CHARMLESS HADRONIC B DECAYS

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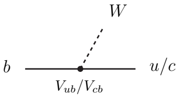
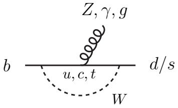
THE ROYAL SOCIETY

In a nutshell

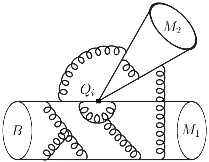
Excellent laboratory to probe the nature of flavour-changing quark transitions





Rich phenomenology: $B \to \pi\pi$, $B \to \pi K$, $B \to \phi K_S$, $B_s \to \phi\phi$, $B_s \to K^* K^*$, ...

The challenge:



The strategies:

- flavour symmetries:

  isospin, SU(3), U-spin, V-spin

- heavy-quark expansions:

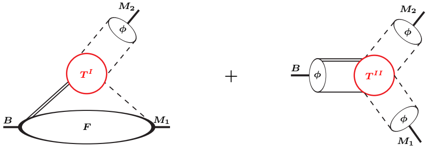
  QCDF, SCET, pQCD

QCD factorisation

[Beneke, Buchalla, Neubert, Sachrajda 99]

Hadronic matrix elements factorise in heavy quark limit $m_b \to \infty$

$$\langle M_1 M_2 | Q_i | \bar{B} \rangle \simeq F^{BM_1}(0) \int du \, T_i^I(u) \, \phi_{M_2}(u) + \int d\omega \, du \, dv \, T_i^{II}(\omega, u, v) \, \phi_B(\omega) \, \phi_{M_1}(v) \, \phi_{M_2}(u)$$



vertex corrections $T^I = 1 + \mathcal{O}(\alpha_s)$ spectator scattering $T^{II} = \mathcal{O}(\alpha_s)$

- valid to all orders in $\alpha_s(m_b)$ and to leading power in Λ_{QCD}/m_b
- strong phases from final-state interactions $\Rightarrow$ $\mathcal{O}(\alpha_s)$, $\mathcal{O}(1/m_b)$
- conceptually QCDF = SCET ≠ pQCD

  but phenomenological implementations are quite different

Comparison

[BBNS: Beneke, Buchalla, Neubert, Sachrajda 99]
[BPRS: Bauer, Pirjol, Rothstein, Stewart 04]
[pQCD: Keum, Li, Sanda 00]

	BBNS (QCDF)	BPRS (SCET)	pQCD
$\alpha_s(\sqrt{\Lambda m_b})$	perturbative	non-perturbative	perturbative
charm loops	perturbative (small phase)	non-perturbative (large phase from fit to data)	perturbative (small phase)
weak annihilation (power correction)	non-perturbative (model, arbitrary phase)	perturbative (zero bins, small phase)	perturbative (large phase)
perturbative calculation	towards NNLO	NLO	essentially LO
hadronic input	from lattice + QCD sum rules	from QCD sum rules + data, model $\xi_J^{BM}(z)$	from QCD sum rules + data, model $\phi_B(x,b)$

- different levels of calculations vs. phenomenological fitting
- no consensus on strong-rescattering phases

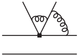
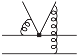
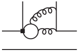
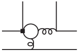
  $\Rightarrow$ theory predictions for direct CP asymmetries can differ a lot!

Perturbative calculation in QCDF

Two hard-scattering kernels for each operator insertion

$$\langle M_1 M_2 | Q_i | B \rangle \;\simeq\; F^{BM_1}\, T_i^I \otimes \phi_{M_2} \;+\; T_i^{II} \otimes \phi_B \otimes \phi_{M_1} \otimes \phi_{M_2}$$

strong phases $\sim \mathcal{O}(\alpha_s) \;\Rightarrow\;$ NNLO is first correction for direct CP asymmetries!

Status	2-loop vertex corrections (T_i^I)		1-loop spectator scattering (T_i^{II})	
Trees		[GB 07, 09] [Beneke, Huber, Li 09]		[Beneke, Jäger 05] [Kivel 06] [Pilipp 07]
Penguins		[in progress]		[Beneke, Jäger 06] [Jain, Rothstein, Stewart 07]

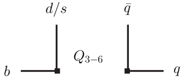
- first NNLO results for tree-dominated observables [GB, Pilipp 09; Beneke, Huber, Li 09]
- no NNLO results for direct CP asymmetries yet
- power-suppressed scalar penguin a_6^p known at NLO

Missing NNLO ingredient

Various contributions to up/charm QCD penguin amplitudes

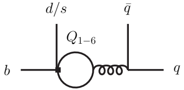
- tree insertions of penguin operators

  2-loop, similar to tree calculation



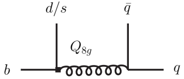
- penguin insertions of current-current and penguin operators

  2-loop, internal charm quark introduces additional scale



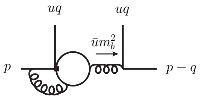
- insertions of magnetic dipole operator

  1-loop, much simpler [Kim, Yoon 11]



$\mathcal{O}(70)$ diagrams at NNLO

- 2 loops, 3 scales (m_b, um_b, m_c), 4 legs
- charm contribution has non-trivial threshold at $\bar{u}m_b^2 \gtrsim 4m_c^2$



Status of calculation

[GB, Beneke, Huber, Li in progress]

Automated reduction to scalar master integrals (IBP, Laporta)

$\Rightarrow$ $\mathcal{O}(20)$ additional master integrals compared to 2-loop tree calculation

Calculation of master integrals

- analytical approach for leading (and some subleading) singularities $\Rightarrow$ 2dHPLs
- two independent numerical implementations for remainder

  Mellin-Barnes + sector decomposition + Feynman parameters $\Rightarrow$ agreement $\sim 10^{-4}$

Current status:

- $Q^u_{1,2}$: complete, fully analytic results
- $Q^c_{1,2}$: numerical results for kernels $T^I_i(u)$, working on $\int du\; T^I_i(u)\, \phi_{M_2}(u)$
- $Q_{3\text{-}6}$: 2-loop matrix elements complete, need UV and IR subtractions

Tree amplitudes

Perturbative structure of colour-allowed and colour suppressed tree to NNLO

$$
\begin{aligned}
\alpha_1(\pi\pi) &= [\,1.008\,]_{V_0} + [\,0.022 + 0.009\,i\,]_{V_1} + [\,0.024 + 0.026\,i\,]_{V_2} \\
&\quad - [\,0.014\,]_{S_1} - [\,0.016 + 0.012\,i\,]_{S_2} - [\,0.008\,]_{1/m_b} \\
&= 1.015^{+0.020}_{-0.029} + (0.023^{+0.015}_{-0.015})\,i
\end{aligned}
$$

$$
\begin{aligned}
\alpha_2(\pi\pi) &= [\,0.224\,]_{V_0} - [\,0.174 + 0.075\,i\,]_{V_1} - [\,0.029 + 0.046\,i\,]_{V_2} \\
&\quad + [\,0.084\,]_{S_1} + [\,0.037 + 0.022\,i\,]_{S_2} + [\,0.052\,]_{1/m_b} \\
&= 0.194^{+0.130}_{-0.095} - (0.099^{+0.057}_{-0.056})\,i
\end{aligned}
$$

- individual NNLO corrections significant but large cancellations
- PT well-behaved at scales m_b and $\sqrt{m_b\Lambda} \simeq 1.5$ GeV
- α_1: stable under radiative corrections, precise prediction
- α_2: real part dominated by spectator scattering $\sim \; \lambda_B^{-1} = \int \frac{d\omega}{\omega}\, \phi_B(\omega)$

  $\Rightarrow$ substantial hadronic uncertainties, but $\arg(\alpha_2/\alpha_1)$ small

$B \to \pi\pi/\pi\rho/\rho\rho$

[GB, Pilipp 09]

CP-averaged branching ratios in units of 10^{-6}

Mode	QCDF	B	Experiment
$\pi^-\pi^0$	$6.22^{+2.37}_{-2.01}$	5.46	$5.59^{+0.41}_{-0.40}$
$\rho_L^-\rho_L^0$	$21.0^{+8.5}_{-7.3}$	21.3	$22.5^{+1.9}_{-1.9}$
$\pi^-\rho^0$	$9.34^{+4.00}_{-3.23}$	10.4	$8.3^{+1.2}_{-1.3}$
$\pi^0\rho^-$	$15.1^{+5.7}_{-5.0}$	11.9	$10.9^{+1.4}_{-1.5}$
$\pi^+\pi^-$	$8.96^{+3.78}_{-3.32}$	5.21	$5.16^{+0.22}_{-0.22}$
$\pi^0\pi^0$	$0.35^{+0.37}_{-0.21}$	0.63	$1.55^{+0.19}_{-0.19}$
$\pi^+\rho^-$	$22.8^{+9.1}_{-8.0}$	13.2	$15.7^{+1.8}_{-1.8}$
$\pi^-\rho^+$	$11.5^{+5.1}_{-4.3}$	8.41	$7.3^{+1.2}_{-1.2}$
$\pi^\pm\rho^\mp$	$34.3^{+11.5}_{-10.0}$	21.6	$23.0^{+2.3}_{-2.3}$
$\pi^0\rho^0$	$0.52^{+0.76}_{-0.42}$	1.64	$2.0^{+0.5}_{-0.5}$
$\rho_L^+\rho_L^-$	$30.3^{+12.9}_{-11.2}$	22.3	$23.6^{+3.2}_{-3.2}$
$\rho_L^0\rho_L^0$	$0.44^{+0.66}_{-0.37}$	1.33	$0.69^{+0.30}_{-0.30}$

B: mimics enhanced colour-suppressed amplitude
(with $\lambda_B \to \lambda_B/2$ and smaller form factors)

- theory uncertainties highly correlated (form factors, $|V_{ub}|$)
- colour-suppressed modes $\pi^0\pi^0/\pi^0\rho^0/\rho^0\rho^0$ uncertain (λ_B and $1/m_b$)
- overall preference for enhanced colour-suppressed amplitude
- Belle update on $A_{CP}(\pi^+\pi^-)$ in better agreement with QCDF

[for a similar analysis cf. Beneke, Huber, Li 09]

Semileptonic ratios

Eliminate dependence on form factors and $|V_{ub}|$ via

$$\mathcal{R}_{M_3}(M_1 M_2) = \frac{\Gamma(B \to M_1 M_2)}{d\Gamma(B \to M_3 \ell \nu)/dq^2|_{q^2=0}}$$

Mode	QCDF	B	Experiment
$\mathcal{R}_\pi(\pi^- \pi^0)$	$0.70^{+0.12}_{-0.08}$	0.95	$0.81^{+0.14}_{-0.14}$
$\mathcal{R}_\rho(\rho_L^- \rho_L^0)$	$1.91^{+0.32}_{-0.23}$	2.38	n.a.
$\mathcal{R}_\rho(\pi^- \rho^0)$	$0.85^{+0.22}_{-0.14}$	1.16	n.a.
$\mathcal{R}_\pi(\pi^0 \rho^-)$	$1.71^{+0.27}_{-0.24}$	2.07	$1.57^{+0.32}_{-0.32}$
$\mathcal{R}_\pi(\pi^+ \pi^-)$	$1.09^{+0.22}_{-0.20}$	0.97	$0.80^{+0.13}_{-0.13}$
$\mathcal{R}_\pi(\pi^+ \rho^-)$	$2.77^{+0.32}_{-0.31}$	2.46	$2.43^{+0.47}_{-0.47}$
$\mathcal{R}_\rho(\pi^- \rho^+)$	$1.12^{+0.20}_{-0.14}$	1.01	n.a.
$\mathcal{R}_\rho(\rho_L^+ \rho_L^-)$	$2.95^{+0.37}_{-0.35}$	2.68	n.a.
$R(\rho_L^- \rho_L^0 / \rho_L^+ \rho_L^-)$	$0.65^{+0.16}_{-0.11}$	0.89	$0.89^{+0.14}_{-0.14}$
$R(\pi^- \pi^0 / \pi^+ \pi^-)$	$0.65^{+0.19}_{-0.14}$	0.98	$1.01^{+0.09}_{-0.09}$

- theory uncertainties largely reduced (still correlated)
- satisfactory description of clean observables
- data on $B \to \rho \ell \nu$ spectrum currently insufficient

B: mimics enhanced colour-suppressed amplitude
(with $\lambda_B \to \lambda_B/2$ and smaller form factors)

Tree-dominated B_s decays

[GB, CKM 2010]

CP-averaged branching ratios in units of 10^{-6}

Mode	QCDF	B	Experiment
$\pi^- K^+$	$8.73^{+5.77}_{-4.60}$	4.88	$5.4^{+0.6}_{-0.6}$
$\pi^0 K^0$	$0.50^{+0.71}_{-0.35}$	1.12	n.a.
$\pi^- K^{*+}$	$15.4^{+8.6}_{-7.0}$	11.0	n.a.
$\pi^0 K^{*0}$	$0.39^{+0.58}_{-0.26}$	0.90	n.a.
$\rho^- K^+$	$22.4^{+14.7}_{-11.6}$	12.5	n.a.
$\rho^0 K^0$	$0.73^{+1.28}_{-0.58}$	2.24	n.a.
$\rho_L^- K_L^{*+}$	$40.7^{+22.4}_{-18.3}$	29.1	n.a.
$\rho_L^0 K_L^{*0}$	$0.70^{+1.07}_{-0.54}$	1.87	n.a.
$\rho^- K^+/\pi^- K^+$	$2.57^{+0.31}_{-0.26}$	2.57	n.a.
$\rho_L^- K_L^{*+}/\pi^- K^{*+}$	$2.64^{+0.31}_{-0.33}$	2.65	n.a.

B: mimics enhanced colour-suppressed amplitude
(with $\lambda_{B_s} \to \lambda_{B_s}/2$ and smaller form factors)

- hadronic parameters less well known (form factors, λ_{B_s})
- much simpler pattern of annihilation contributions
- clean ratios of colour-allowed modes may be used to test charming penguins [Zhu 10]

Key hadronic parameter: λ_B

What do we know about $\lambda_B^{-1}(\mu) = \int \frac{d\omega}{\omega}\, \phi_B(\omega;\mu)$?

- scale dependence known to one-loop order [Lange, Neubert 03]

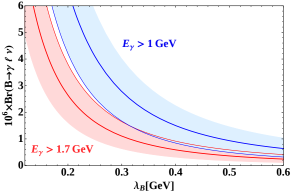
  expansion in eigenfunctions of evolution kernel [GB, Feldmann, Wang, Yip 13]

- QCD sum rule + OPE-based estimates [Braun, Ivanov, Korchemsky 03; Lee, Neubert 05; Kawamura, Tanaka 08]

  $\Rightarrow\ \lambda_B(1\text{GeV}) \simeq (350 - 500)$ MeV

- but $\pi\pi/\pi\rho/\rho\rho$ branching ratios seem to prefer $\sim$ 200 MeV ?

λ_B can be measured in $B \to \gamma\ell\nu$ decays with energetic photons



- state-of-the-art analyses [Beneke, Rohrwild 11; Braun, Khodjamirian 12]

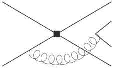
  (NLL, ”tree” $1/m_b$-corrections)

- Babar 09 data $\Rightarrow\ \lambda_B(1\text{GeV}) > 115$ MeV

  good prospects at super flavour factory

Weak annihilation

Power-suppressed annihilation amplitudes introduce model dependence



$\Rightarrow \quad X_A = \left(1 + \rho_A\, e^{i\phi_A}\right) \ln \frac{m_B}{\Lambda_h}$ IR-cutoff $\Lambda_h = 0.5$ GeV

Two-parameter model: $\rho_A \leq 1$ and arbitrary soft-rescattering phase ϕ_A (universal)

Insights from pure annihilation decays

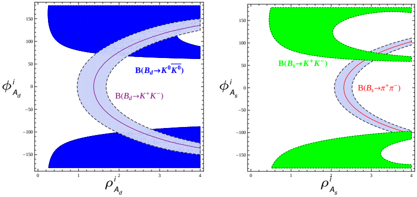
	"QCDF" (S4)	
$10^6\,\mathrm{Br}(B_d \to K^+ K^-) = 0.12 \pm 0.05$	0.07	($\Delta D = 1$, exchange topology)
$10^6\,\mathrm{Br}(B_s \to \pi^+ \pi^-) = 0.73 \pm 0.14$	0.16	($\Delta S = 1$, penguin annihilation)

$\Rightarrow$ challenges universal annihilation model

global $\pi\pi/\pi K/KK$ analysis gives

$\rho_A^f \sim 1.6, \;\; \rho_{A_d}^i \sim 2.5, \;\; \rho_{A_s}^i \sim 3.0$

[Wang, Zhu 13]



$B \to VV$

Angular analysis of $B \to V(\to PP)V(\to PP)$ gives access to helicity amplitudes $\mathcal{A}_0, \mathcal{A}_\pm$

- power counting (for $\bar{B}$) $\mathcal{A}_0 : \mathcal{A}_- : \mathcal{A}_+ \sim 1 : \frac{\Lambda_{QCD}}{m_b} : \left(\frac{\Lambda_{QCD}}{m_b}\right)^2$

  (violated by QED effects, relevant for neutral vector mesons) [Beneke, Rohrer, Yang 05]

- $\mathcal{A}_0$ on same theoretical footing as PP/PV decays

  $\Rightarrow$ clean description of $B \to V_L V_L$ decays

- $\mathcal{A}_\pm$ (or $\mathcal{A}_{||}, \mathcal{A}_\perp$) do not factorise

  $\Rightarrow$ predictions for polarisation observables are model dependent

- in practise fit transverse penguin amplitude to data (e.g. $B \to \phi K^*$) [Beneke, Rohrer, Yang 06]

  $\Rightarrow$ predict other polarisation observables using QCDF

- transverse penguin amplitude sensitive to weak annihilation [Kagan 04]

  $\Rightarrow$ possible explanation of the polarisation puzzle

$B_s \to \phi\phi$

Pure $b \to s$ penguin decay $\Rightarrow$ CP asymmetries are clean null tests in SM

- robust SM test with few theory input [Bartsch, Buchalla, Kraus 08]

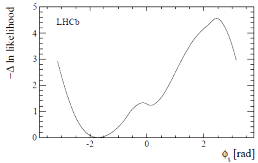
  (uses QCDF only to predict difference of charm and up penguin in $B_s \to \phi_L \phi_L$)

  $$\Rightarrow \quad S_{\phi\phi}, |C_{\phi\phi}| \lesssim \lambda^2 \eta \left[\frac{\text{Br}(B_s \to \phi_L \phi_L)}{15 \cdot 10^{-6}} \right]^{-1/2} \lesssim 0.02$$

  first LHCb measurement (1fb^{-1}):

  $$\phi_s \in [-2.46, -0.76] \text{ at 68\% CL}$$

  (p-value of SM prediction 16%)



- polarisation observables

  $$f_L = 0.329 \pm 0.033 \pm 0.017 \text{ (LHCb)} \qquad f_L = 0.48^{+0.26}_{-0.27} \text{ (QCDF)}$$

  triple product asymmetries consistent with SM expectations

Conclusion

We are about to complete the NNLO calculation in hadronic B decays —
phenomenlogocial update of all $B \to PP/PV/VV$ observables largely overdue

Lattice and QCD sum rule calculations provide valuable input for our predictions —
happy to see progress on decay constant and form factor determinations

Experiments can help to further strengthen our predictions —
examples include $B_d \to K^+K^-$, $B_s \to \pi^+\pi^-$, $B \to \gamma\ell\nu$ and $B \to \rho\ell\nu$

Backup slides