Applied String Theory

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Durham Annual Theory Meeting 2012

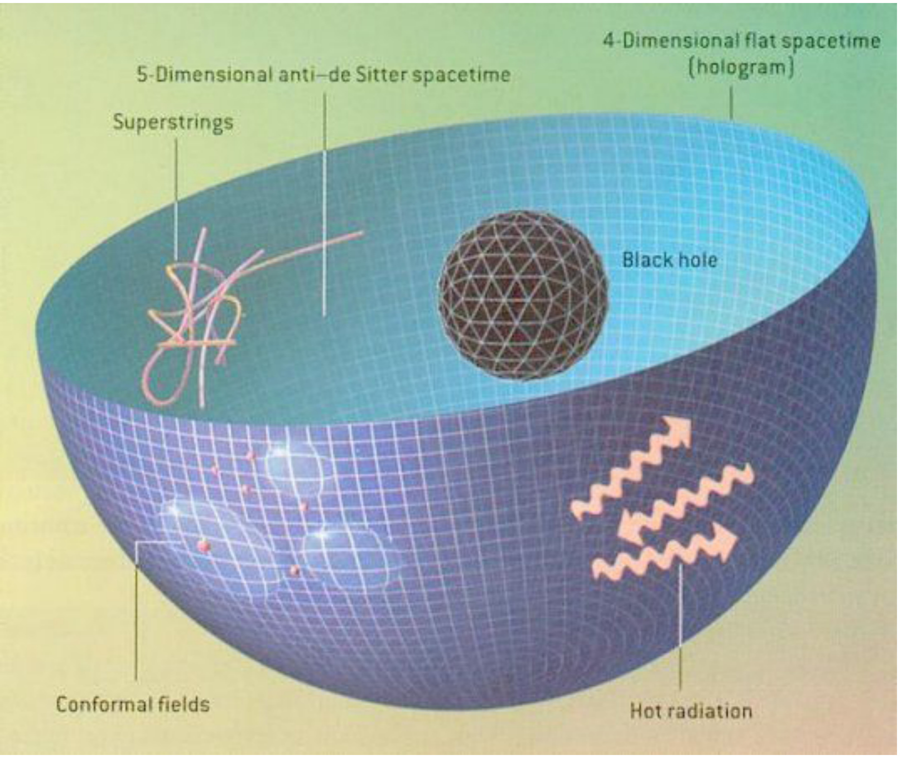
UNIVERSITY OF CAMBRIDGE

Science & Technology Facilities Council

erc

Holography (AdS/CFT)

Strongly interacting quantum field theory ⟷ Gravity in (at least) one dimension higher



Many Explicit Examples

e.g

d=3+1 *SU(N)* Yang-Mills
+ scalars and fermions
(with lots of supersymmetry)

$\Longleftrightarrow$

d=9+1 Type IIB string theory
on $AdS_5 \times S^5$

d=2+1 *SU(N)* x *SU(N)*
Chern-Simons + matter
(with lots of supersymmetry)

$\Longleftrightarrow$

d=9+1 Type IIA string theory
on $AdS_4 \times CP^3$

Two Major Directions

- Explore details of individual supersymmetric theories
- Study physical properties of generic theories

Physics in the Unrealistic Regime

<u>Why you shouldn't care</u>

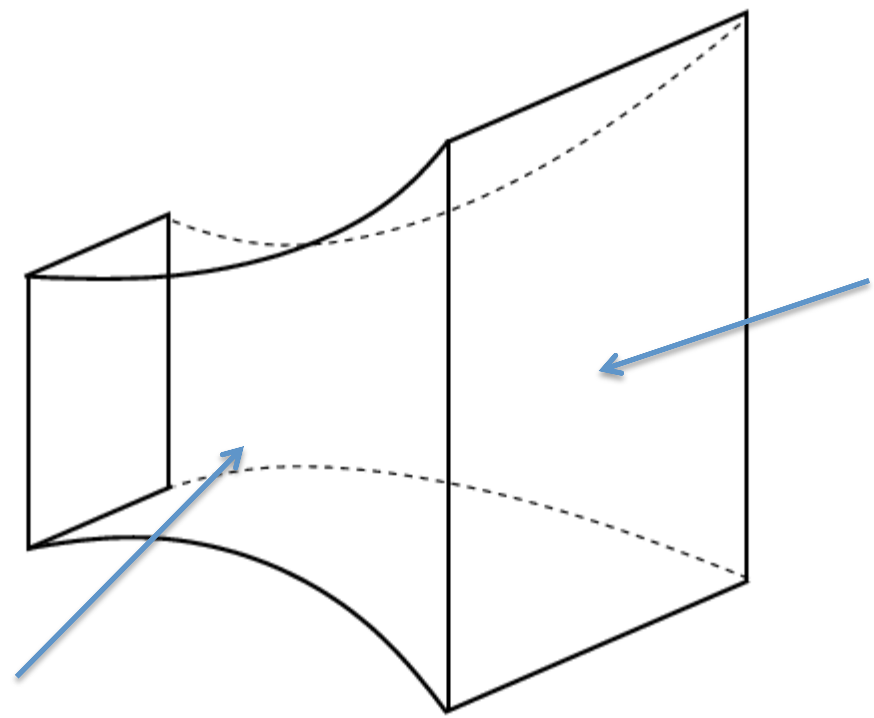
- Complicated theories
- Large N
- (Supersymmetric)

<u>Why you should</u>

- Strongly coupled
- Completely solvable
- Universal behaviour

Universal Behaviour

The Vacuum State

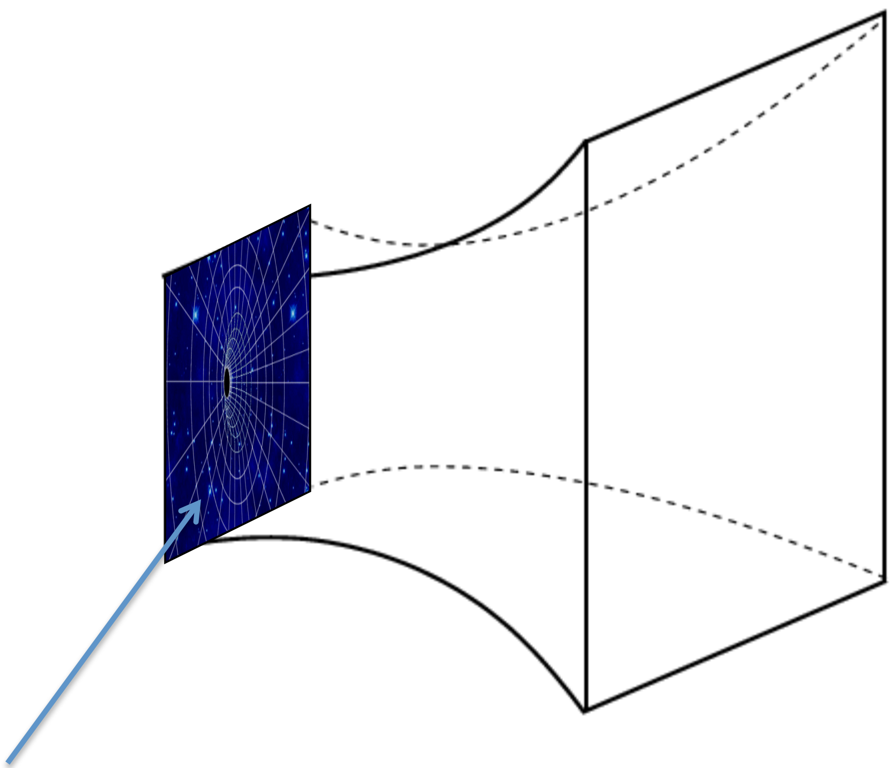


Boundary field theory

My attempt at drawing
Anti de-Sitter spacetime

$$ds^2 = \frac{L^2}{r^2}\left(dr^2 + \eta_{\mu\nu}dx^\mu dx^\nu\right)$$

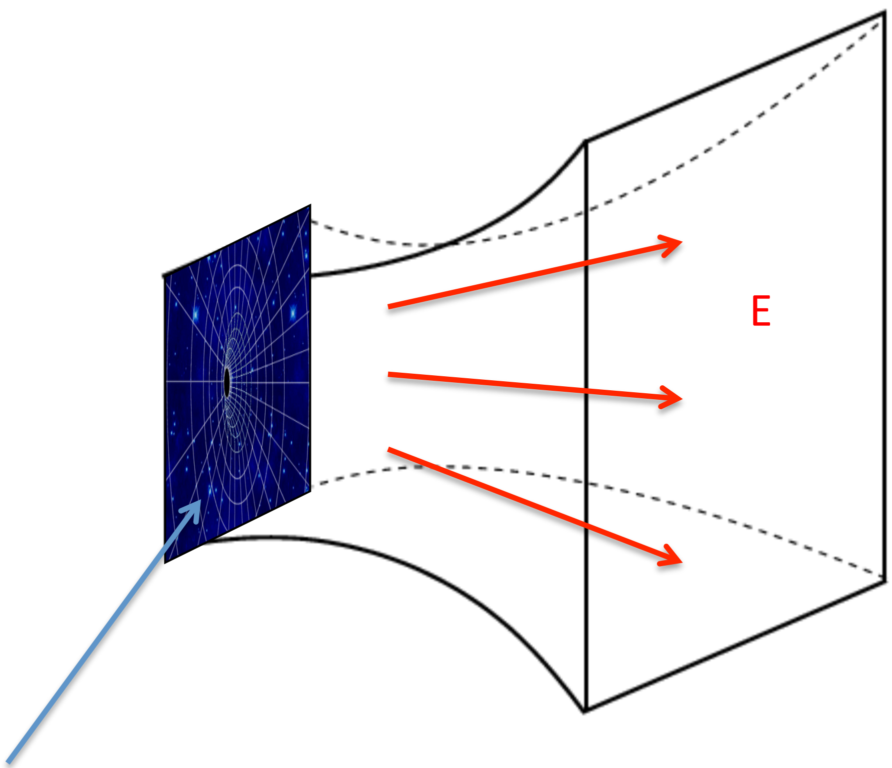
Heating up the Boundary Theory



Boundary field theory

- Black hole
- Hawking radiation = finite temperature

Finite Density Matter



Boundary field theory

- Reissner-Nordstrom black hole
- Hawking radiation = finite temperature
- Electric field = chemical potential

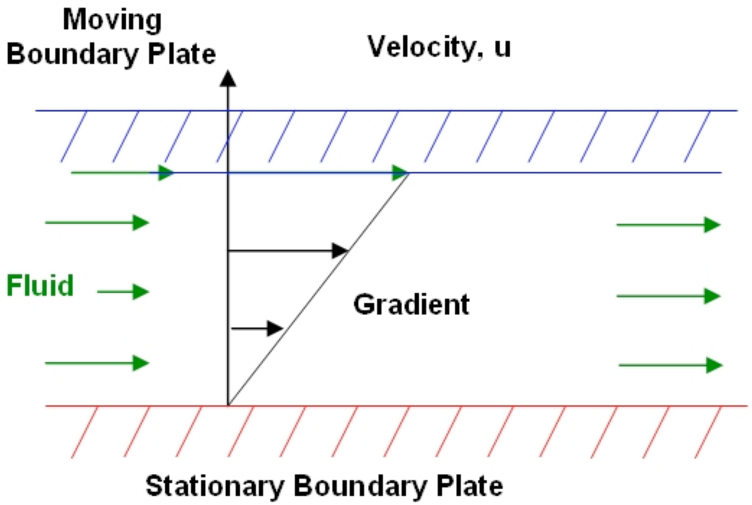
Three Stories

Story 1: Fluid Mechanics

Ultimate Low Energy Effective Theory

$\rho(\vec{x},t)$ $\quad$ $T(\vec{x},t)$ $\quad$ $\vec{u}(\vec{x},t)$

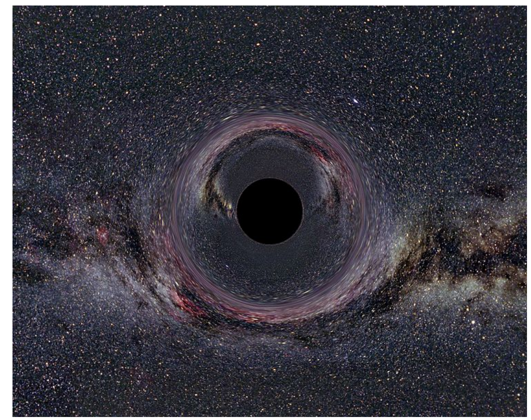
Shear Viscosity



$$\frac{\eta}{s} = \frac{1}{4\pi}$$

Kovtun, Policastro, Son, Starinets (2003)

Fluid-Gravity Correspondence



Navier-Stokes Equations

Einstein Equations

Battacharya, Hubeny, Minwalla, Rangamani (2009)

Happy Corollary

Extra terms in hydrodynamics

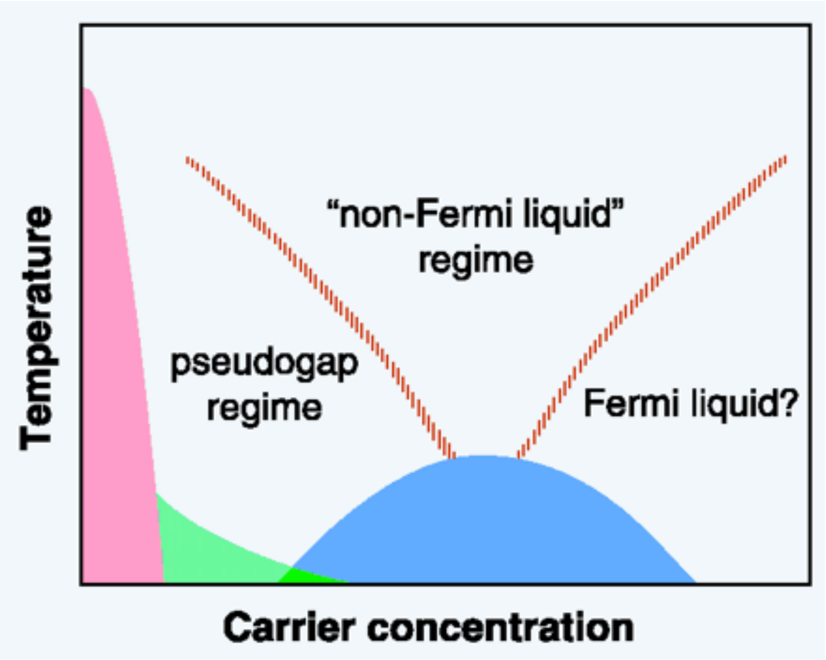
Minwalla et al. (2011)
Son and Surowka (2009)

For the Future

Singularities? Turbulence?

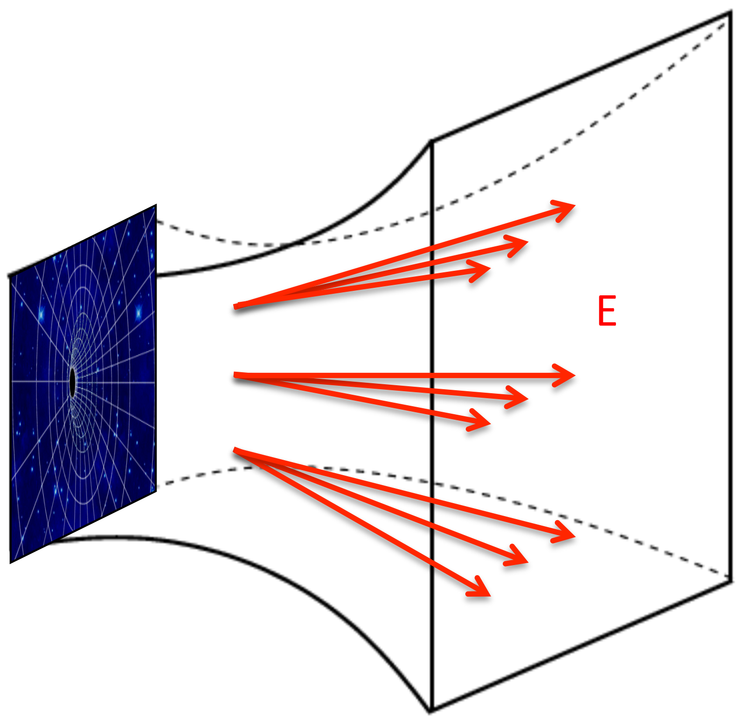
Story 2: Conductivity

Unconventional Superconductors



Cleanest examples in *d=2+1* dimensional planes

Ohm's Law



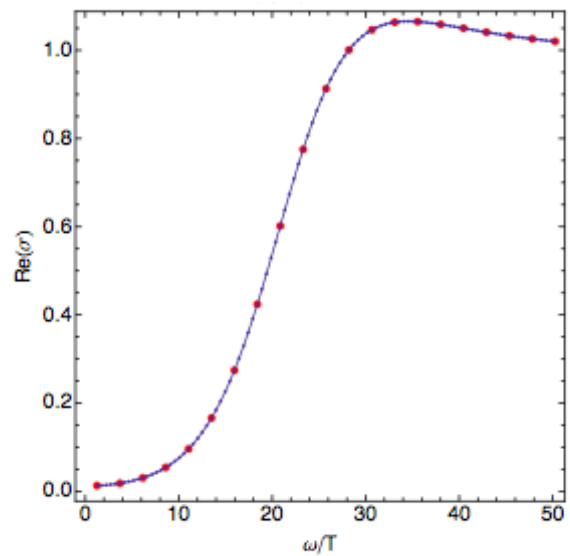
Boundary field theory
d=2+1

$$j(\omega) = \sigma(\omega) E(\omega)$$

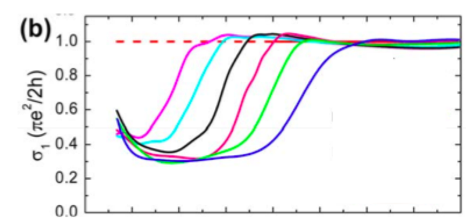
Optical Conductivity in d=2

Herzog, Kovtun, Sachdev and Son (2007); Hartnoll (2008)

$$j(\omega) = \sigma(\omega) E(\omega)$$



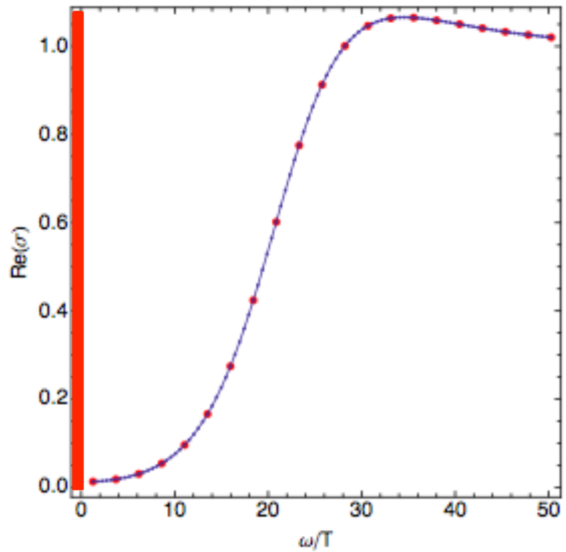
c.f. graphene



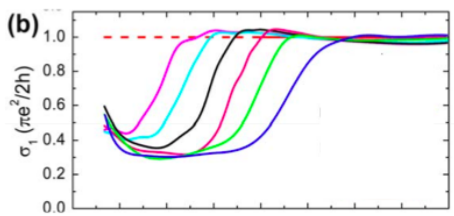
Optical Conductivity in d=2

Herzog, Kovtun, Sachdev and Son (2007); Hartnoll (2008)

$$\mathrm{Re}\,\sigma(\omega) \sim K\,\delta(\omega)$$



c.f. graphene



Because our theory is missing an underlying lattice

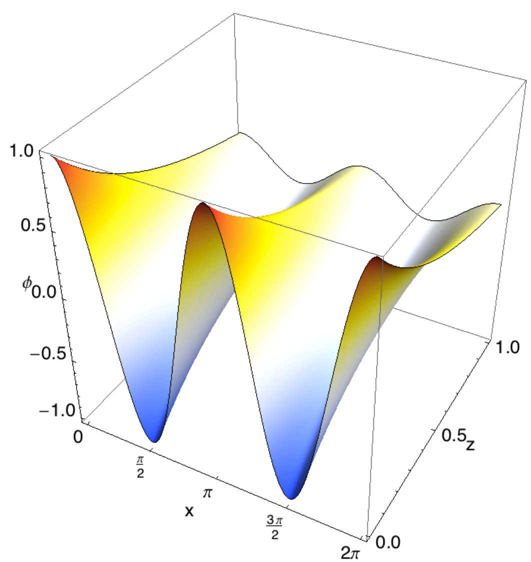
The lattice black hole

Horowitz, Santos, Tong (2012)

$$\mathcal{L} = \mathcal{L}_{\mathrm{QFT}} + \mu(x, y)\mathcal{Q}$$

Spatially dependent chemical potential

$$\mu = \mu[1 + Acos(k_L x)]$$

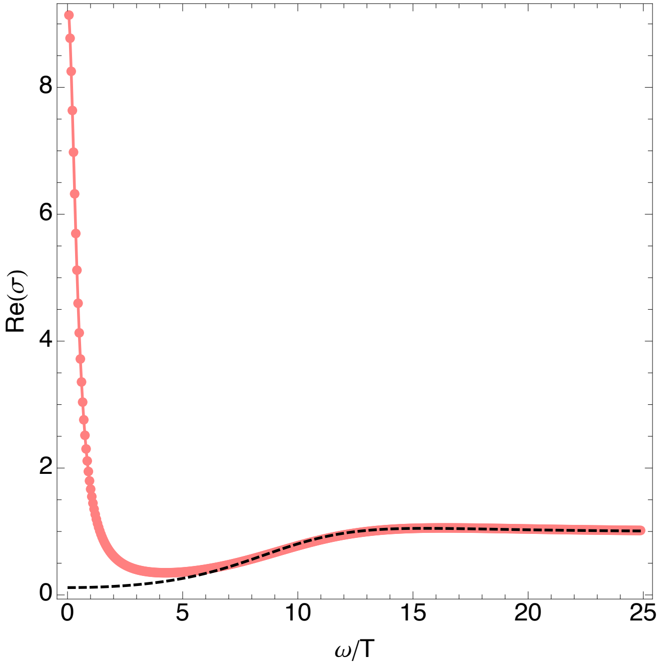


<u>Parameters:</u> $T,\ \mu,\ k_L,\ A$

$$ds^2 = \frac{L^2}{z^2}\Big[- g_{tt}(z,x)dt^2 + g_{zz}(z,x)dz^2 + g_{xx}(z,x)(dx + a(z,x)dz)^2 + g_{yy}(z,x)dy^2 \Big]$$

Optical Conductivity

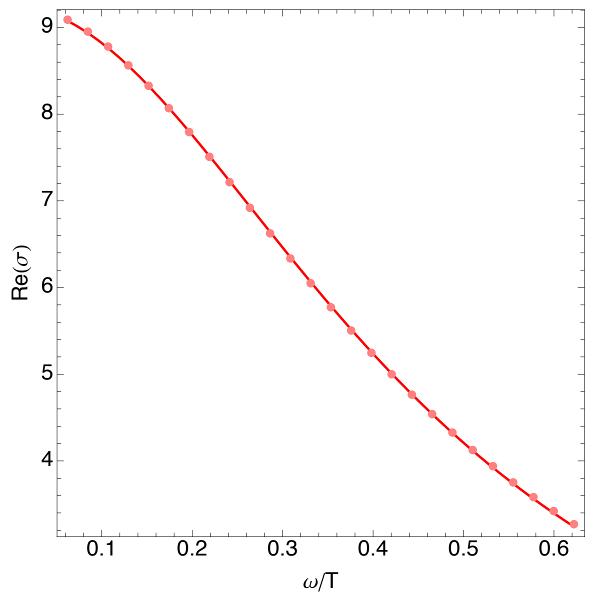
$$j(\omega) = \sigma(\omega) E(\omega)$$



Delta function spreads out

Low Frequency Behaviour

$$\omega \lesssim T$$

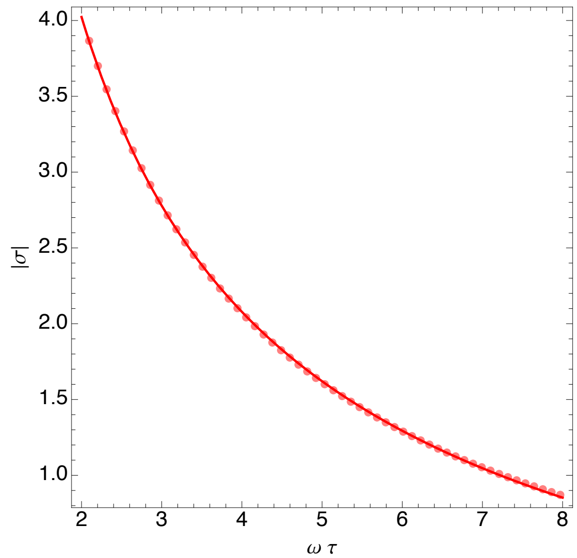


$$\sigma(\omega) = \frac{K\tau}{1 - i\omega\tau}$$

Classical Drude model from general relativity

Mid-Frequency Behaviour

$$\omega \gtrsim T$$

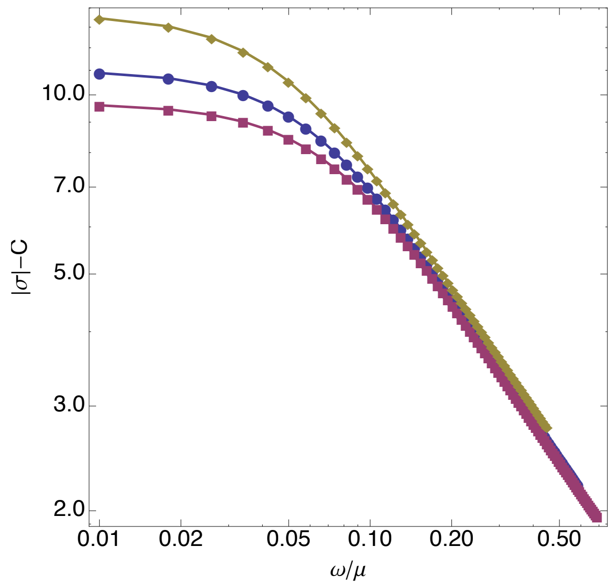


$$|\sigma(\omega)| = \frac{B}{\omega^{2/3}} + C$$

Surprising power-law behaviour

Robust Power-Law

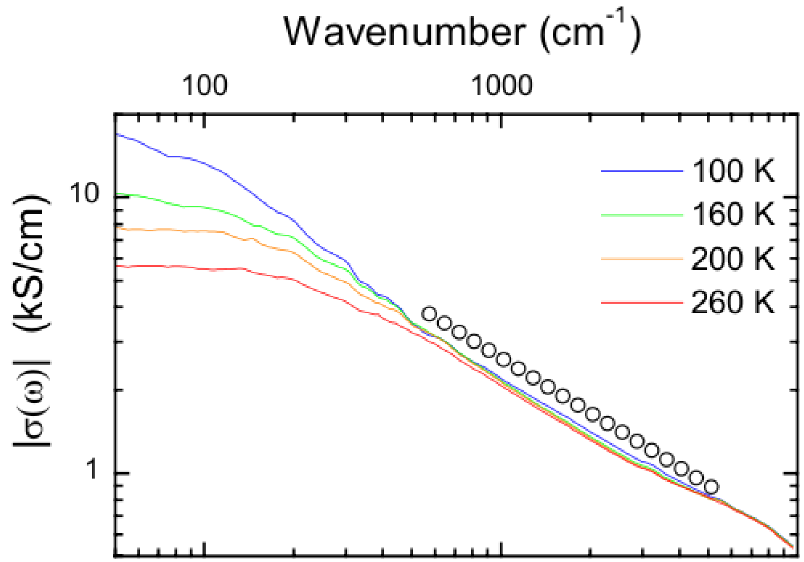
Log-log plots



$$|\sigma(\omega)| = \frac{B}{\omega^{2/3}} + C$$

B temperature independent

Scaling in High T_C Superconductors



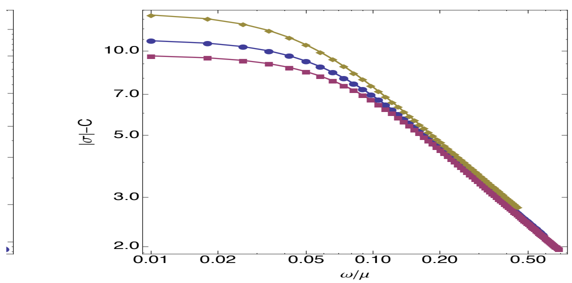
$$|\sigma(\omega)| = \frac{B}{\omega^{2/3}}$$

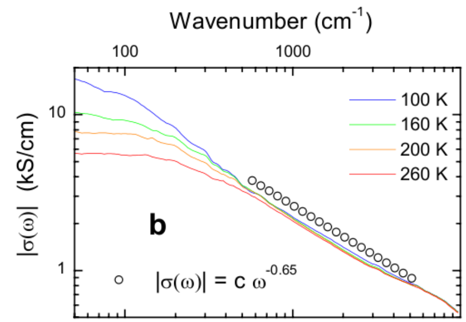
$Bi_2Sr_2Ca_{0.92}Y_{0.08}Cu_2O_{8+\delta}$

van der Marel et al. (2003)

For the Future

Why?





$$|\sigma(\omega)| \sim \omega^{-2/3}$$

Story 3: Non-Fermi Liquids

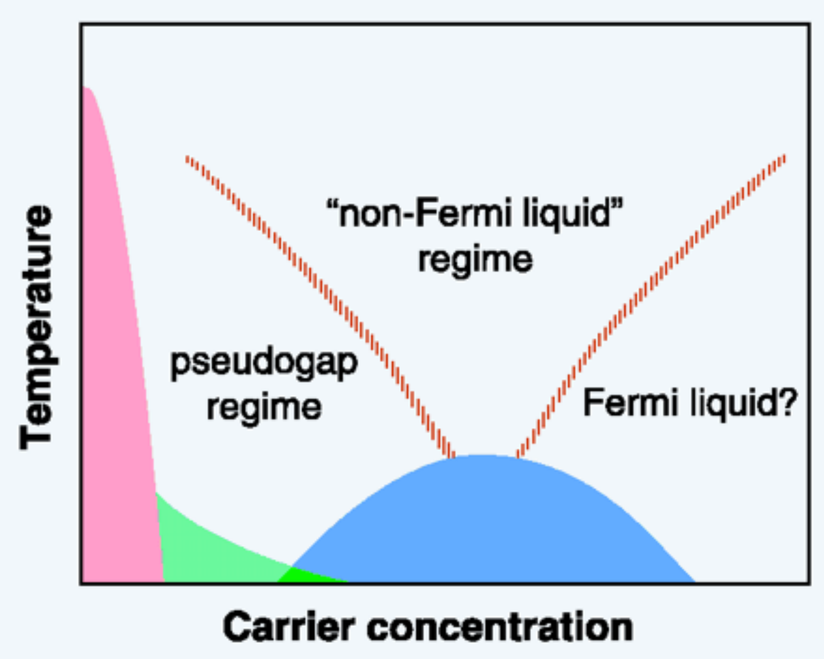
Landau Fermi Liquid Theory

Interacting Fermions
at density

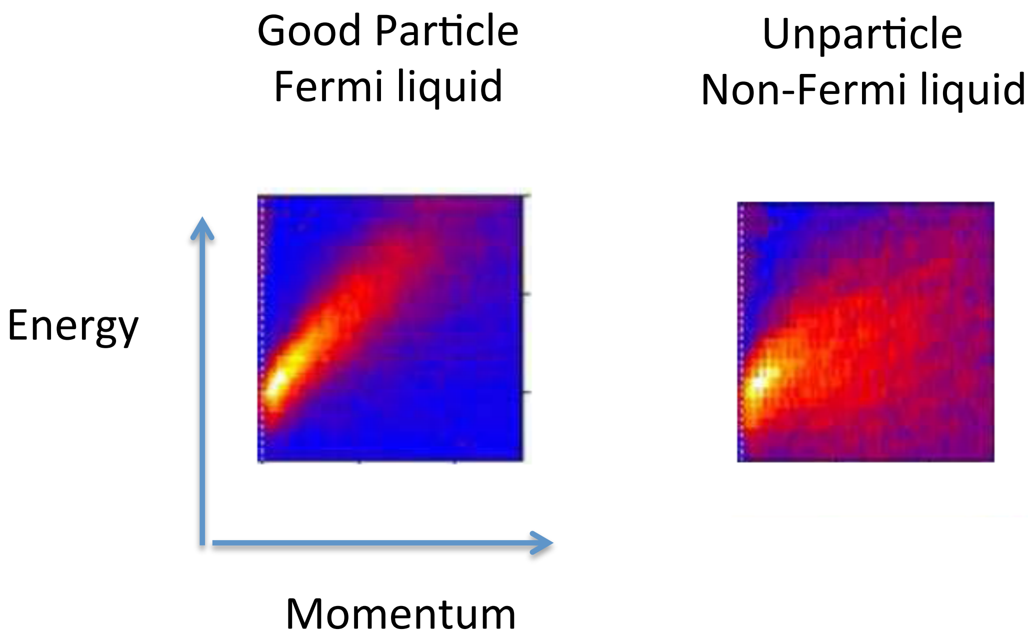
RG flow

Free fermions with
Fermi surface

Except…

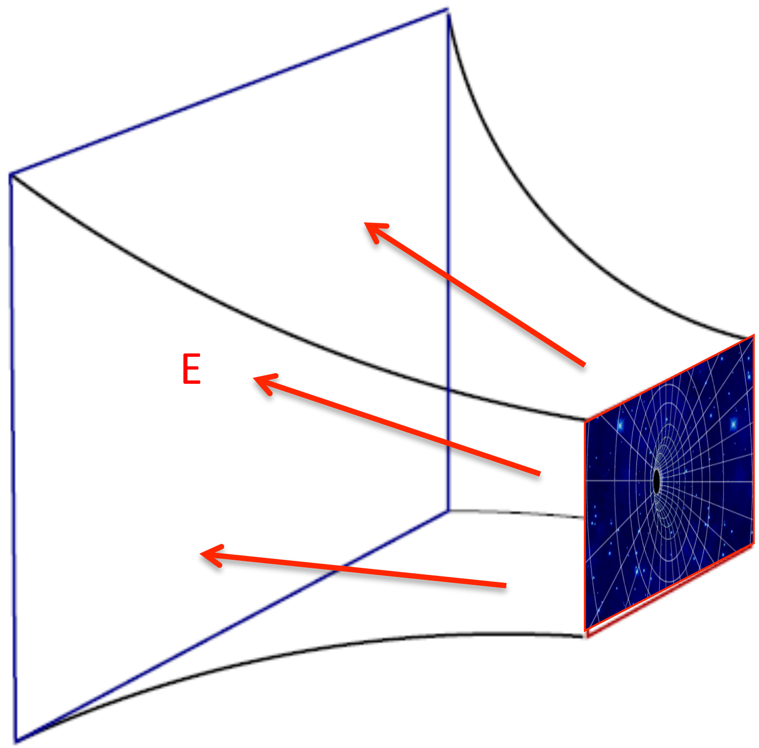

"non-Fermi liquid"
regime
pseudogap
regime
Fermi liquid?
Temperature
Carrier concentration

No Quasi-Particles



Major open problem in theoretical physics

Fermions in AdS/CFT



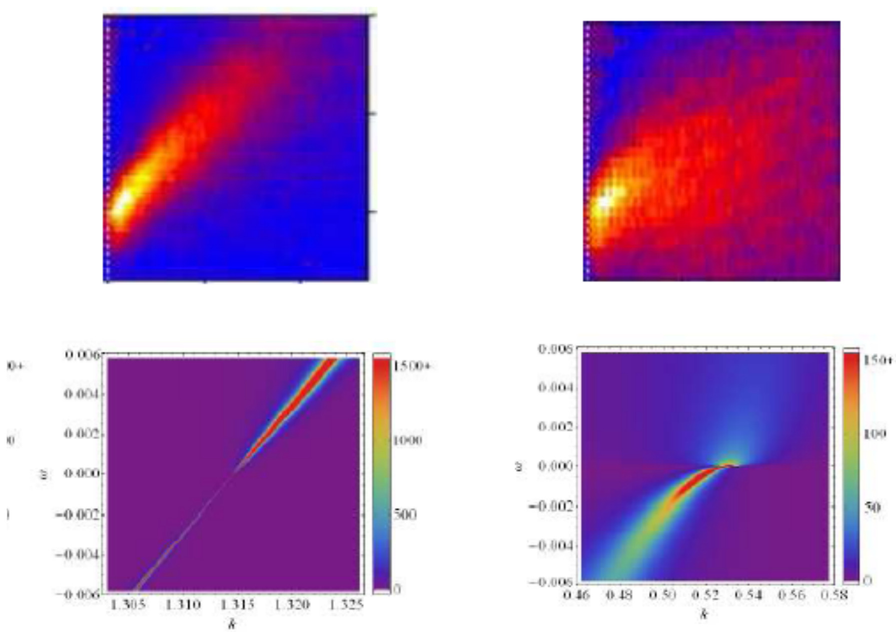
Solve the Dirac equation in this background: $\not{D}\psi - m\psi = 0$

Sung-Sik Lee (2009)

Non-Fermi Liquids

- Fermi Surface
  - Zero energy excitations at non-zero momentum

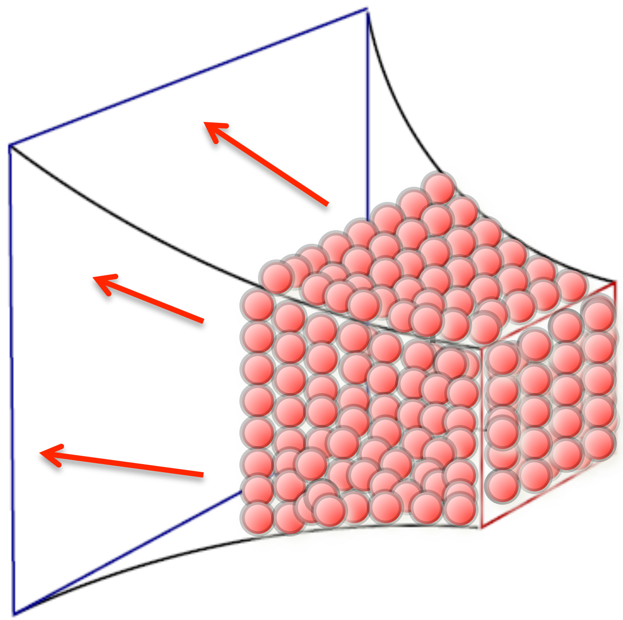
- Excitation spectrum depends on various parameters
  - Fermion mass, background geometry



Cubrovic et al. (Leiden)
Liu et al. (MIT) (2009)

But there's an instability…

Hartnoll, Polchinski, Siverstein and Tong (2009)



…and a star forms in the bulk.

This is an *electron star*. The low energy physics appears to be that of a Landau Fermi liquid. ???

Explored in detail by Hartnoll and collaborators.

For the Future

Understand the electron star in more detail

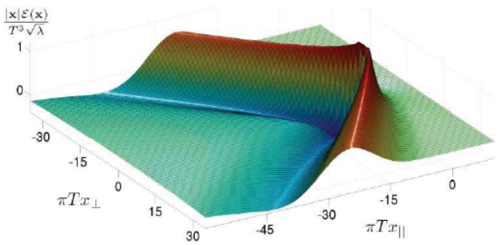
Summary

Holography very useful tool to explore aspects of strongly interacting matter

Much Much More…

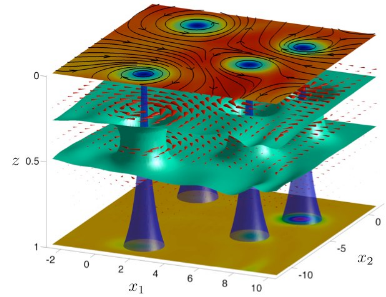
Jet Quenching

Chesler and Yaffe



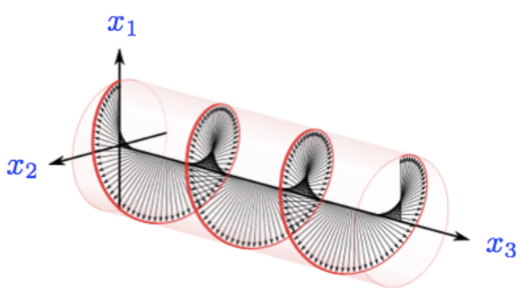
Quantum Turbulence

Adams, Chesler and Liu



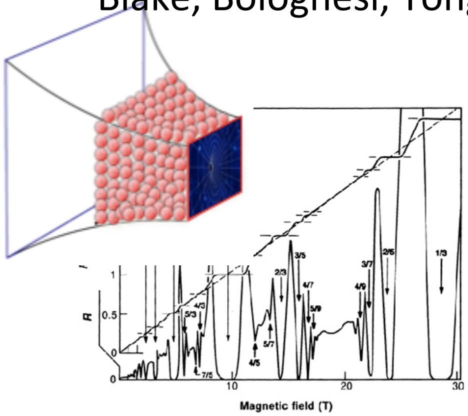
Crystal Formation

Donos, Gauntlett + many others



Lowest Landau Level Physics

Blake, Bolognesi, Tong, Wong



The End

Thank you for your attention

Extra Material

DC Resistivity

$$\rho \sim T^{2\nu-1}$$

- Exponent depends on lattice spacing: $\nu = \frac{1}{2}\sqrt{5 + 2(k/\mu)^2 - 4\sqrt{1 + (k/\mu)^2}}$

- This is characteristic of a *locally critical* theory i.e. $z \to \infty$

Hartnoll, Hofman (2012)

- More recently, a mechanism suggested to drive this to linear resistivity.

Donos, Hartnoll (2012)

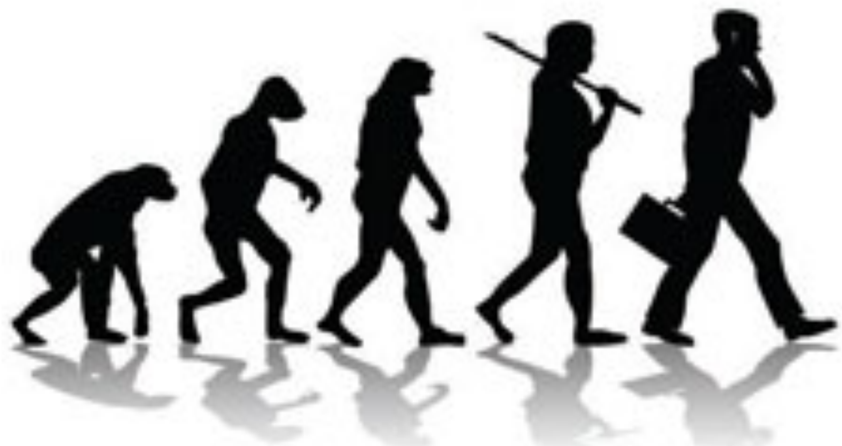
Navier-Stokes Equations

$\rho(\vec{x},t) \qquad T(\vec{x},t) \qquad \vec{u}(\vec{x},t)$

$$\partial^\mu T_{\mu\nu} = 0 \qquad\qquad T_{\mu\nu} = \ldots$$

$$\partial^\mu J_\mu = 0 \qquad\qquad J_\mu = \ldots$$

Evolution of Exact Results at Strong Coupling



- ‘70s and ‘80s: Quantities protected by symmetries and anomalies
    - Chiral Lagrangian
- ’90s and ’00s: Quantities protected by supersymmetry
    - BPS quantities, superpotentials
- ‘00s and ’10s: Anything you like
    - But only in very particular, large N, theories