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A Review of Lepton Flavor Violating Processes

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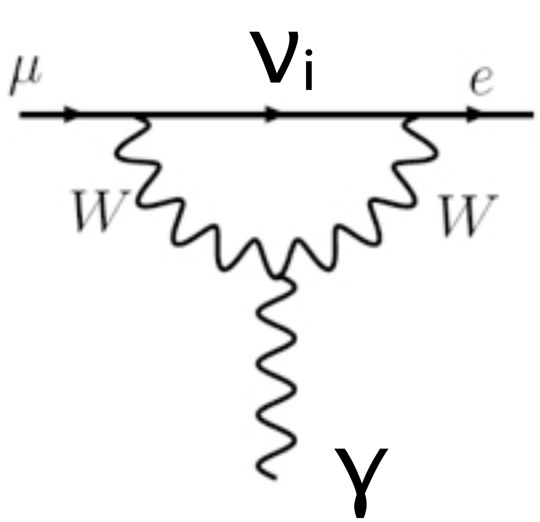
LFV: general considerations

- ν oscillations imply that individual lepton family numbers are not conserved (after all $L_{e,\mu,\tau}$ are "accidental" symmetries of SM)

- In SM + massive "active" ν, effective CLFV vertices are tiny (GIM-suppression), resulting in un-observably small rates, e.g.



$$Br(\mu \to e\gamma) = \frac{3\alpha}{32\pi} \left| \sum_{i=2,3} U^*_{\mu i} U_{ei} \frac{\Delta m^2_{1i}}{M^2_W} \right|^2 < 10^{-54}$$

Petcov '77, Marciano-Sanda '77 ....

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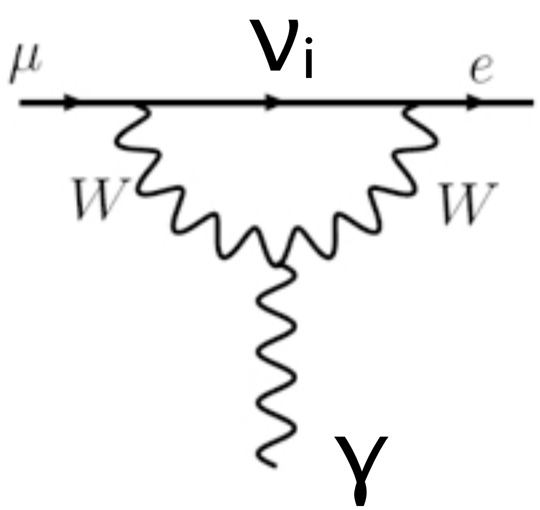

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Petcov '77, Marciano-Sanda '77 ....

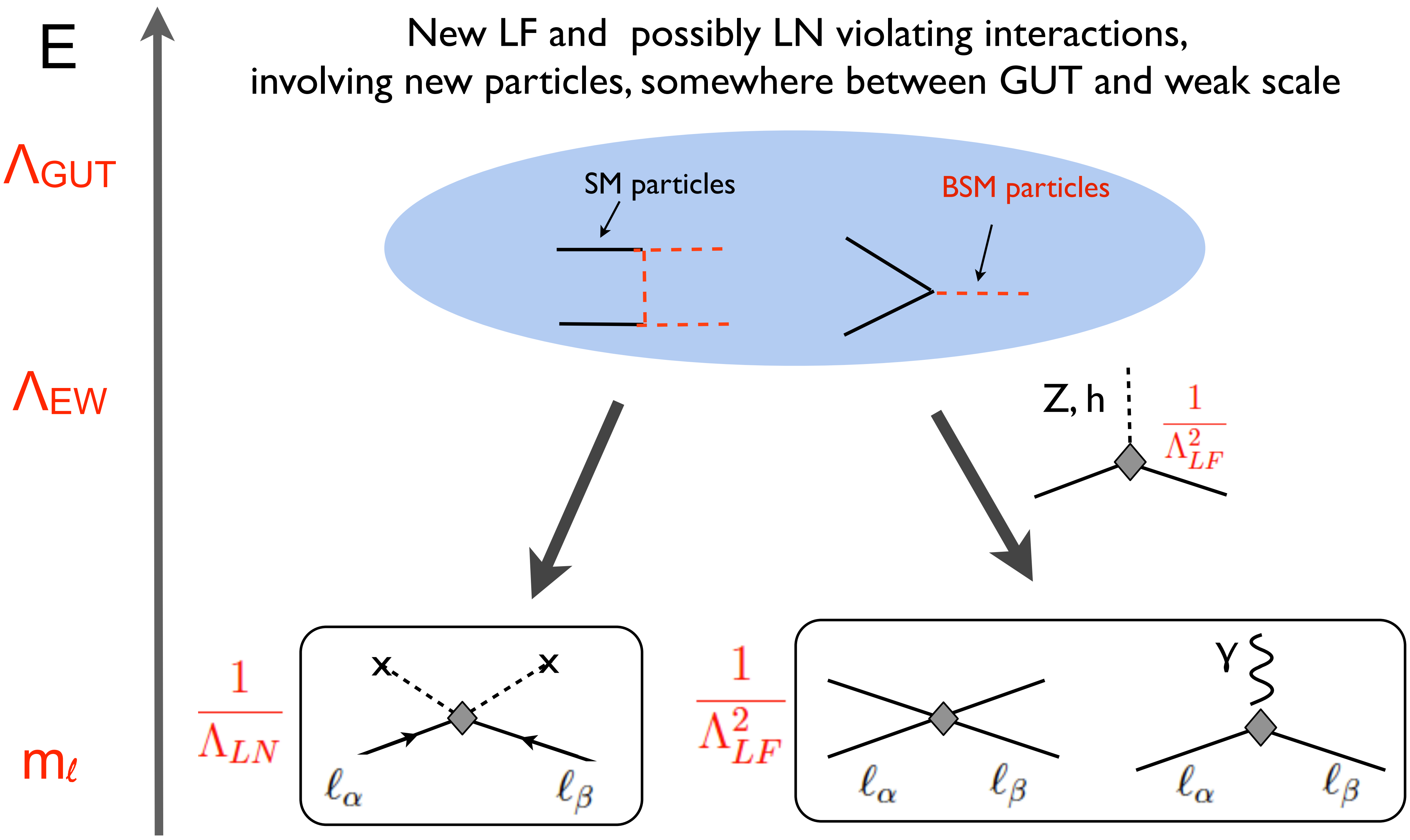
- Extremely clean probe of "BνSM" physics

dim-4 Dirac or dim5 Majorana

$$\mathcal{L}_{\nu \mathrm{SM}} = \mathcal{L}_{\mathrm{SM}} + \mathcal{L}_{\nu-\mathrm{mass}}$$

LFV: big picture

New LF and possibly LN violating interactions,
involving new particles, somewhere between GUT and weak scale



Each scenario generates specific pattern of weak-scale and low-energy operators,
controlling ν mass (dim5) and LFV processes (dim6).
We can probe the underlying physics up to very high scales by a combination of
low-energy and collider searches

LFV: probes

- Low energy: rare decays of μ and τ , strongest probes (sensitive to scales beyond LHC reach)

$$\mu \to e\gamma, \quad \mu \to e\bar{e}e, \quad \mu\,(A,Z) \to e\,(A,Z)$$

$$\tau \to \ell\gamma, \quad \tau \to \ell_\alpha \bar{\ell}_\beta \ell_\beta, \; \tau \to \ell\, Y \qquad Y = P, S, V, P\bar{P}, \ldots$$

- High Energy: can compete in τ ↔ μ and τ ↔ e sector

LHC

$$pp \;\to\; R \to \; \ell_\alpha \bar{\ell}_\beta + X \qquad\qquad R = Z', h, \tilde{\nu}, \ldots.$$

$$pp \;\to\; \ell_\alpha \bar{\ell}_\beta + X$$

EIC (?)

$$e\,p \;\to\; \ell + X$$

Discovering and Diagnosing

- Redundancy of searches is very important at this stage, as various probes serve as:
- Discovery tools (observation $\Rightarrow$ BSM physics)
- Diagnosing tools: reconstruct the underlying dynamics
  - What type of mediator? (operator structure)
    LHC vs $\mu \to 3e$ vs $\mu \to e\gamma$ vs $\mu \to e$ conversion
    (and similarly for tau decays)
  - What sources of flavor breaking? (pattern of LFV rates)
    $\mu \to e$ vs $\tau \to \mu$ vs $\tau \to e$

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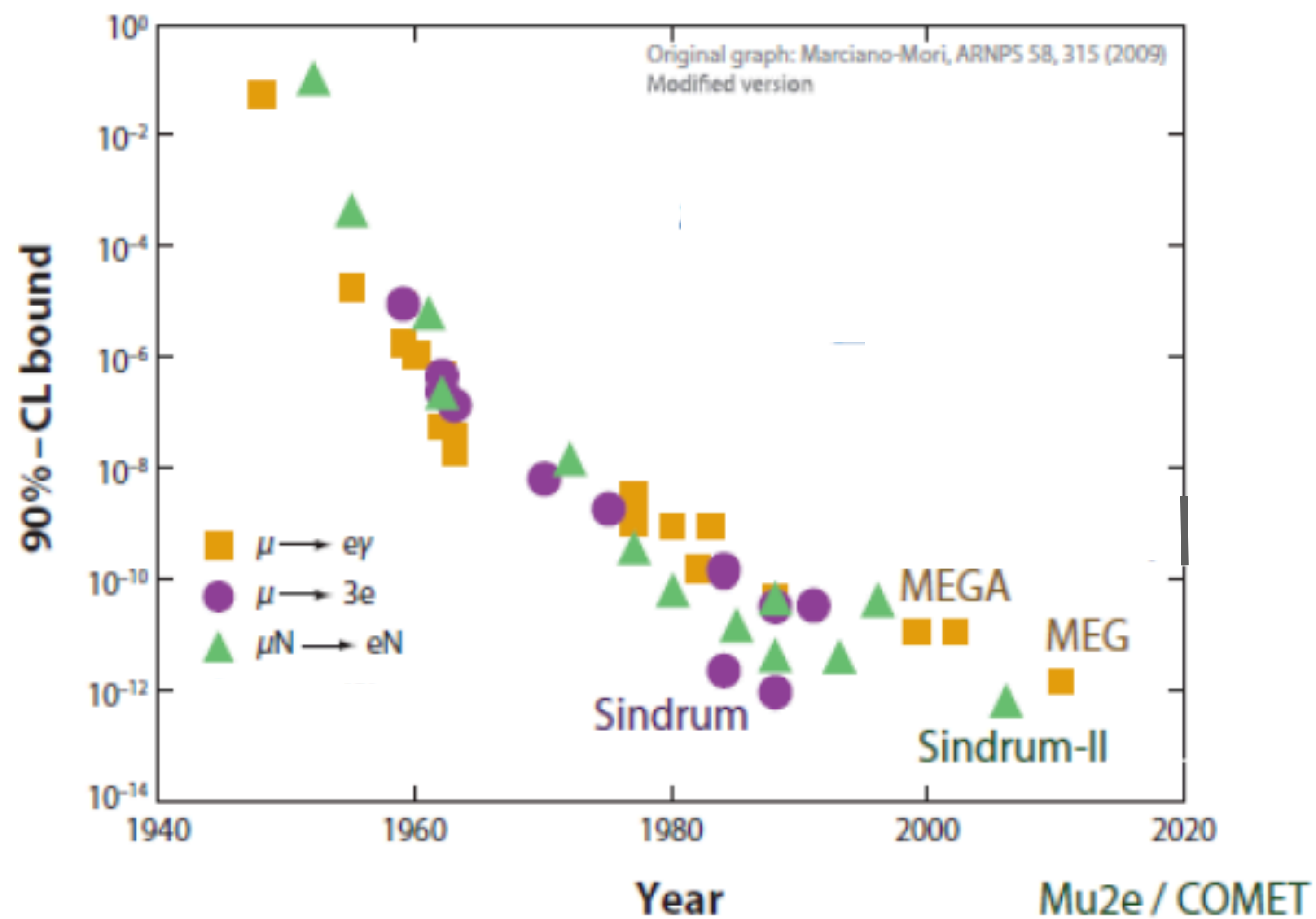
Outline

- Low Energy probes $\nearrow$ Discovery potential
- High Energy probes $\searrow$ Diagnosing power

Low energy probes

Experiment: status and prospects

- Muon processes :

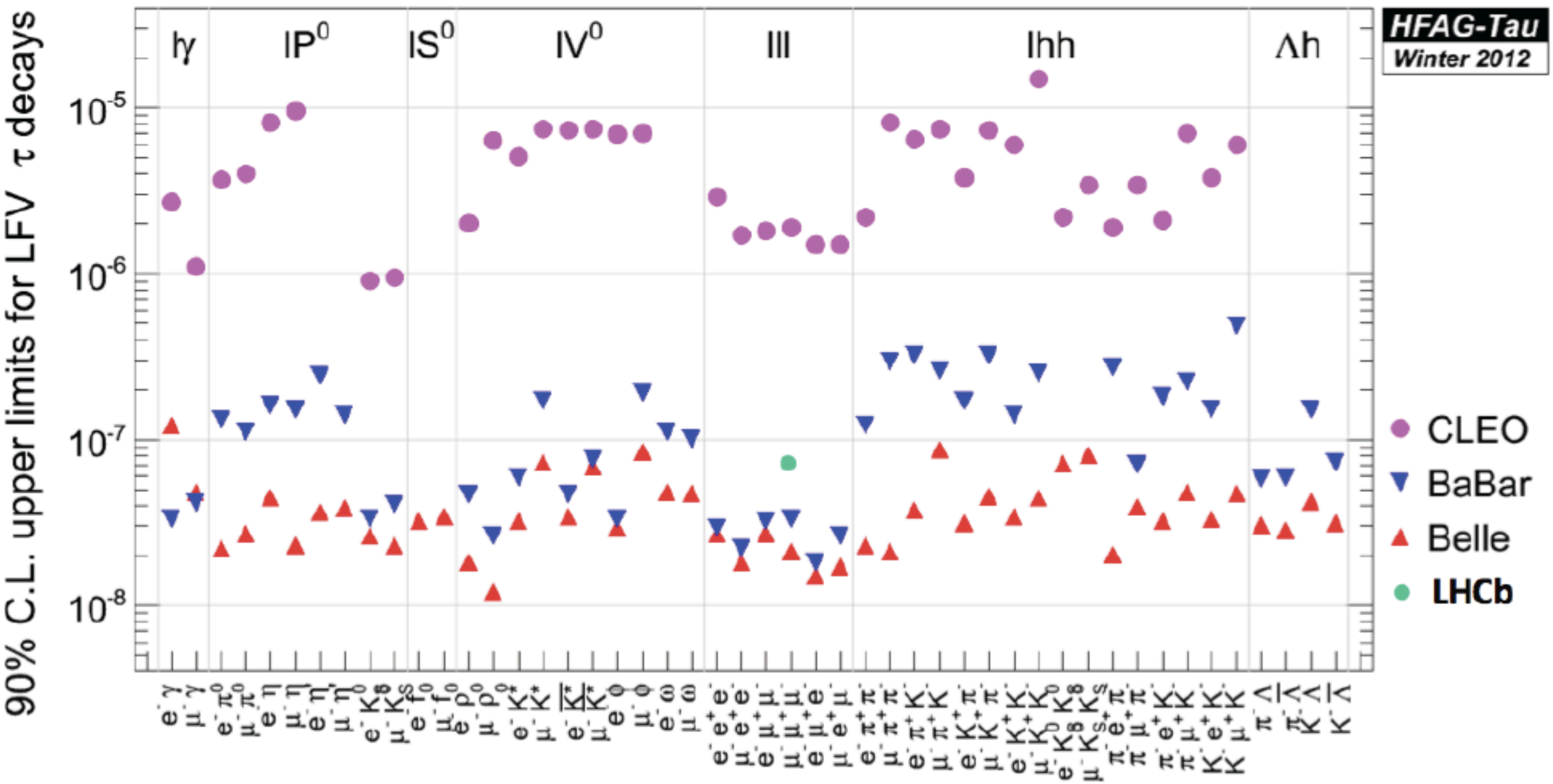


$B_{\mu\to e\gamma} < 5.7\times 10^{-13}$ ⟶ $10^{-/14}$ (MEG at PSI)

$B_{\mu\to 3e} < 1.0\times 10^{-12}$ ⟶ $10^{-15/16}$ (PSI)

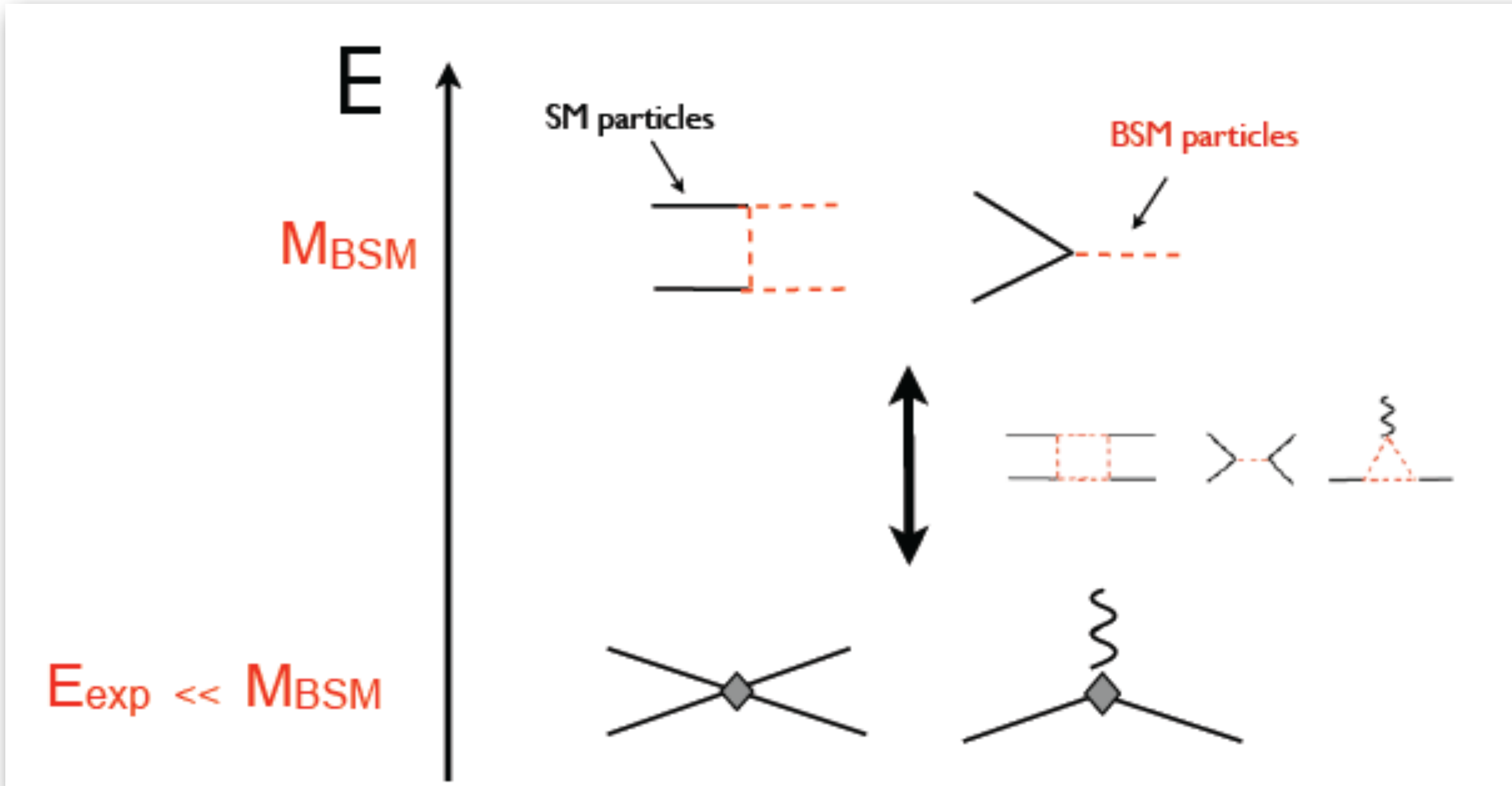
$B^{Ti}_{\mu-e} < 4.3\times 10^{-12}$ ⟶ $10^{-16/17\to -18}$ (Mu2e, COMET)

- Tau decays:



10^{-9} sensitivities at Belle-II (KEK), LHCb

Low energy phenomenology: EFT



- At low energy, BSM dynamics described by local operators

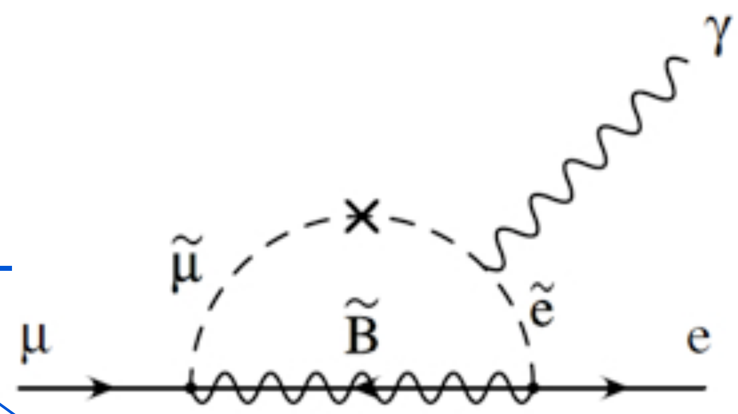
$$\boxed{\mathcal{L} = \mathcal{L}_{SM} + \frac{C^{(5)}}{\Lambda} O^{(5)} + \sum_i \frac{C_i^{(6)}}{\Lambda^2} O_i^{(6)} + \dots}$$

$$\Lambda \leftrightarrow M_{\rm BSM} \qquad\qquad C_i \; [g_{\rm BSM}, \; M_a/M_b]$$

- LFV processes sensitive to scale and flavor structure of couplings

- Several operators generated at dim6: rich phenomenology

Dominant in SUSY-GUT and SUSY see-saw scenarios



$$\mathcal{L}_{eff} \supset \frac{[\alpha_D]^{ij}}{\Lambda^2} \varphi^\dagger \, \bar{e}^i_R \sigma_{\mu\nu} \ell^j_L \, F^{\mu\nu}$$

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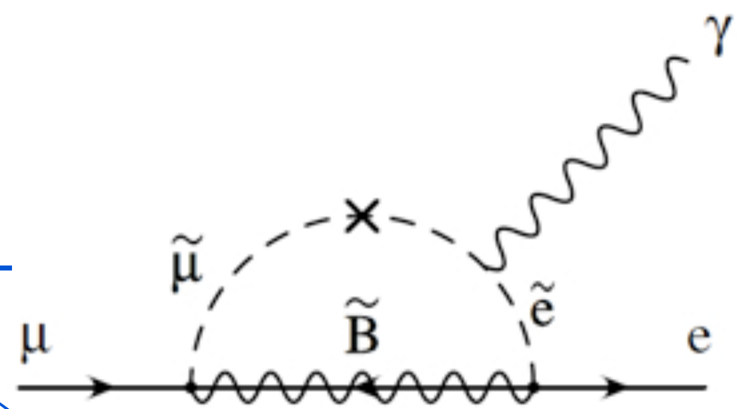
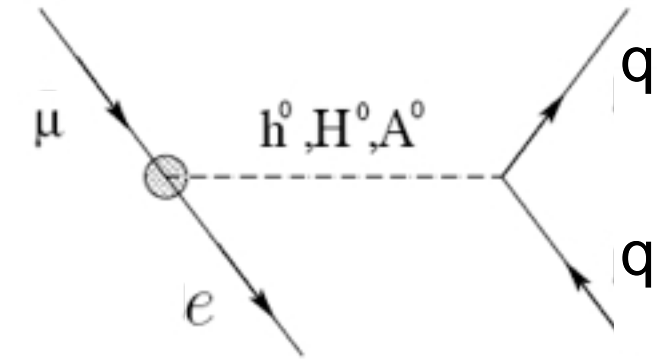


Dominant in RPV SUSY and RPC SUSY for large tan(β) and low m_A



$$\mathcal{L}_{eff} \supset \frac{[\alpha_D]^{ij}}{\Lambda^2}\,\varphi^\dagger\;\bar{e}^i_R\sigma_{\mu\nu}\ell^j_L\,F^{\mu\nu}\;+\;\frac{[\alpha_S]^{ij}}{\Lambda^2}\;\bar{e}^i_R\ell^j_L\;\bar{q}_L d_R$$

Dominant in RPV SUSY

- Several operators generated at dim6: rich phenomenology

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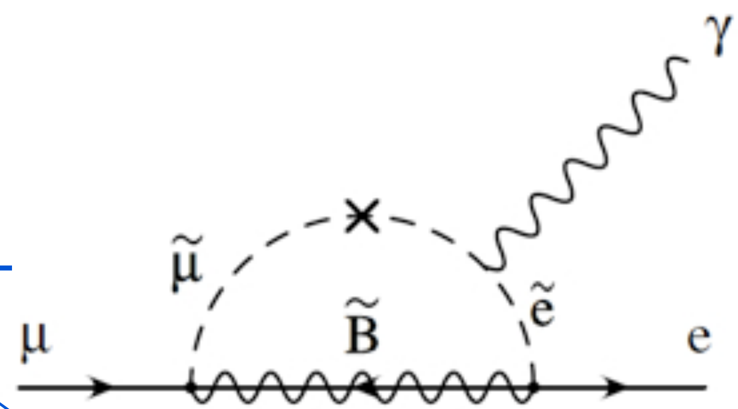

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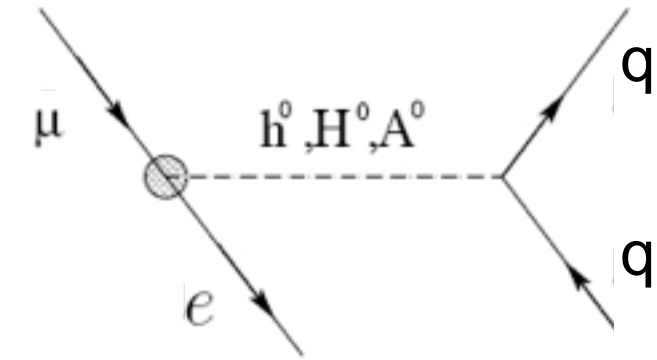

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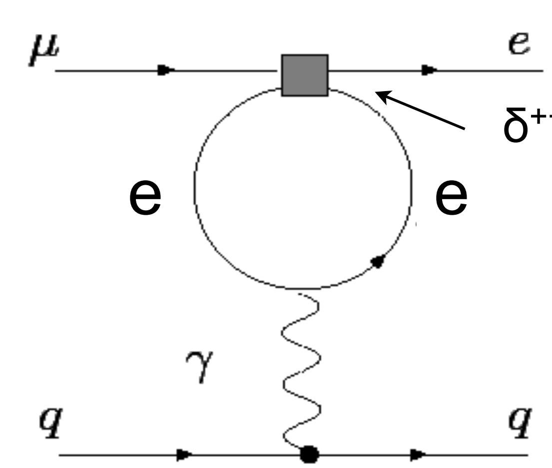
$$+ \frac{[\alpha_{V(z)}]^{ij}}{\Lambda^2}\, \bar{\ell}^i_L \gamma_\mu \ell^j_L\, \varphi^\dagger D^\mu \varphi + \frac{[\alpha_{V(\gamma)}]^{ij} e_q}{\Lambda^2}\, \bar{\ell}^i_L \gamma_\mu \ell^j_L\, \bar{q}_L \gamma^\mu q_L + \ldots$$

Z-penguin

(Type III seesaw, ..)

Enhanced in triplet models (Type II seesaw), Left-Right symmetric models



... + 4-lepton operators

What can we extract from data

- Ask questions on LFV dynamics without choosing a specific model (answers will help discriminating among models)

  - What is the sensitivity to the effective scale Λ?
    What is the relative sensitivity of various processes?

    Discovery potential

  - What is relative the strength of various operators (α_D vs α_S ...)? $\rightarrow$ Mediators

  - What is the flavor structure of the couplings ($[\alpha_D]^{e\mu}$ vs $[\alpha_D]^{\tau\mu}$...)? $\rightarrow$ Sources of flavor breaking

    Diagnosing power

Sensitivity to NP scale

- What combination of scale Λ + couplings produces observable rates?

$$\boxed{BR_{\alpha\to\beta} \sim (v_{EW}/\Lambda)^4 * (\alpha_n)_{\alpha\beta}{}^2}$$

Observable CLFV @ $10^{-1?}$ ⇔ new physics between weak and GUT scale

- Current limit from $\mu \to e\gamma$ implies

$$\boxed{\Lambda/\sqrt{[\alpha_D]^{e\mu}} > 2 \times 10^4\ \mathrm{TeV}}$$

$\downarrow$ even after taking into account loop factors

New physics at TeV scale already quite constrained

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- What about other processes? Relative sensitivity depends on the model: each process probes a different combination of operators

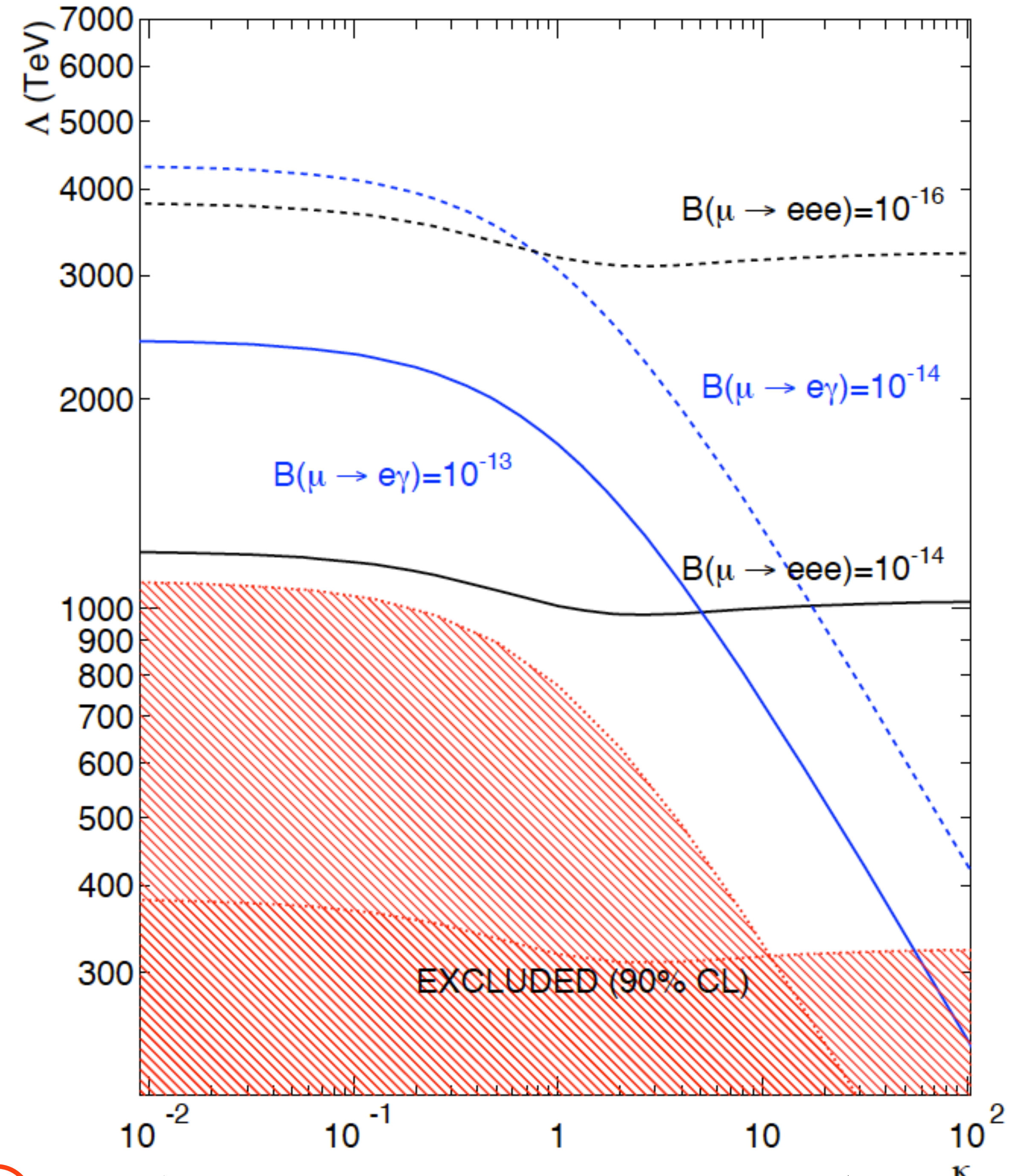
$\mu \to e\gamma$ vs $\mu \to 3e$

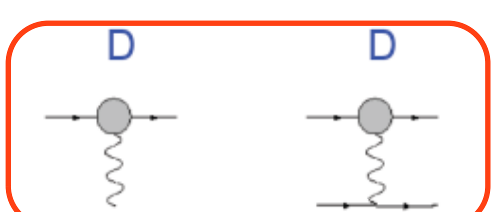
- A simple example with two operators

De Gouvea, Vogel 1303.4097

$$\mathcal{L}_{\rm CLFV} = \frac{m_\mu}{(\kappa+1)\Lambda^2}\bar{\mu}_R \sigma_{\mu\nu} e_L F^{\mu\nu} + h.c. \\ \frac{\kappa}{(1+\kappa)\Lambda^2}\bar{\mu}_L \gamma_\mu e_L \left(\bar{e}\gamma^\mu e\right) + h.c.\,.$$

- κ controls relative strength of dipole vs vector operator





dipole

vector

μ → eγ vs μ → e conversion

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$$\mathcal{L}_{\rm CLFV} = \frac{m_\mu}{(\kappa+1)\Lambda^2}\bar{\mu}_R\sigma_{\mu\nu}e_L F^{\mu\nu} + h.c.$$
$$\frac{\kappa}{(1+\kappa)\Lambda^2}\bar{\mu}_L\gamma_\mu e_L\left(\bar{u}_L\gamma^\mu u_L + \bar{d}_L\gamma^\mu d_L\right) + h.c.\,.$$

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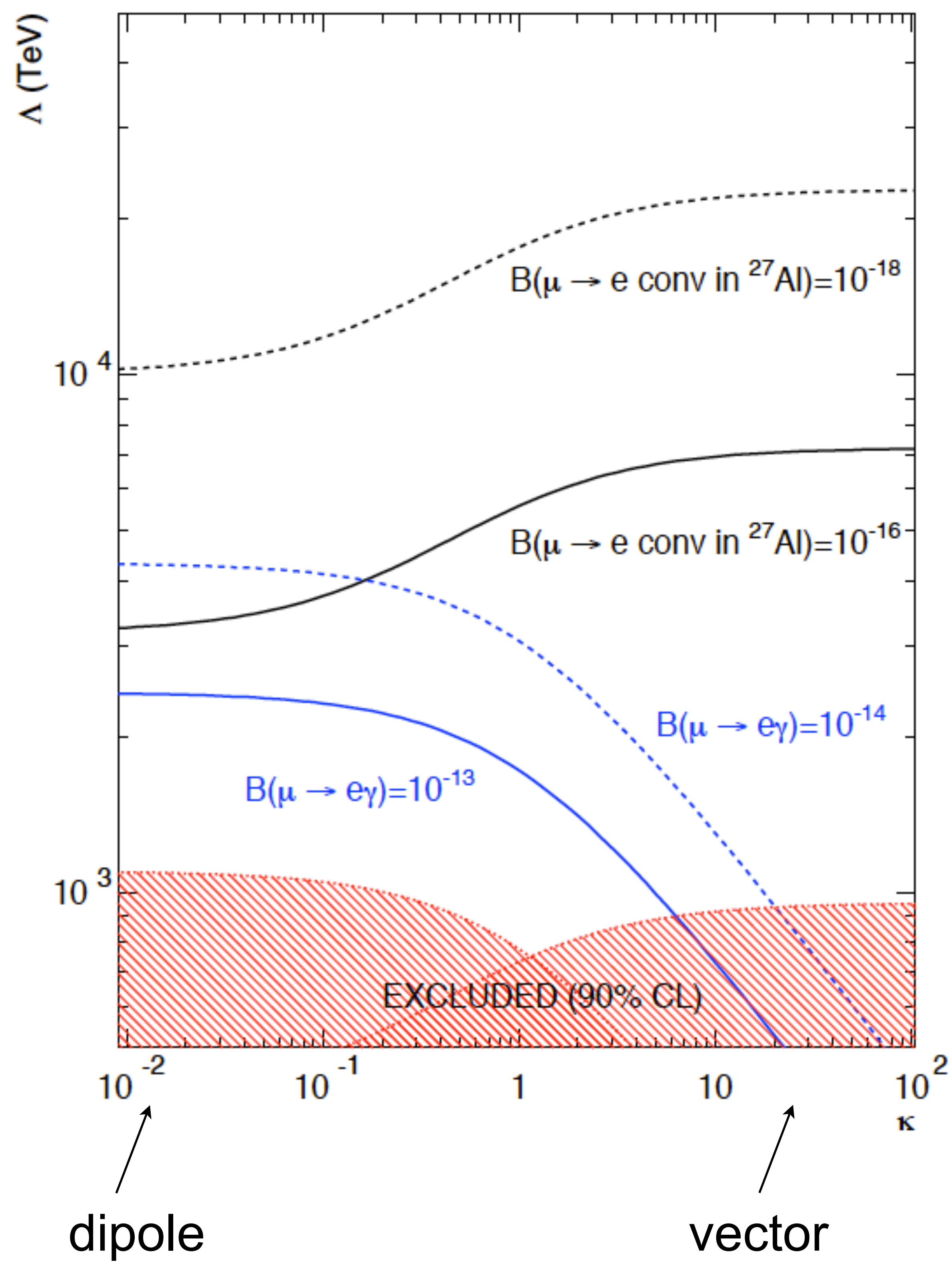
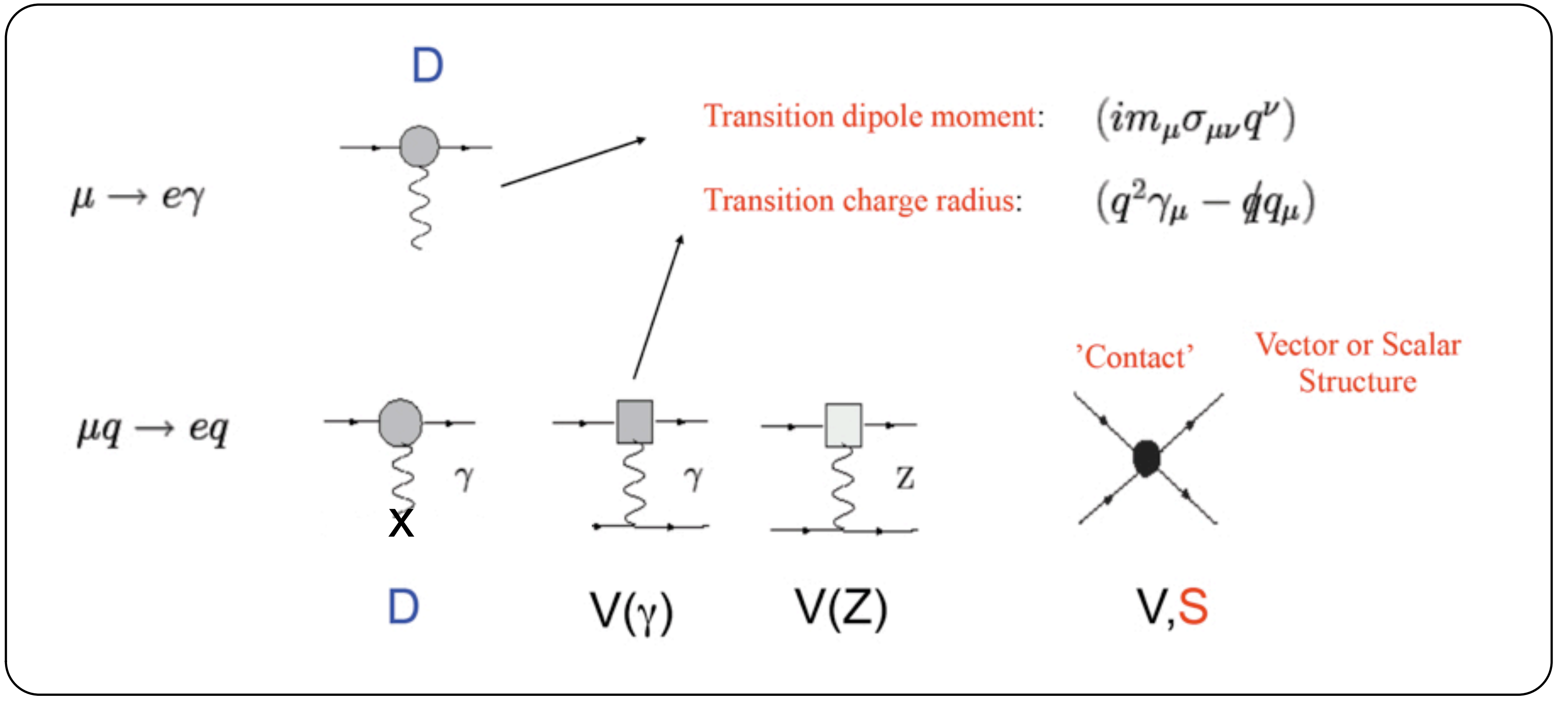


Sensitivity to operators

- $\mu \to e\gamma$ and $\mu \to e$ conversion: powerful diagnostic tool
- By measuring $B(\mu\to e,Z)/B(\mu\to e\gamma)$ and $B(\mu\to e, Z_1)/B(\mu\to e, Z_2)$, we can infer the relative strength of effective operators

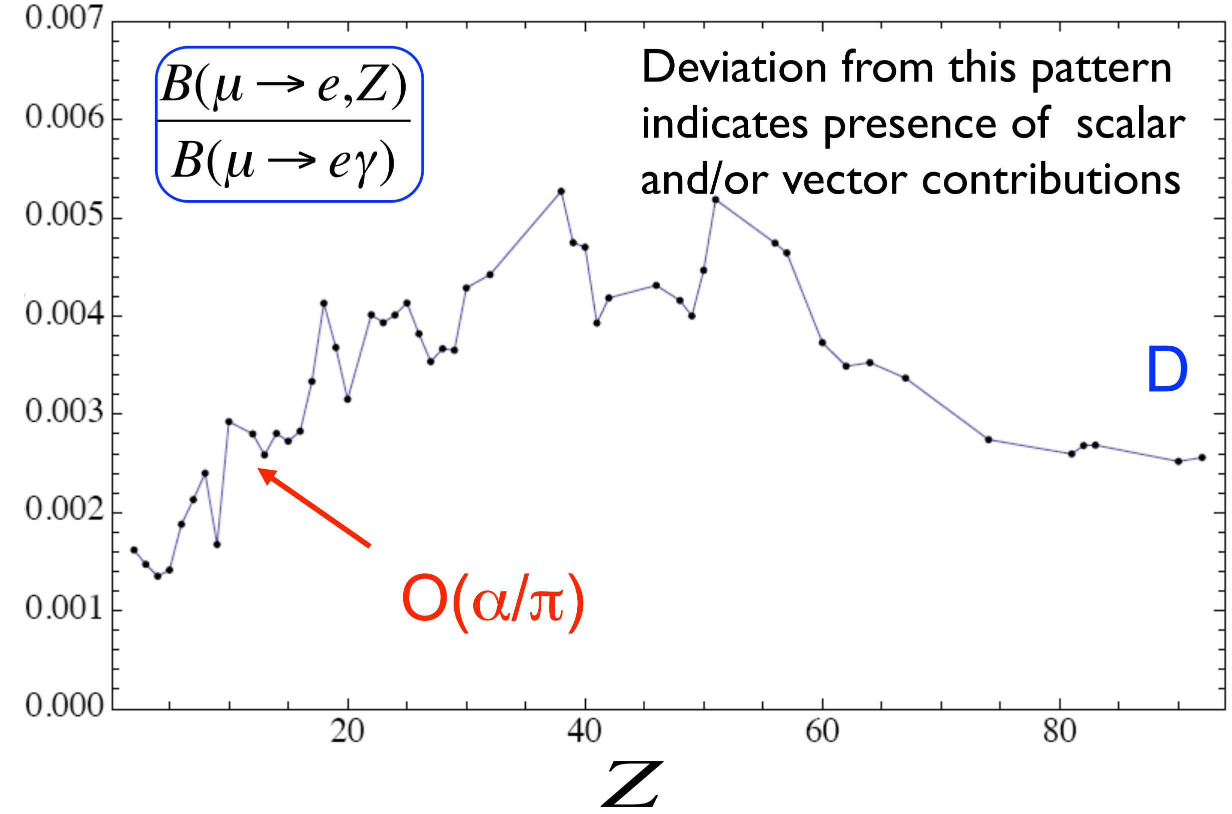


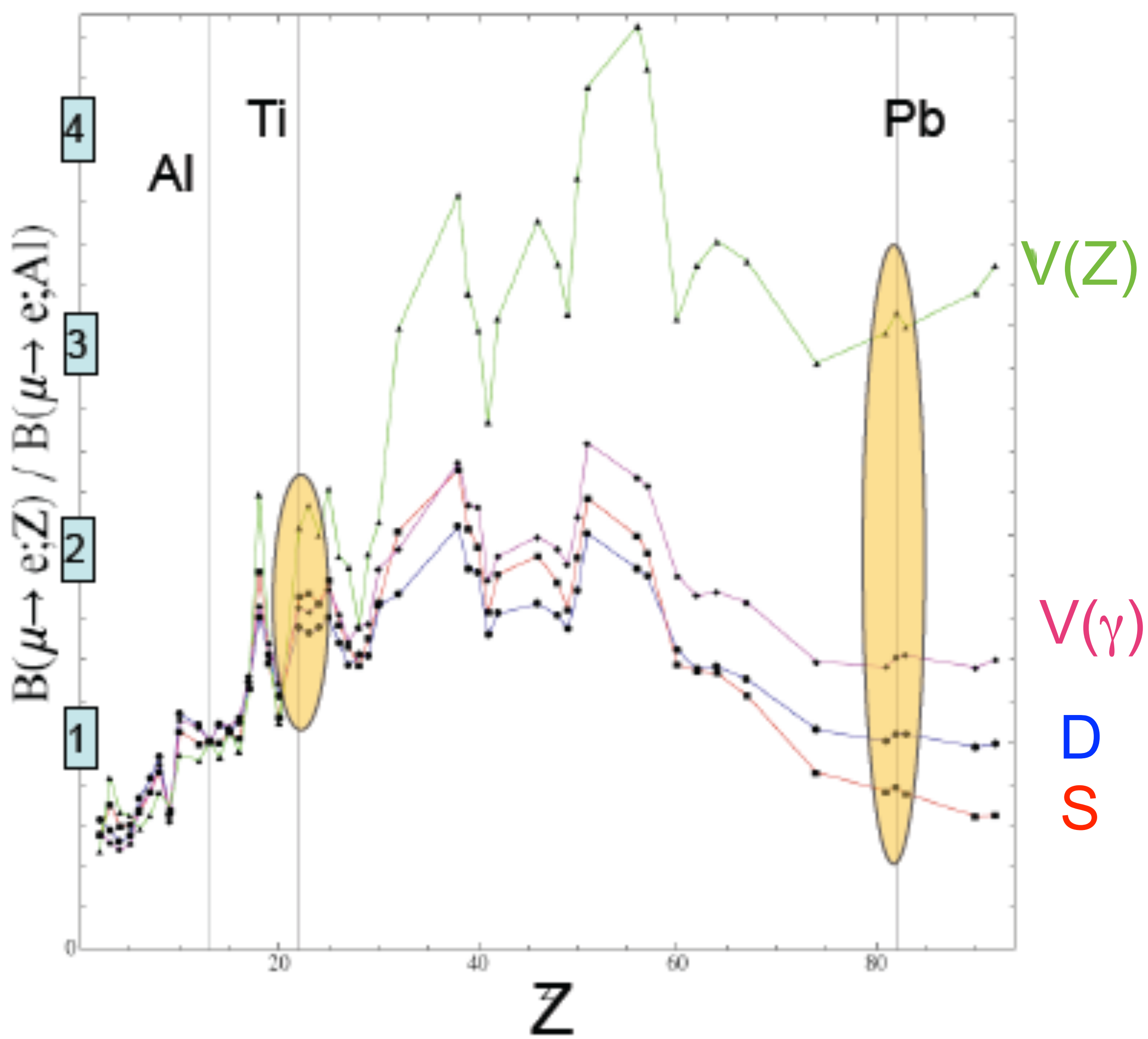
- Similarly, one can use Dalitz plot analysis of $\mu\to 3e$, $\tau\to 3l$

VC-Kitano-Okada-Tuzon '09

- $\mu \to e\gamma$ vs $\mu \to e$ conversion: probe non-dipole operators

$$B_{\mu \to e} = \frac{\Gamma(\mu^- + (Z,A) \to e^- + (Z,A))}{\Gamma(\mu^- + (Z,A) \to \nu_\mu + (Z-1,A))}$$





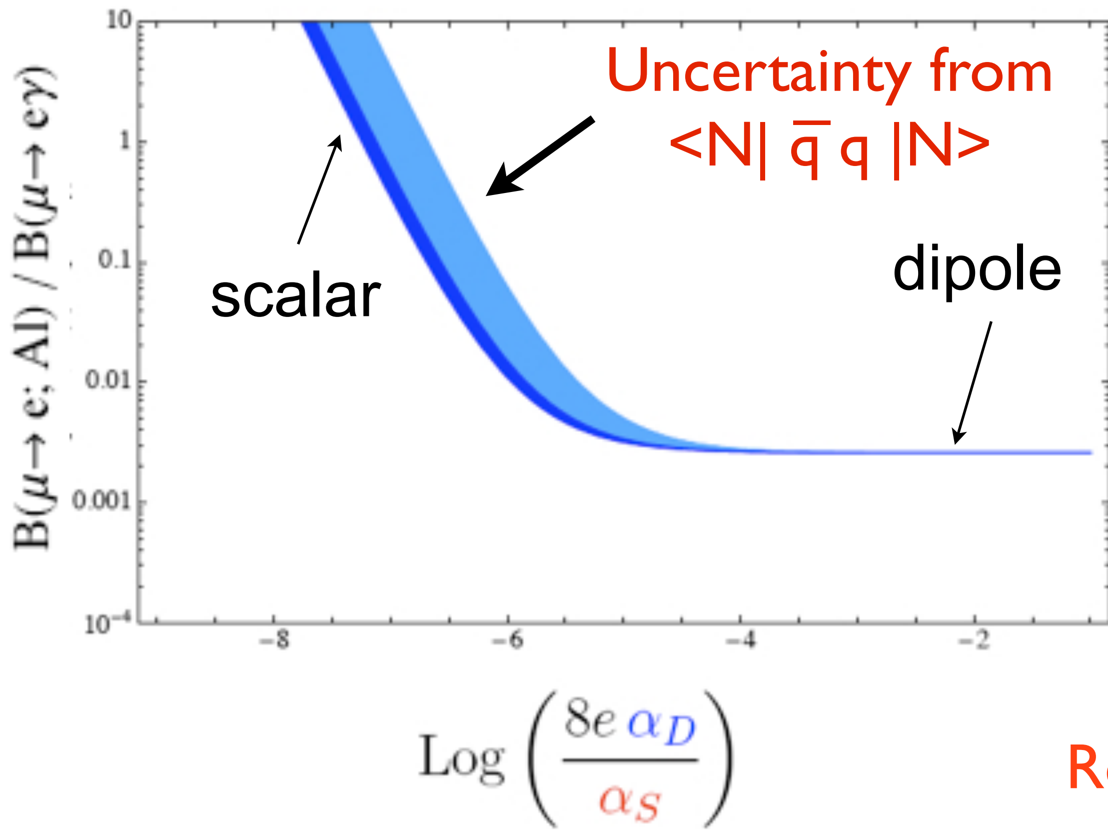
- Conversion amplitude has non-trivial dependence on target, that distinguishes D,S,V underlying operators

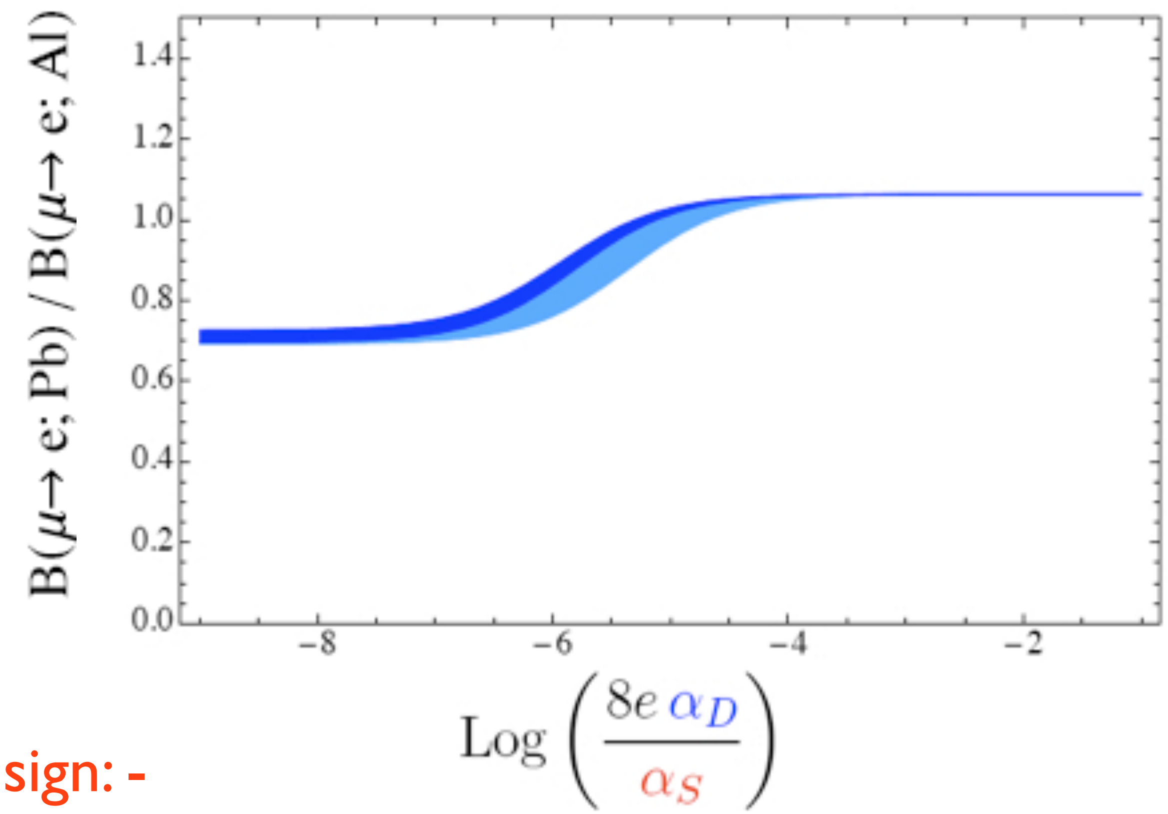
- Discrimination: need 5% measure of Ti/Al or 20% measure of Pb/Al

VC-Kitano-Okada-Tuzon '09

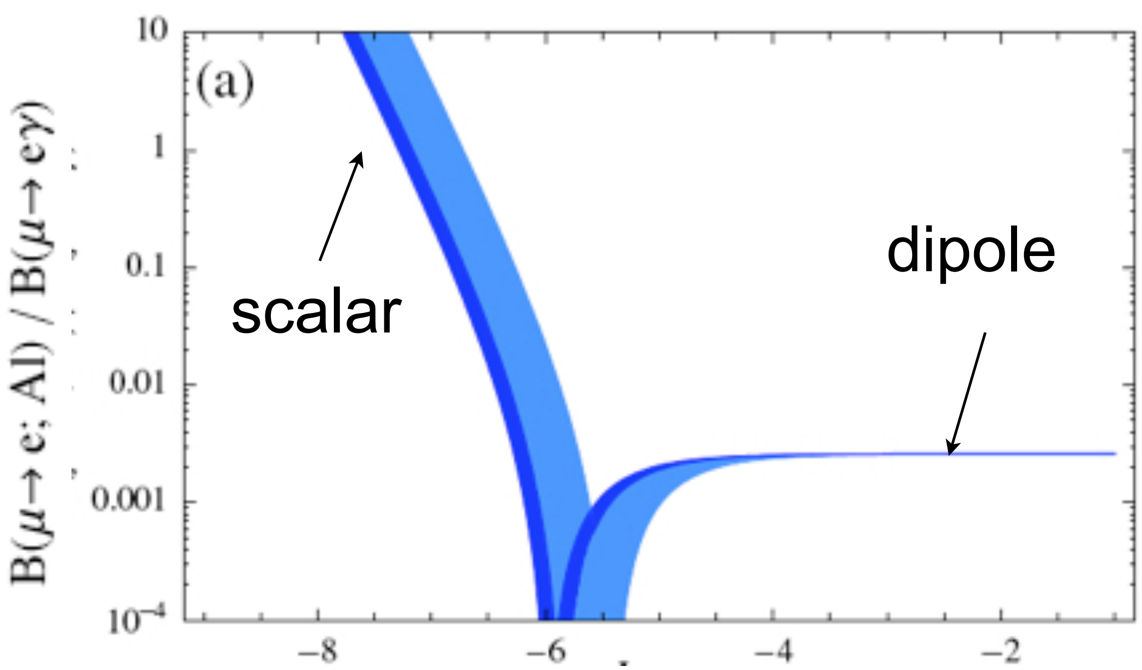
- Beyond single operator dominance: S and D

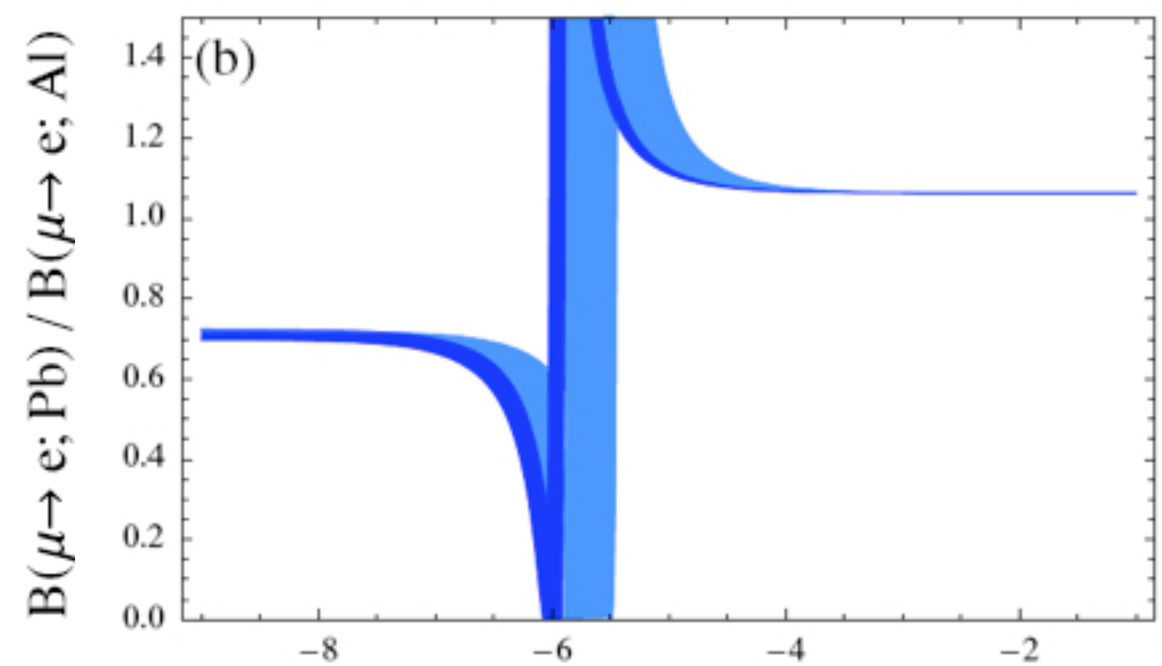
Relative sign: +





Relative sign: -



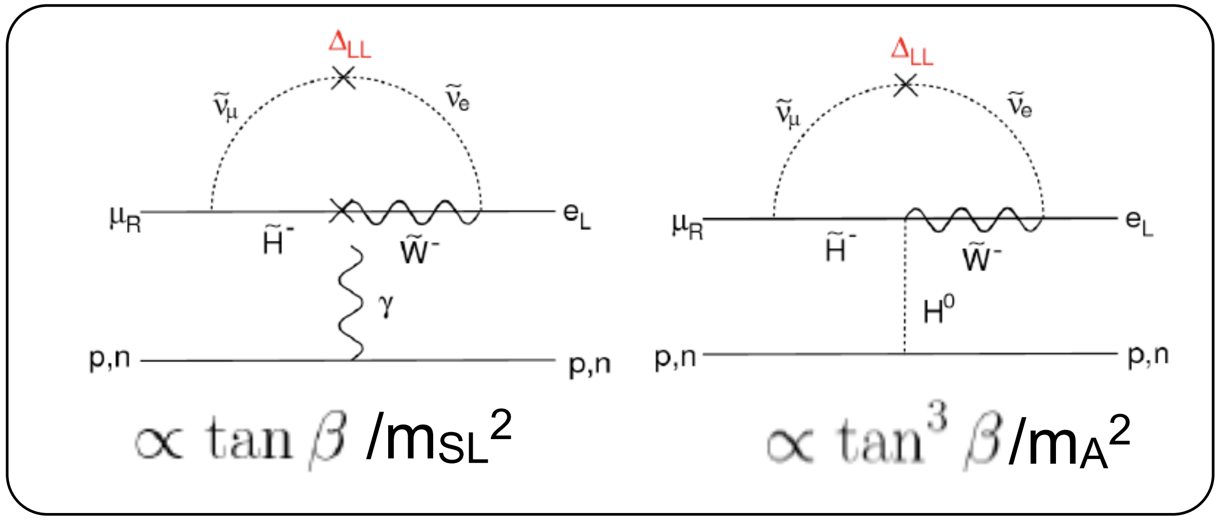


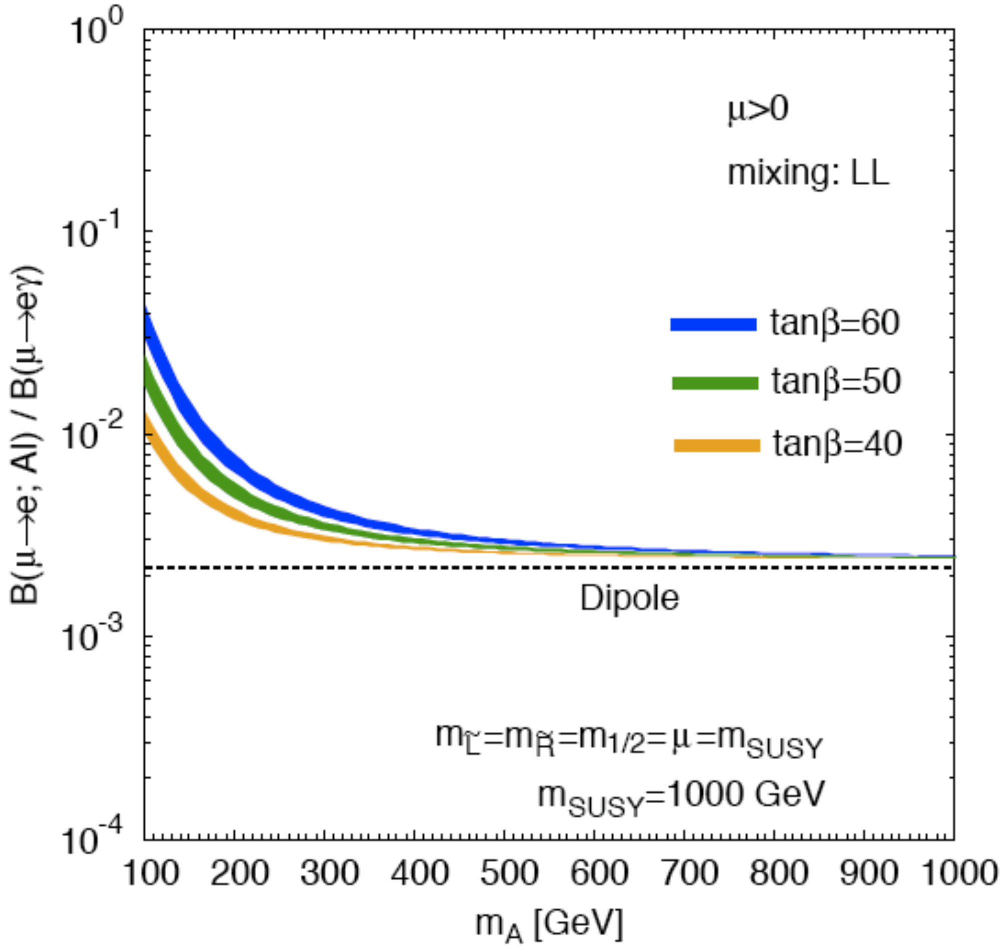
VC-Kitano-Okada-Tuzon '09

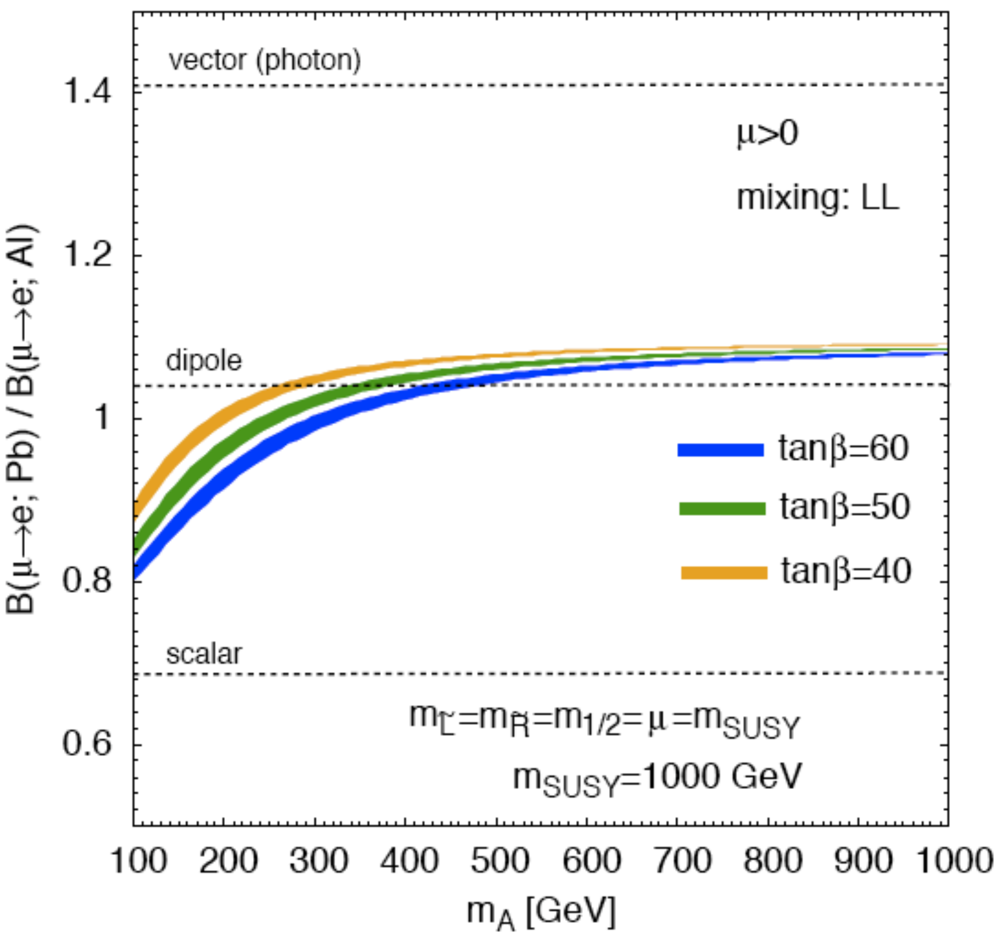
- Explicit realization in a SUSY scenario

Kitano-Koike-Komine-Okada 2003

- Dipole vs scalar operator (mediated by Higgs exchange) in SUSY see-saw models



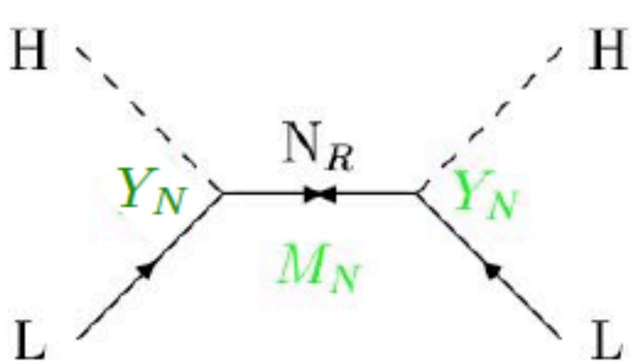




- Explicit realization: see-saw models

Type I: Fermion singlet

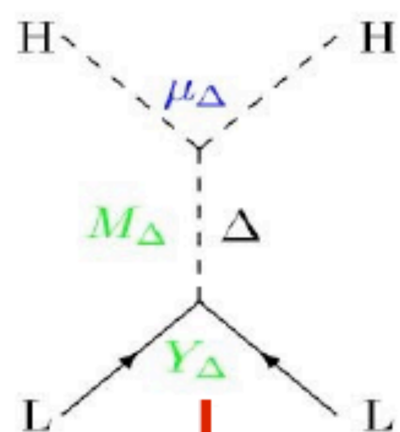
N_{R_i}



$$m_\nu = Y_N^T \frac{1}{M_N} Y_N v^2$$

Type II: Scalar triplet

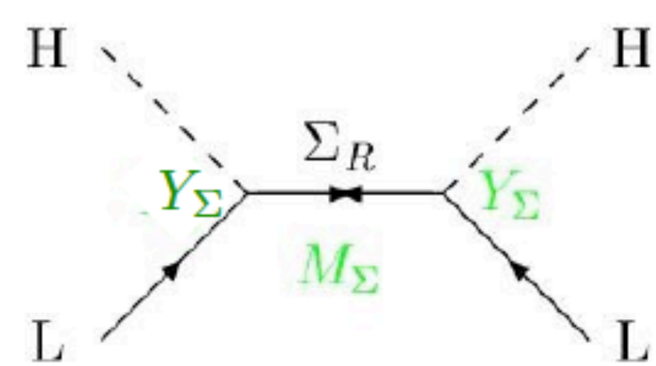
$\Delta \equiv (\Delta^{++}, \Delta^{+}, \Delta^{0})$



$$m_\nu = Y_\Delta \frac{\mu_\Delta}{M_\Delta^2} v^2$$

Type III: Fermion triplet

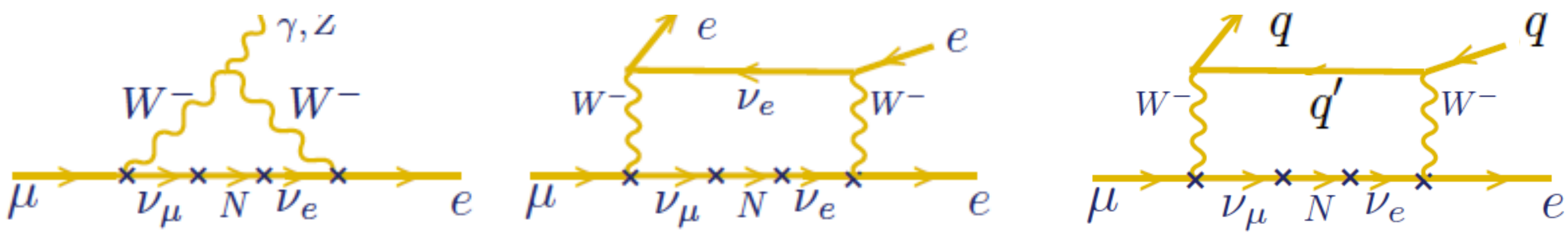
$\Sigma_i \equiv (\Sigma_i^{+}, \Sigma_i^{0}, \Sigma_i^{-})$



$$m_\nu = Y_\Sigma^T \frac{1}{M_\Sigma} Y_\Sigma v^2$$

- Observable CLFV if see-saw scale low (with protection of LN)
- Each model leads to specific CLFV pattern

- CLFV in Type I seesaw: loop-induced D,V operators, coefficients controlled by N_i masses



$$\Gamma(\mu \to e\gamma) = \sum_{N_i} \frac{|Y_{N_{ie}} Y^\dagger_{N_{i\mu}}|^2}{m^4_{N_i}} \cdot [\, c + c' \log(m^2_{N_i}/m^2_W)\,]^2$$

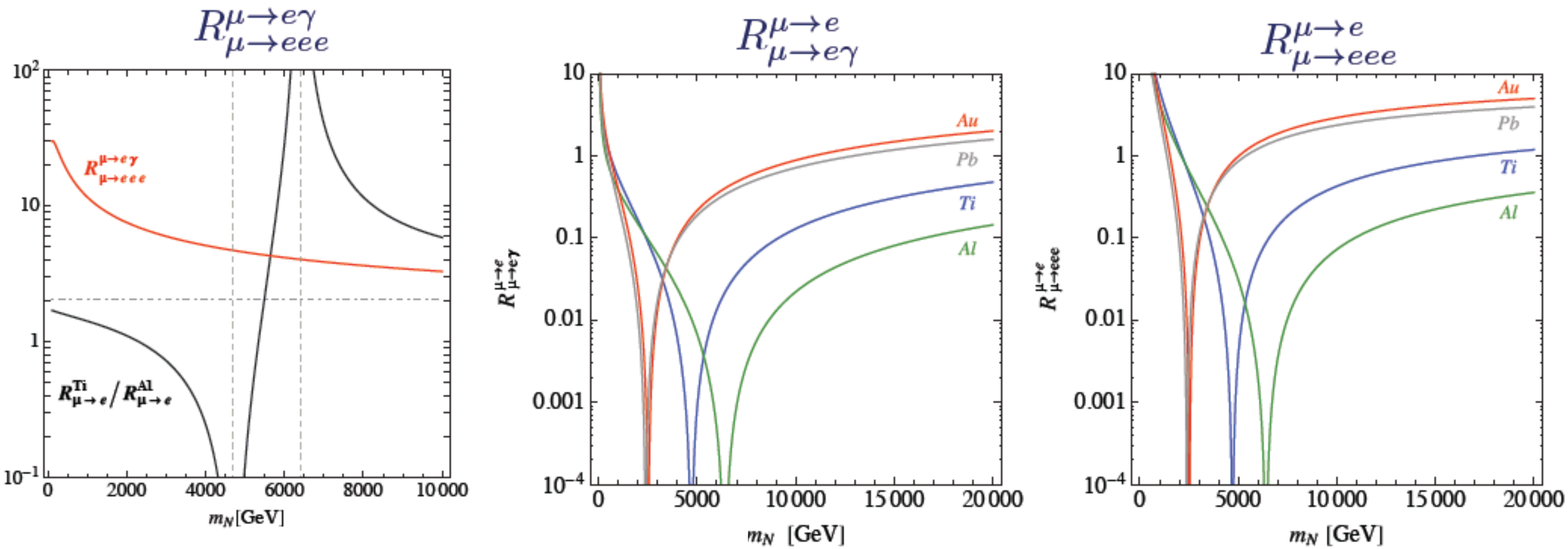
$$\Gamma(\mu \to eee) = \sum_{N_i} \frac{|Y_{N_{ie}} Y^\dagger_{N_{i\mu}}|^2}{m^4_{N_i}} \cdot [\, d + d' \log(m^2_{N_i}/m^2_W)\,]^2$$

$$R^N_{\mu\to e} = \sum_{N_i} \frac{|Y_{N_{ie}} Y^\dagger_{N_{i\mu}}|^2}{m^4_{N_i}} \cdot [\, b^N + b'^N \log(m^2_{N_i}/m^2_W)\,]^2$$

- For ~degenerate N_i masses (suppressed LNV), ratio of 2 rates with same flavor transition depends only on seesaw scale

Alonso-Dhen-Gavela-Hambye '13

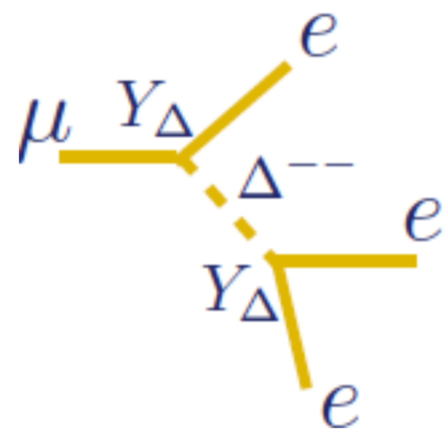
- CLFV in Type I seesaw: loop-induced D,V operators, coefficients controlled by N_i masses

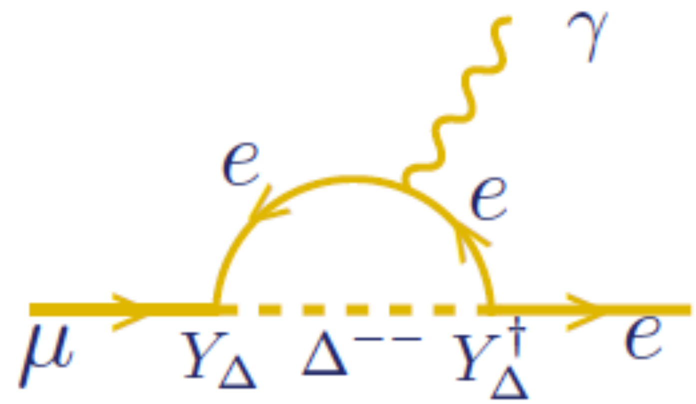


- With three rate measurements (2 ratios):
  - determine seesaw scale or
  - rule out scenario

Alonso-Dhen-Gavela-Hambye ‘13

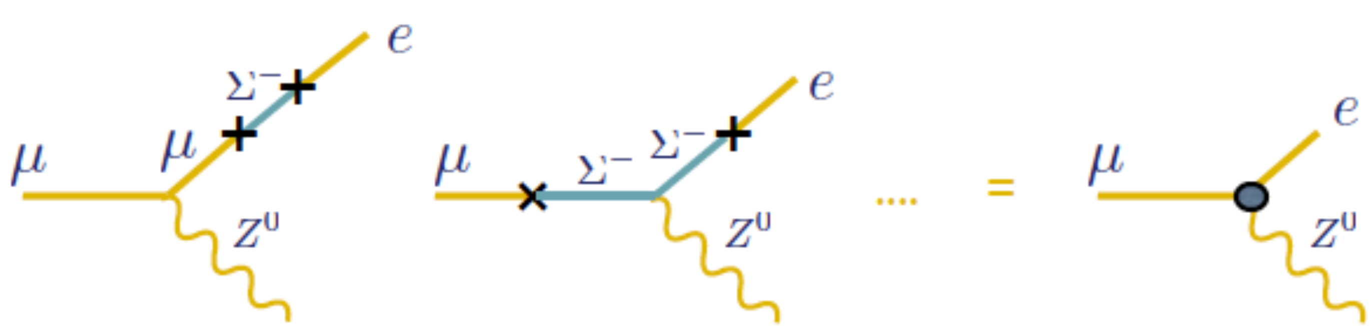
- CLFV in Type II seesaw: tree-level 4L operator (D,V at loop) → 4-lepton processes most sensitive





- CLFV in Type III seesaw: tree-level LFV couplings of Z ⇒

μ →3e and μ →e conversion at tree level, μ →eγ at loop



Abada-Biggio-Bonnet-Gavela-Hambye '07, '08

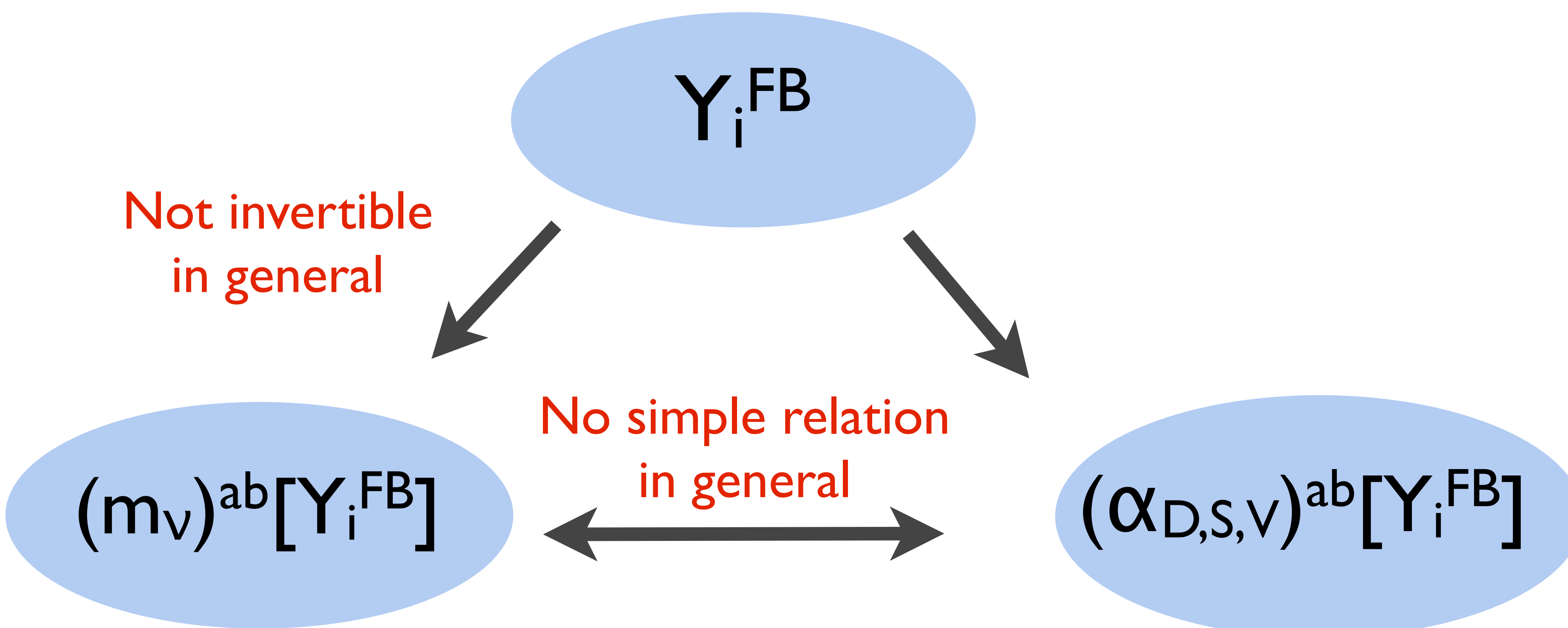
- Ratios of 2 processes with same flavor transition are fixed

$$
\begin{aligned}
Br(\mu \to e\gamma) &= 1.3 \cdot 10^{-3} \cdot Br(\mu \to eee) = 3.1 \cdot 10^{-4} \cdot R^{\mu\to e}_{Ti} \\
Br(\tau \to \mu\gamma) &= 1.3 \cdot 10^{-3} \cdot Br(\tau \to \mu\mu\mu) \\
Br(\tau \to e\gamma) &= 1.3 \cdot 10^{-3} \cdot Br(\tau \to eee)
\end{aligned}
$$

Sensitivity to flavor structures

- Each model has its flavor group (← field content) and sources of flavor breaking Y_i^{FB} (Yukawa-type, mass matrices of heavy states, ...)

- Y_i^{FB} leave imprint in m_ν and CLFV effective couplings $\alpha_{D,V,S,...}$



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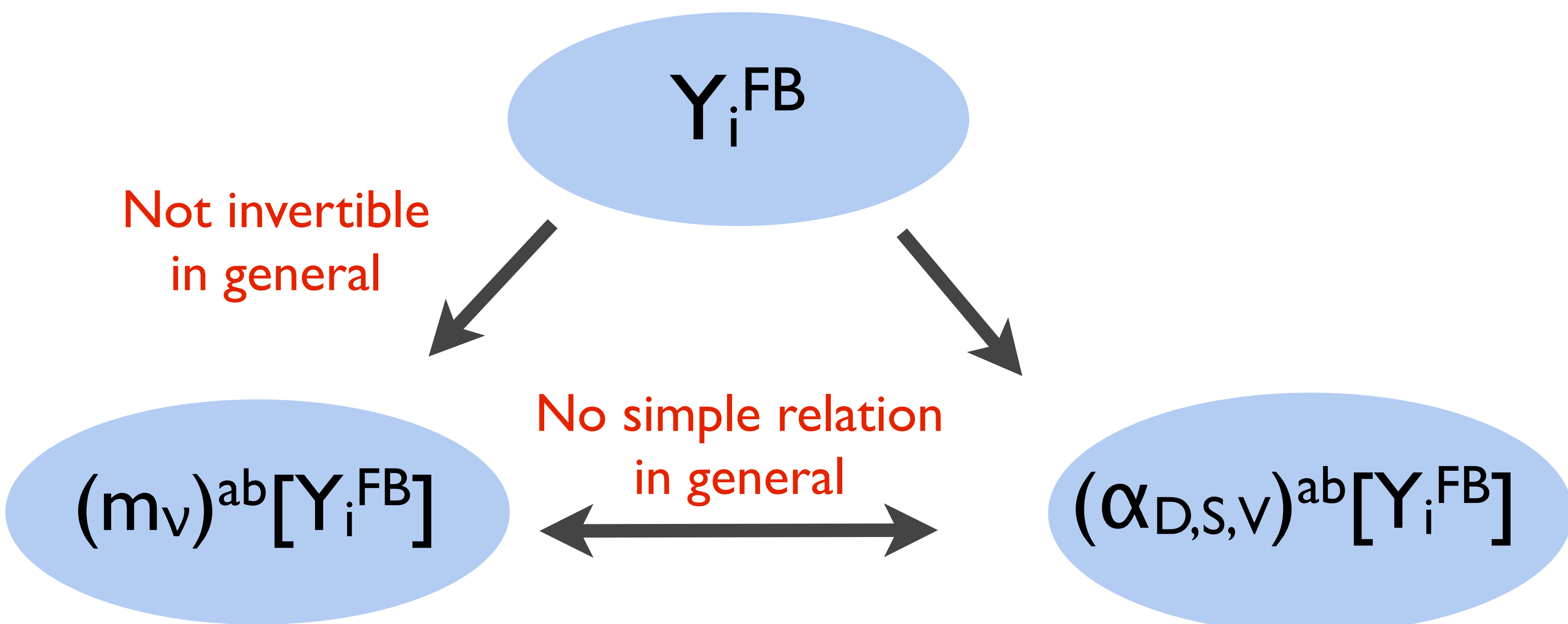

Minimal Lepton Flavor Violation
tries to remedy this issue.
No unique realization

VC-Grinstein-Isidori-Wise '05
Davidson-Palorini '06
Gavela-Hambye-Hernandez-Hernandez '09
Alonso-Isidore-Merlo-Munoz-Nardi '11
..

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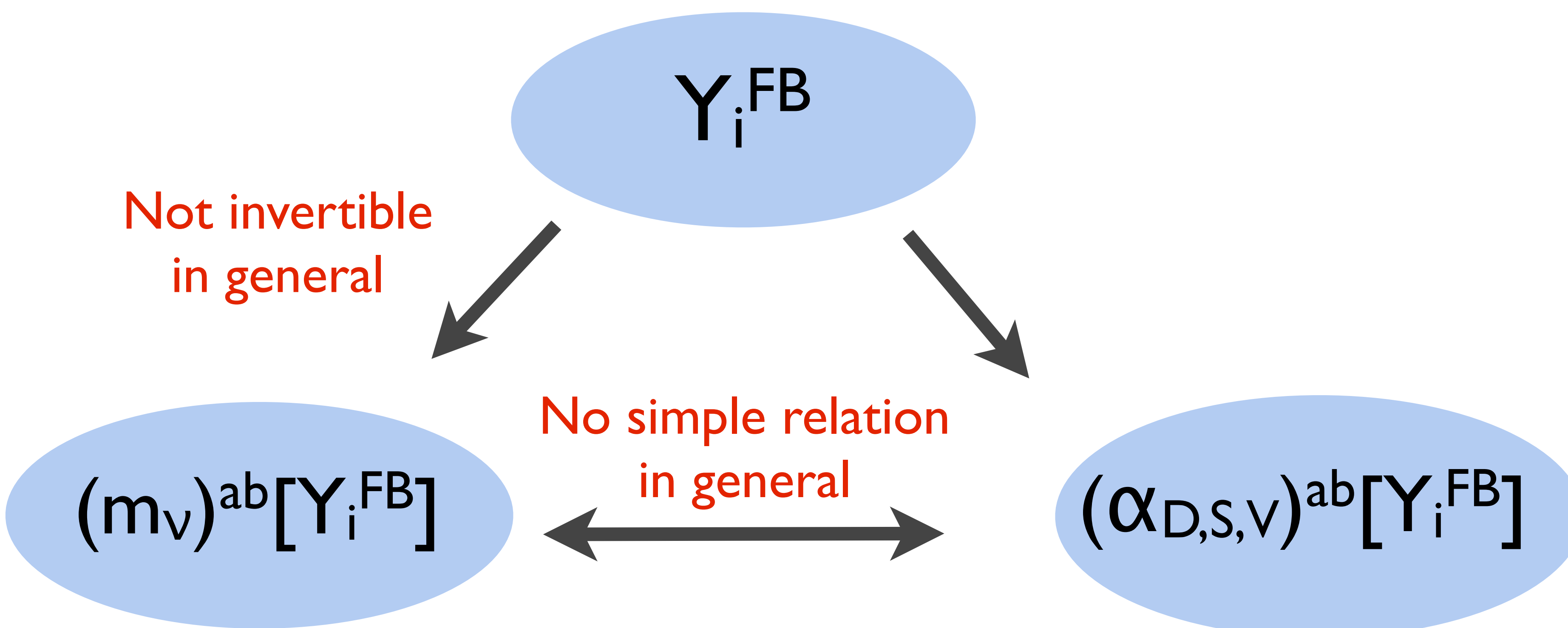
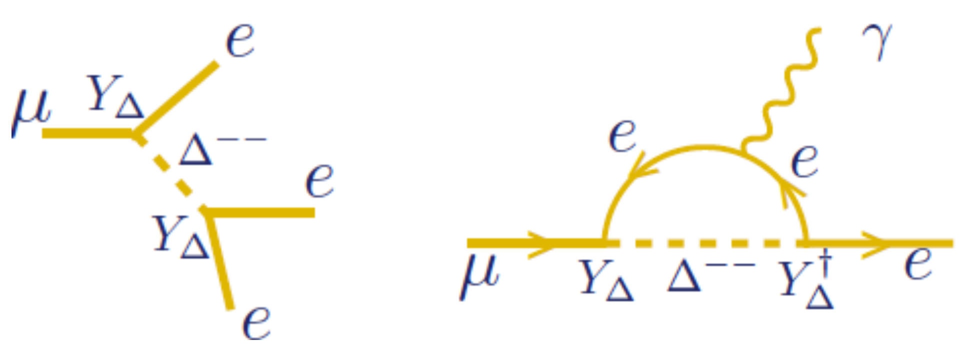


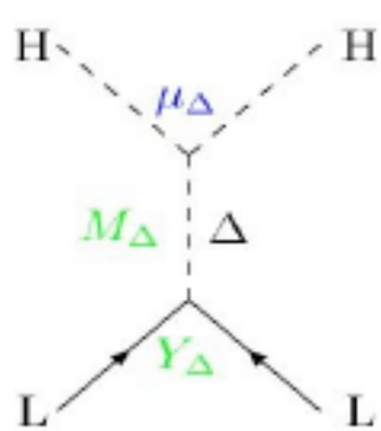
- No general statement, but CLFV provides non-trivial tests of any given model ansatz for the nature and structure of Y_i^{FB}.
  Cleanest test-ground: $\mu \to e\gamma$ vs $\tau \to \mu\gamma$ ($\tau \to e\gamma$)

- Example: Type II seesaw model (scalar triplet)
  Explicit realization of Minimal Lepton Flavor Violation

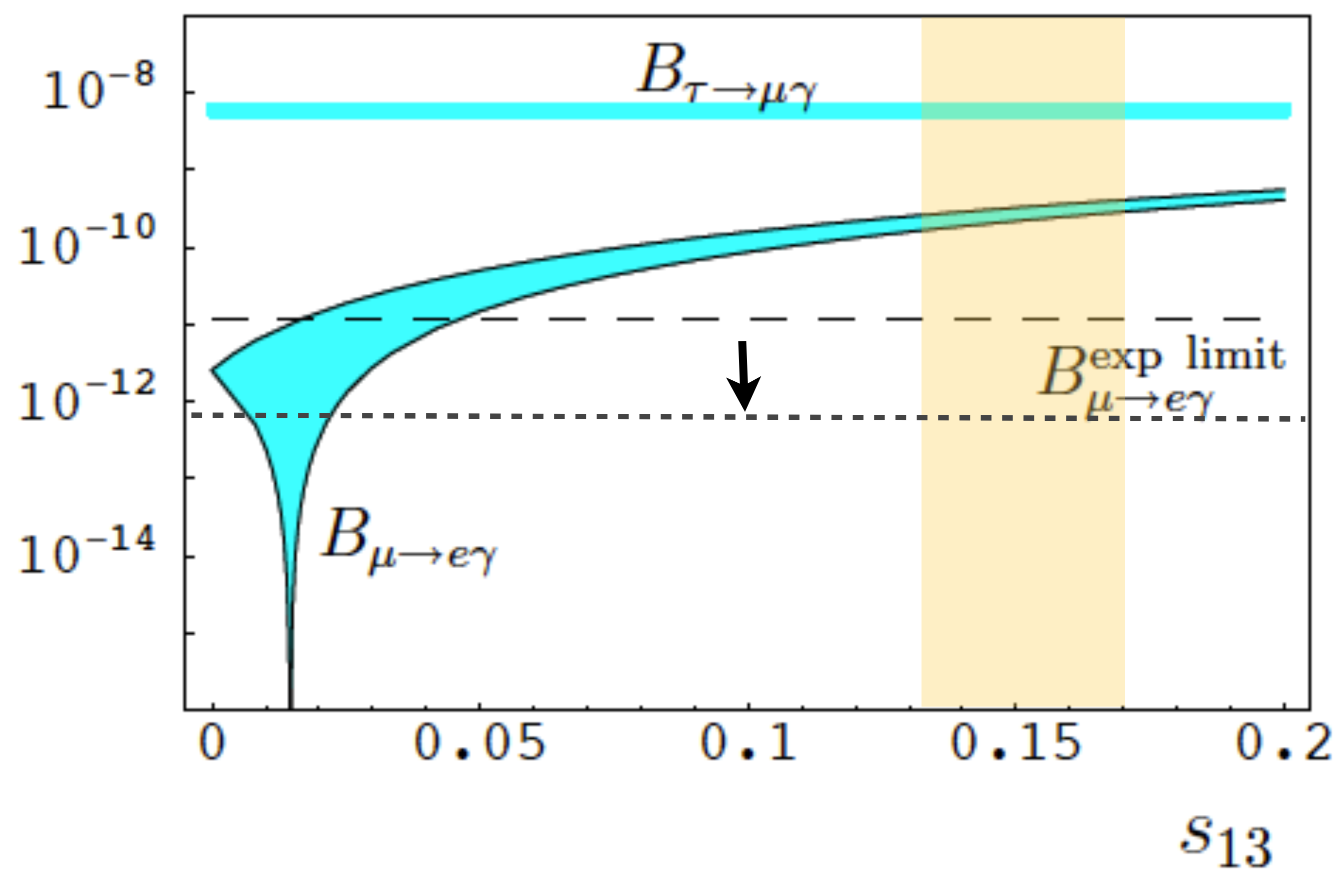
CLFV controlled by

$Y_\Delta \propto m_\nu$





Rossi '02, VC-Grinstein-Isidori-Wise '05



$\tau \to \mu\gamma$ not observable at (super-)B factories

- A different example: SU(5) GUT models (with ~ degenerate N_i)
- Two competing structures:

$$\boxed{\begin{array}{l} \frac{v}{\Lambda^2}\, \bar{e}_R^i \left(\lambda_e \lambda_\nu^\dagger \lambda_\nu\right)^{ij} \sigma^{\mu\nu} e_L^j \; F_{\mu\nu} \\ \\ \frac{v}{\Lambda^2}\, \bar{e}_R^i \left(\lambda_U \lambda_U^\dagger \lambda_D^T\right)^{ij} \sigma^{\mu\nu} e_L^j \; F_{\mu\nu} \end{array}}$$

$\rightarrow$ PMNS mixing pattern $\quad M_\nu > 10^{12}$ GeV

$\rightarrow$ CKM mixing pattern $\quad M_\nu < 10^{12}$ GeV

[~ Barbieri-Hall-Strumia '95]

- CKM $\Rightarrow$ more hierarchical pattern of BRs: $\tau \to \mu\gamma$ is within reach of (super-)B factories

$$10 - 100 \;:\; 1 \;:\; 1$$

$$B(\tau \to \mu\gamma) \;:\; B(\tau \to e\gamma) \;:\; B(\mu \to e\gamma) \quad \begin{cases} \mathrm{Min}\left[s_{13}^{-2}\,,\; \dfrac{\Delta m^2_{\mathrm{atm}}}{\Delta m^2_{\mathrm{sol}}}\right] \;:\; 1 \;:\; 1 \\ \\ \lambda_C^{-6} \;:\; \lambda_C^{-4} \;:\; 1 \end{cases}$$

$\lambda_C \equiv V_{us}$

$$10^4 \;:\; 500 \;:\; 1$$

High energy probes

High scale LFV mediators

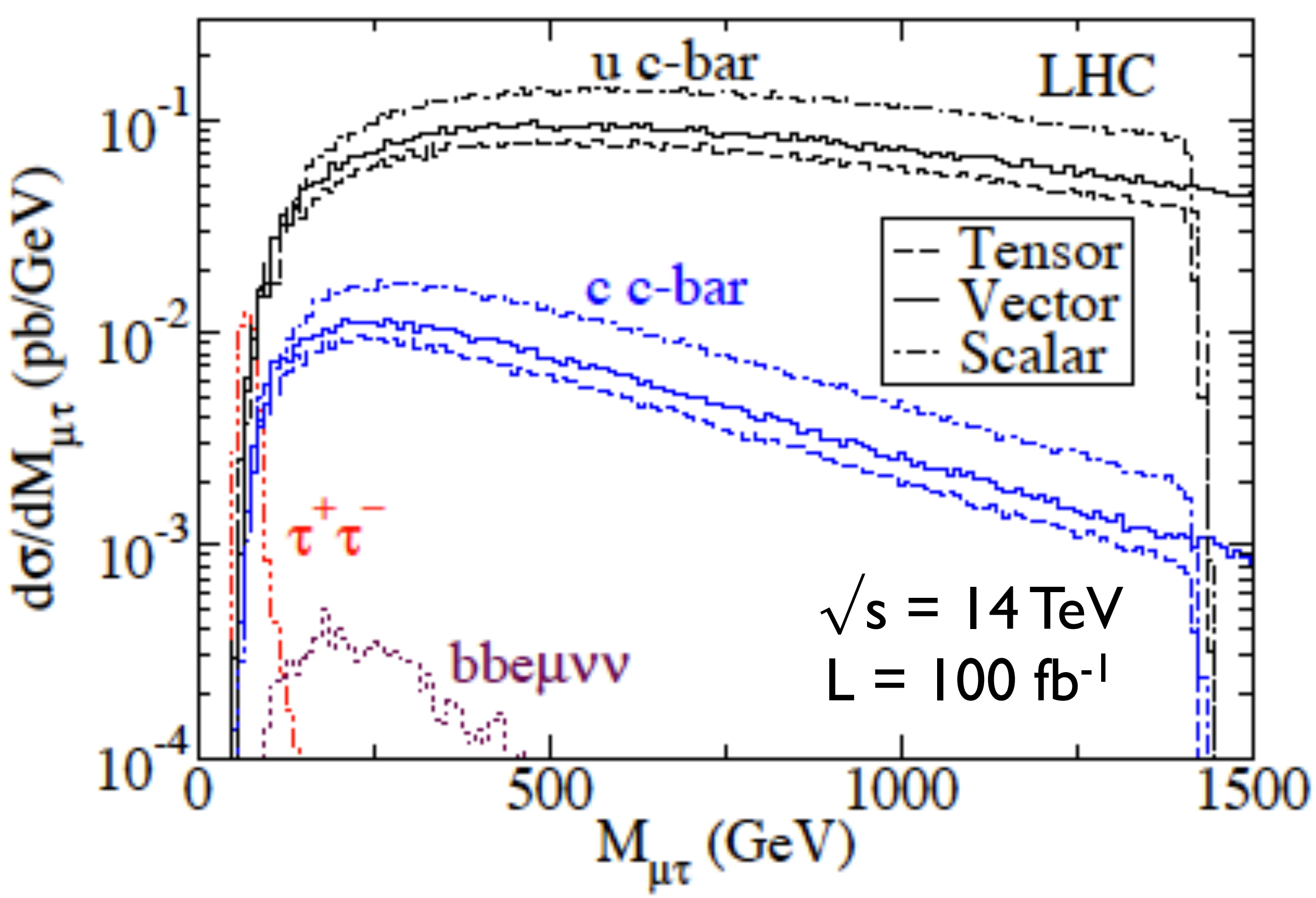
- If Λ_{FV} >> TeV, EFT description is still appropriate at colliders
- 4-fermion operators mediate $p\overset{(-)}{p} \rightarrow \ell_\alpha \bar{\ell}_\beta + X$
- Can collider compete with rare decays? Yes, in the μτ sector

Han-Lewis-Sher 2010

$$\boxed{\sigma(\bar{q}_i q_j \rightarrow \mu\tau) \propto \frac{s}{\Lambda^4}}$$

$$\frac{C^j_{\alpha\beta}}{\Lambda^2}\left(\bar{\mu}\,\Gamma_j\,\tau\right)\left(\bar{q}^\alpha\,\Gamma_j\,q^\beta\right)$$

$$C^j_{\alpha\beta} = 4\pi\,\mathcal{O}(1)$$



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Λ_{NP} (TeV)	2σ sensitivity			5σ discovery		
Coupling	$1,\gamma_5$	$\gamma_\mu,\gamma_\mu\gamma_5$	$\sigma_{\mu\nu}$	$1,\gamma_5$	$\gamma_\mu,\gamma_\mu\gamma_5$	$\sigma_{\mu\nu}$
$u\bar{u}$	18	19	21	14	15	17
$d\bar{d}$	16	17	19	12	13	15
$s\bar{s}$	9.0*	9.6*	11	7.1*	7.6**	8.6
$d\bar{s}$	13	14	16	10	11*	13
$d\bar{b}$	12	13	14	9.7	10	11
$s\bar{b}$	8.7	9.2	10	6.8	7.3	8.2
$u\bar{c}$	15	16	18	12	13	14
$c\bar{c}$	7.2	7.6	8.6	5.7	6.0	6.8
$b\bar{b}$	5.8	6.2	7.0	4.6	4.9	5.5

$\sqrt{s}$ = 14 TeV   L = 100 fb^{-1}

Direct searches at the LHC

- If $\Lambda_{FV} \sim$ TeV, then can study LFV couplings of the mediator at the LHC and at low-energy

- LFV decays of new resonances. Vast literature. Examples:
    - $Z' \to \ell_a \bar{\ell}_b$
    - $\tilde{\nu} \to \ell_a \bar{\ell}_b$ (and related channels motivated by RPV SUSY)
    - Higgs
    - ...

- Here discuss LFV couplings of the Higgs

Higgs LFV couplings

- Non-standard (LFV) couplings of the Higgs arise in several models
- Conveniently parameterized by effective interaction:

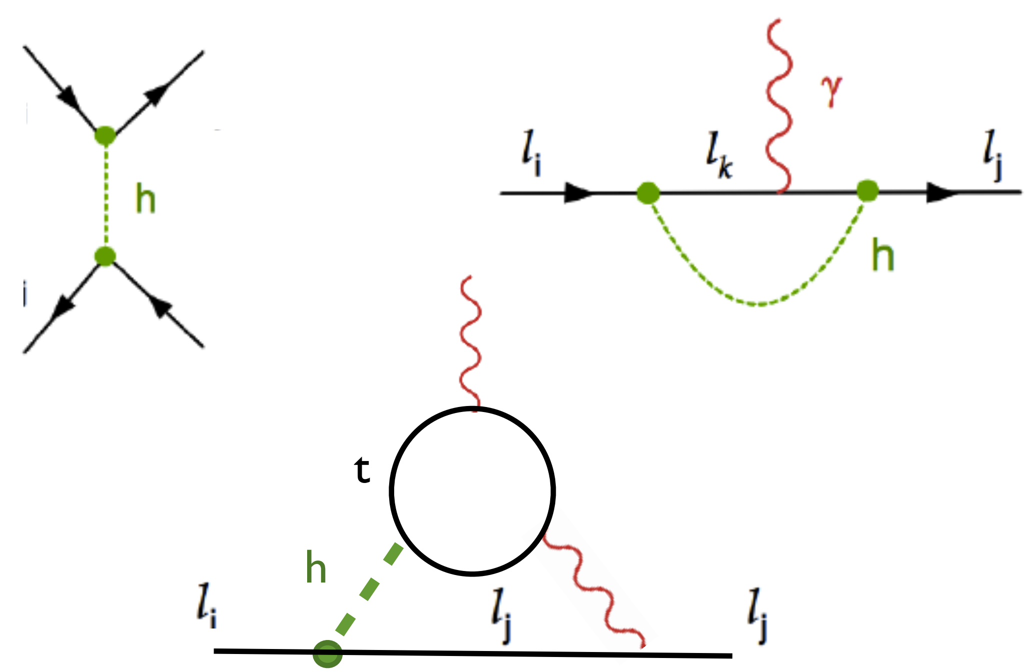
Goudelis-Lebedev-Park '11
Davidson-Grenier '10
...

$$\boxed{\mathcal{L}_Y = -m_i \bar{f}_L^i f_R^i - Y_{ij}(\bar{f}_L^i f_R^j)h + h.c. + \cdots}$$

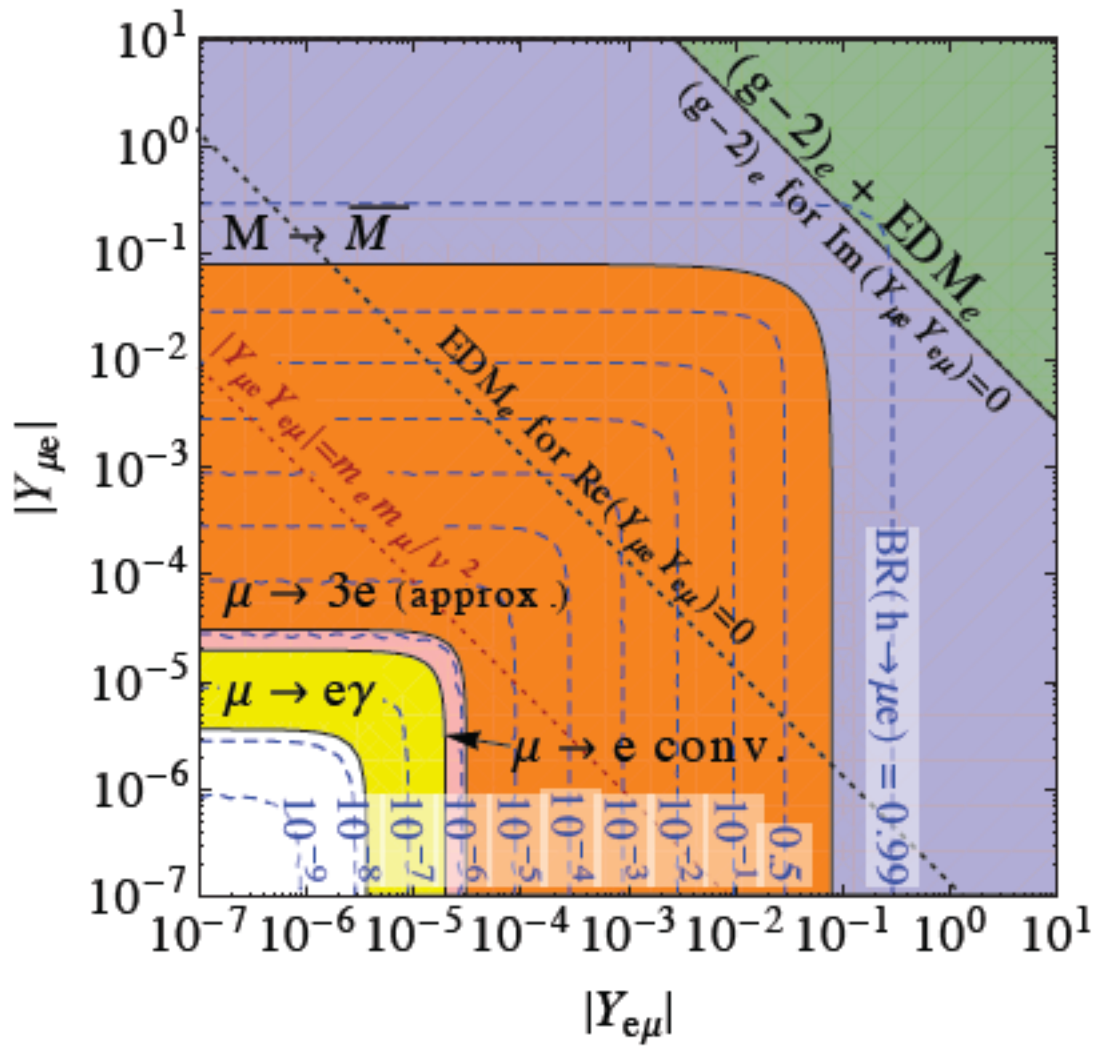
Harnik-Kopp-Zupan '12
Blankenburg-Ellis-Isidori 12
McKeen-Pospelov-Ritz '12

$$\Delta\mathcal{L}_Y = -\frac{\lambda'_{ij}}{\Lambda^2}(\bar{f}_L^i f_R^j)H(H^\dagger H) + h.c.$$

- $\mathcal{L}_Y$ mediates LFV Higgs decays & generates at low-energy scalar 4f operators (tree), dipole (loops).



- Constraints: Higgs decays vs low-energy LFV and LFC observables
- μe sector: low-energy constraints very powerful

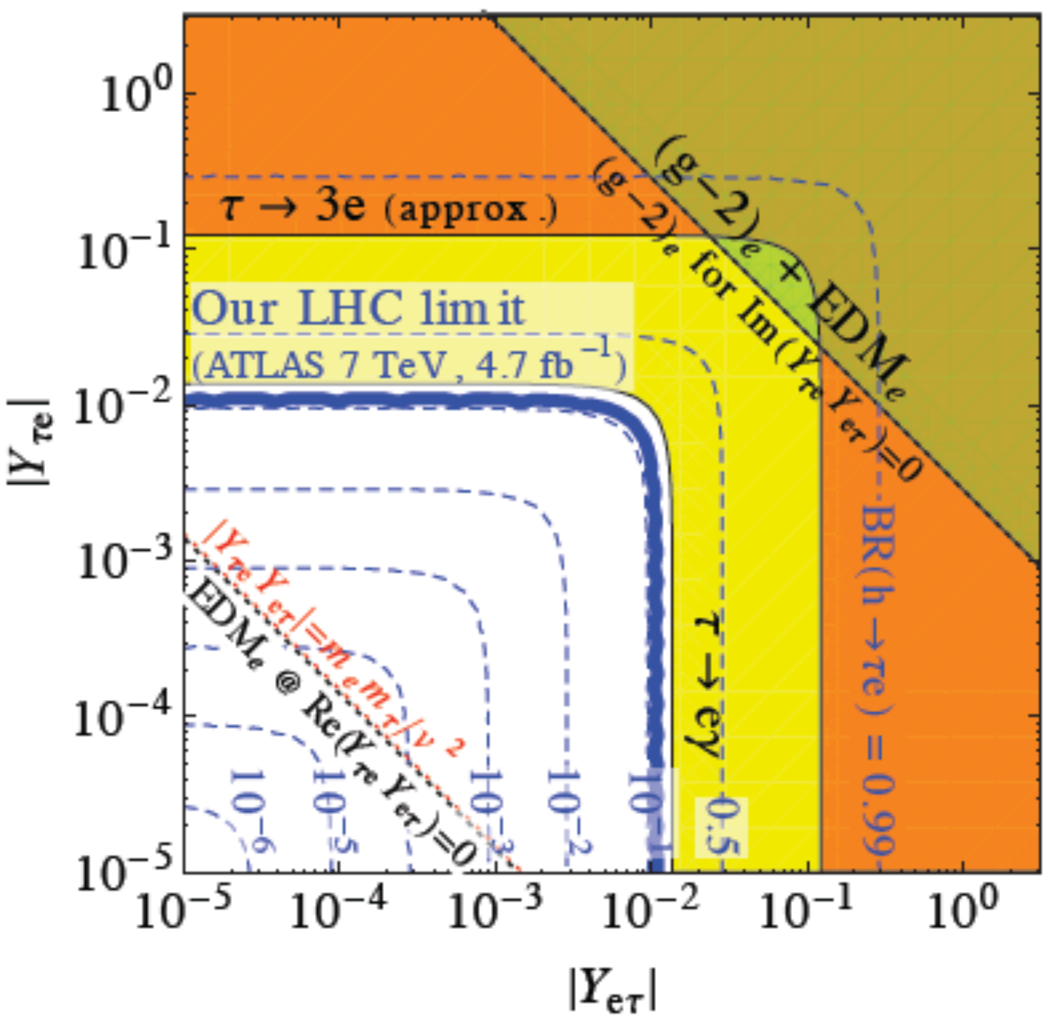


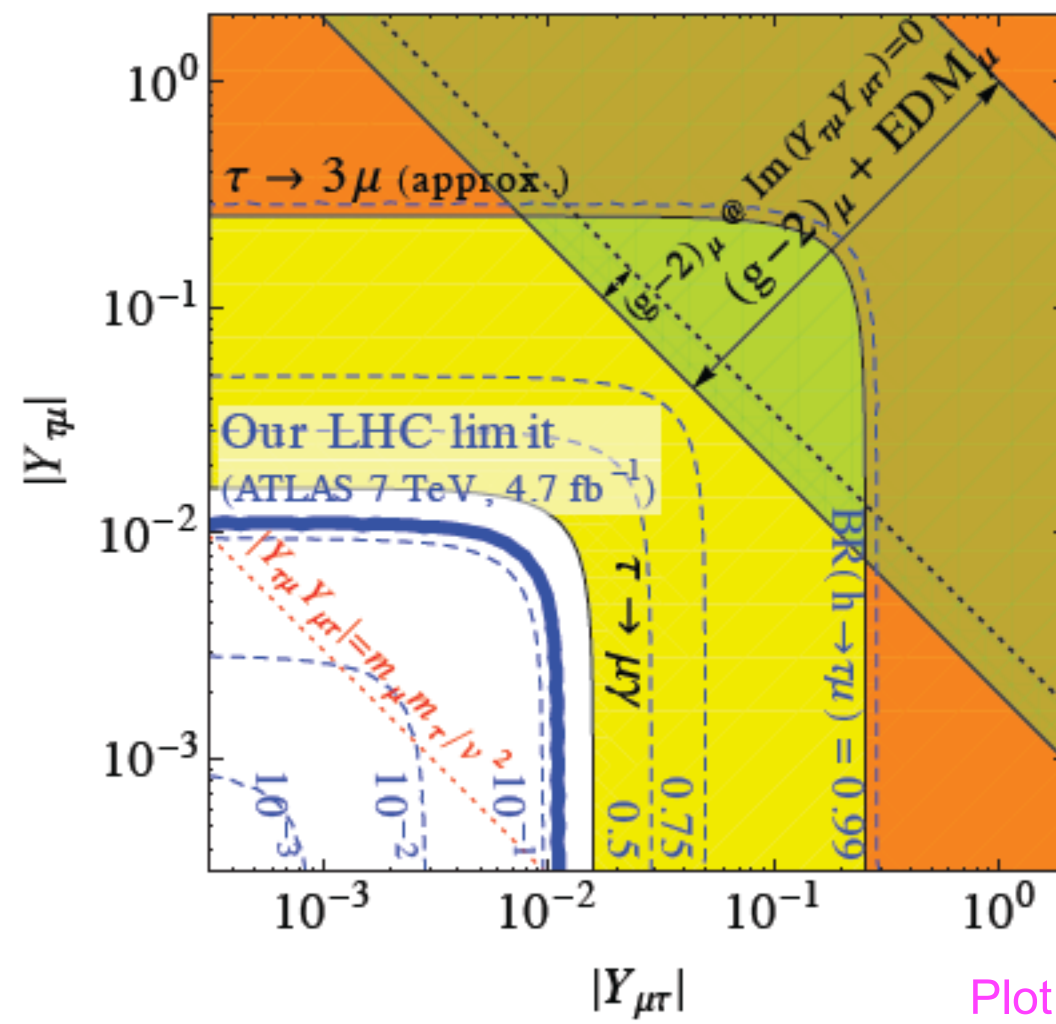
* Diagonal couplings set to SM value

Plot from Harnik-Kopp-Zupan '12

- Constraints: Higgs decays vs low-energy LFV and LFC observables
- μτ and eτ sectors: large LFV BRs possible (strongest constraints from Higgs decay)

* Diagonal couplings set to SM value





Plot from
Harnik-Kopp-Zupan '12

- This strongly motivates a dedicated search at the LHC

Conclusions

- Charged LFV: deep probes of physics BSM
- "Discovery" tools: clean, high scale reach
- "Model-discriminating" tools (with and without the LHC)

  - Operator structure $\rightarrow$ mediators
  - μe vs $\tau\mu$ vs τe $\rightarrow$ sources of flavor breaking

Exciting prospects in the next 5-10 years:

★ 3-4 orders of magnitude improvement in μ processes

★ 1-2 orders of magnitude improvement in τ processes

★ LHC can play a significant role!

Backup Slides

Omissions / discussion topics?

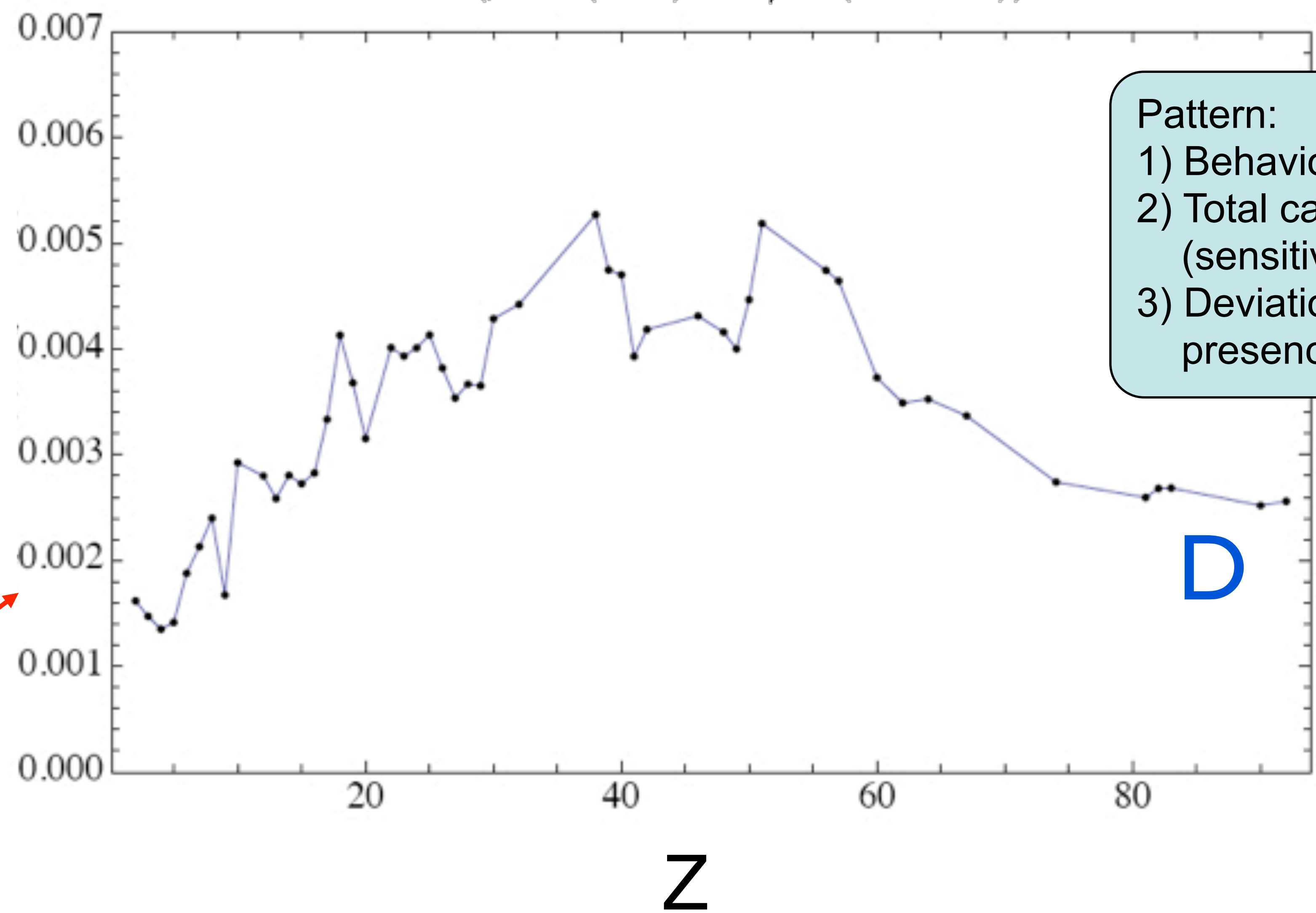
- Connection to flavor models (other talks)
- Neutrino "NSI" (Non Standard Interactions) and CLFV
- Hadronic tau decays ($\tau \to \mu\pi\pi$, etc.)
- ...

- $\mu \to e\gamma$ vs $\mu \to e$ conversion: probe existence non-dipole operators

Kitano-Koike-Okada ‘02
VC-Kitano-Okada-Tuzon ‘09

$$B_{\mu\to e} = \frac{\Gamma(\mu^- + (Z,A) \to e^- + (Z,A))}{\Gamma(\mu^- + (Z,A) \to \nu_\mu + (Z-1,A))}$$

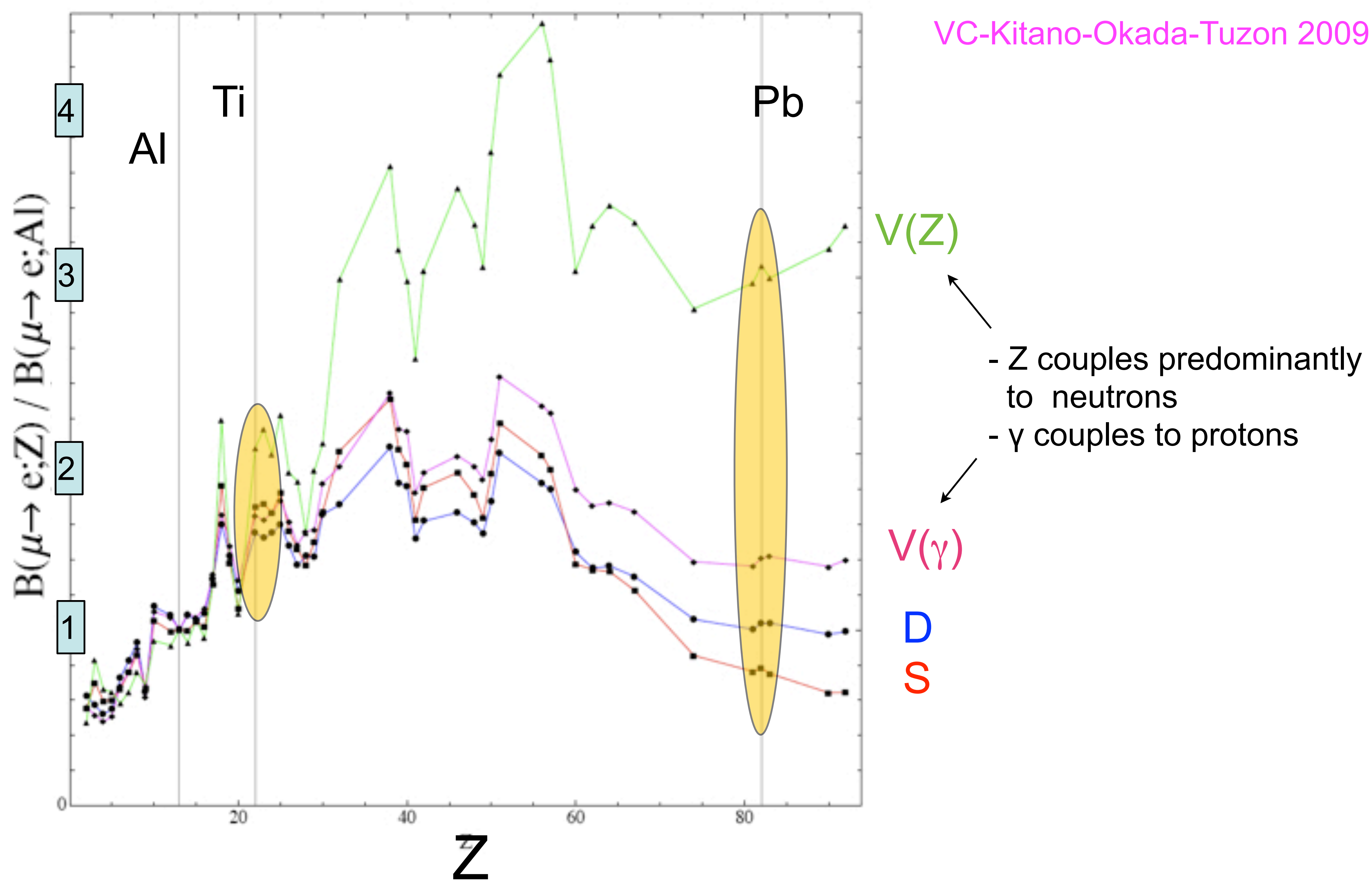
$$\boxed{\frac{B(\mu \to e, Z)}{B(\mu \to e\gamma)}}$$



Pattern:
1) Behavior of overlap integrals**
2) Total capture rate
(sensitive to nuclear structure)
3) Deviations would indicate
presence of scalar / vector terms

$O(\alpha/\pi)$

- μ→e conversion amplitude has non-trivial dependence on target, that distinguishes D,S,V underlying operators



- Essentially free of theory uncertainty (largely cancels in ratios)
- Discrimination: need ~5% measure of Ti/Al or ~20% measure of Pb/Al
- Ideal world: use Al and a large Z-target (D,V,S have largest separation)

Target dependence of mu-to-e

- How does this work? Conversion amplitude has non-trivial dependence on target nucleus, that distinguishes D,S,V underlying operators

$$M_{fi} \sim \langle e^-; A, Z| \int d^3x \, \hat{O}_\ell(x) \, \hat{O}_q(x) \; |\mu^-; A, Z\rangle$$

$$\sim \int d^3x \, \bar{\psi}_e O_\ell \psi_\mu \; \langle A, Z|\hat{O}_q|A, Z\rangle$$

Czarnecki-Marciano-Melnikov

Kitano-Koike-Okada

- Lepton wave-functions in EM field generated by nucleus

- Relativistic components of muon wave-function give different contributions to D,S,V overlap integrals. For example:

$$\bar{\psi}_e \gamma_0 \psi_\mu = \bar{\psi}_e \, \psi_\mu + O(v_\mu/c)$$

- Expect largest discrimination for heavy target nuclei

- Sensitive to <u>hadronic</u> and <u>nuclear</u> properties

$$\langle A, Z| \; \bar{q}\Gamma q \; |A, Z\rangle$$

$$\downarrow$$

$$f^{(q)}_{\Gamma N} \langle A, Z| \; \bar{\psi}_N \Gamma \psi_N \; |A, Z\rangle$$

$$\downarrow$$

$$\begin{aligned} \langle A, Z| \; \bar{\psi}_p(\gamma_0)\psi_p \; |A, Z\rangle &= Z \rho^{(p)} \\ \langle A, Z| \; \bar{\psi}_n(\gamma_0)\psi_n \; |A, Z\rangle &= (A - Z) \rho^{(n)} \end{aligned}$$

- Dominant sources of uncertainty:

  - Scalar matrix elements $\langle i|\, m_q q\bar{q}\, |i\rangle = \sigma_q^{(i)} \bar{\psi}_i \psi_i$

$$\sigma_{\pi N} = \frac{m_u + m_d}{2} \langle p|\bar{u}u + \bar{d}d|p\rangle \rightarrow 53\ ^{+21}_{-10}\ \text{MeV} \qquad \boxed{(45 \pm 15)\ \text{MeV}}$$

ChPT | JLQCD 2008 | Lattice range 2012 (Kronfeld 1203.1204)

$$y = \frac{2\ \langle p|\bar{s}s|p\rangle}{\langle p|\bar{u}u + \bar{d}d|p\rangle} \in [0, 0.4] \rightarrow [0, 0.05] \qquad \boxed{[0.04, 0.12]}$$

  - Neutron density (heavy nuclei)

** Qualitative behavior of overlap integrals

$\phi_e(x)$ → free outgoing electron wf

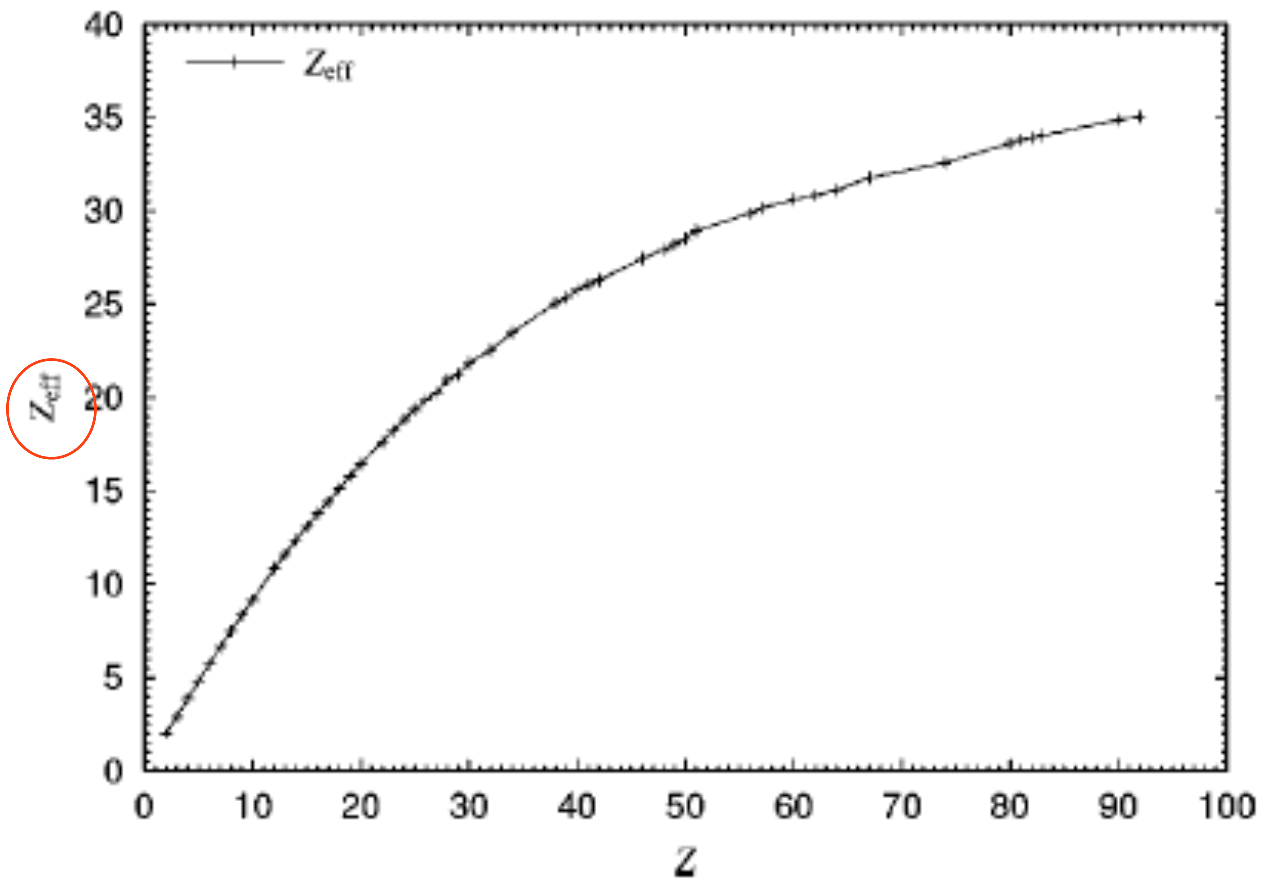
$\phi_\mu(x) \to \langle \phi_\mu(x) \rangle$ (average value)

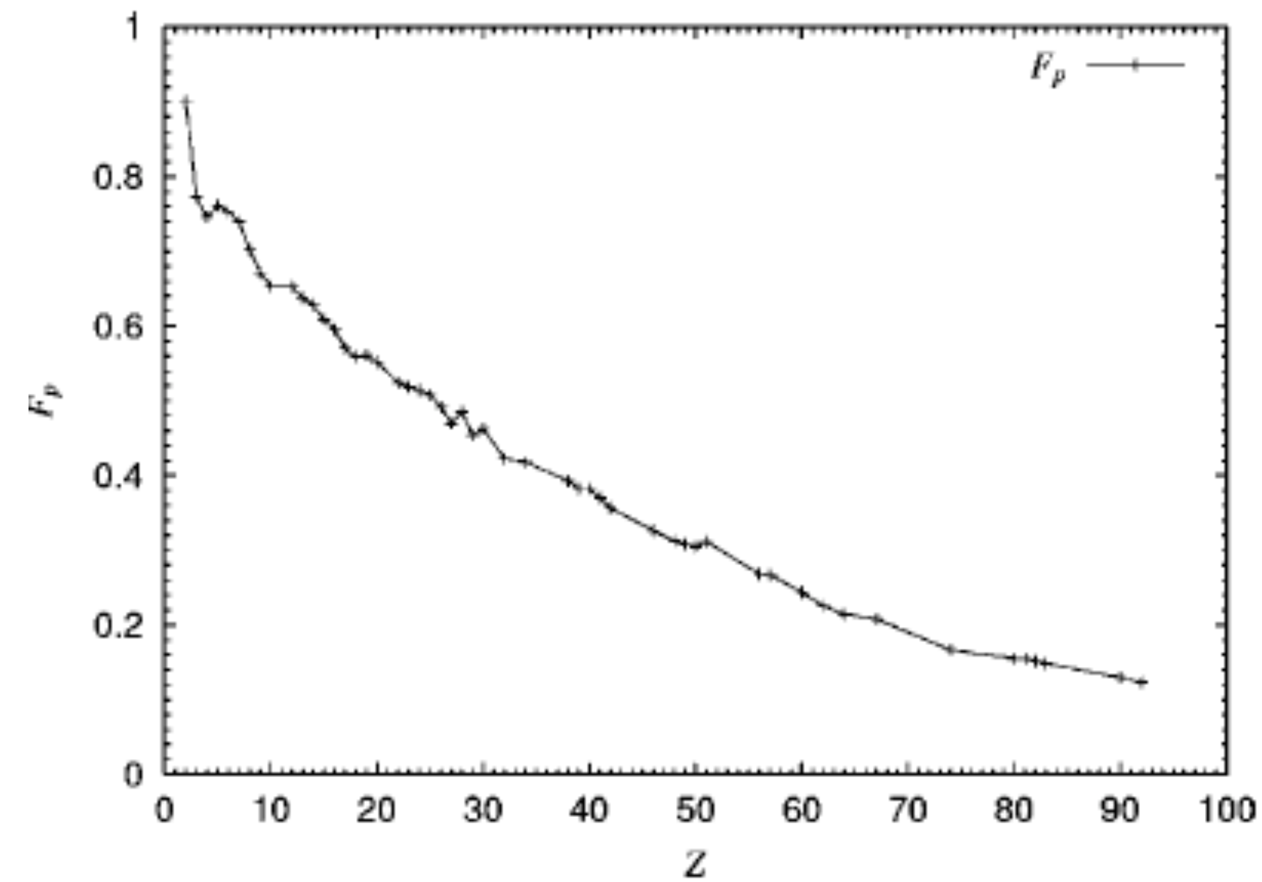
$$\boxed{I \sim \int d^3x \, \phi_e^*(x) \phi_\mu(x) \, \rho_p(x) \quad \to \quad \langle \phi_\mu \rangle \, F_p}$$

$p \sim m_\mu$

$$\langle \phi_\mu \rangle^2 = \int_0^\infty dr 4\pi r^2 (g_\mu^2 + f_\mu^2) \rho^{(p)} = \frac{4 m_\mu^3 \alpha^3 Z_{\rm eff}^4}{Z}$$

$$F_p = \int_0^\infty dr 4\pi r^2 \rho^{(p)} \frac{\sin m_\mu r}{m_\mu r}$$



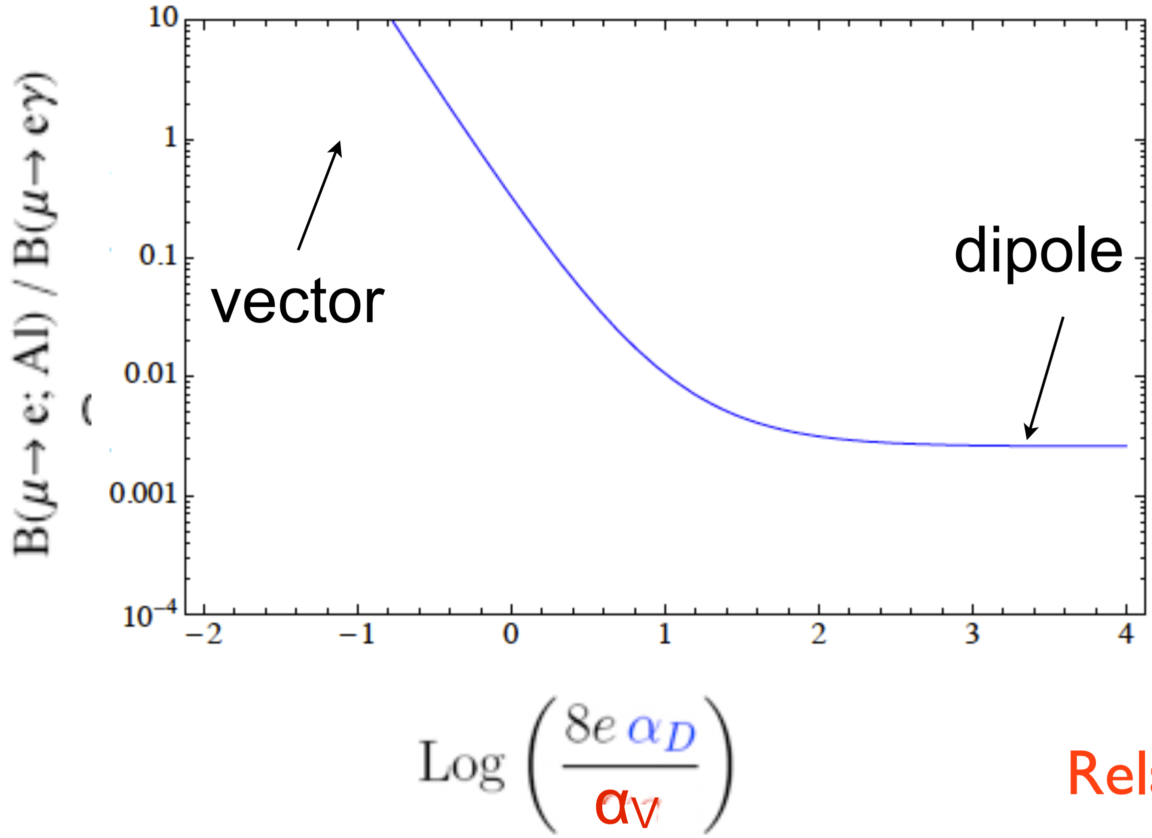


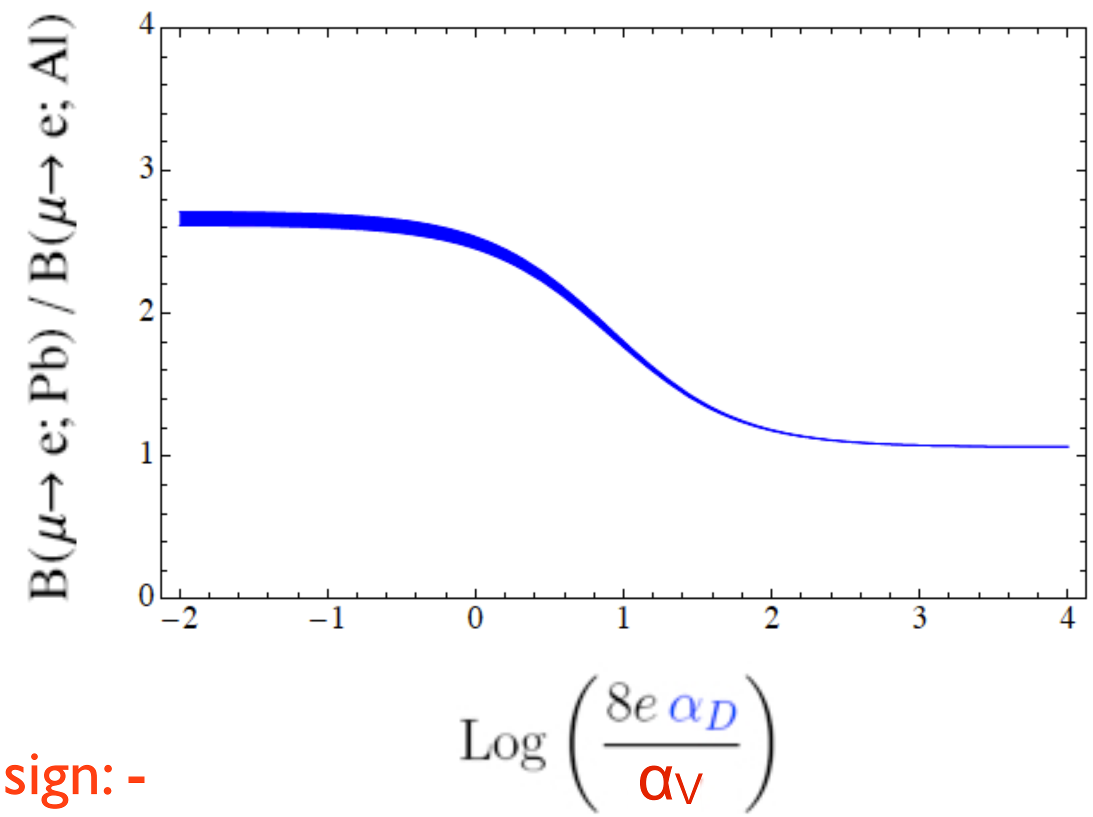
Kitano-Koike-Okada

VC-Kitano-Okada-Tuzon '09

- Beyond single operator dominance: V and D

Relative sign: +





Relative sign: -