Handling astrophysical uncertainties on direct detection experiments

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- Astrophysical uncertainties
  - i) observations
  - ii) simulations
- Consequences
- Strategies
  - i) integrate out
  - ii) marginalise over
- Parameterising the speed distribution

Introduction

Differential event rate for elastic scattering:

(assuming spin-independent coupling and $f_p=f_n$)

$$\frac{\mathrm{d}R}{\mathrm{d}E} = \frac{\sigma_{\mathrm{p}}\rho_0}{\mu_{\mathrm{p},\chi}^2 m_\chi} A^2 F^2(E) \int_{v_{\mathrm{min}}}^{\infty} \frac{f(v)}{v}\,\mathrm{d}v \qquad v_{\mathrm{min}} = \left(\frac{E(m_A + m_\chi)^2}{2 m_A {m_\chi}^2} \right)^{1/2}$$

Particle physics parameters:

WIMP mass and cross-section, $m_\chi \quad \sigma_{\mathrm{p}}$

Astrophysical input:

local DM density and speed distribution $\rho_0 \quad f(v)$

Experimental constraints on σ-m_χ plane usually calculated using 'standard halo model':

isotropic, isothermal sphere, with Maxwell-Boltzmann speed distribution

$$f(\mathbf{v}) \propto \exp\left(-\frac{3|\mathbf{v}|^2}{2\sigma^2} \right) \qquad \sigma = \sqrt{\frac{3}{2}} v_{\mathrm{c}}$$

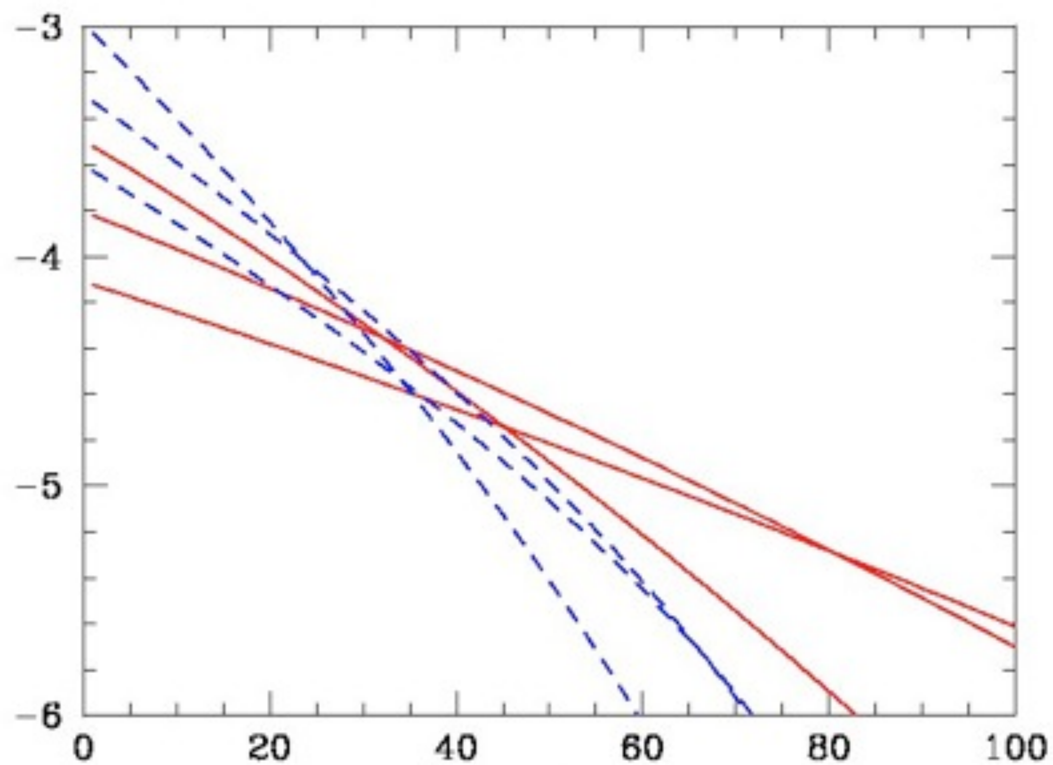
with v_c=220 km s^{-1} and local density ρ_0=0.3 GeV cm^{-3}

Energy spectrum

Energy spectrum has characteristic energy which depends on the WIMP mass, target mass and velocity dispersion:

$$E_{\mathrm{R}} = \frac{2\mu_{A\chi}^2 v_{\mathrm{c}}^2}{m_A} \qquad \begin{array}{ll} \propto m_\chi^2 & m_\chi \ll m_A \\ \sim \mathrm{const} & m_\chi \gg m_A \end{array}$$

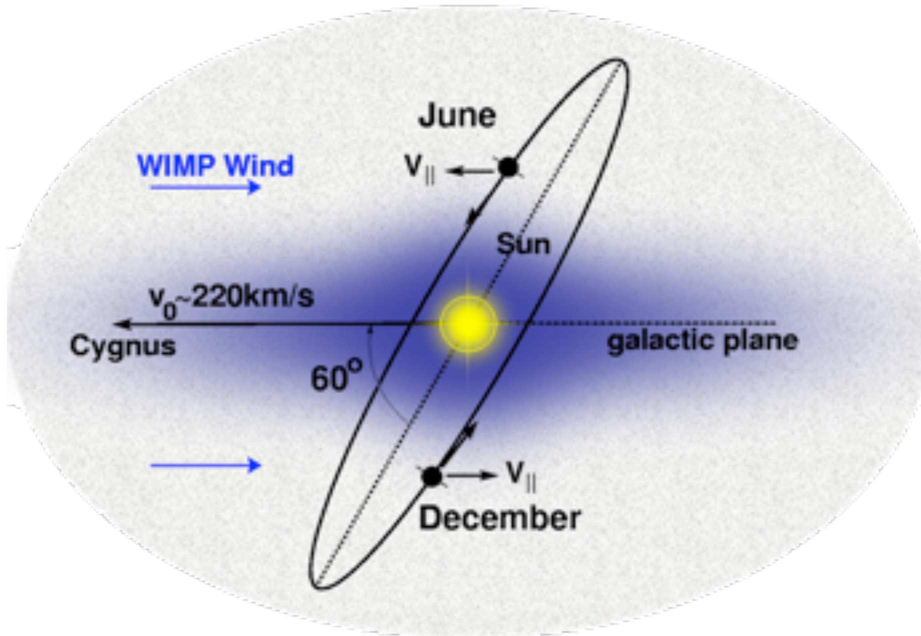
$\log_{10}\left(\frac{\mathrm{d}R}{\mathrm{d}E}\right)$



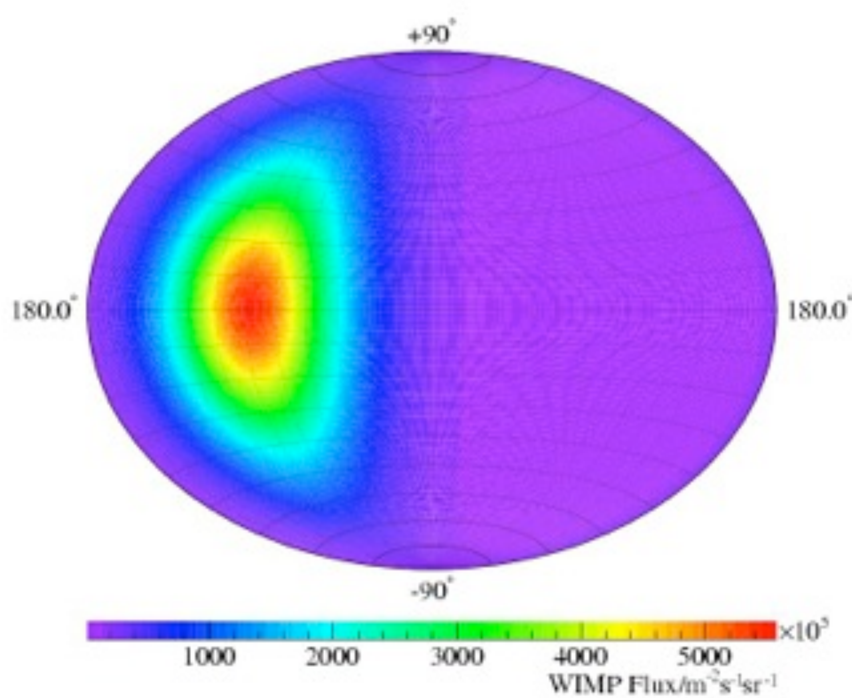
$E/(1\,\mathrm{keV})$

Differential event rate:
Ge and Xe m_χ = 50, 100, 200 GeV

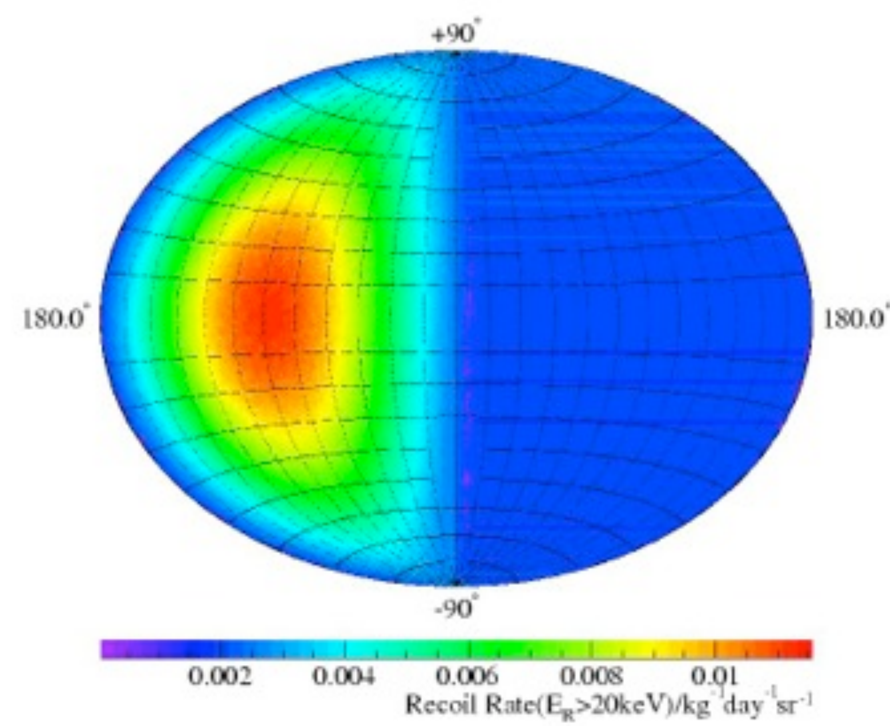
Direction dependence Spergel



Sheffield DM group



WIMP flux

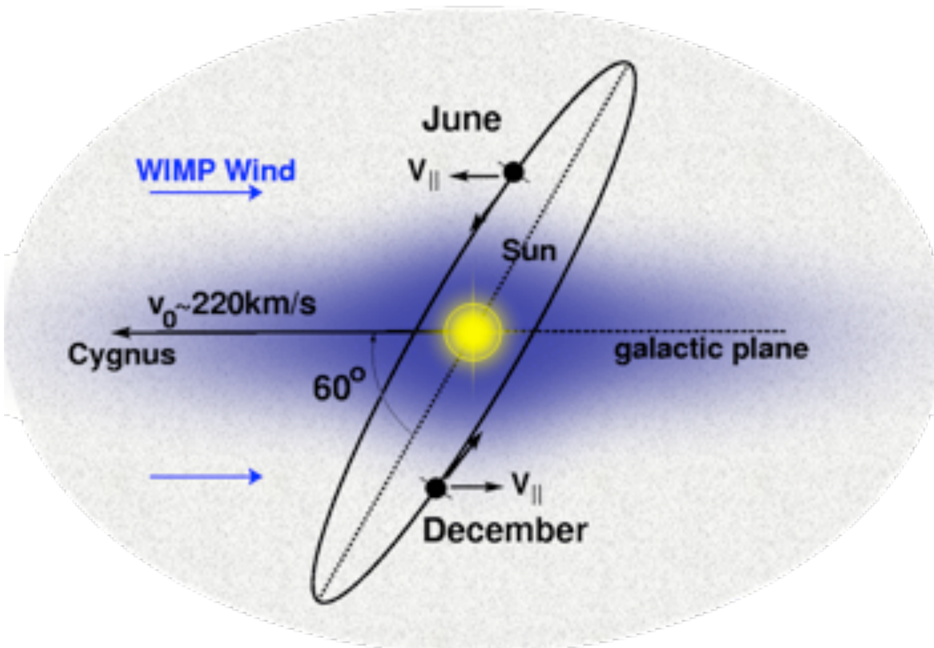


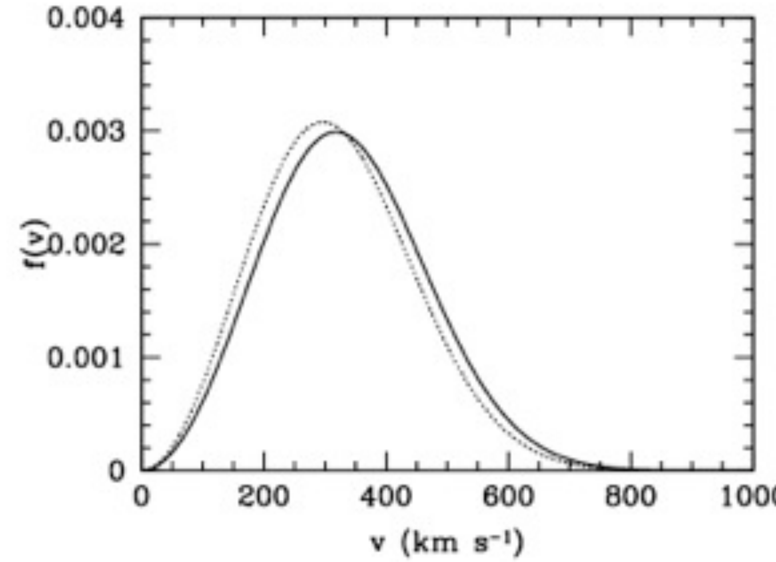
Recoil rate

Recoil rate largest in direction opposite to direction of Solar motion.

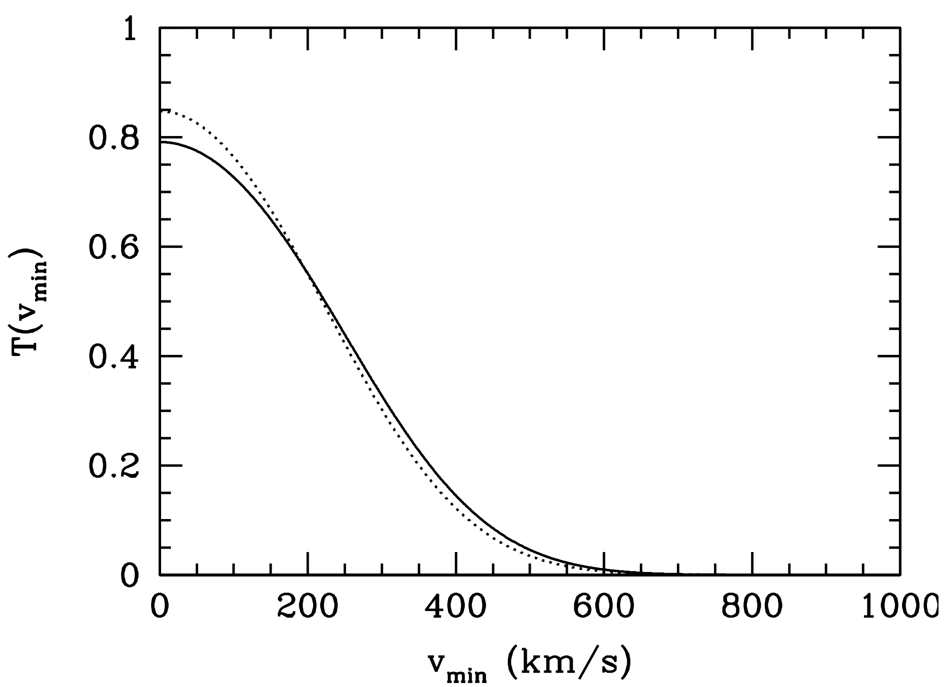
Ratio of rates in rear and forward directions is large.

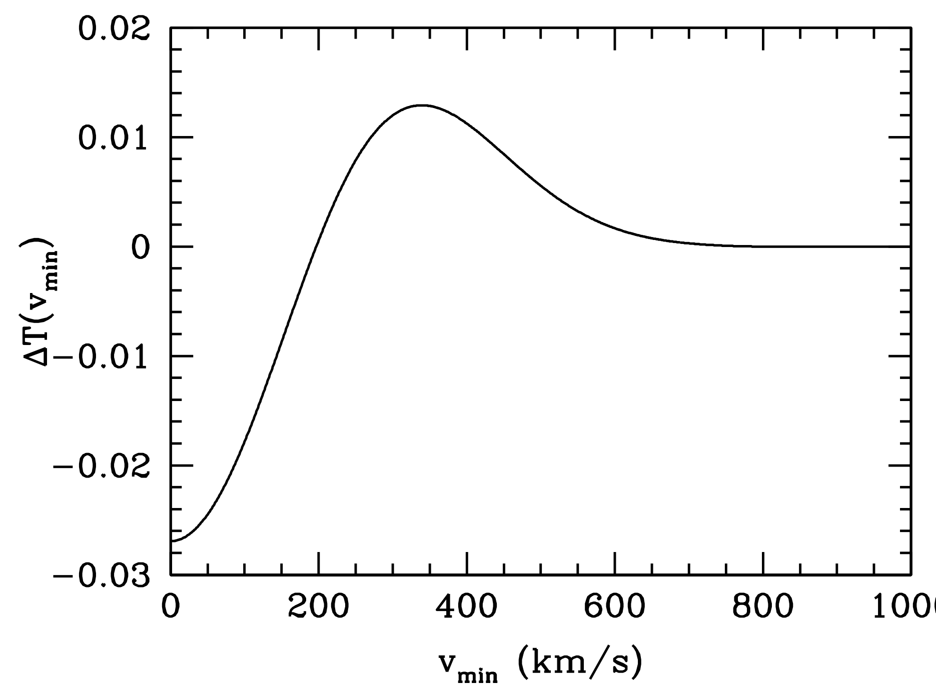
Annual modulation Drukier, Freese & Spergel





Maxwell-Boltzmann speed dist.
detector rest frame (summer and winter)





Signal <O(10%)

Astrophysical uncertainties i) observations

Local density:

Mass modelling: e.g. Widrow et al., Catena & Ullio, Weber and de Boer, Fornasa & Green in prep model for the MW (luminous components + halo) + multiple data sets (rotation curve, velocity dispersions of halo stars, local surface mass density, total mass...).

~10% statistical errors, central values vary in range $\rho_0 = (0.3 - 0.4)\,\mathrm{GeV\,cm^{-3}}$

Model independent/minimal assumption methods e.g. Salucci et al. Gabari et al. give consistent values, but with significantly larger errors.

Local circular speed:

Reid & Brunthaler proper motion of Sgr A*: $v_{\phi,\odot} \sim (250 \pm 10)\,\mathrm{km\,s^{-1}}$

Bovy et al. APOGEE data (l.o.s. v of 3000 stars): $v_{\phi,\odot} = (242^{+10}_{-3})\,\mathrm{km\,s^{-1}}$

$v_{\mathrm{c}} = (218 \pm 6)\,\mathrm{km\,s^{-1}}$

implies ϕ component of Sun's motion wrt Local Standard of Rest (LSR) larger than thought or LSR orbit non-circular.

McMillan & Binney dropping flat rotation curve assumption: $v_{\mathrm{c}} = (200 - 280)\,\mathrm{km\,s^{-1}}$

n.b. Standard halo has one-to-one relationship between circular speed and velocity dispersion & peak speed, but in general this isn't the case.

Local escape speed:

Smith et al, high velocity stars from the RAVE survey
assume $f(|\mathbf{v}|) \propto (v_{\rm esc} - |\mathbf{v}|)^k$ with 2.7< k<4.7 (motivated by simulations).
$498\,{\rm km\,s^{-1}} < v_{\rm esc} < 608\,{\rm km\,s^{-1}}$ median likelihood: $v_{\rm esc} = 544\,{\rm km\,s^{-1}}$

Summary of observations of MW properties:

Traditional values of circular speed and local density (v_c=220 km s^{-1} and ρ_0=0.3 GeV cm^{-3}), are fairly consistent with recent determinations, which have ~10% statistical errors (but systematic uncertainties from modelling are still significantly larger).

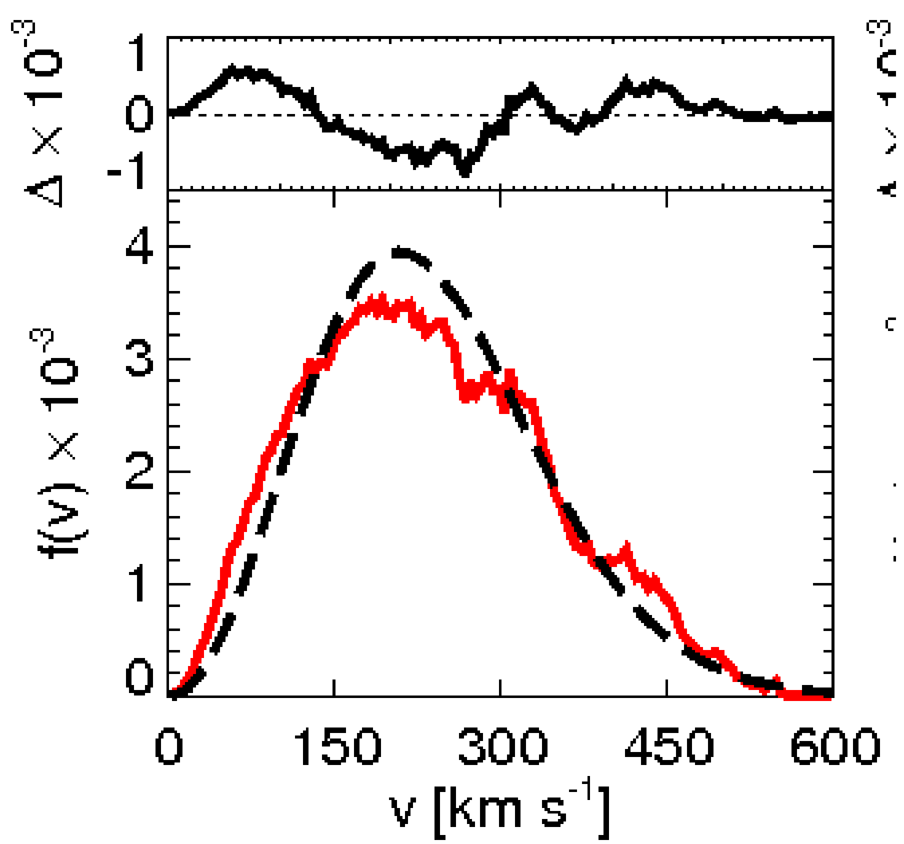
ii) simulations

Systematic deviations from multi-variate gaussian: more low speed particles, peak of distribution lower/flatter.

Features in tail of dist, 'debris flows', incompletely phased mixed material. Lisanti & Spergel; Kuhlen, Lisanti & Spergel

Deviations less pronounced in lab frame than Galactic rest frame.

Vogelsberger et al.

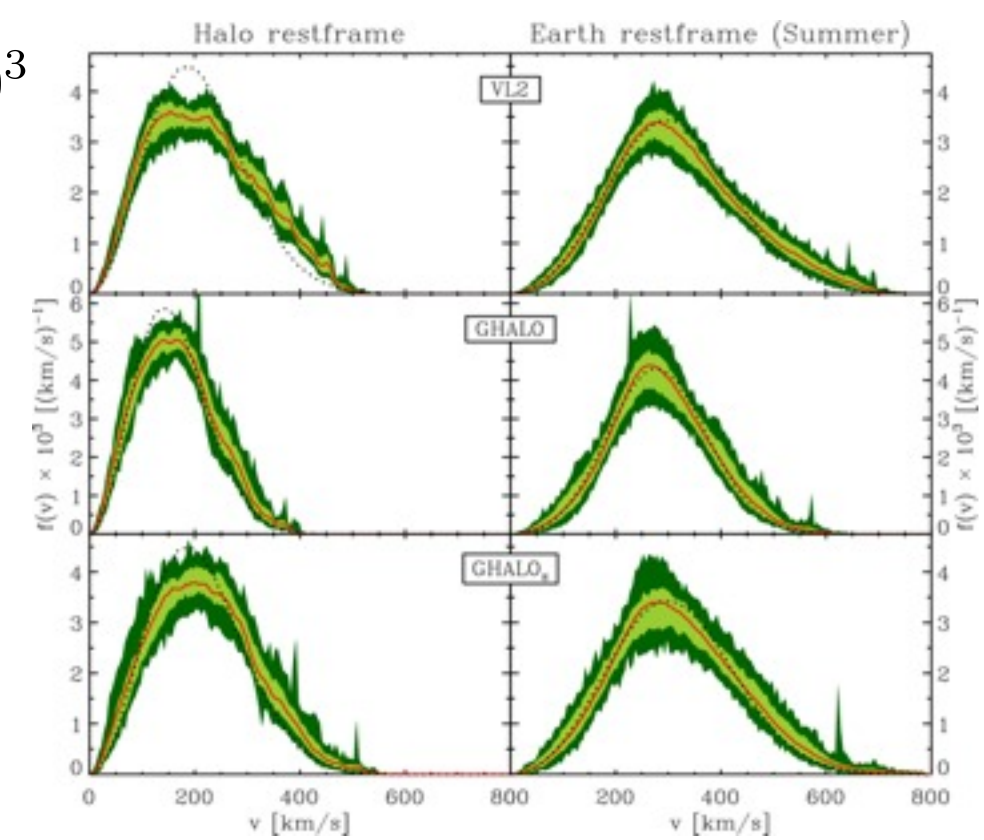


Aquarius simulation data,
best fit multi-variate Gaussian

Kuhlen et al.

halo rest frame Earth rest frame

$f(v) \times 10^3$



VL2

GHALO

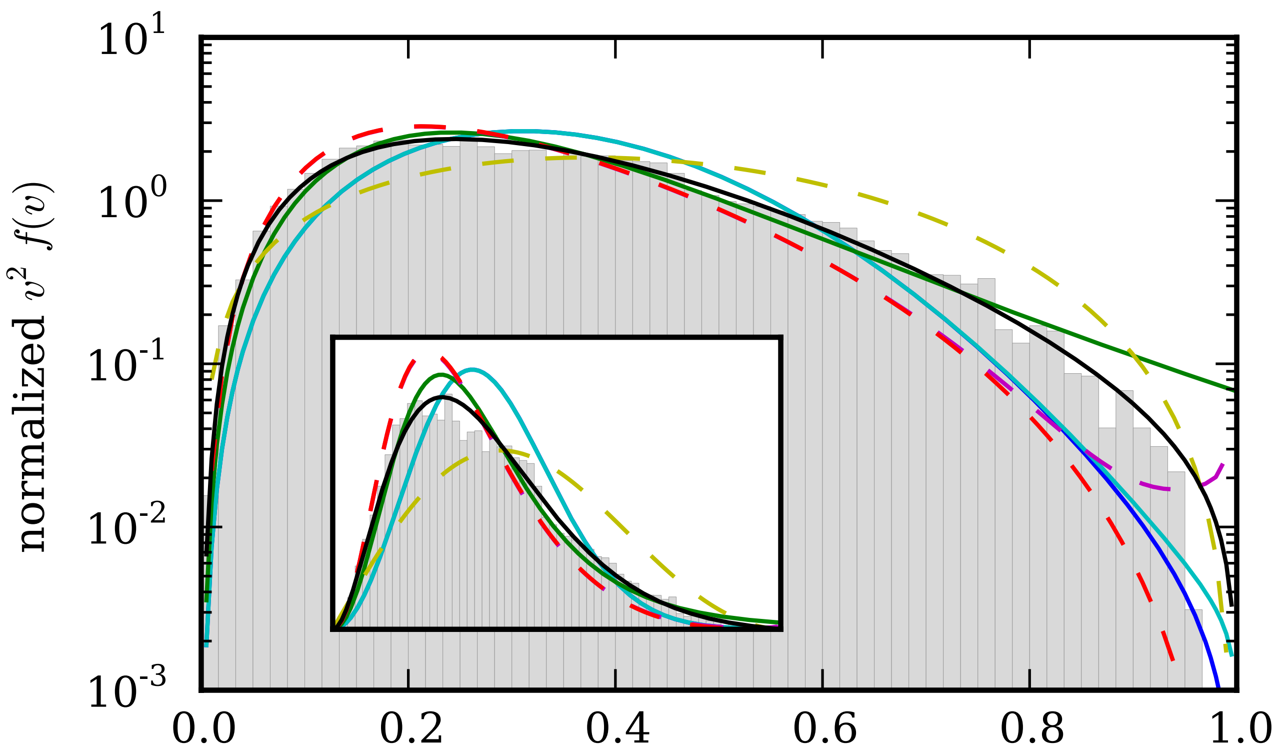
GHALO scaled

$v[\mathrm{km/s}]$

Various functional forms for f(v) proposed.

Hard to fit shape of bulk of distribution and tail with a single, simple function:

$v^2 f(v)$



$v/v_{\rm esc}$

data from one simulation

Mao, Strigari & Wechsler

SHM

Lisanti et al. double power law

Tsallis

Eddington

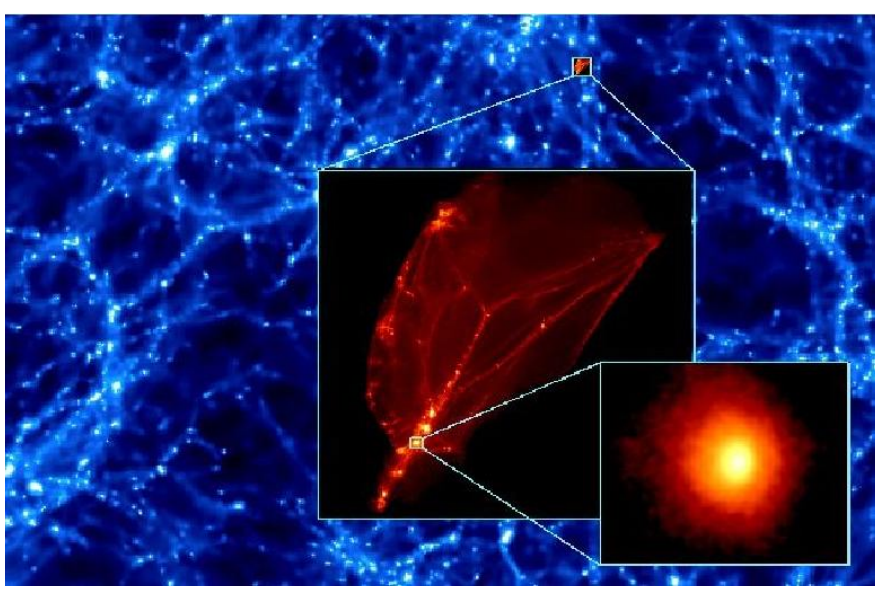
Osipkov-Merritt

β=0.5

Caveats:

a) scales resolved by simulations are many orders of magnitude larger than those probed by direct detection experiments

~300 kpc ~30 kpc ~0.3 mpc



microhalo simulation
Diemand, Moore & Stadel

Resolution of best Milky Way simulations is many orders of magnitude larger than the mass of the first WIMP microhalos to form

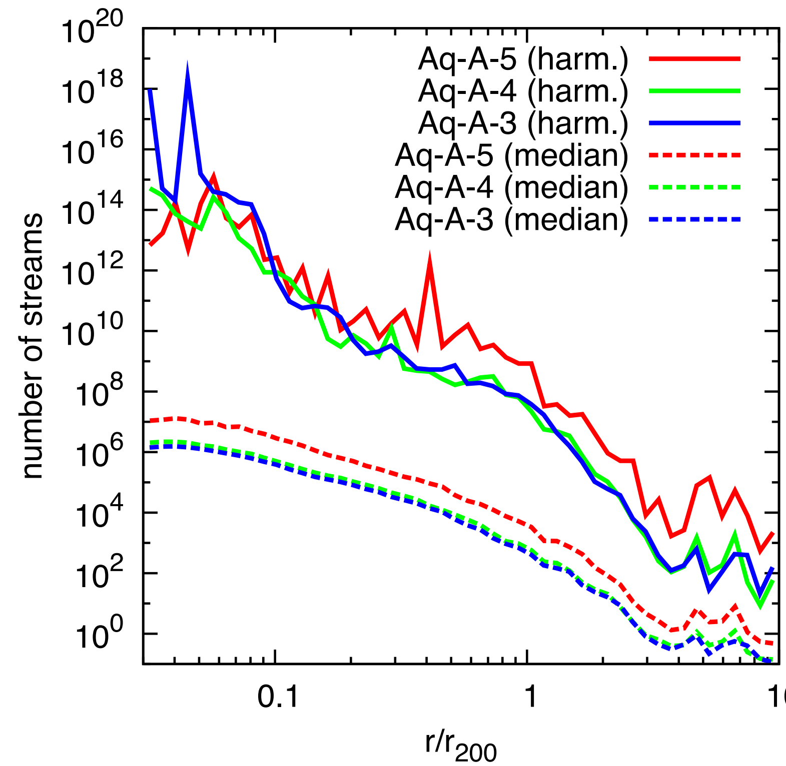
fine structure in ultra-local DM velocity distribution?

Vogelsberger & White:

Follow the fine-grained phase-space distribution, in Aquarius simulations of Milky Way like halos.

From evolution of density deduce ultra-local DM distribution consists of a huge number of streams (but this assumes ultra-local density= local density).

At solar radius <1% of particles are in streams with $\rho > 0.01\rho_0$.



number of streams as a function of radius
calculated using harmonic mean/median stream density

Schneider, Krauss & Moore:

Simulate evolution of microhalos. Estimate tidal disruption and heating from encounters with stars, produces 10^2-10^4 streams in solar neighbourhood.

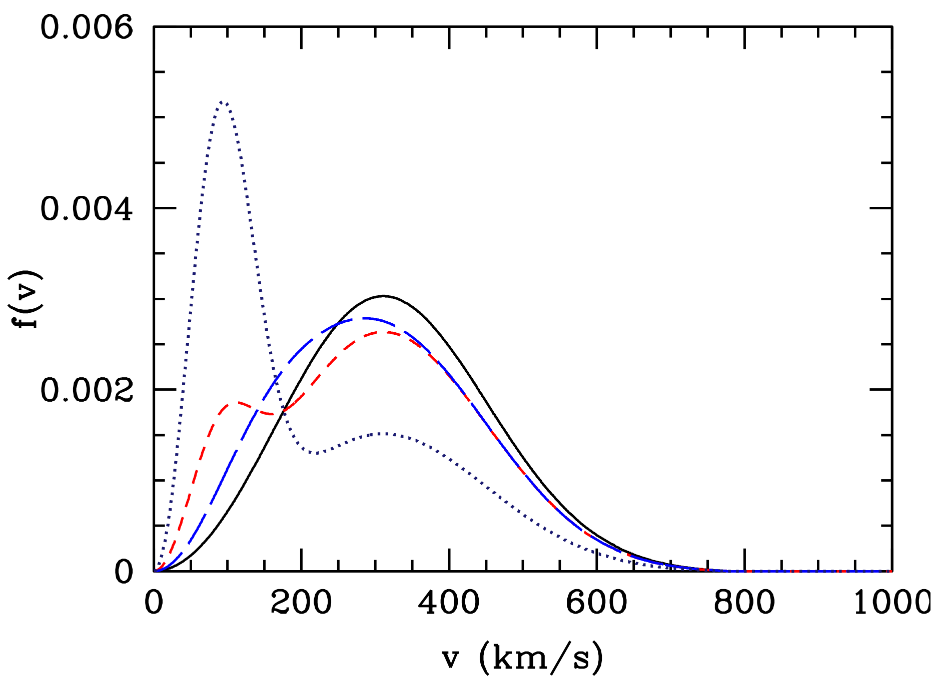
not-so fine structure:

Purcell, Zentner & Wang DM component of Sagittarius leading stream may pass through the solar neighbourhood (as originally suggested by Freese, Gondolo & Newberg).

b) effect of baryons on DM speed distribution?

Sub-halos merging at z<1 preferentially dragged towards disc, where they're destroyed leading to the formation of a co-rotating dark disc. Read et al., Bruch et al., Ling et al.

Could have a significant effect if density is high and velocity dispersion low.



SH

SH + high density $\rho_D = \rho_H$, low dispersion DD

SH + lower density $\rho_D = 0.15\rho_H$, low dispersion DD

SH + lower density, high dispersion DD

Properties of dark disc are uncertain (simulating baryonic physics and forming Milky Way-like galaxies is hard).

Purcell, Bullock & Kaplinghat to be consistent with observed properties of thick disc, MW's merger history must be quiescent compared with typical ΛCDM merger histories, hence DD density must be relatively low, $< 0.2\,\rho_H$. Also dispersion larger than stellar thick disk.

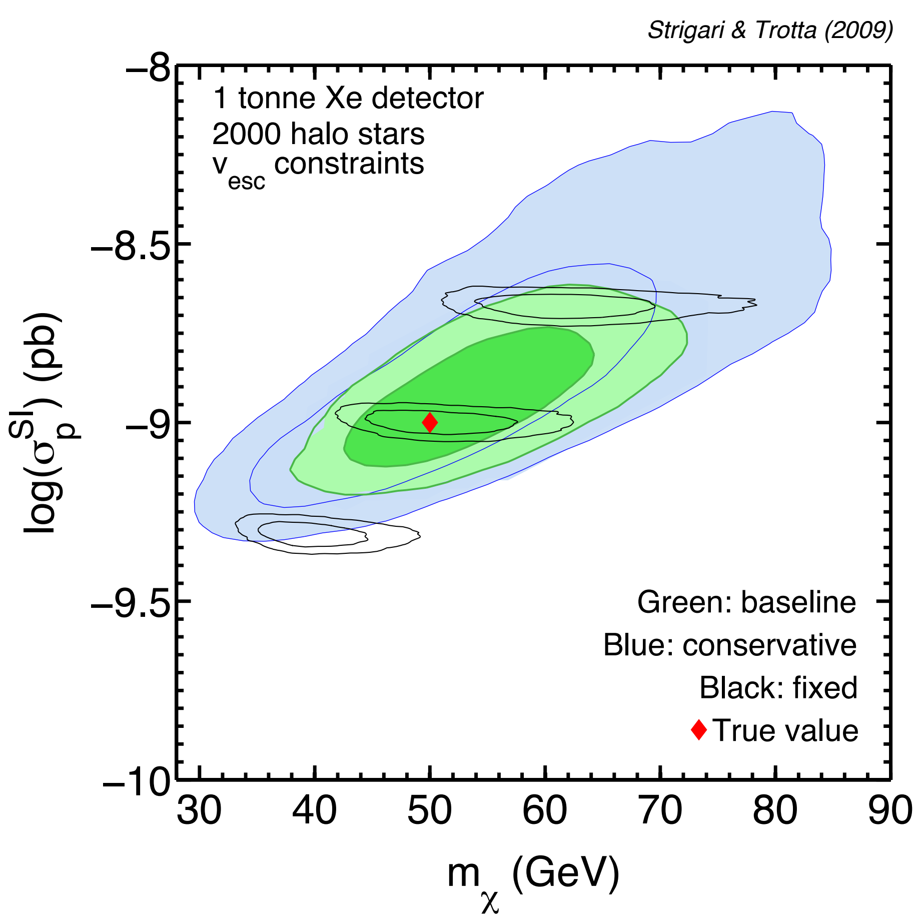
Consequences

Realisation that uncertainties in f(v) will affect signals goes right the way back to the early direct detection papers in the 1980s (e.g. Drukier, Freese & Spergel).

Density:

Event rate proportional to product of σ and ρ, therefore uncertainties in ρ translate directly into uncertainties in σ, same for all DD experiments (but affects comparisons with e.g. collider constraints on σ).

Strigari & Trotta uncertainty leads to bias in determination of WIMP mass:



<u>Circular speed (standard halo):</u>

Shifts exclusion limits, similar, but not identical, effect for all experiments.

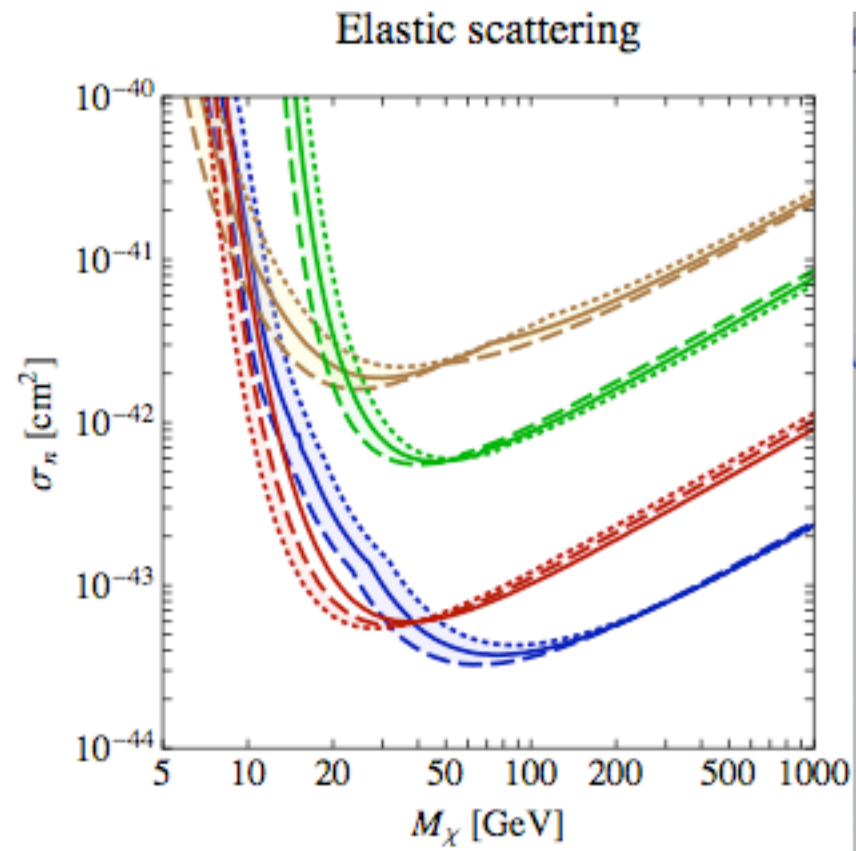
McCabe

.... v_c=195 km/s

____ v_c=220 km/s

- - - v_c=255 km/s

(old)CDMSII Si, CDMSII Ge
CRESST, ZENON 10



Bias in future WIMP mass determination:

$$E_{\rm R} = \frac{2\mu_{A\chi}^2 v_{\rm c}^2}{m_A}$$

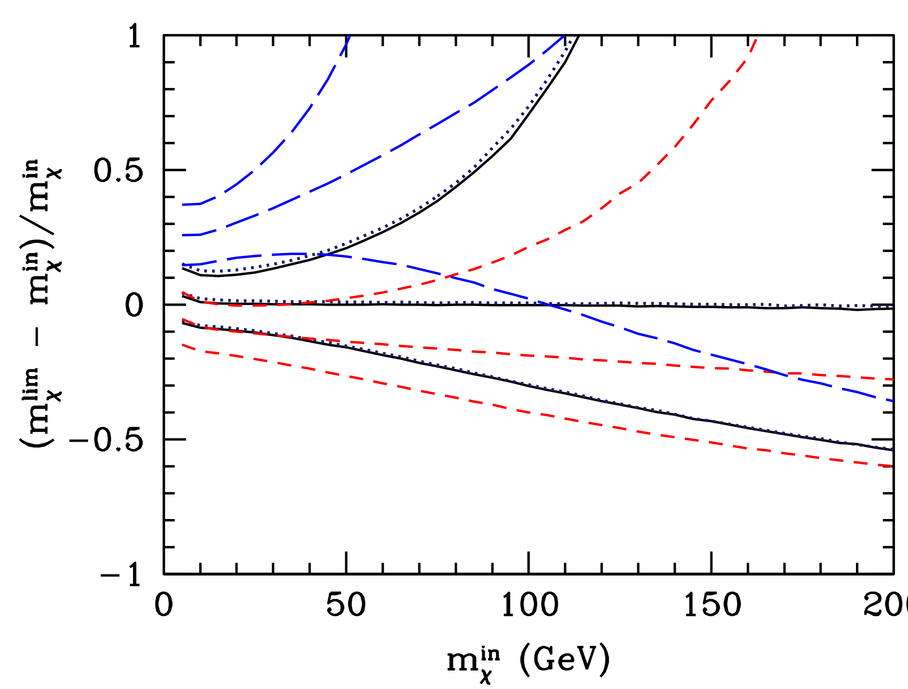
$$\frac{\Delta m_\chi}{m_\chi} = [1 + (m_\chi / m_{\rm A})] \frac{\Delta v_{\rm c}}{v_{\rm c}}$$

________ v_c = 220 km/s

--------- 200 km/s

_ _ _ _ _ 280 km/s

fractional mass limits from a simulated ideal Ge experiment, $\sigma = 10^{-8}$ pb



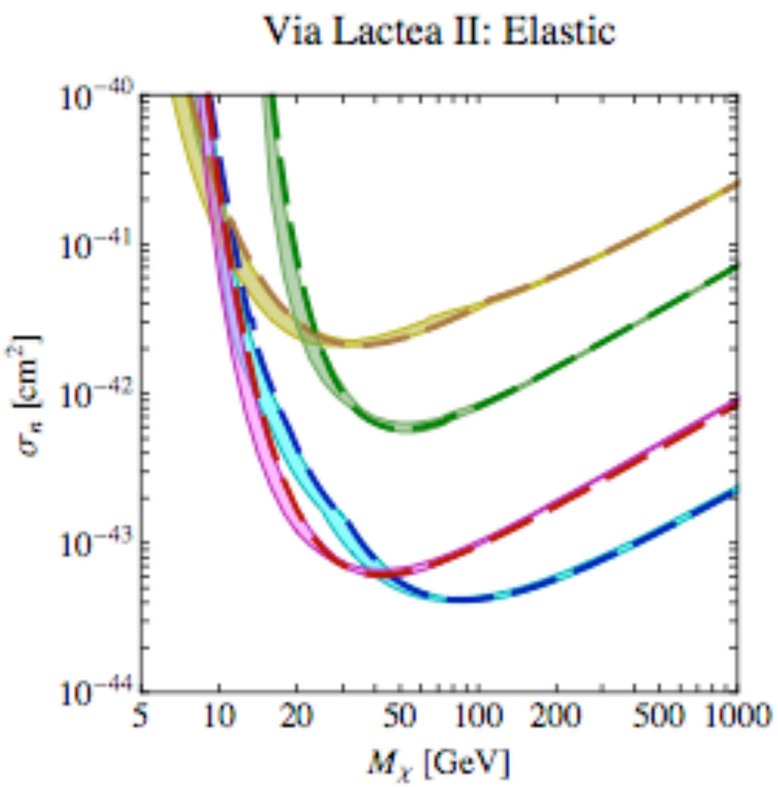
Shape of velocity distribution

Differential event rate is proportional to integral over speed distribution so exclusion limits are relatively insensitive to exact shape of velocity distribution:

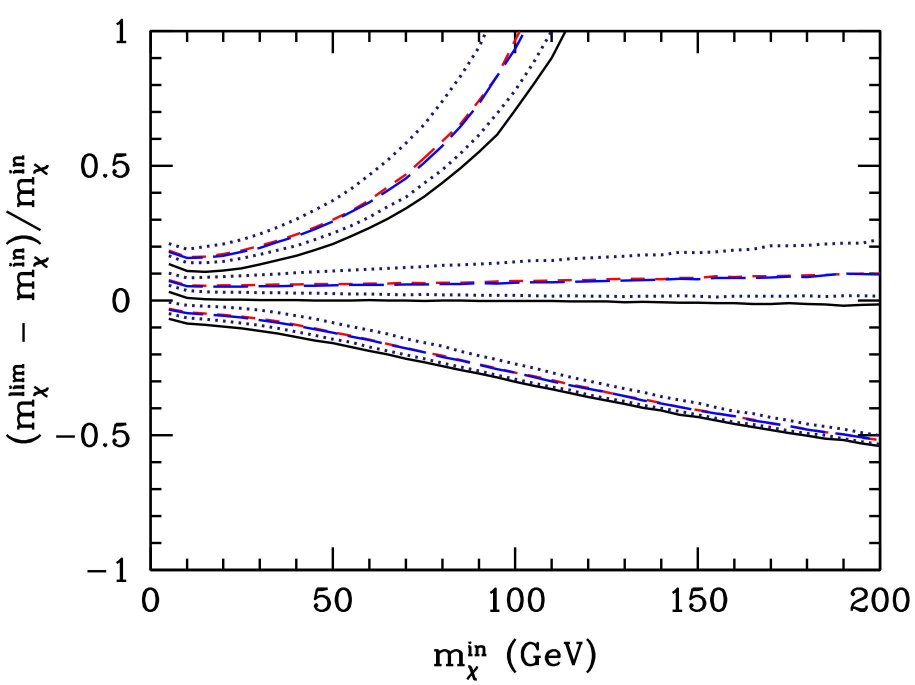
(smallish) change in shape/stochastic uncertainty in exclusion limits.

McCabe

(old)CDMSII Si, CDMSII Ge
CRESST, XENON 10

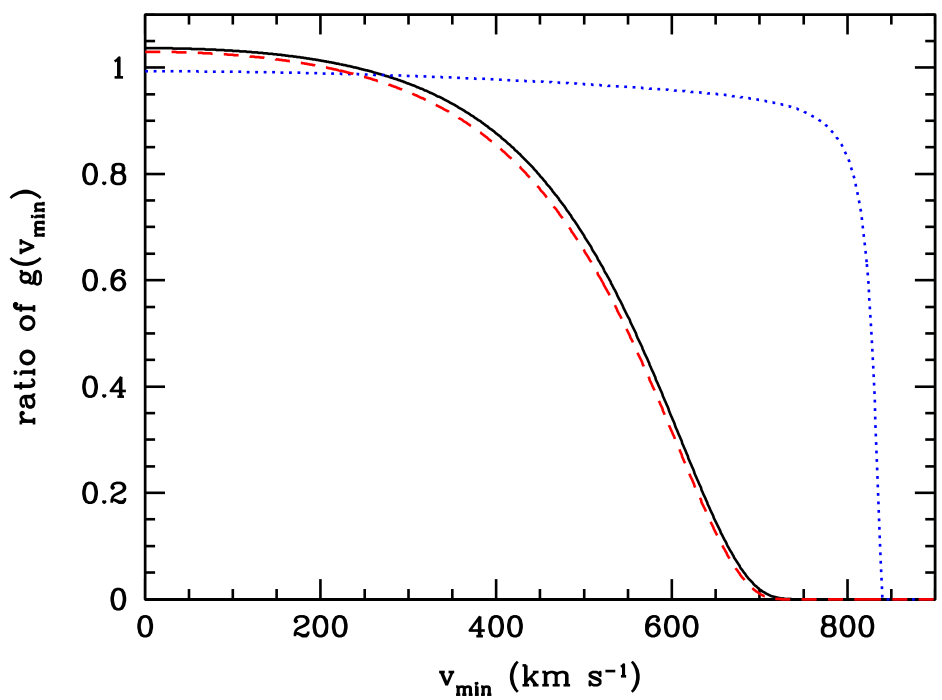


2-5% bias in future WIMP mass determination.



Escape speed & shape of high v tail

Can have significant effect on event rates/exclusion limits for light WIMPs:

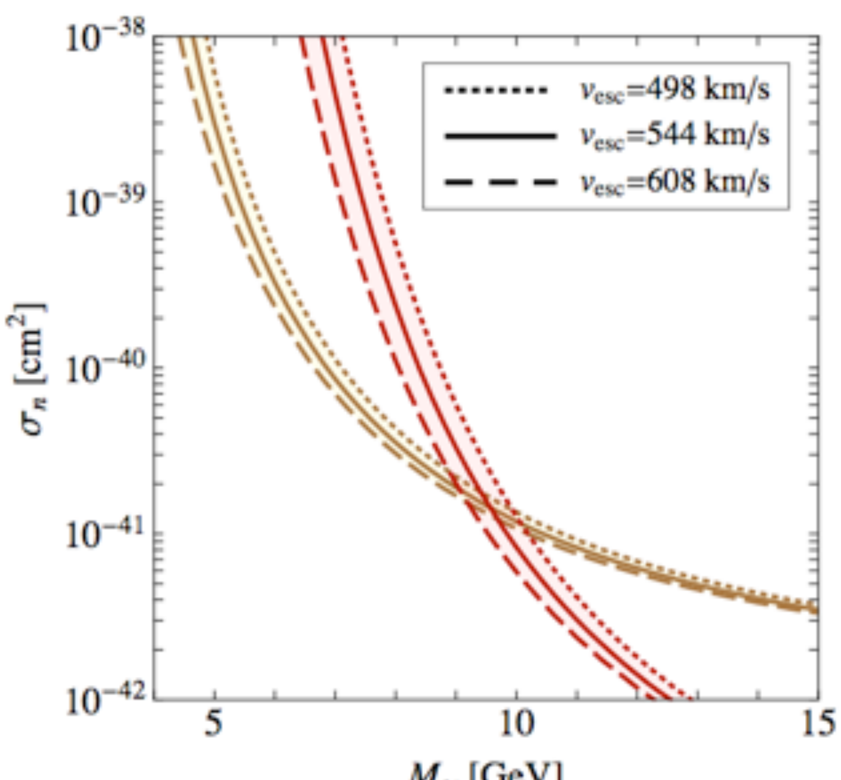


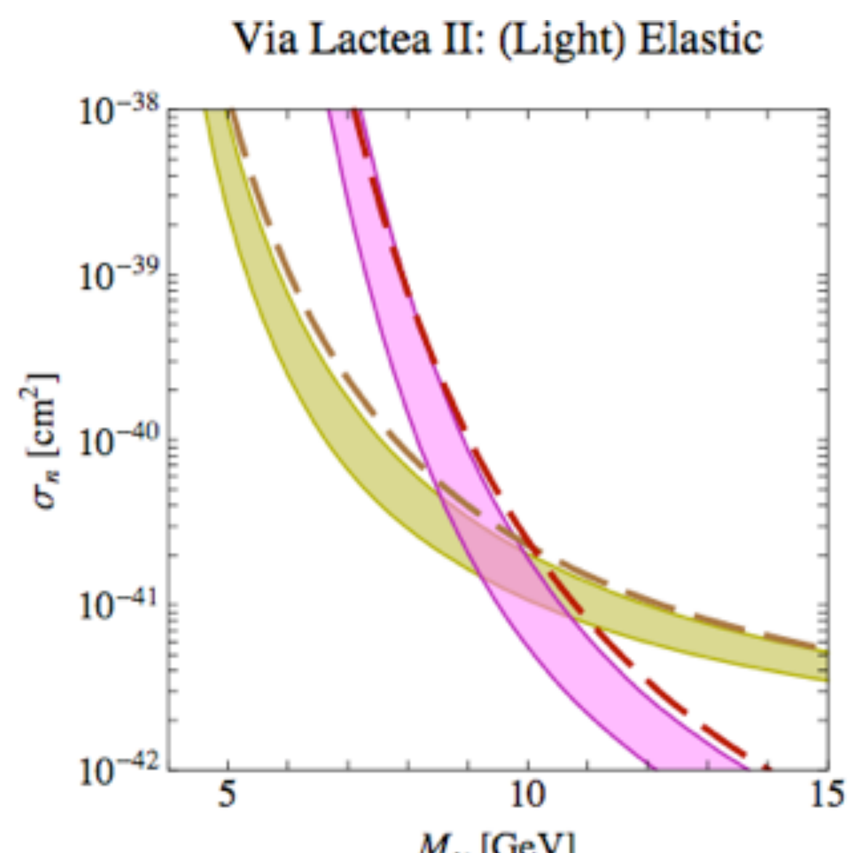
Ratio of speed integral to that of Maxwellian with sharp cut-off at $v_{\rm esc} = 608 \, {\rm km \, s^{-1}}$:

same f(v) neglecting Earth's orbit

Lisanti et al. k=1.5 $v_{\rm esc} = 498 \, {\rm km \, s^{-1}}$

Lisanti et al. neglecting Earth's orbit



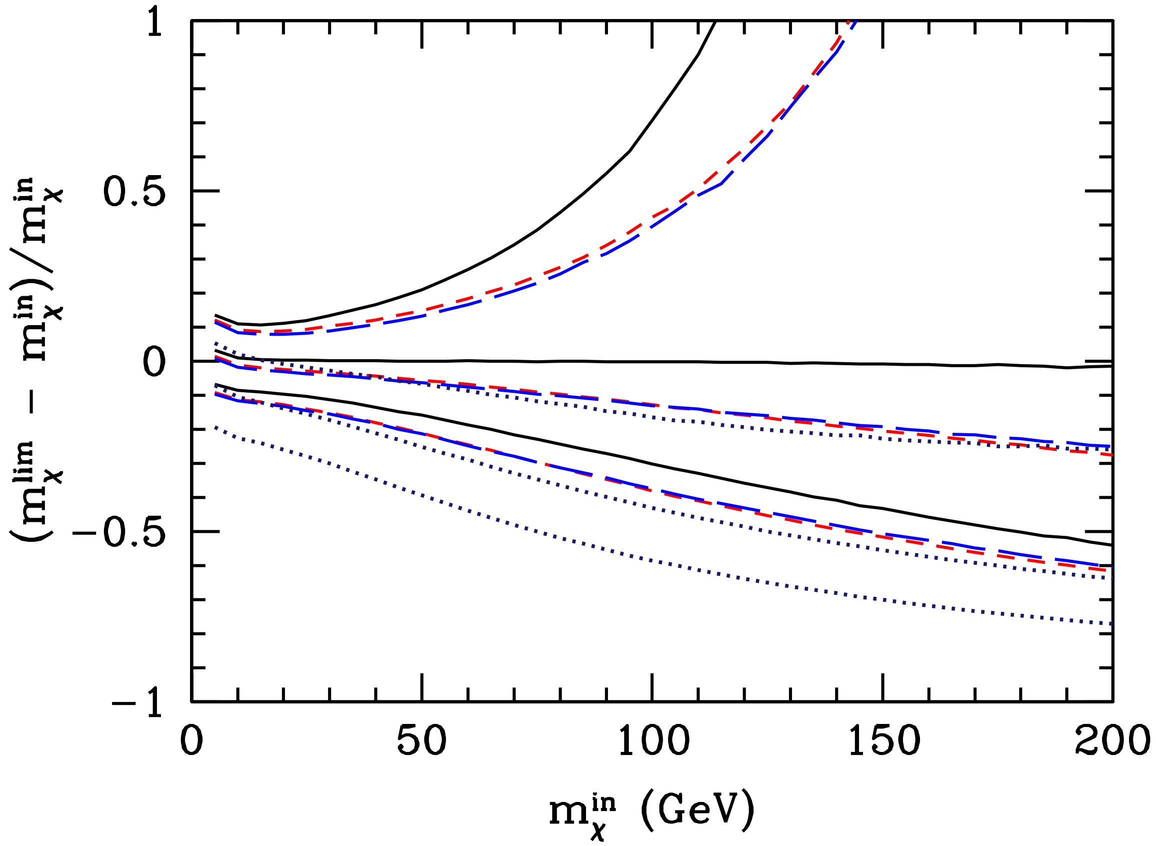


McCabe

(old)CDMSII Si,
XENON 10

Dark disc

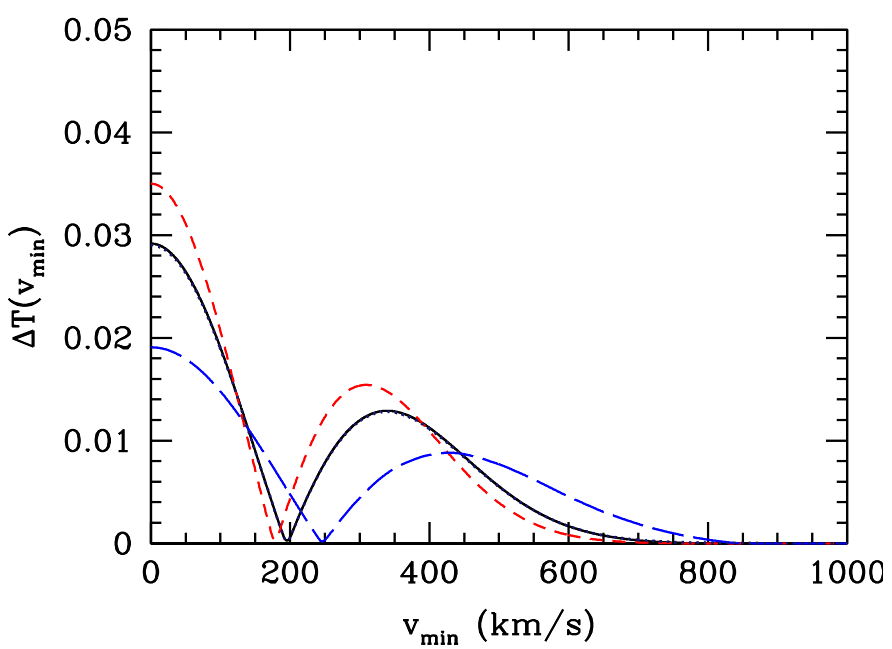
Could significantly bias mass determination, if density sufficiently high and/or velocity dispersion low.



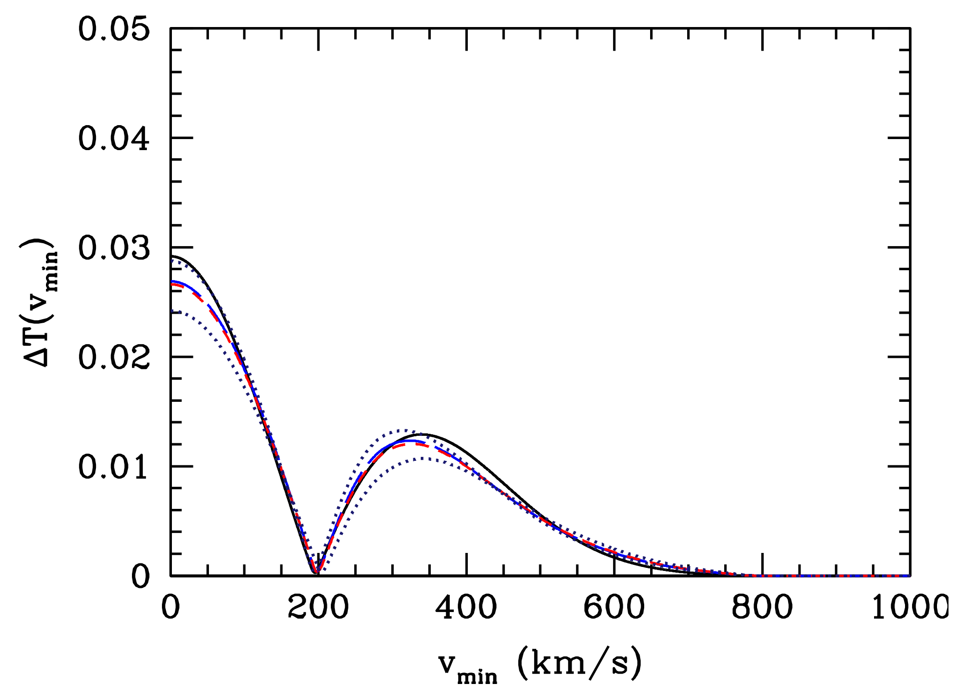
Annual modulation

Arises from small shift in speed distribution due to Earth's orbit.

Amplitude (and phase) sensitive to detailed shape of speed distribution.



SHM varying v_c



varying shape of f(v)

Direction dependence

Rear-front directional asymmetry is robust, but peak direction of high energy recoils can change. Kuhlen et al.

Strategies i) integrate out

Fox, Liu & Weiner

Compare experiments in $g(v_{min})$ space:

$$g(v_{\rm min}) = \int_{v_{\rm min}}^{\infty} \frac{f(v)}{v}\,{\rm d}v \qquad\qquad v_{\rm min} = \left(\frac{E(m_A + m_\chi)^2}{2 m_A m_\chi^2}\right)^{1/2}$$

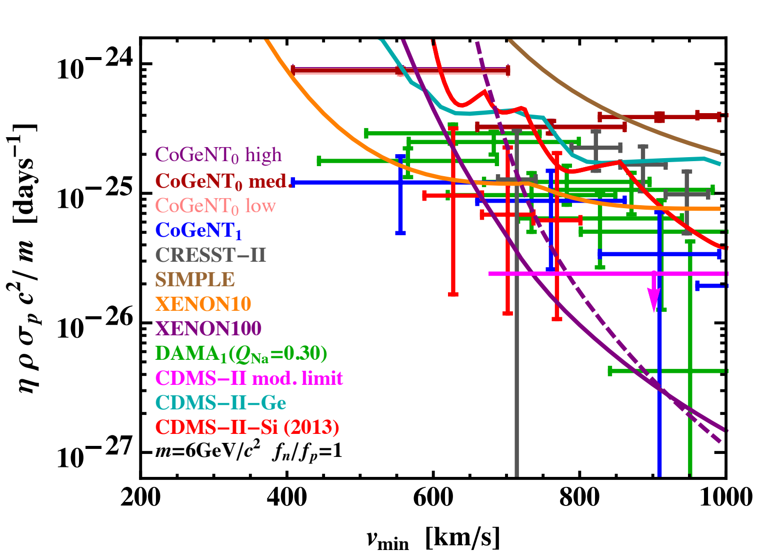
v_{min} values probed by each experiment depend on, unknown, WIMP mass, therefore need to do comparison for each mass of interest.

Can incorporate experimental energy resolution and efficiency Gondolo & Gelmini, and also annual modulation signals. Frandsen et al.; Herrero-Garcia, Schwetz & Zupan.

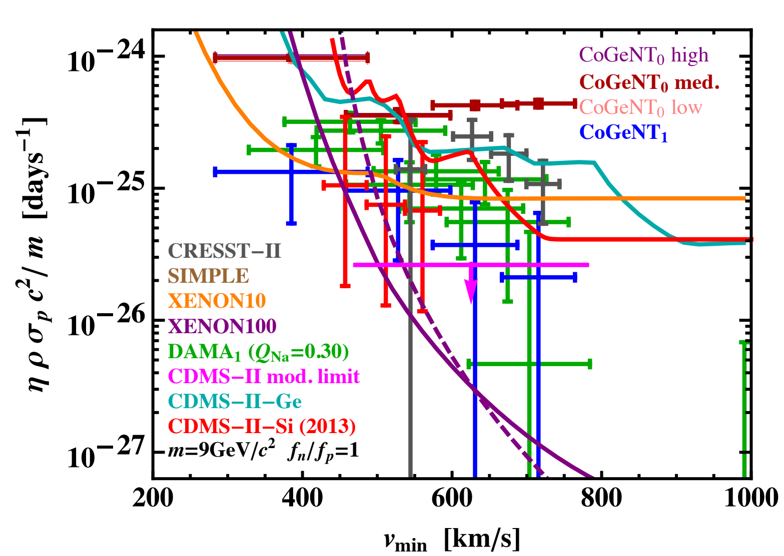
Extremely powerful for checking consistency of signals and exclusion limits. Frandsen et al.; Del Nobile, Gelmini, Gondolo & Huh.

Normalised $g(v_{min})$ versus v_{min} Del Nobile, Gelmini, Gondolo & Huh

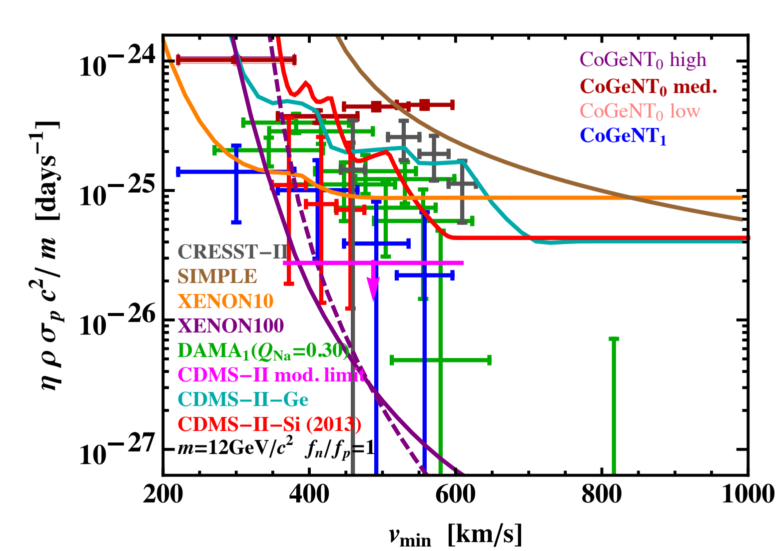
$m_\chi = 6\,{\rm GeV}$



$m_\chi = 9\,{\rm GeV}$



$m_\chi = 12\,{\rm GeV}$



Strategies ii) marginalise over

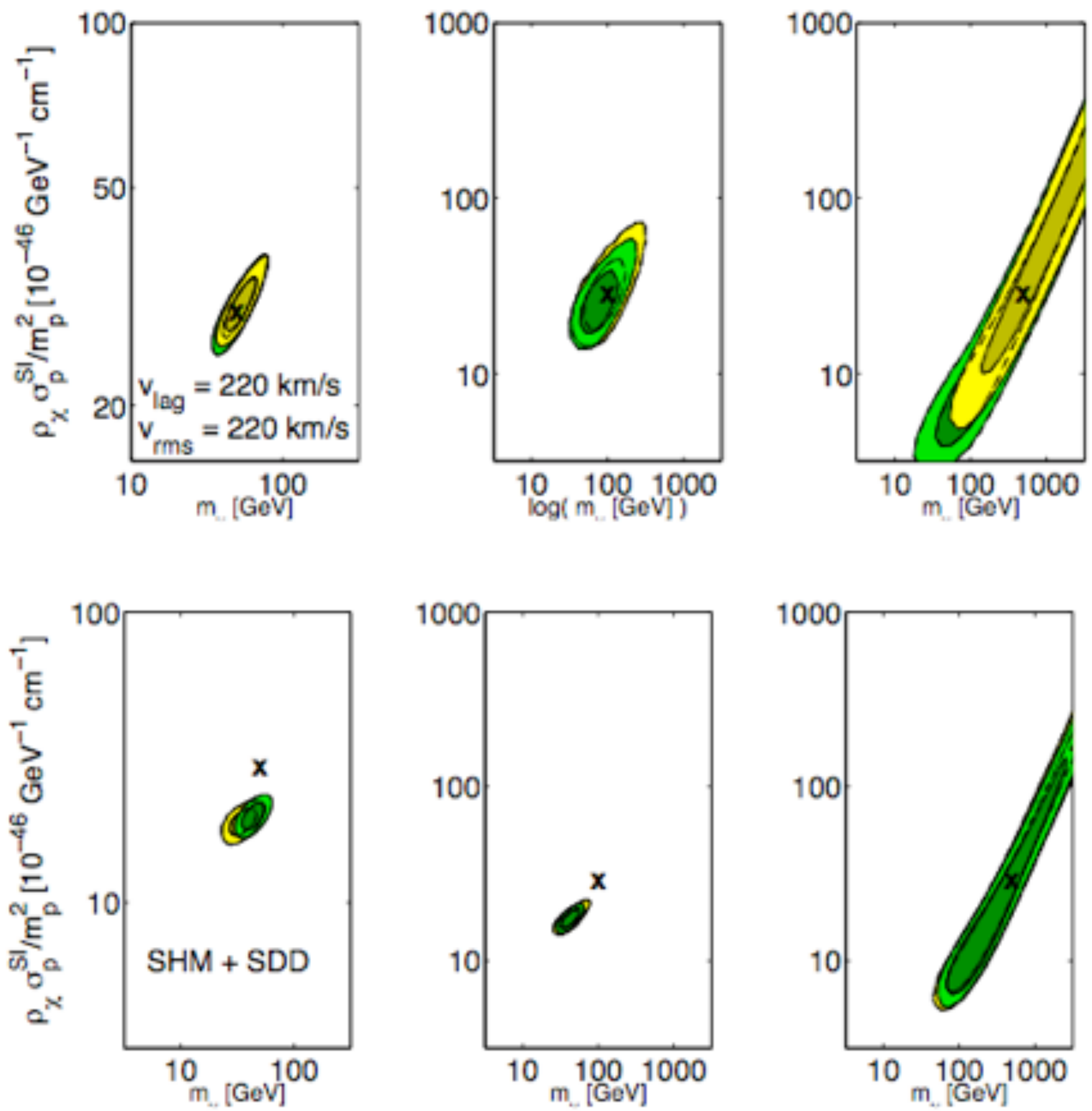
Parameterize f(v) and/or Milky Way model and marginalise over these parameters, possibly including astrophysical data too e.g. stellar kinematics.

Strigari & Trotta; Peter x2; Pato et al. x2; Lee & Peter; Billard, Meyet & Santos; Alves, Hedri & Wacker; Kavanagh & Green x2; Friedland & Shoemaker

If actual shape of f(v) is similar to assumed shape this works well, but if not can get significant biases:

$m_\chi = 50\,\mathrm{GeV}$ $\quad 100\,\mathrm{GeV}$ $\quad 500\,\mathrm{GeV}$

$$D = \frac{\rho_0 \sigma_\mathrm{p}}{m_\chi^2}$$



m_χ

Peter simulated data from future tonne scale Xe, Ar & Ge expts, analysed assuming standard halo model (allowing v_{lag} & v_{rms} to vary).

standard halo model in

standard halo model + dark disc in

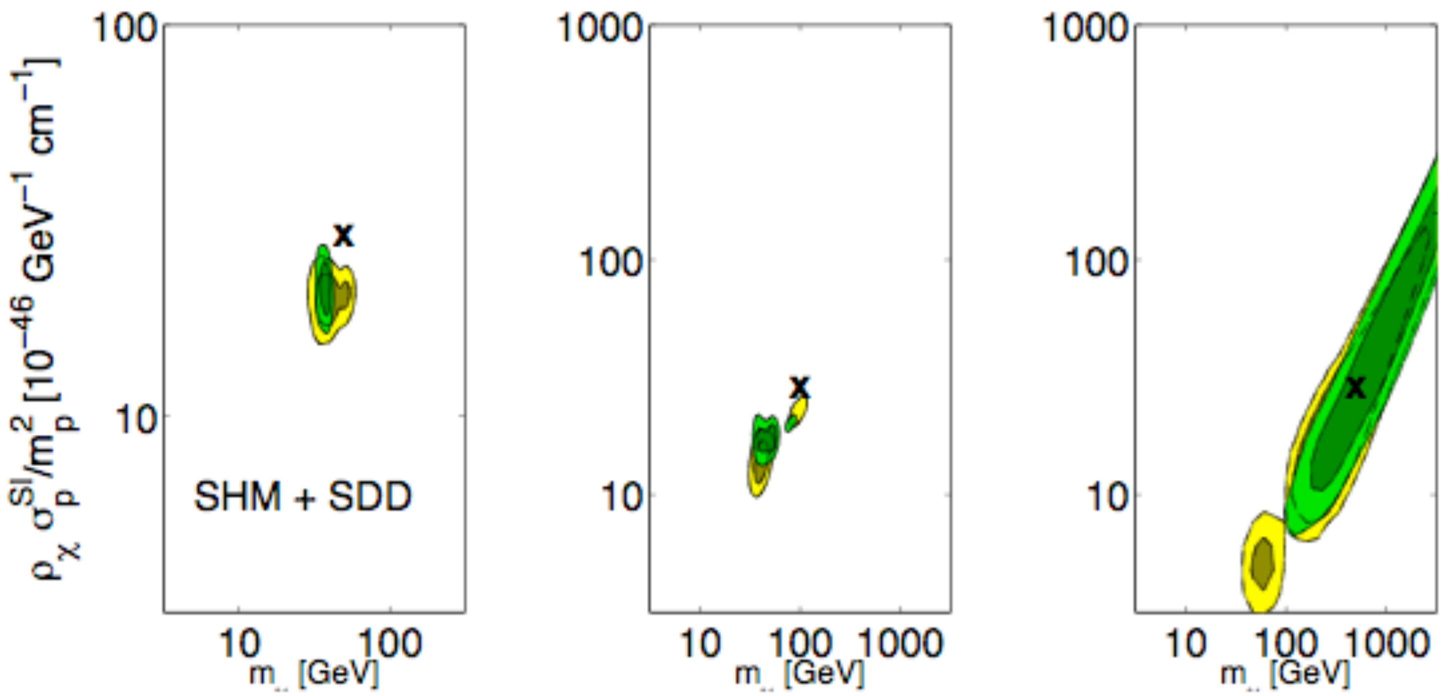
Parameterizing speed distribution

With a single experiment can't say anything about the WIMP mass without making assumptions about f(v) (recoil energies depend on speeds and mass).

But with multiple experiments can break this degeneracy. Drees & Shan; Peter

Peter Use empirical parameterization of f(v), and constrain its parameters along with mass & cross-section.

First approach: piece-wise constant in bins



standard halo model + dark disc in

Better than assuming wrong f(v), but m_χ & σ both biased.

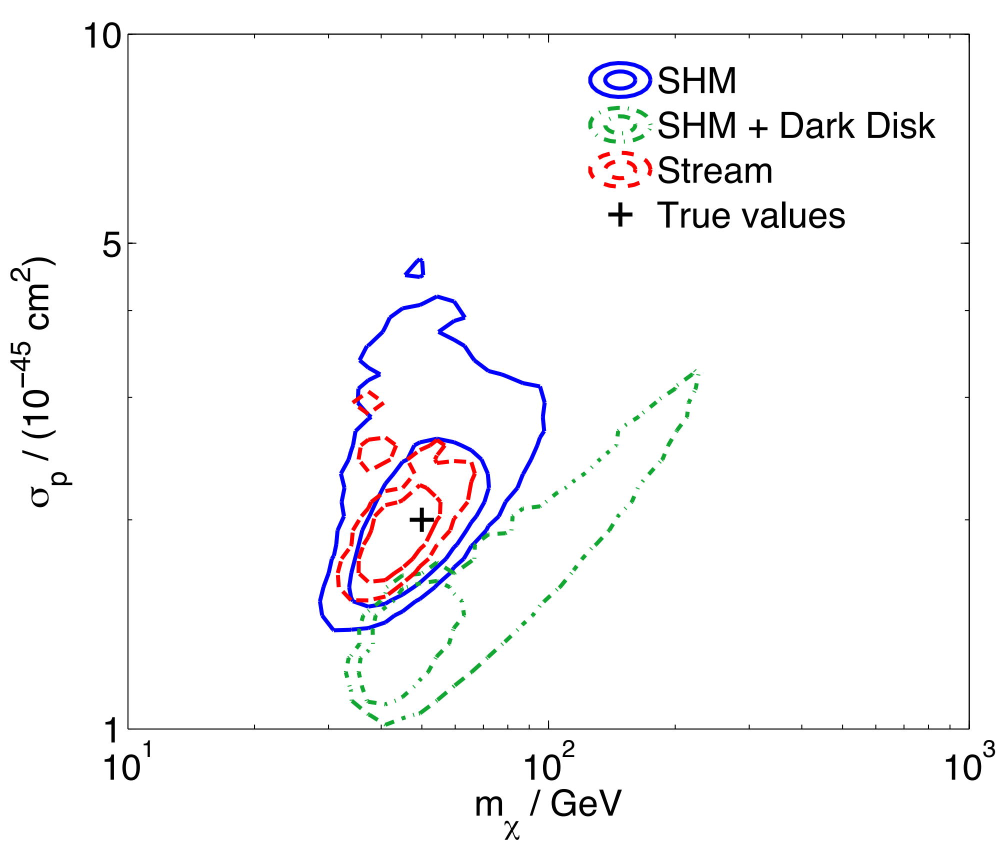
Kavanagh & Green

Want parameterisation without fixed scales, and with ability to accommodate features in speed distribution.

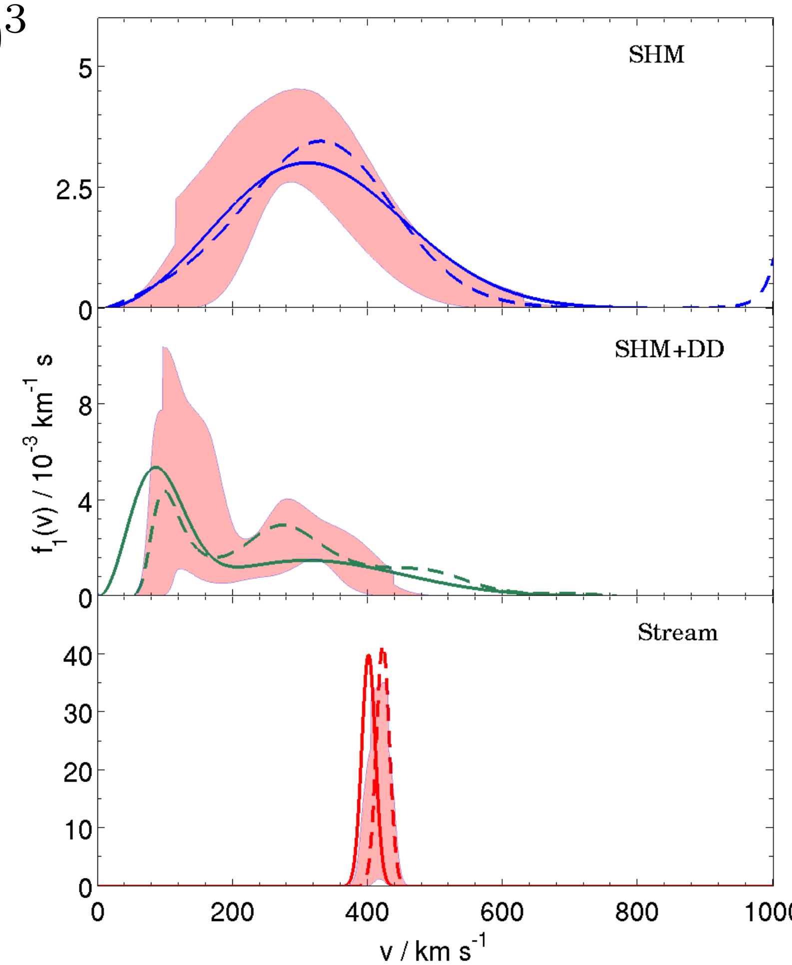
Since f(v) ≥ 0, parameterise log of f(v) in shifted Legendre polynomials:

$$f(v) \propto \exp\left\{ -\sum_{k=0}^{N} a_k \bar{P}_k(v/v_{\rm max}) \right\}$$

Gives good reconstruction of WIMP mass even for extreme input f(v) (stream or dark disc), and allows f(v) to be reconstructed:



$f(v) \times 10^3$



SHM

SHM+DD

Stream

Summary

- Direct detection energy spectrum depends on the local dark matter density, ρ_0, and velocity distribution, f(v):

    local DM density → normalisation of event rate, and hence σ
    velocity dispersion → characteristic scale of energy spectrum and hence m_χ
    shape of WIMP velocity distribution → event rate for light WIMPs and amplitude and phase of annual modulation signal

- Determinations of ρ_0 and v_c have ~10% statistical errors, but systematic errors are larger.

- Can assess compatibility of signals/exclusion limits in speed integral, $g(v_{min})$, space ('integrating out the astrophysics').

- Parameterising f(v)/Milky Way model and marginalising works well **if** actual shape of f(v) is close to assumed shape.

- For unbiased mass measurement use a suitable empirical parameterisation (e.g. shifted Legendre polynomials), and probe f(v) too.