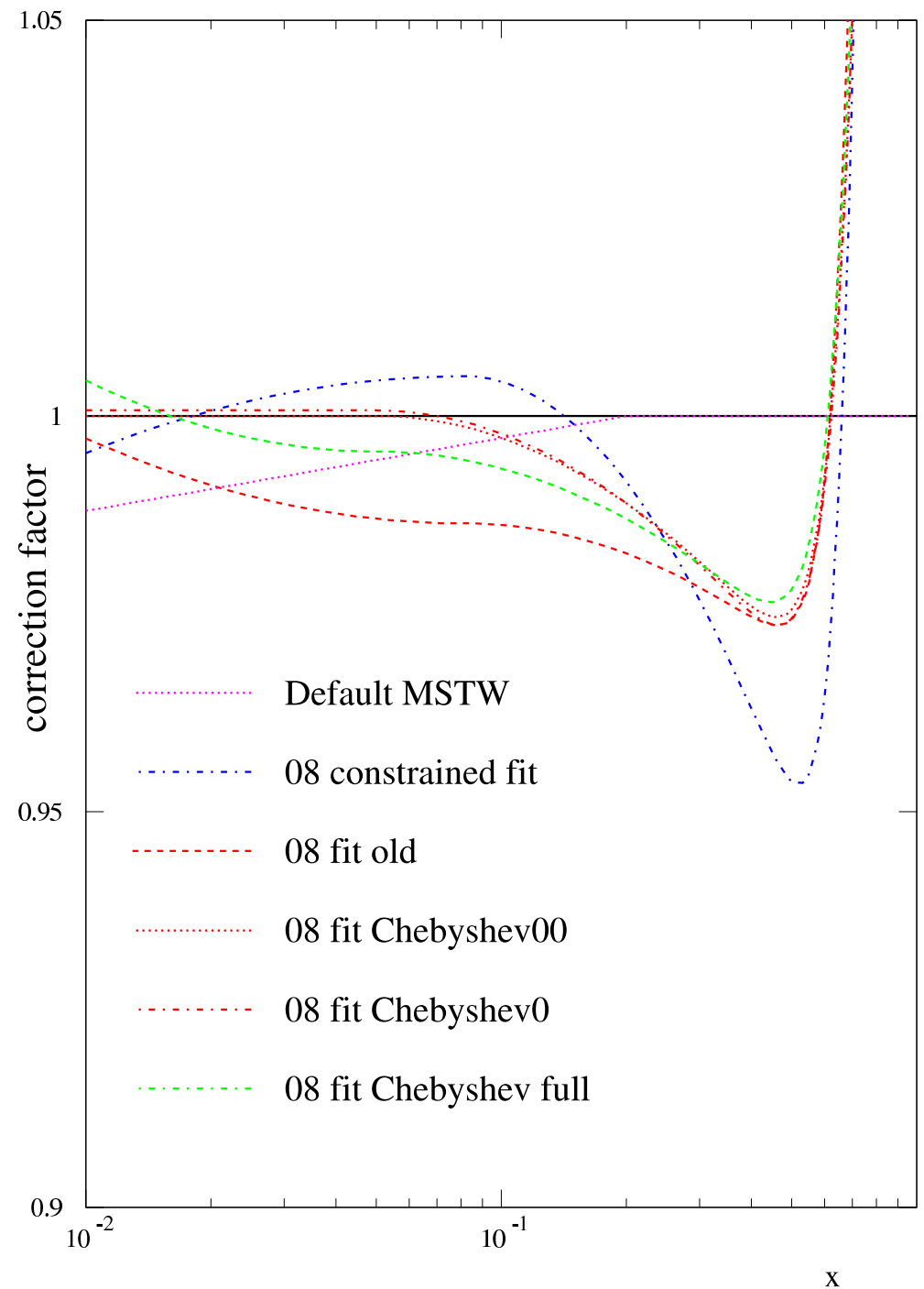


Given previous relationship between **Tevatron** asymmetry and deuterium corrections where partial success was noted revisit with extended parameterisation.

Default for **MSTW** some shadowing for $x < 0.01$.

Previously big improvement in fit, but “unusual” corrections.

Now improvement again but much more stable, and sensible for deuterium corrections. (No shadowing favoured though.)



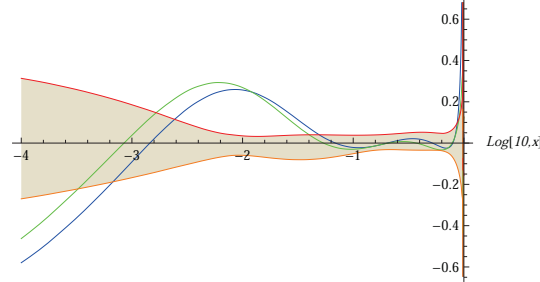
Now also get variation in $d_V(x)$ for higher x due to deuterium correction (seen before) and $x \leq 0.03$ due to parameterization and corrections.

Prelim. MSTW2008Cpdeut PDFs.

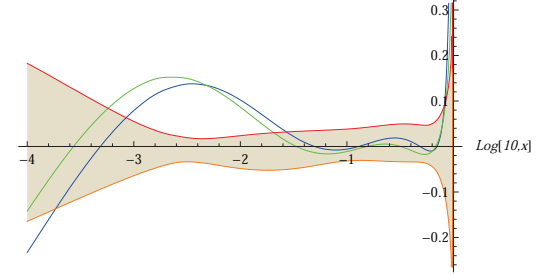
Fit to ATLAS W, Z rapidity data at NLO improves to 49/30 for MSTWCp and 46/30 for MSTWCpeut.

— CP, — CPdeut

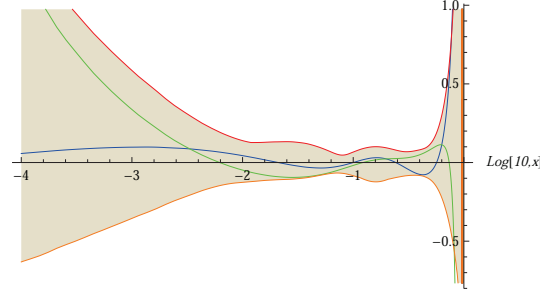
Fractional difference and Fractional uncertainty in xu_v at $Q^2 = 1 \text{ GeV}^2$



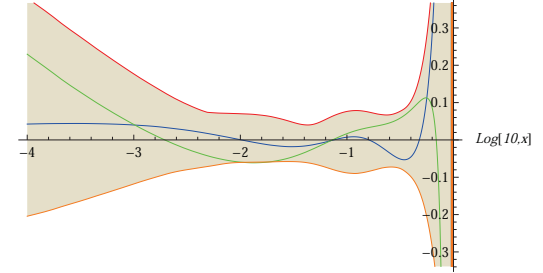
Fractional difference and Fractional uncertainty in xu_v at $Q^2 = 10^4 \text{ GeV}^2$



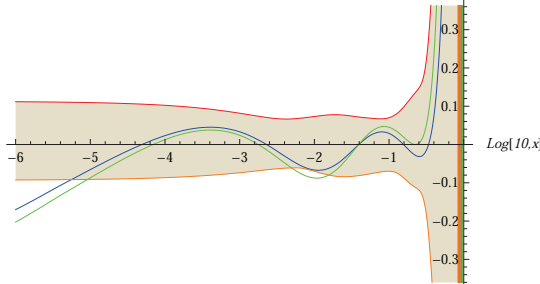
Fractional difference and Fractional uncertainty in xd_v at $Q^2 = 1 \text{ GeV}^2$



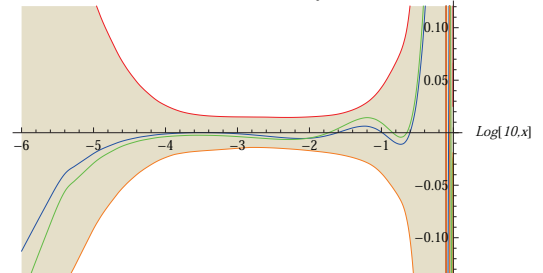
Fractional difference and Fractional uncertainty in xd_v at $Q^2 = 10^4 \text{ GeV}^2$



Fractional difference and Fractional uncertainty in xS at $Q^2 = 1 \text{ GeV}^2$



Fractional difference and Fractional uncertainty in xS at $Q^2 = 10^4 \text{ GeV}^2$

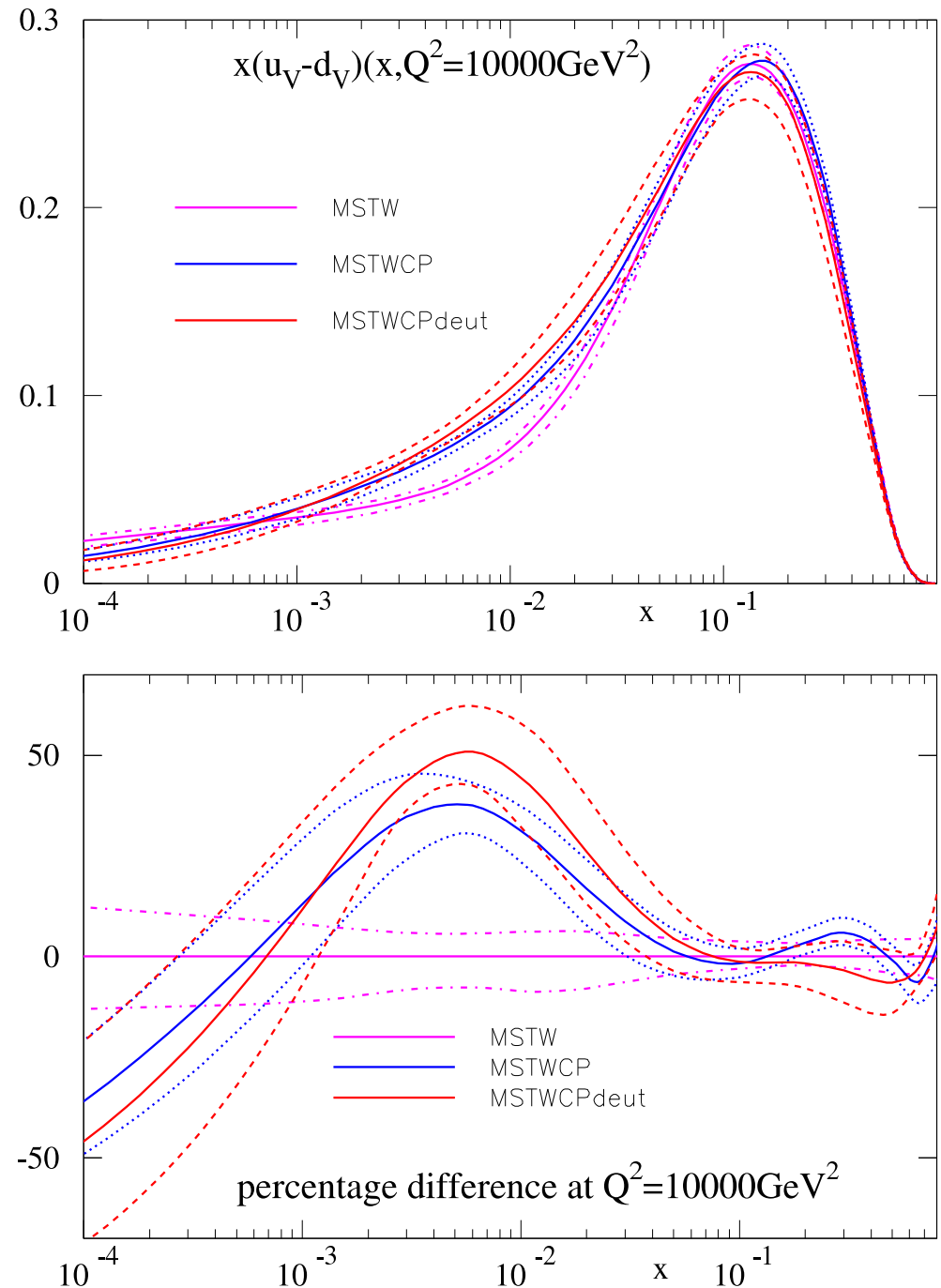


Preliminary uncertainty sets have 23 eigenvectors (20 in MSTW2008).

Main effect in uncertainty an increase in $d_V(x, Q^2)$ due to deuterium correction uncertainties, and minor valence uncertainty increase from extra parameter.

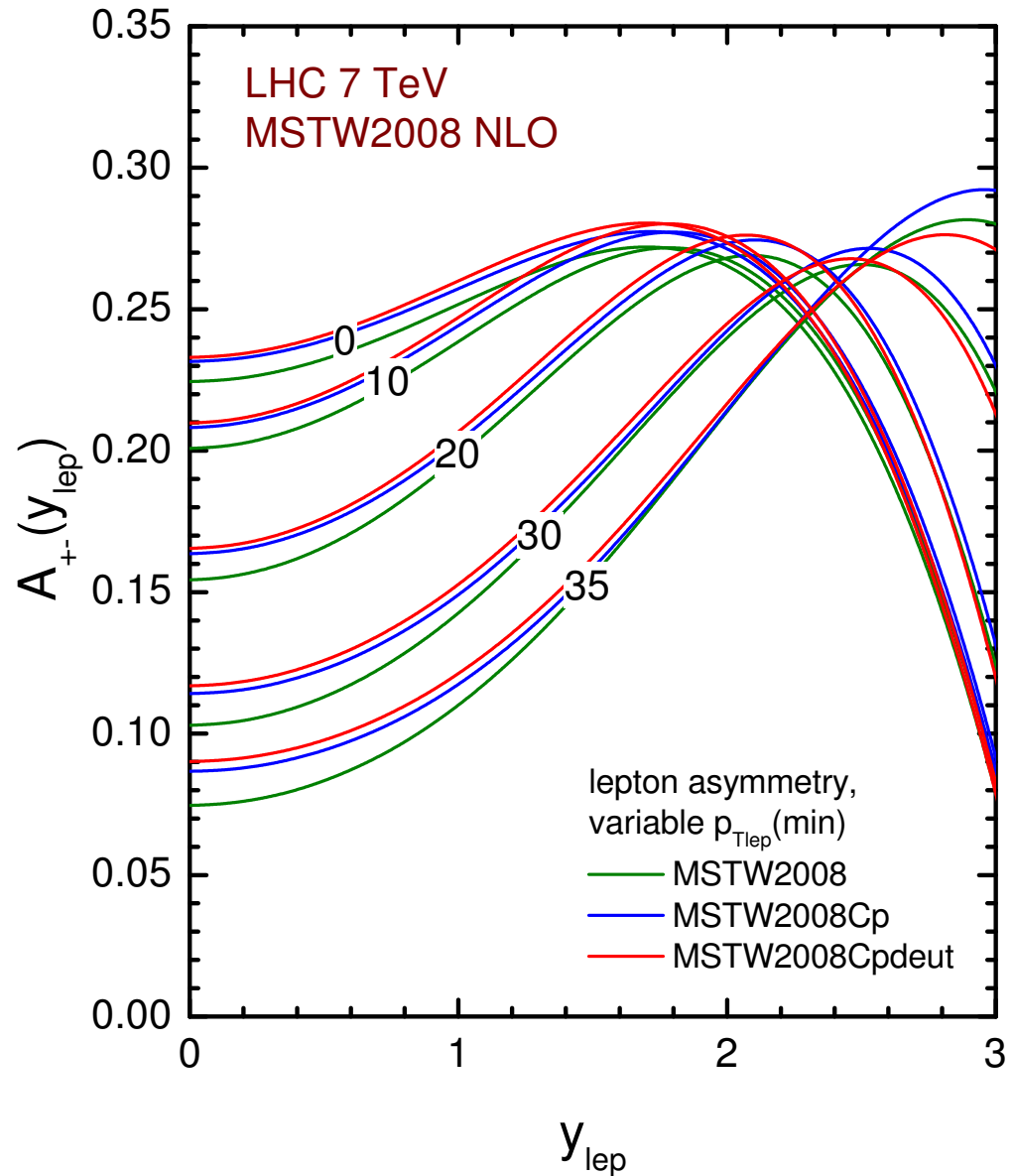
Shown is change in central value and uncertainty for $u_V(x) - d_V(x)$ at $Q^2 = 10,000 \text{ GeV}^2$.

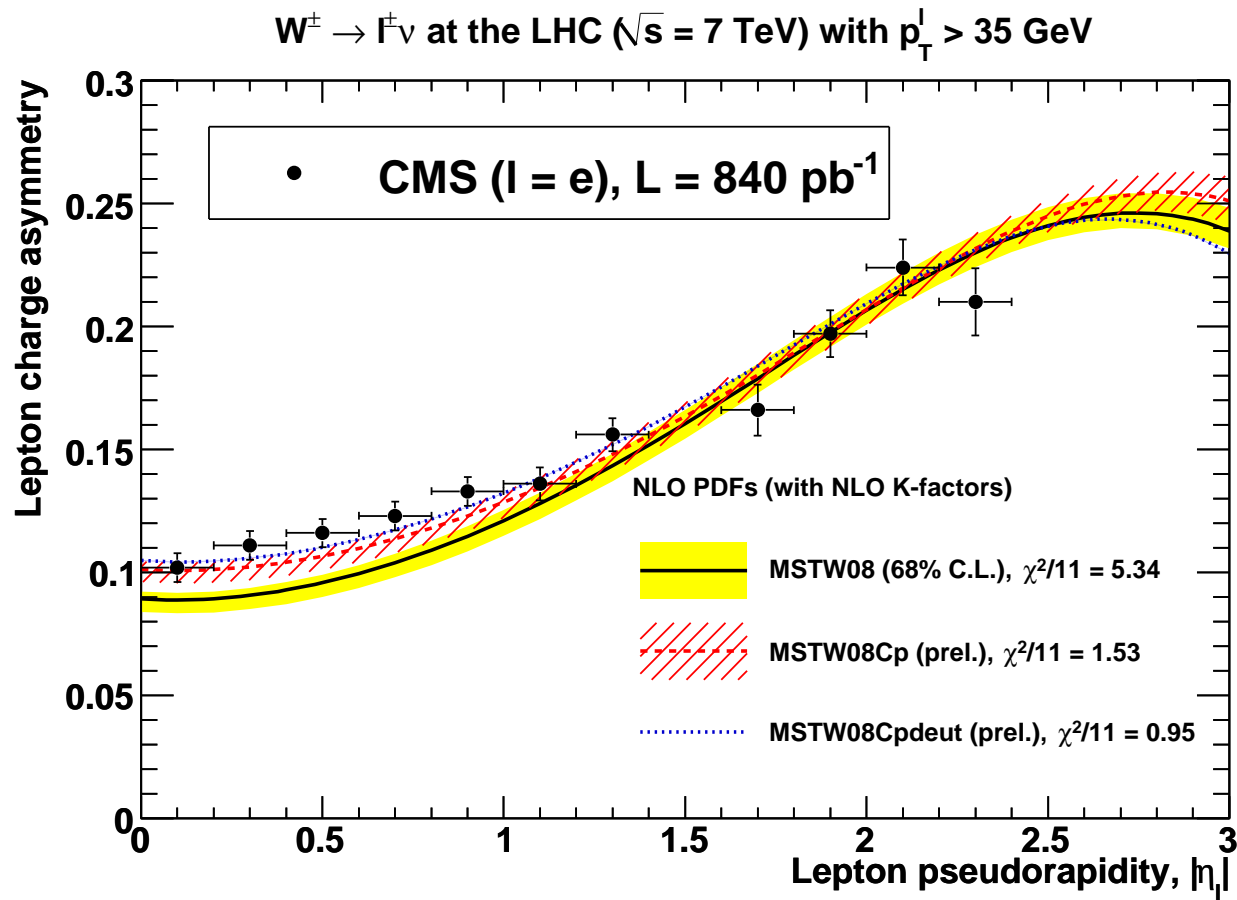
Biggest effect at lower x than probed at the LHC (yet).



Increases lepton asymmetry, but very preferentially for high p_T cut. (Curves made here with LO calculations).

Most of the effect already obtained for parameterisation extension, but some from deuterium study.





Prediction for $p_T > 35$ GeV CMS asymmetry data using MCFM (G. Watt).

Note no change to data fit, just parameterisation and some from deuterium corrections.

Main deuterium effect absence of shadowing in default fit.

Can try reweighting approach and dependence on eigenvectors using modified MSTW2008 sets (B. Watt).

No significant changes in fits to jet data at all.

For W, Z rapidity data eigenvectors preferred mainly alter gluon shape and fine details of u_V and d_V still. Small preference for eigenvectors with higher strange.

Effective number of sets now much higher, ~ 500 out of 1000.

After reweighting get $\chi^2 = 39.5/30$ and $\chi^2 = 38.5/30$.

No noticeable pull on strange.

Big change in high p_T cut asymmetry, but very specifically sensitive to $u_V(x, Q^2) - d_V(x, Q^2)$. What about other quantities? Other PDFs changed little. α_S free but tiny change. Expect little variation.

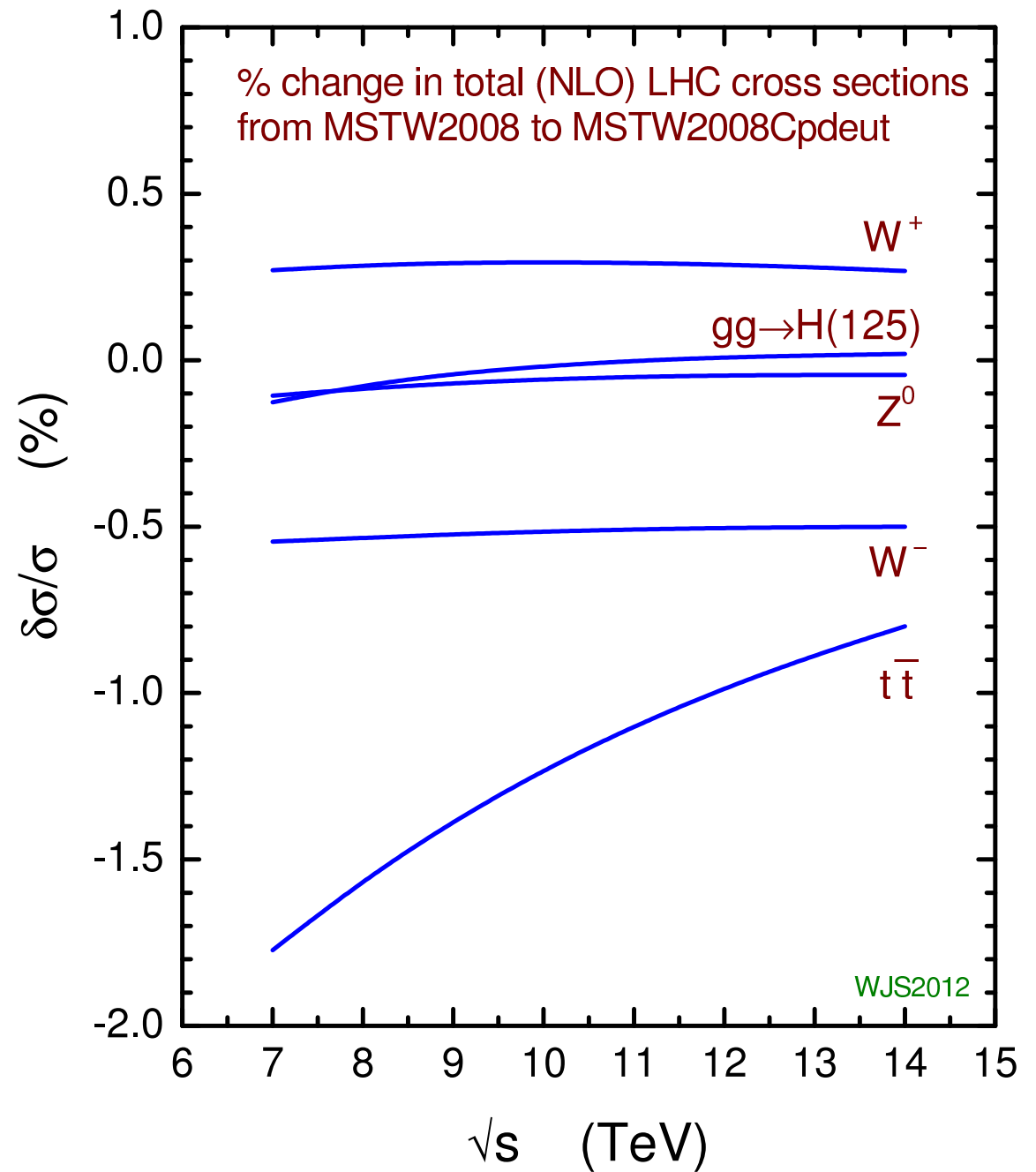
The % change in the cross sections ($M_H = 120\text{GeV}$).

	MSTWCP	MSTWCPdeut
W Tev	+0.6	+0.1
Z Tev	+0.8	+0.7
W^+ LHC (7TeV)	+0.7	+0.3
W^- LHC (7TeV)	-0.7	-0.4
Z LHC (7TeV)	+0.0	-0.1
W^+ LHC (14TeV)	+0.6	+0.3
W^- LHC (14TeV)	-0.6	-0.5
Z LHC (14TeV)	+0.1	-0.1
Higgs TeV	-0.5	-1.8
Higgs LHC (7TeV)	+0.2	-0.1
Higgs LHC (14TeV)	+0.1	+0.1

Extreme stability in total cross sections, all far inside uncertainties. Even $\sigma(W^+)/\sigma(W^-)$ barely more than 1%.

Seen clearly on plot.

Note – uncertainty on $\sigma_{t\bar{t}} \sim 5 - 6\%$ from PDFs + $\alpha_S(M_Z^2)$ at 7 TeV.



Conclusions

Monte Carlo approach to using PDFs based on best fit and eigenvectors is straightforward. Good accuracy obtained with similar number of sets to the case of eigenvector approaches. Allows different PDFs to be combined just by sampling random PDFs from each.

An **NLO** fit using a **FFNS** shows qualitative differences to all **GM-VFNS** variations and tendency for smaller $\alpha_S(M_Z^2)$.

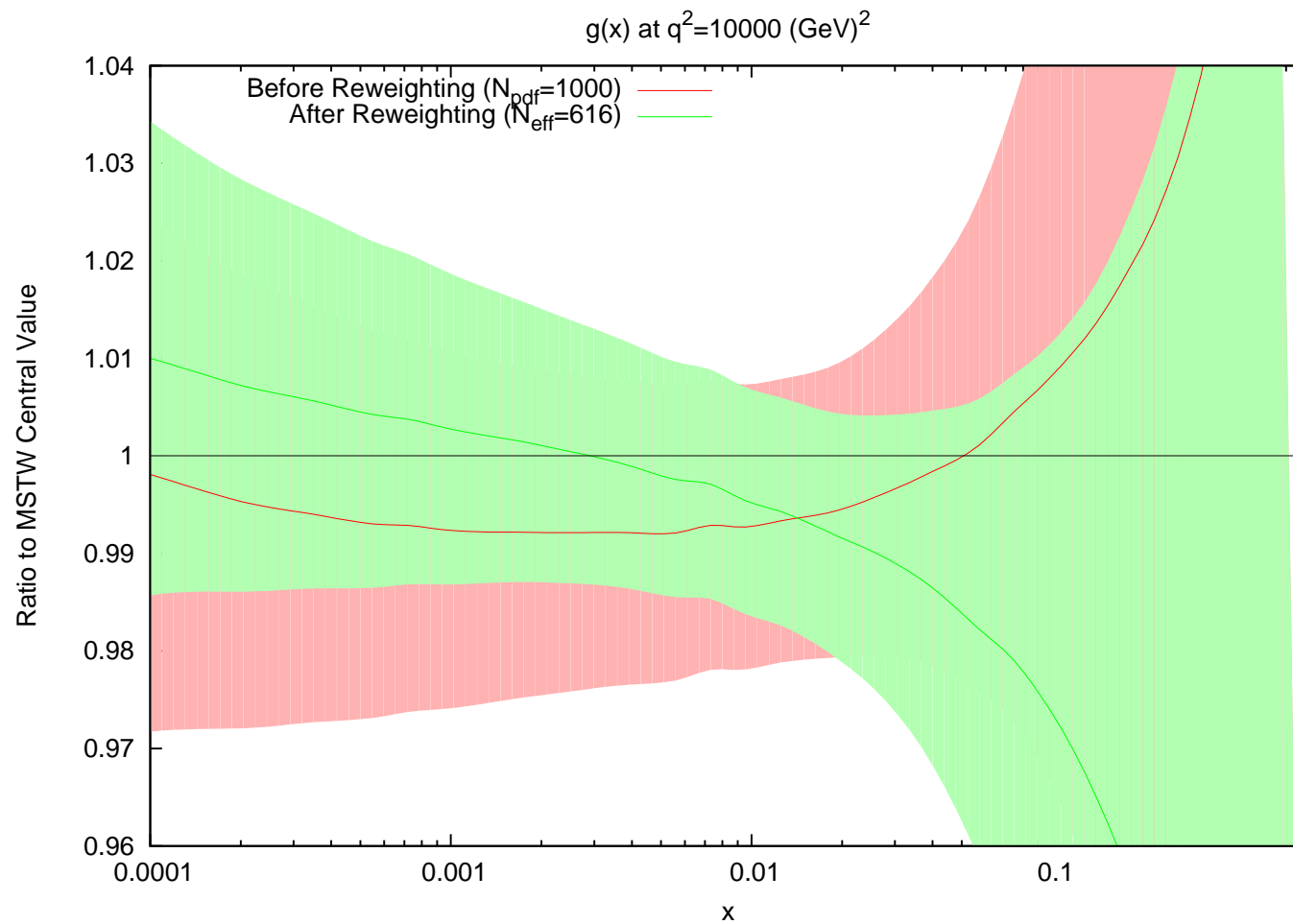
MSTW08 fits current **LHC** data as well, or better than other sets, with exception of (particularly high- p_T) lepton asymmetry. In the main need more data for constraints.

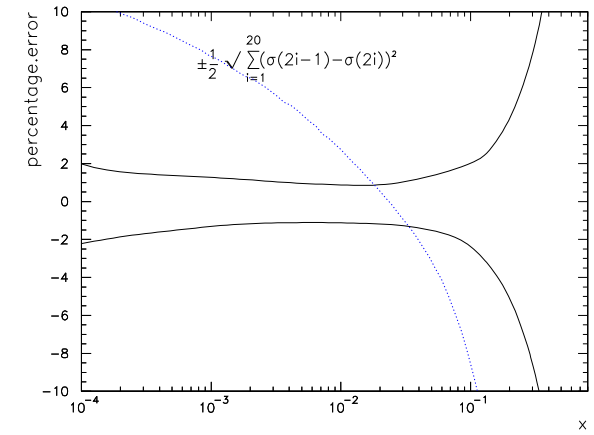
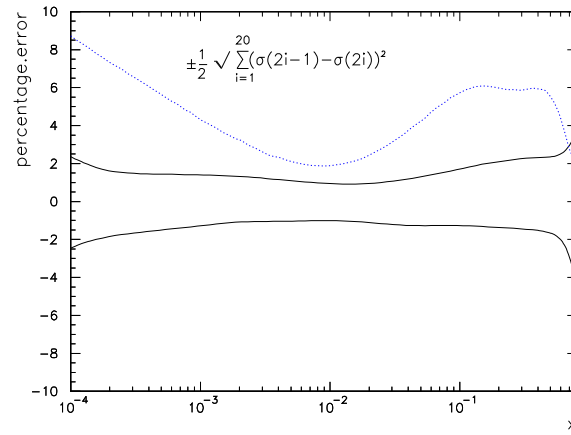
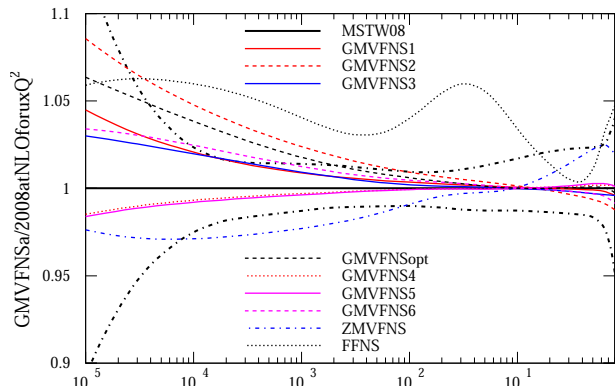
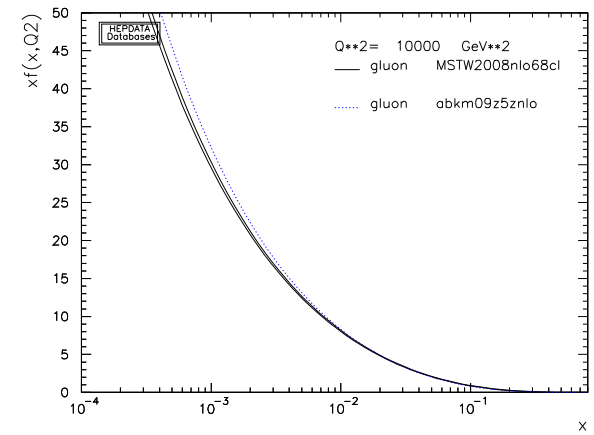
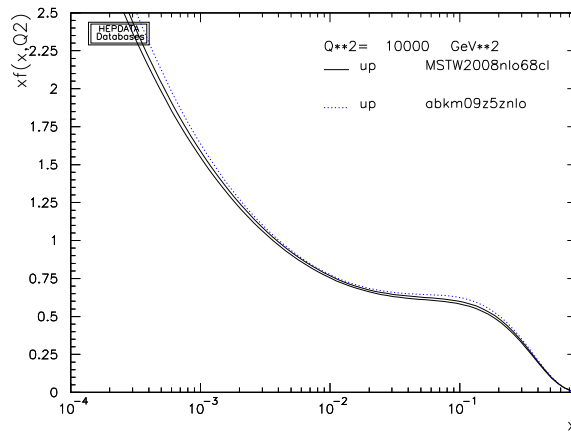
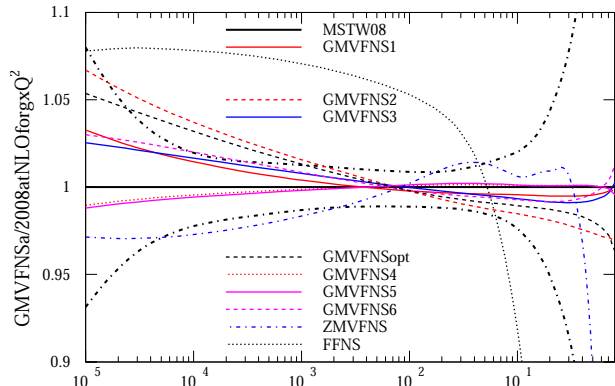
Studies of parameterisation dependence suggest ~ 4 terms in a Chebyshev polynomial about the maximum needed for very high precision. Backs up conclusion that in current **MSTW** fits the only need for an extended parameterisation is for small- x valence quarks.

Automatically improves comparison to **LHC** lepton asymmetry data. Makes fit with deuterium corrections much more stable, and these lead to further slight improvements. Most cross sections practically unchanged.

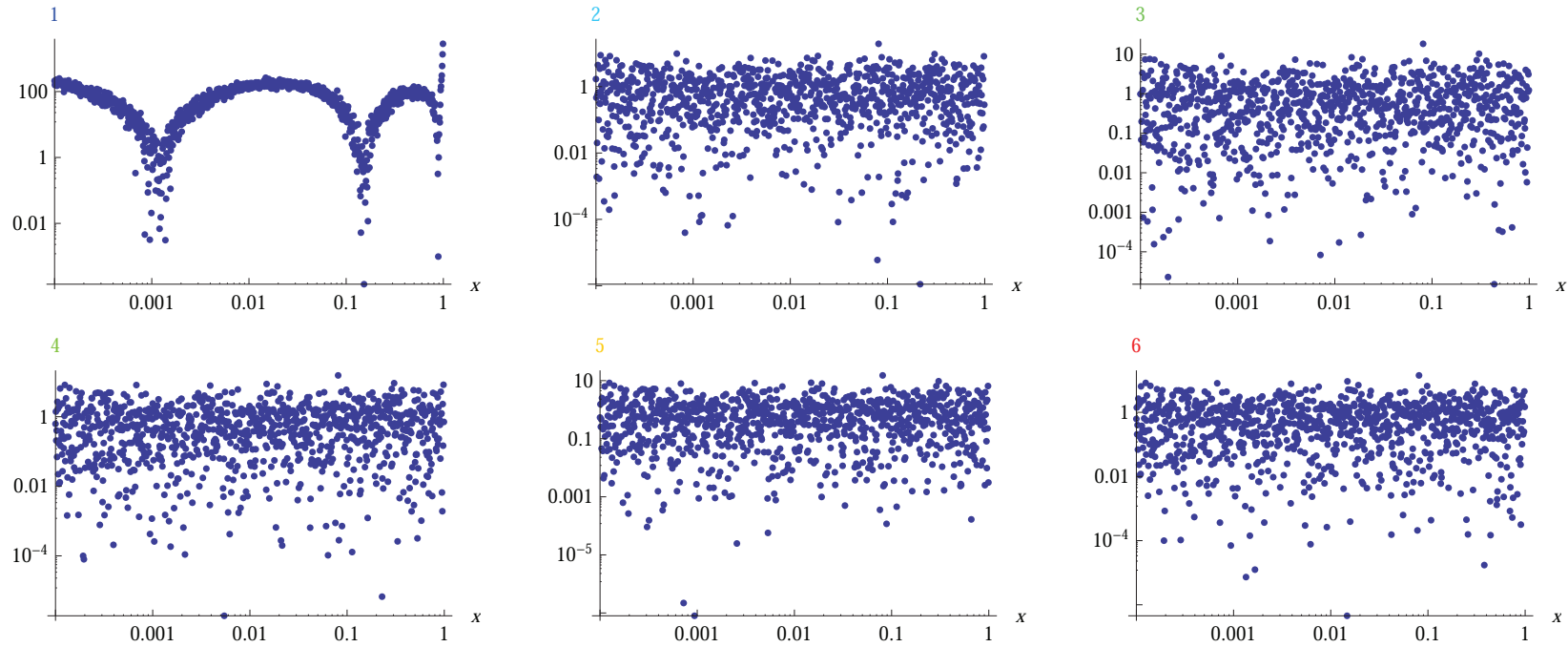
Can see effect of data on the gluon using reweighting technique, $R = 0.4$.

Clearly little pull with present data.





Contributions to χ^2 .



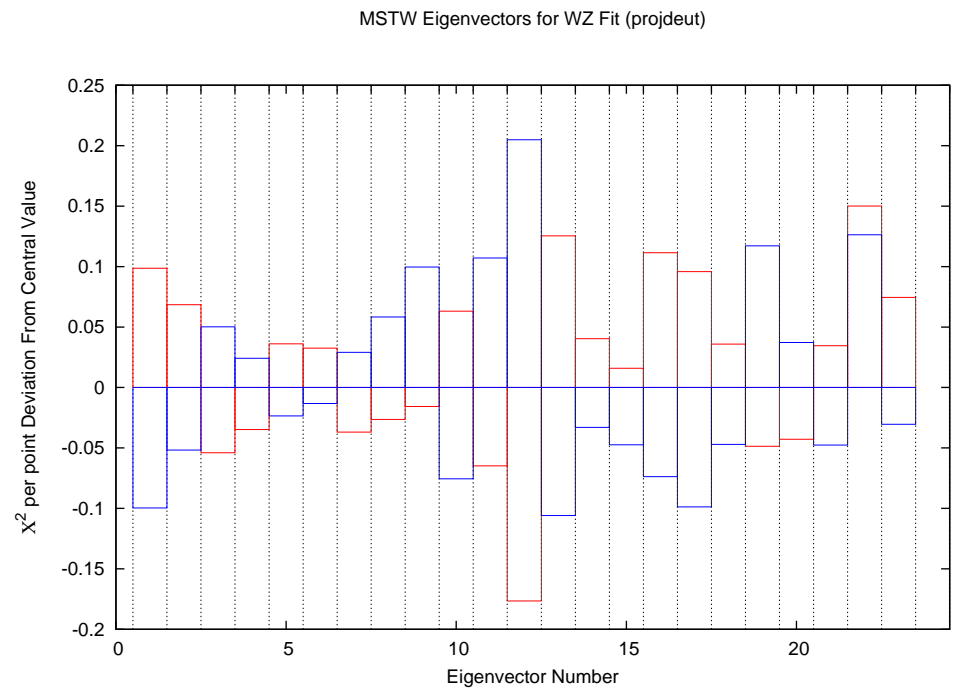
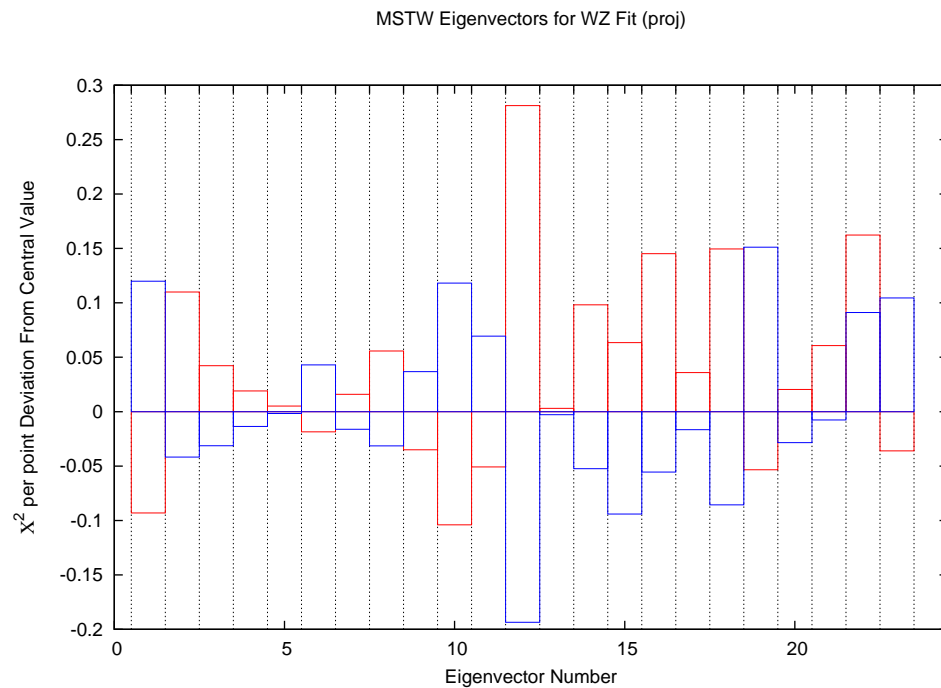
After 5 – 6 polynomials start fitting noise, i.e. χ^2 lower than real function.

Conclude 4 parameters fine. (Note first 2 just re-expression of standard MSTW parameterisation.)

Can try reweighting approach and dependence on eigenvectors using modified MSTW2008 sets (B. Watt).

No significant changes in fits to jet data.

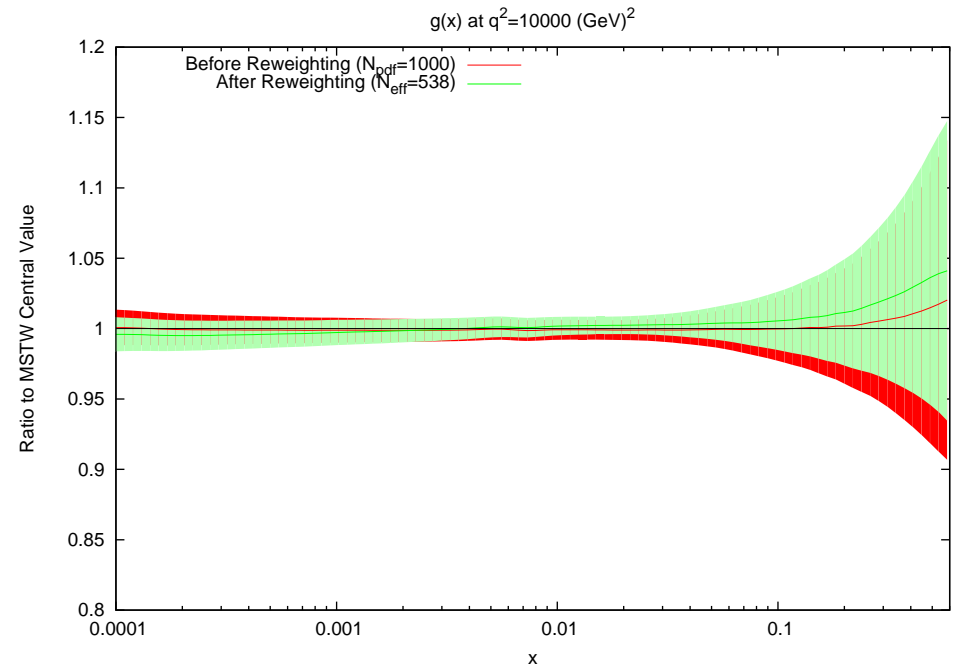
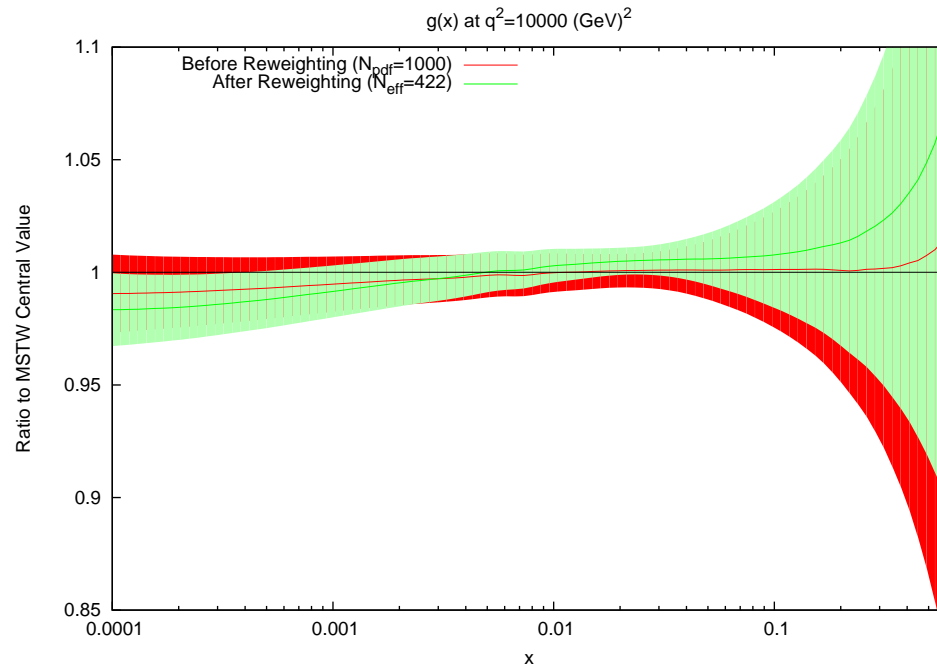
For W, Z rapidity data eigenvectors preferred mainly alter gluon shape and details of u_V and d_V still. Small preference for eigenvectors with higher strange.



Effective number of sets now much higher.

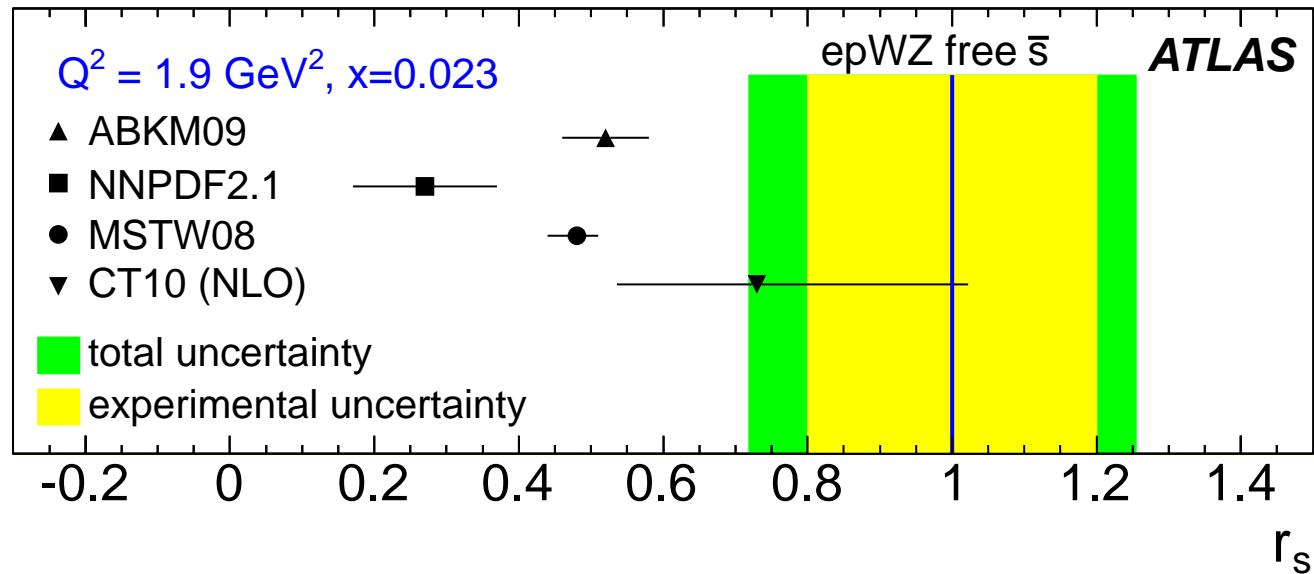
After reweighting get $\chi^2 = 39.5/30$ and $\chi^2 = 38.5/30$.

No noticeable pull on strange.



Strange Quark

Recently suggested by [ATLAS](#) study that strange quark fraction at $x \sim 0.01$ much larger than generally suggested - though there is quite a lot of variation.



Mostly determined in many fits by dimuon data

$$\nu_\mu \rightarrow \mu^- + W^+, \quad W^+ + s \rightarrow c$$

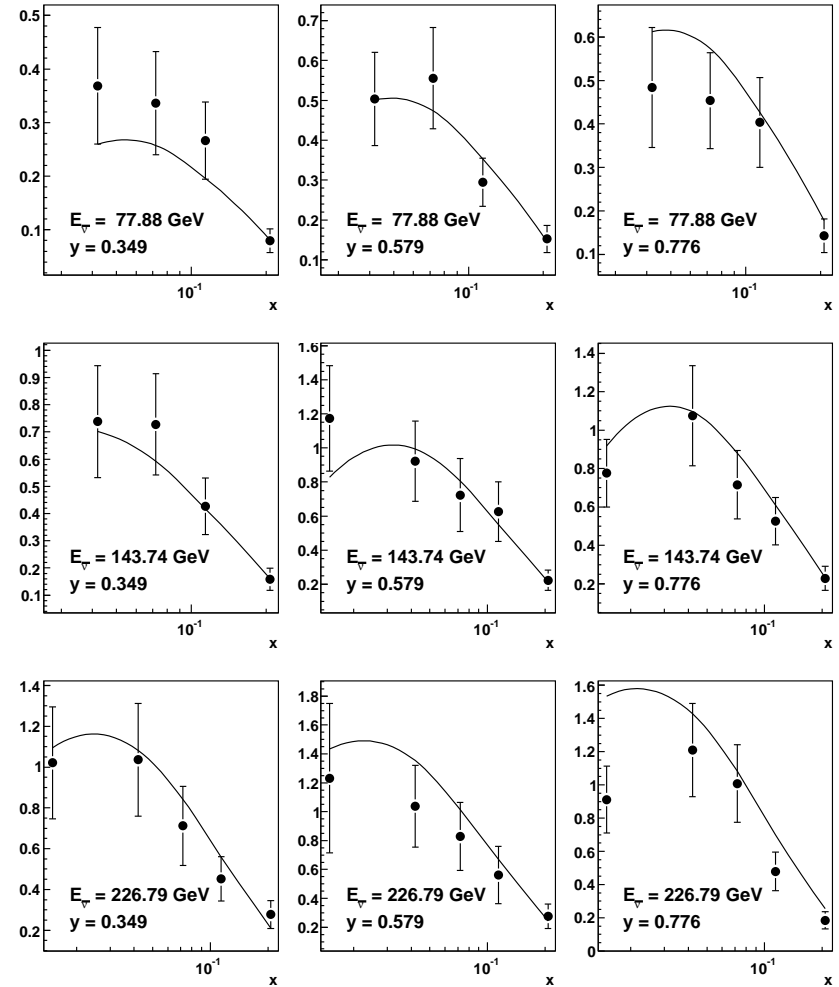
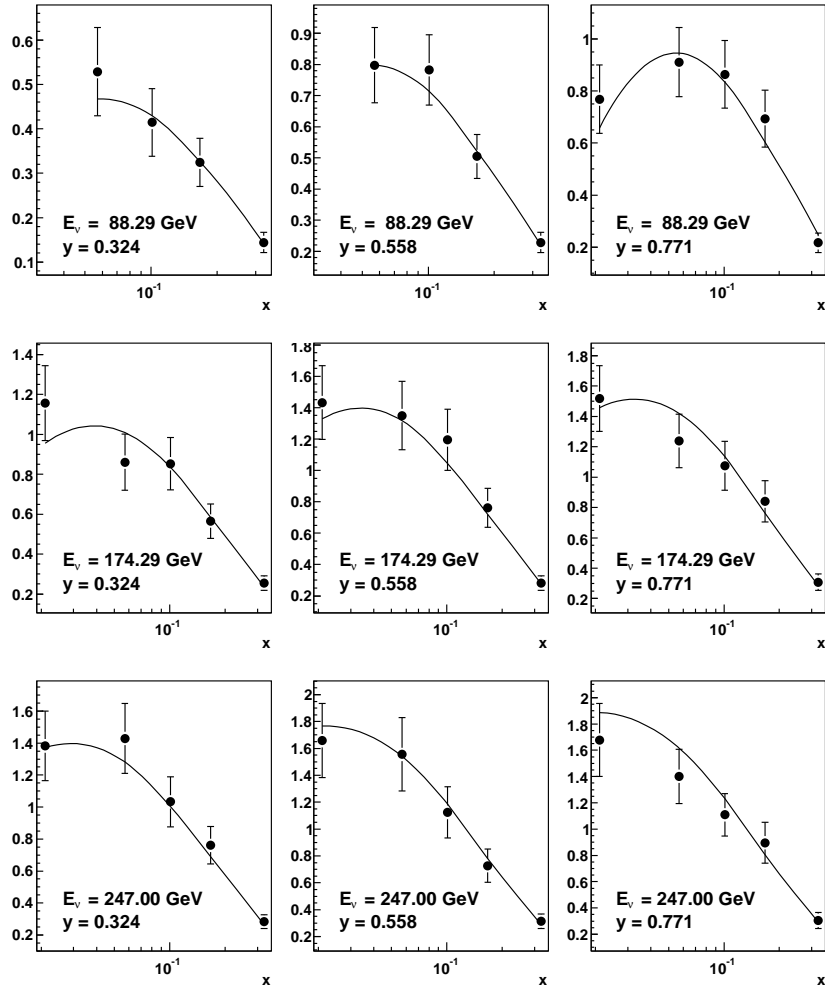
where the charm meson decays to a muon. From [CCFR](#), [NuTeV](#), the latter being more constraining.

$$\text{NuTeV } \frac{100\pi}{G_F^2 M_N E_\nu} \frac{d\sigma}{dx dy} (\nu_\mu N \rightarrow \mu^+ \mu^- X) \text{ in GeV}^{-2}$$

MSTW 2008 NNLO PDF fit, $\chi^2 = 13$ for 21 DOF

$$\text{NuTeV } \frac{100\pi}{G_F^2 M_N E_\nu} \frac{d\sigma}{dx dy} (\bar{\nu}_\mu N \rightarrow \mu^+ \mu^- X) \text{ in GeV}^{-2}$$

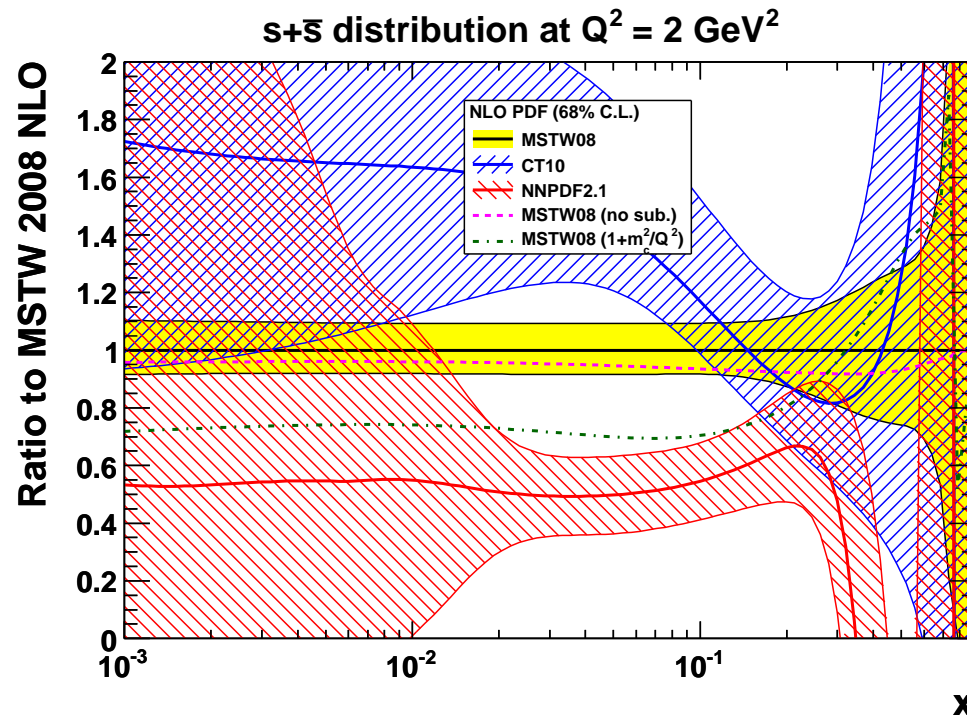
MSTW 2008 NNLO PDF fit, $\chi^2 = 32$ for 19 DOF



1cm

Where $Q^2 = 2m_p x E_\nu$. At $x \sim 0.02$ $Q^2 \sim 2 - 5 \text{ GeV}^2$. Lowest x bin usually $Q^2 = 2 - 3 \text{ GeV}^2$.

Significant variation in PDFs (ABM similar to MSTW). Maybe partially explained by Q^2 cuts (MSTW 2GeV^2 , NNPDF 3GeV^2 , CT10 4GeV^2). Strange almost unchanged if MSTW cut 5GeV^2 .



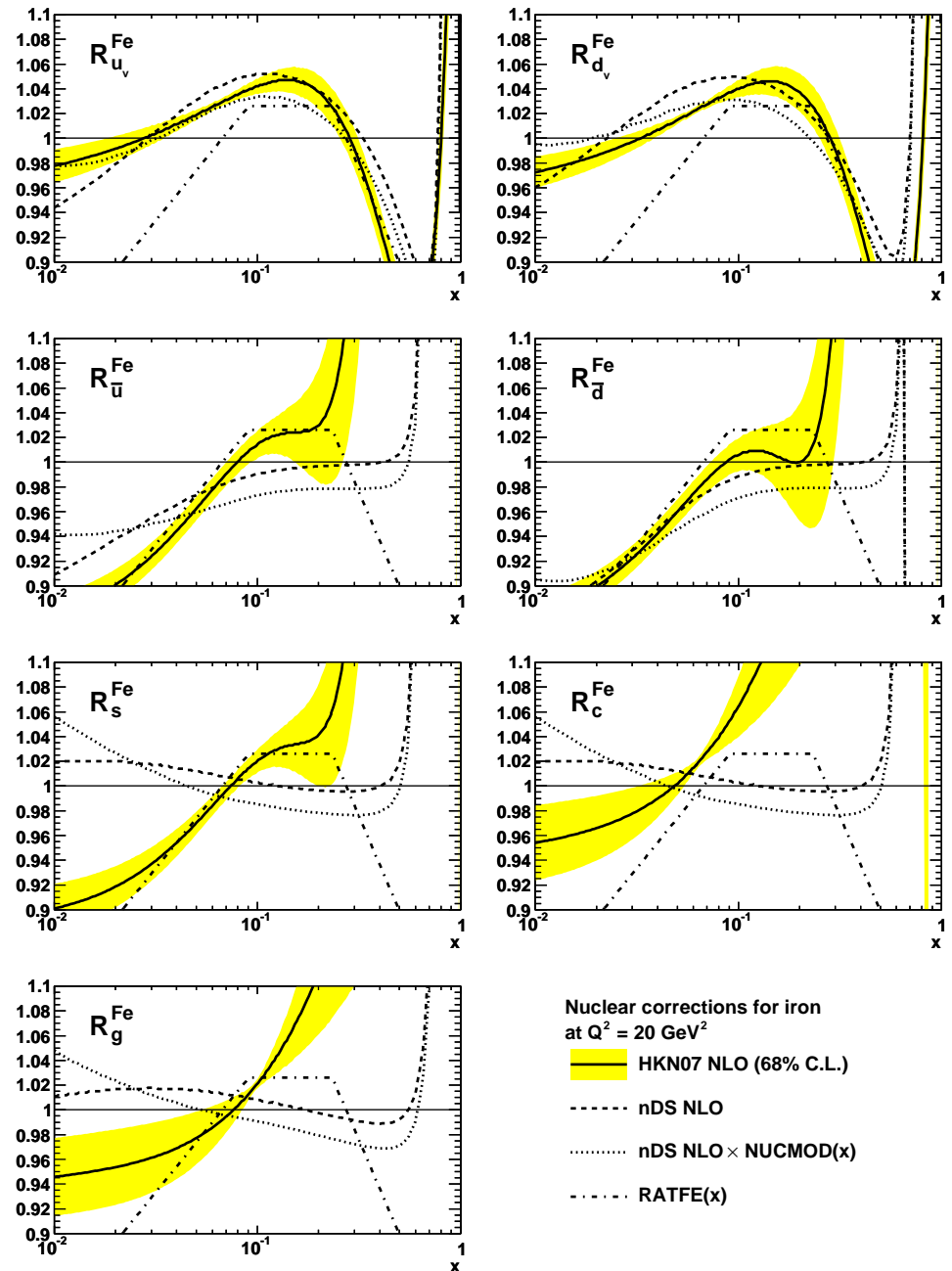
Factor of $(1 + m_c^2/Q^2)$ in NNPDF2.1 lowers MSTW a little - cuts different.

Correction of contribution from initial state charm quarks/subtraction from gluon ($\sigma \propto s + (1 - y)^2 \bar{c}$, $y = 0.3 - 0.7$) to be consistent with acceptance corrections moves MSTW down very slightly (smaller $y \rightarrow$ smaller charm). Plot by G. Watt.

Requires use of nuclear corrections.

Can vary by $\sim 10\%$ at $x \sim 0.01$. A little more at low Q^2 .

MSTW allow no penalty variation in nuclear corrections with three parameters (normalisation, low x shape and high x shape).



Try various fits changing strange parameterisation. General form

$$s(x, Q_0^2) + \bar{s}(x, Q_0^2) = A(1-x)^\eta(1 + \epsilon x^{0.5} + \gamma x)x^\delta, \quad Q_0^2 = 1\text{GeV}^2.$$

where δ set equal to light sea. Fix ϵ and γ because the fit finds no improvement if left free. A leads to suppression and η slightly greater than for light sea.

Try raising strange at low x by setting A so that $s(x, Q_0^2) + \bar{s}(x, Q_0^2)$ is a third of the total sea at input at low x . Try 4 variations.

- $k=1$ where $k = (s + \bar{s})/(\bar{u} + \bar{d})$ - all other parameters fixed. Strange exactly 1/3 of sea at input. $\Delta\chi^2 = -10$ for ATLAS, W,Z data.
- $k=1\ 1p$ - η free. $\Delta\chi^2 = -11$ for ATLAS, W,Z data.
- $k=1\ 2p$ - η, γ free. $\Delta\chi^2 = -10$ for ATLAS, W,Z data.
- $k=1\ 3p$ - η, γ, ϵ free. $\Delta\chi^2 = -4$ for ATLAS, W,Z data.

$k = 1$ - $\Delta\chi^2 = 1200$. NuTeV dimuon χ^2 25 times worse. All nuclear data and Drell Yan data (E866 and Tevatron) much worse.

$k = 1 1p$ - $\Delta\chi^2 = 190$. NuTeV dimuon χ^2 120 worse. Nuclear and Drell Yan data worse. Nuclear correction modified.

$k = 1 2p$ - $\Delta\chi^2 = 55$. NuTeV dimuon χ^2 42 worse. Nuclear and Drell Yan data slightly worse. (Similar to CT10 strange)

$k = 1 3p$ - $\Delta\chi^2 = 43$. NuTeV dimuon χ^2 17 worse. Nuclear and Drell Yan data slightly worse.

Does not resolve issues. Some pull from ATLAS data.

Much more from $W+c$ data (see Stirling and Vryonidou study).

