At what mass scales is the New Physics responsable for neutrino masses?
and
How to measure them?

(Scales at the origin of neutrino masses)

Sacha Davidson
IPN de Lyon/CNRS

I have nothing to say (feel free to read your email)

What does gravity do with axions?

(I don't know much about axions — but we could discuss what I am confused about :))

Sacha Davidson (+ M. Elmer + G. Raffelt)
IPN de Lyon/CNRS, France

1. the gravitational interactions of axions as $Cold$ Dark Matter should
   - contribute to U expansion
   - grow density fluctuations into observed structures

2. Does gravity distinguish axions from WIMPs?
   Is a classical scalar field $really$ the same as a dust of particles?
   (a) ? axion Bose Einstein condensation (by "gravitational thermalisation")? Sikivie etal
   $\Rightarrow$ BEC supports vortices $\Rightarrow$ caustics in galactic DM $\Rightarrow$ $axions \neq WIMPs!$

   (b) QFT calculation (Newton in Minkowski) of graviton emmision rate by axions Saikawa+Yamaguchi

Outline

1. reminders about axions (as cold dark matter)
2. How to distinguish axions from WIMPs?
   - Sikivie's answer
3. summary ? ? ? ? ?
4. confusion clearing house
   - gravity, thermalisation and all that...
5. calculate something
   - in classical field theory: axion viscosity due to gravitational interactions
6. discussion

Particle physics motivation for the axion: "strong CP problem"

QCD Lagrangian : $-\frac{1}{4g^2}F^A_{\mu\nu}F^{\mu\nu A}$ $\quad A : 1..8$

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($\vec{E}^2 + \vec{B}^2$ $\quad\quad$ $\vec{E}\cdot\vec{B}$)

- θ is CPV — neutron edm $\Rightarrow \theta \lesssim 10^{-10}$

Pich deRafael
Pospelov, Ritz

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Recall(!), the chiral anomaly :

$$\partial_\mu j_5^\mu \propto F\widetilde{F}$$

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Peccei Quinn

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SSB the U(1) at $f_{PQ} \gtrsim 10^8$ GeV : $\rho = f_{PQ}e^{i\phi}/\sqrt{2}$
$\Rightarrow$ goldstone $a = f_{PQ}\phi \simeq$ only NP at accessible energies ($[a] = m$)

Peccei Quinn

$$\text{mixes to pion} \quad : \quad m_a \sim \frac{m_\pi f_\pi}{f_{PQ}} \simeq 6\times 10^{-5}\,\frac{10^{11}\ \text{GeV}}{f_{PQ}}\ \text{eV}$$

...
Srednicki NPB85

$$\text{couplings to SM} \propto \frac{1}{f_{PQ}} \propto m_a$$

The axion in cosmology: a field and/or particles

The Peccei-Quinn Phase Transition: SSB of the U(1) $\rightarrow f_{PQ} e^{ia/f_{PQ}}$
a random in $-\pi \rightarrow \pi$ from one horizon to next
one string/horizon

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PQPT before inflation

$a(x,t) \sim a_0(t) + \delta a(x,t)$, a_0 random
(isocurvature) fluctuations on all scales
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PQPT before inflation	inflation before PQPT
$a(x,t) \sim a_0(t) + \delta a(x,t)$, a_0 random	$\langle a^2 \rangle_{U\ today} \sim \pi^2 f_{PQ}^2/3$
(isocurvature) fluctuations on all scales	$\delta\rho \sim \rho$ on horizon-scale
one string in the U	one string per horizon

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QCD Phase Transition ($T \sim 200$ MeV):
strings go away (radiate axions $\Rightarrow f_{PQ} \lesssim$ few 10^{11} GeV)
$m_a(t) : 0 \to f_\pi m_\pi / f_a$. When $m_a > H$:

$$\ddot{a} + 3H\dot{a} - \nabla^2 a + m_a^2 a = 0 \qquad \Rightarrow a(x,t) \simeq \frac{a(t_{QCDPT})}{R^{3/2}(t)} e^{i(mt - xH_{QCDPT})}$$

$$\Rightarrow \rho_a(t) = m_a^2 a^2 \simeq \frac{m^2 a^2(t_{QCDPT})}{R^3(t)}$$

redshifts like CDM — solution to strong CP problem is a DM candidate!

Constraints on the QCD axion (one parameter BSM)

$$m_a \sim \frac{m_\pi f_\pi}{f_{PQ}} \simeq 6 \times 10^{-5} \frac{10^{11} \text{ GeV}}{f_{PQ}} \text{ eV} \qquad \text{couplings to SM } \propto \frac{1}{f_{PQ}}$$

cosmology: for PQPT after inflation avoid isocurvature bds, have strings, interesting wrt Sikivie
axion field oscillations + (cold) axion particles from string decay not overclose U:

$$\rho_a \sim m_a^2 f_{PQ}^2 \ , \text{ redshifts once } m_a > H \qquad \Rightarrow \qquad m_a \gtrsim 10^{-4} \text{ eV} \ \ (f_{PQ} \lesssim 10^{11} \text{ GeV})$$

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searches: $aF\widetilde{F} \Rightarrow$ axion conversion to photon in a $\vec{B}$ field ...resonant in a cavity.
ADMX: CDM axions, sensitive to $10^{-6} < m_a < 10^{-5}$ eV
CAST: axions from the sun, sensitive $0.1 \lesssim m_a <$ eV

stellar energy loss: (upper bd on cplg)

$$m_a \lesssim 10^{-2} \text{ eV} \qquad\qquad (f_{PQ} \gtrsim 10^9 \text{ GeV})$$

$$10^{-2} \text{ eV} \gtrsim m_a \gtrsim 10^{-4} \text{ eV}$$

The axion as a Cold Dark Matter candidate

- expand the U the same
- grow large scale linear fluctuations the same $(P \sim \sigma \sim 0...)$ Ratra, Hwang+Noh

...at the QCD PT, axion energy density inherits adiabatic fluctuations in plasma...

$\Rightarrow$ How to tell WIMPs from axions?

to tell axions from WIMPs?

- expand the U the same
- grow large scale linear fluctuations the same
- ?? could non-linear structure formation be different?? (**Umm...** non-linear/N-body is hard!)

Sikivie:

1. **axions form a Bose-Einstein Condensate** (= almost all axions in lowest energy mode...the case for PQPT before inflation)
2. BEC can support vortices (?WIMP halo not?)
3. vortices allow caustics in the galactic DM distribution ⇔ axion DM signature?

BEC galactic halos:
Rindler-Daller+Shapiro

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Suppose PQ PT after inflation. The classical axion field can be represented as a coherent state of axion particles (of momentum $\lesssim H_{QCDPT}$).
QFT rate for axions (momentum $\vec{k}$) to emit gravitons:

$$i\frac{\partial \hat{n}_k}{\partial t} = \left[\hat{H}_{int}, \hat{n}_k\right] \simeq \frac{G_E}{H(t)^2}\rho_a^2 \gg H(t) n_k$$

Saikawa+Yamaguchi

(evaluated in coherent state $\Leftrightarrow$ classical field caln.)

Sikivie interprets as gravitational thermalisation rate: hugely occupied low-$\vec{p}$ modes, equilibrium after $T_\gamma \lesssim$ keV, $\rightarrow$ BE condensate.

But are some of those gravitons expanding the U, and some growing fluctuations?

Why is that a thermalisation rate??

Summary

1. Axions = motivated DM candidate — how to confirm/distinguish from WIMPs?
   1− find axion in terrestrial searches (CAST, ADMX...)
   2− *are axions different from WIMPs in structure formation?*
   Axions can bose-einstein condense...
   ...selon Sikivie, they *do* form BEC due to "gravitational thermalisation"!
   (...but is that gravitational interaction rate a thermalisation rate?)
   galactic halo of bose-einstein condensate could have observable caustics in x-space density

2. I learned in kindergarten: *New Physics is HEAVY*...
   ⇒ m_ν from seesaw, DM is a WIMP
   ⇒ some people model build, and phenomenologists do Effective Field Theory...

But maybe New Physics is light? m_ν is Dirac and DM is the axion?

- what is the equivalent of EFT for NP that is light and feebly coupled?

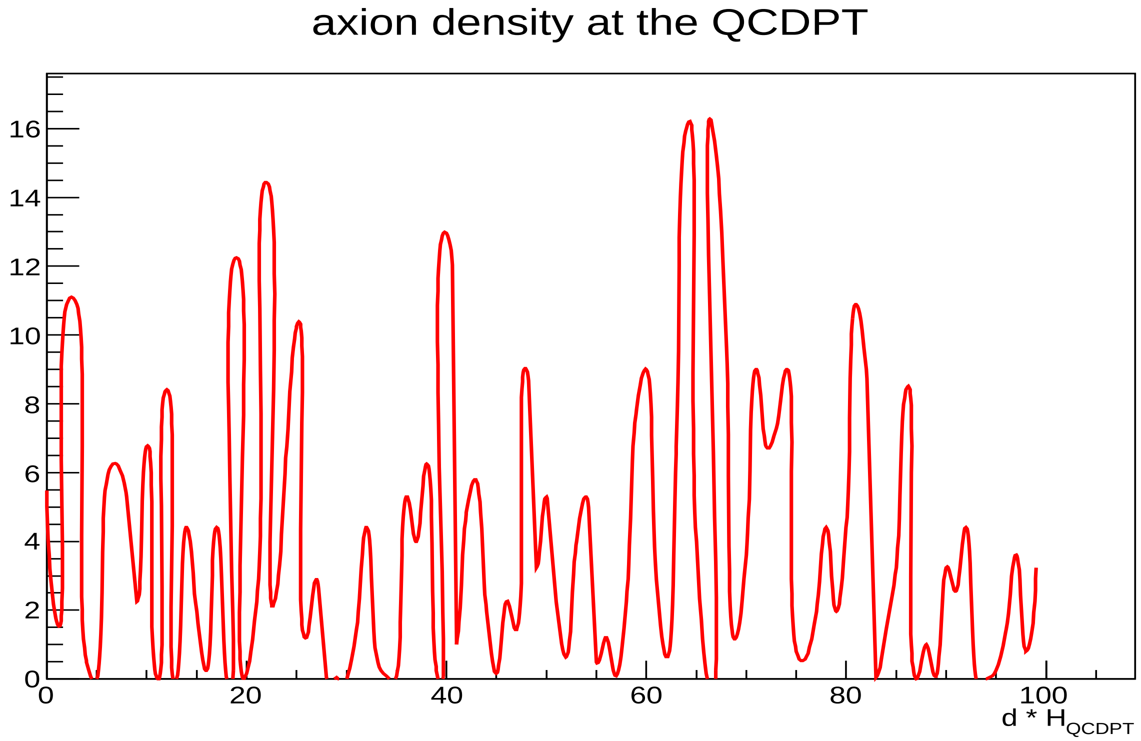
Outline

1. reminders about axions (as dark matter)
2. summary = ? ? ? ? ? + discussion ?
3. confusion clearing house
   - ...but the density is not homogeneous!
   - eqns of motion: axion field vs density fluctuations
   - thermalisation in closed unitary systems — what is the bath? **(go to 4)**
   - particles vs fields?
   - gravity and the second law — can gravity "thermalise" anything?
   - whats a Bose-Einstein condensate? in particle physics, field theory?
4. calculate something
   - find, in linearised Einstein's Eqns, gravitational interactions *not* contributing to inhomogeneity growth

     $\Leftrightarrow$ **anisotropic stress** = viscosity
   - estimate diffusion length due to gravitational viscosity...verry short

Inhomogeneities are $\mathcal{O}(1)$ on the QCD horizon scale

$a(\vec{x}, t)$ random from one horizon(~ 5km) to next; $\rho_a(\vec{x}, t) \simeq m_a^2 a^2(\vec{x}, t)$



$\Rightarrow$ its *not* a spatially homogeneous distribution of particles various momenta

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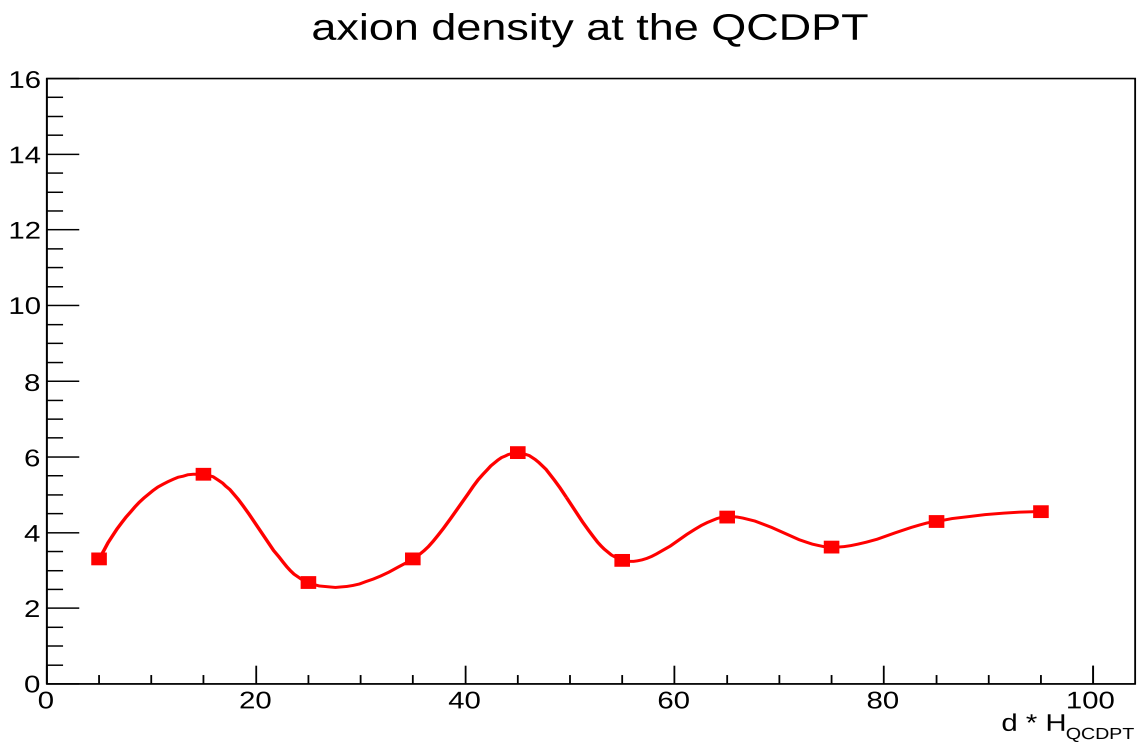

Not a problem for LSS (on scales $\gtrsim 10^{10}$ longer!); field smoothes on averaging...

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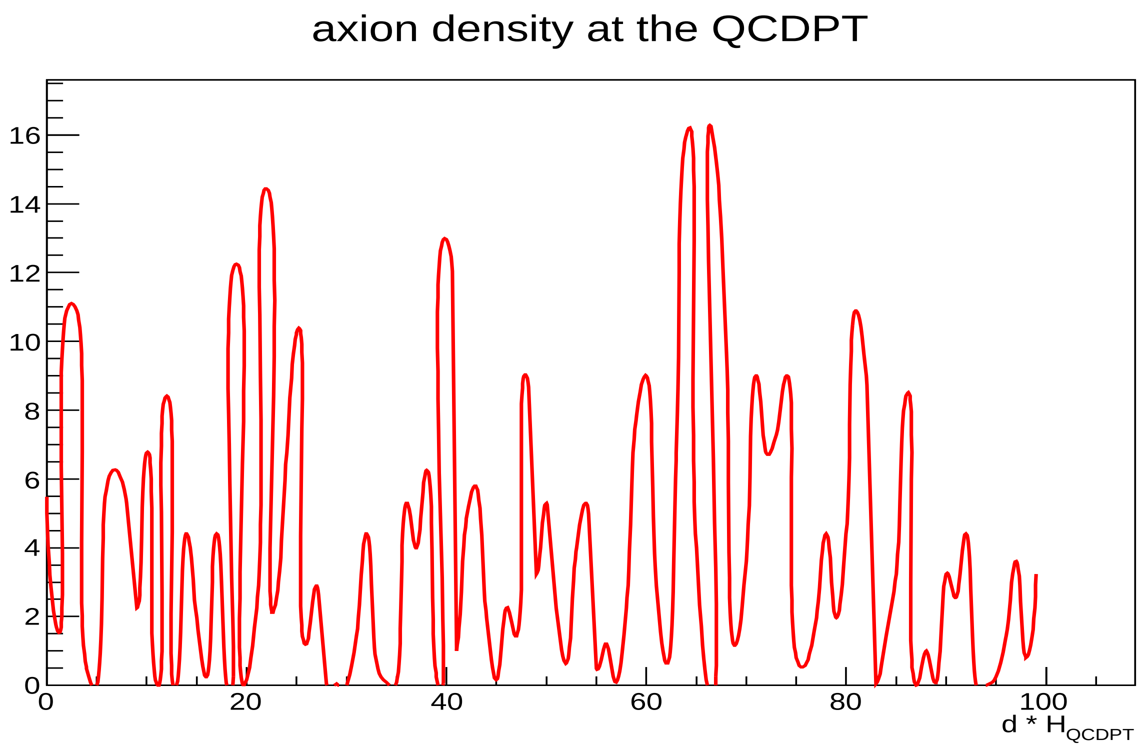

But how can axions form a *homogeneous-on-QCD-horizon-scale* bose-einstein condensate = zero mode of field? ??

$v = H_{QCDPT}/m_a \lesssim 10^{-6} c$...not "free-stream" QCD-horizon distance before t_{eq}:

$$d(t) = \int^t \frac{H_{QCDPT}}{m_a R(t')} dt' \sim \frac{H_{QCDPT}}{m_a} \frac{1}{H(t)R(t)} = \frac{R(t)}{m_a} \ll \frac{R(t)}{H_{QCDPT}}$$

(RD U, $R(t)$ =1@QCDPT)

Eqns of motion — axion field vs density fluctuations?

Inside the horizon, can study axion evolution by

1. get Eqns of motion of the field, from $T^{\mu\nu}_{\ \ ;\nu} = 0$

$$\ddot{a} + 3H\dot{a} - \nabla^2 a + m_a^2 a = -2m^2 a\phi$$

   and combine with Poisson: $-\nabla^2\phi = 4\pi G_E \delta\rho$. (This corresponds to single graviton exchange)

   But $\delta\rho_a \propto a^2$, so its non-linear.

2. Or get Eqns of motion for a fluid with scalar perturbations from $T^{\mu\nu}_{\ \ ;\nu} = 0$, and combine with Poisson:

$$\ddot{\delta} + 2H\dot{\delta} - 4\pi\rho\delta + \frac{c_s^2}{R^2(t)}\nabla^2\delta = 0$$

   for $\delta = \delta\rho/\rho$. Can *solve* (in fourier space).

Are these the same thing?
From 1), can get rate for axions (labelled $\vec{k}$) to emit a graviton of any wavelength.
From 2), learn that gravitons of wavelength $\vec{p}$ grow $\delta(\vec{p}, t)$.
where $\delta(\vec{p}, t) \sim \sum_q a(q + k, t) a^*(q, t)$ is non-lin in a
Is it the same gravitons?

thermalisation in closed unitary systems?

$$\text{entropy} = \sum_{states\ s} P_s \ln P_s \quad \text{increases}$$

- unitary evolution creates no entropy $\Leftrightarrow$ NO entropy generation in closed systems ... BUT... can calculate "effective" thermalisation: a subset of observables evolve towards equilibrium expectations
  $\Rightarrow$ the "rest" of the system is the bath??

- ex: couple two SHOs. Solve one, substitute into Eqns of second, and find dissipation.

- ...$K - \bar{K}$ evolution is non-unitatry, because not also follow 2π 3π states...

? $\Rightarrow$ divide axions+gravity into

1. U expansion + structure growth
2. other fluctuations which are the bath?

Looking for gravitational interactions that aren't growing fluctuations

(suppose that density fluctuations grow on scale H^{-1}_{QCDPT}(= 5 km comoving))

anisotropic stress $= T^i_j \sim (1-2\phi)\frac{p_i p_j}{m^2}\rho_a$ (ϕ is Newtonian pot: $\nabla^2\phi = -4\pi G_E \delta\rho$)

- gauge invariant
- not included in obtaining eqns for density fluc growth (small:$p \sim H_{QCDPT} \ll m_a$)
- leading contribution is axion "free-streaming" (neglect)

...equate to viscosity of imperfect fluid

$\Rightarrow$ (physical) damping scale for axion density fluctuations, due to "gravitational viscosity": (Jeans length $= 1/\sqrt{H(t)m_a}$; density fluc on shorter distances oscillate due to pressure)

$$\ell^2_{damp}(t) <, \ll \frac{1}{H(t)m_a}$$

This is $\ll$ QCD horizon scale:

$$\frac{1}{H(t)m_a} \ll \left(\frac{1}{H_{QCDPT}}\frac{T_{QCDPT}}{T}\right)^2 = \frac{1}{H(t)H_{QCDPT}}$$

gravity and the second law

1. undergraduate memories say that gravitational collapse of a gas cloud to a star respects the second law...

2. story of $\Omega_{baryon} = 1$ U

   (a) quasi-homogeneous dust clouds collapse
   (b) ...generations of stars, supernovae, black holes...
   (c) ... ... proton decays...
   (d) venerable homogeneous and isotropic U full of photons and gravitons

3. so gravitational thermalisation of axions will happen.
   But does it happen before the U a year old?

Particles vs fields

Develop field operator

$$\hat{a}(t,\vec{x}) = \frac{1}{[R(t)L]^{3/2}} \int \frac{d^3k}{(2\pi)^3} \left\{ \hat{b}_{\vec{k}} \frac{\chi(t)}{\sqrt{2\omega}} e^{i\vec{k}\cdot\vec{x}} + \hat{b}^\dagger_{\vec{k}} \frac{\chi^*(t)}{\sqrt{2\omega}} e^{-i\vec{k}\cdot\vec{x}} \right\}$$

then write the coherent state:

$$|a(\vec{x},t)\rangle \quad \propto \quad \exp\left\{ \int \frac{d^3p}{(2\pi)^3} a(\vec{p},t) b^\dagger_{\vec{p}} \right\} |0\rangle$$

which satisfies $\hat{b}_{\vec{q}}|a(\vec{x},t)\rangle = a(\vec{q},t)|a(\vec{x},t)\rangle$ (can check $\hat{b}_{\vec{q}}\{1 + \int \frac{d^3p}{(2\pi)^3} a(\vec{p},t) b^\dagger_{\vec{p}}\}|0\rangle = a(\vec{q},t)|0\rangle$)
where the classical field is

$$a(t,\vec{x}) = \frac{1}{[R(t)L]^{3/2}} \int \frac{d^3k}{(2\pi)^3} \left\{ a(\vec{k},t) \frac{\chi(t)}{\sqrt{2\omega}} e^{i\vec{k}\cdot\vec{x}} + a^*(\vec{q},t) \frac{\chi^*(t)}{\sqrt{2\omega}} e^{-i\vec{k}\cdot\vec{x}} \right\}$$

Bose-Einstein condensates

1. for particles in stat mech

   ...

2. in equilibrium thermal field theory

   Kapusta, Haber+Weldon
   ...

   scalar field at finite T with density of conserved charge n_Q.
   Quadratic term in potential is $\propto (m^2 - \mu^2 + \mathcal{O}(T^2))\phi^\dagger\phi$. For low T, cannot accomodate charge density in the plasma $\Rightarrow$ phase transtion

$$n_Q = m\langle\phi\rangle^2 + \int \frac{d^3k}{(2\pi)^3}(n_k - n_{\bar{k}})$$

3. in non-equilibrium field theory

   Calzetta+Hu
   Berges+Serreau
   ...

   Occurs for $O(N)$ scalar theories. Analytically using 2PI effective action. And numerically...overpopulation of low-$\vec{p}$ modes (like for axions!), can migrate to the IR!

   does dn_k/dt of S+Y give this? I can't tell...

what is quantum ? Classical?

Can obtain, in classical field theory

$$i\frac{\partial}{\partial t}|\widetilde{a}(\vec{q},t)|^2 \simeq -16\pi m G_E \sum_k \frac{R^2(t)}{|\vec{k}|^2}\delta\rho(\vec{k},t)\left\{\widetilde{a}^*(\vec{q}+\vec{k},t)\widetilde{a}(\vec{q},t) - \widetilde{a}^*(\vec{q},t)\widetilde{a}(\vec{q}-\vec{k},t)\right\} \quad .$$

... quantum in the SY result is to identify $|\widetilde{a}(\vec{q},t)|^2/\sum_p |\widetilde{a}(\vec{p},t)|^2$ as a number density of axion particles?

The distribution of $\hbar$s in the Lagrangian depends on whether the classical limit should be fields or particles— for field, can take $[m] = 1/L$. So to define the particle number of a classical field configuration requires $\hbar$..