

# Isospin $B \rightarrow K^{(*)} l+l$ in and beyond the SM

- a) Generic remarks  $B \rightarrow K^{(*)} l+l$  and isospin
- b) What drives isospin violation
- c) Isospin asymmetries in and beyond SM
- d) Remarks on low recoil region (high  $q^2$ )

-- CHARM-INTERMEZZO --

CP<sup>3</sup> - Origins

Particle Physics & Origin of Mass

The Higgs Centre  
for Theoretical Physics

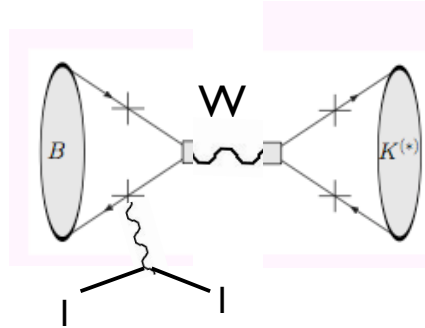
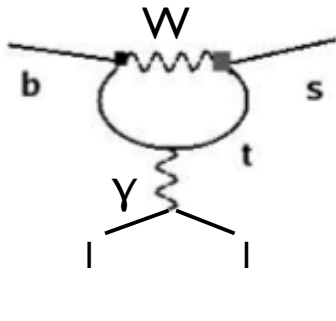
Roman Zwicky

Higgs-centre for theoretical physics -- Edinburgh University  
in collaboration James Lyon & Maria Dimou

UK flavour workshop --- 5-7 Sep 2013 Durham IPPP

# Why $B \rightarrow K^{(*)} l_+ l_-$ ? And what it is.

- 1) It's an FCNC ( $b \rightarrow s$ -transition); thus loop suppressed in SM
- 2) It's measured at experimental facilities (currently LHCb future KEK2 past: Belle/BaBar/CDF)



Wilson coefficient (UV-physics SM & **BSM?**)      operator (IR-physics)

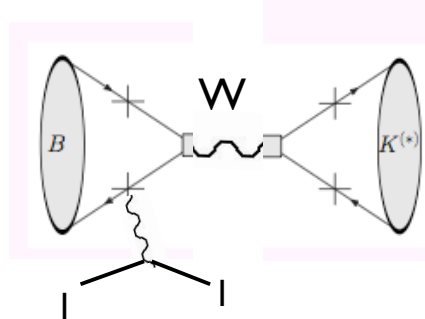
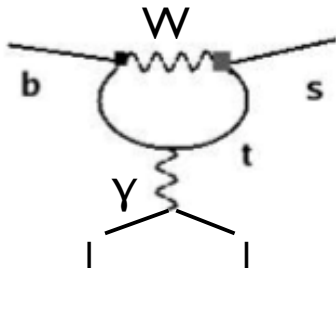
$$H_{\text{eff}} = \sum_i C^i(\mu_F) O^i(\mu_F)$$

$$\mathcal{A} = \langle Vll | H_{\text{eff}} | B \rangle = \sum_i C_i(m_b) \langle Vll | O_i(m_b) | B \rangle$$

**Amplitude**

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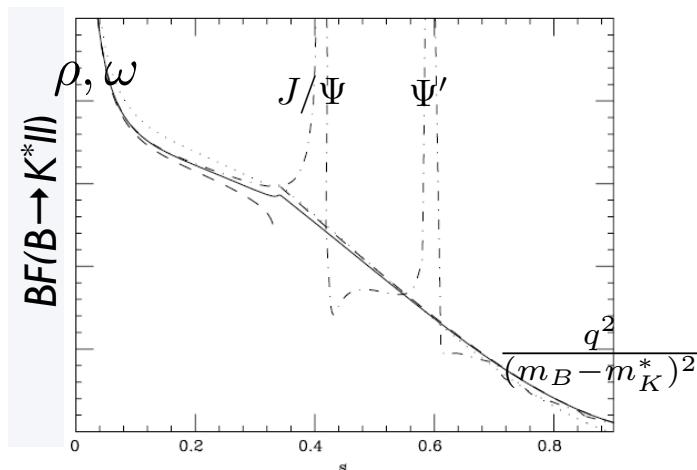
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Amplitude

non-perturbative



**low  $q^2$**  (large recoil)  $E_{K^*} \gg \Lambda_{\text{QCD}}$

- $\Rightarrow$  light-cone dynamics
- QCD-factorization/SCET & LCSR -- form factor LCSR

**high  $q^2$**  (low recoil)  $E_{K^*} \approx \Lambda_{\text{QCD}}$

- OPE (*Grinstein, Pirjol'04, Beylich et al'11*)  $1/m_b \sqrt{q^2}$
- form factors Lattice

this talk  
comments end



# Definition of isospin asymmetries

Lyon & RZ'13

- Experimental definition (Recall:  $q^2$  lepton pair momentum squared)

$$\frac{dA_I^{\bar{0}-}}{dq^2} \equiv \frac{d\Gamma[\bar{B}^0 \rightarrow \bar{K}^{*0} l^+ l^-]/dq^2 - d\Gamma[B^- \rightarrow K^{*-} l^+ l^-]/dq^2}{d\Gamma[\bar{B}^0 \rightarrow \bar{K}^{*0} l^+ l^-]/dq^2 + d\Gamma[B^- \rightarrow K^{*-} l^+ l^-]/dq^2}$$

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- above isospin linear effect -- interference with isospin neutral part
  - compute SM asymmetry
  - extend the basis to include most generic isospin sensitive dimension 6 operators (N.B. do not extend SM isospin neutral part; as “know” to be small by rate)

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**How do we extend the basis?**

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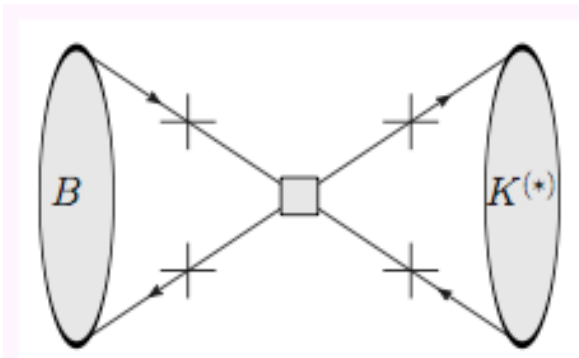
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Weak Annihilation  
(WA)

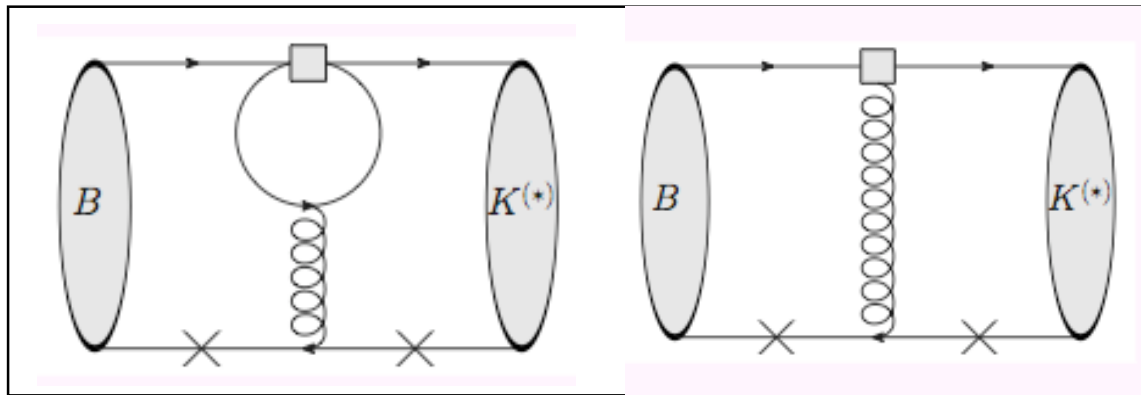
Quark-loop spectator  
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Chromomagnetic-operator  
( $O_8$ )



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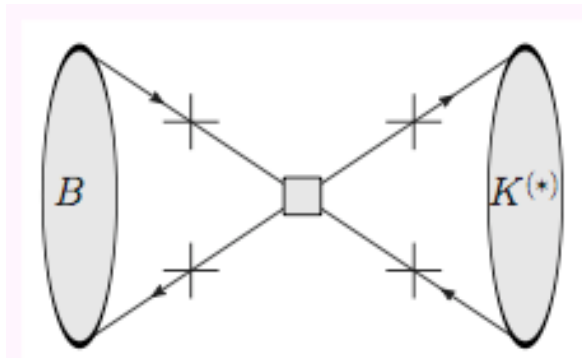
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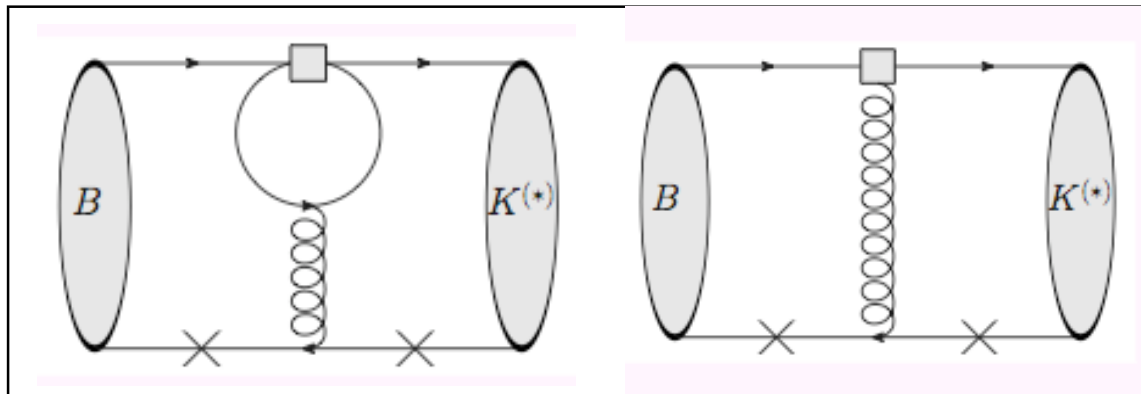
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Answer title Q: IR **QED**-effects & **BSM UV isospin violation** manifested in WA

- very specific operators  $\Rightarrow$  answers the question: “**why isospin asymmetries?**”

# A rough overview of what we did.

- **WA:** 1) extend (*Khodj.&Wyler, Ali&Braun'95*) to  $q^2 \neq 0$  within Light-cone sum rules  
2) introduce most general dimension 6  $H^{\text{eff}}$  at  $O(\alpha_s^0)$

$$\mathcal{H}^{\text{WA},q} = -\frac{G_F}{\sqrt{2}} \lambda_t \sum_{i=1}^{10} a_i^q O_i^{\text{WA}}$$

$$O_9^{\text{WA}} \equiv \bar{q} \sigma_{\mu\nu} b \bar{s} \sigma^{\mu\nu} q \quad O_{10}^{\text{WA}} \equiv \bar{q} \sigma_{\mu\nu} \gamma_5 b \bar{s} \sigma^{\mu\nu} q$$

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We resort to QCDF as LCSR involves 2-loops and complicated analytic structure ..

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10 operators ( $m_q=0$ )

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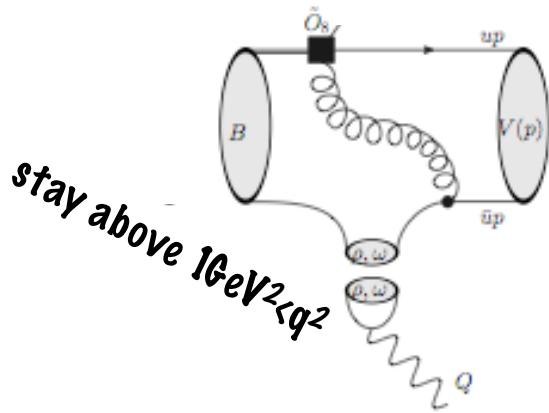
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- **O<sub>8</sub>:** earlier work (*Dimou, Lyon & RZ'12*) BSM: flipped chirality  $\Rightarrow$  trivial

# Hadronic contributions & strong phases

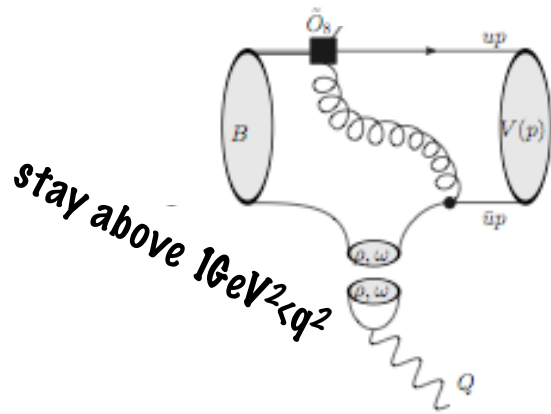
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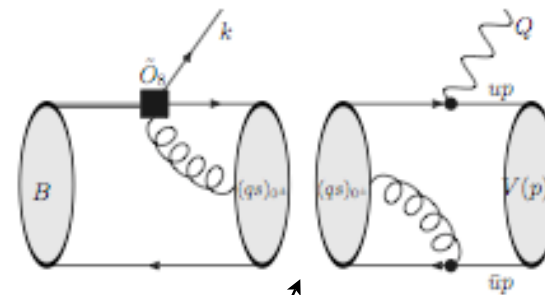


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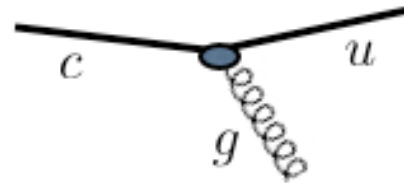
ok as cut at  $m_B^2$   
not in resonance region

- Multihadron state  $(\bar{s}q)_{0\pm}$  q-number and momentum squared  $m_B^2$

## Charm-physics intermezzo

- recent interest  $\Delta A_{CP}(D \rightarrow KK/\pi\pi)$  ---  
one suspect: enhanced chromomagnetic operator new weak phase  
*Isidori, Kamenik, Ligeti, Perez et al '11*

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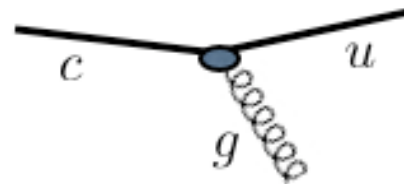


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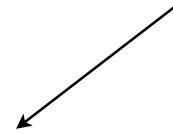
- Appears in  $A_{CP}[D \rightarrow V\gamma]$  *Isidori, Kamenik'12* .....

# $D \Rightarrow V\gamma$ under the microscope

- Consensus:  $B \Rightarrow V\gamma$  **short** distance (**SD**) dominated (penguin)

$D \Rightarrow V\gamma$  **long** distance (**LD**) dominated

our view



*Why? -CKM-hierarchies  
-large top mas*

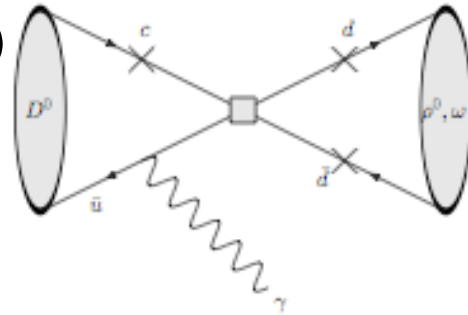
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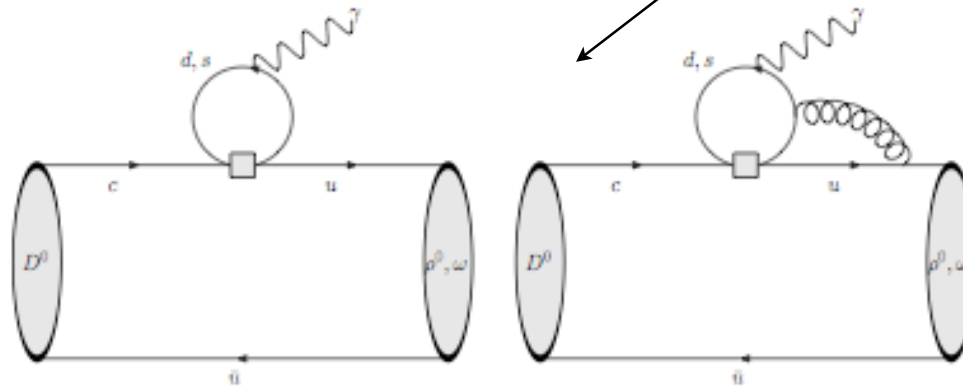
A)



Weak Annihilation (WA)

tree

1



Quarkloops (QL)

1-loop

0 gauge invariance

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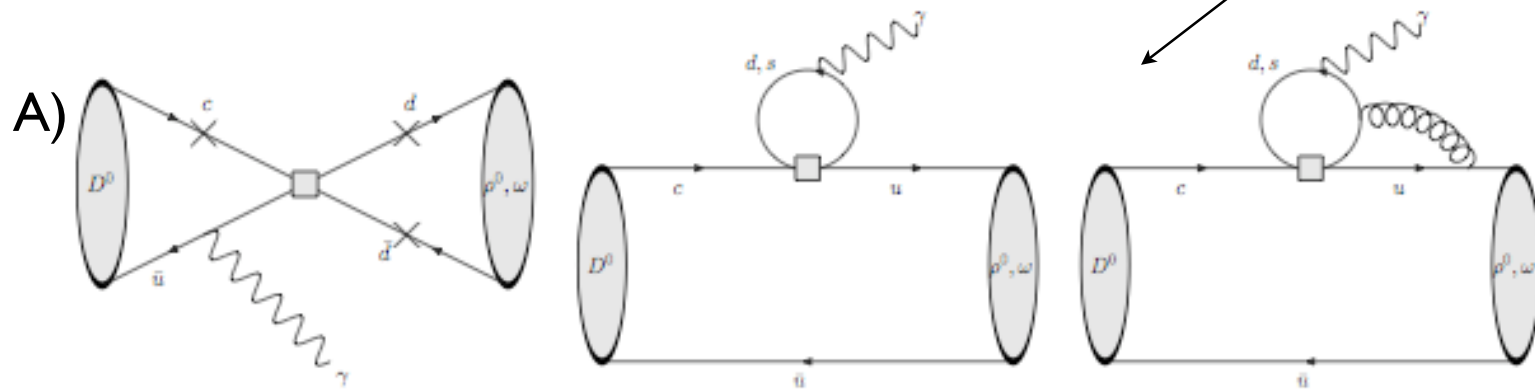
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B) Computation LO  $\alpha_s$  WA: 1) approx. saturates known rates  $D^0 \Rightarrow \bar{K}^{*0}\gamma$ ,  $D^0 \Rightarrow \phi\gamma$

2) chiral limit & no strong phase by virtue of  $\partial \cdot J_{\text{weak}} = 0$

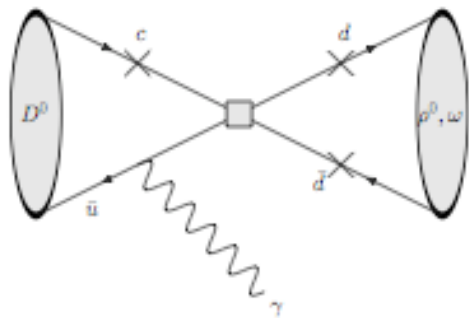
*delicate colour suppressed  
(enhanced NLO?)*





## Comment topic: $A_{CP}[D^0 \rightarrow \rho^0 \gamma]$

- Strong phase of  $O_8$  and WA (dominant) interfere to CP-violation in  $D^0 \rightarrow \rho^0 \gamma$



$\times \text{Im}[C_8]O_8$

*Lyon & RZ 12*

$LD e^{i\delta(\text{strong})} \times \text{Im}[C_8]O_7$

*RG-mixing*

*First: Isidori & Kamenik 12*

*main difference: IK: LD not specified depends sizeable strong phase*

*LZ: LD=WA no strong phase at leading order – strong phase through  $O_8$*

**END OF Charm-physics intermezzo**

# Fun with dispersion relations

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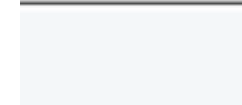
**Cauchy's thm**

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**"usually"**

Physical Region

$m_b^2$



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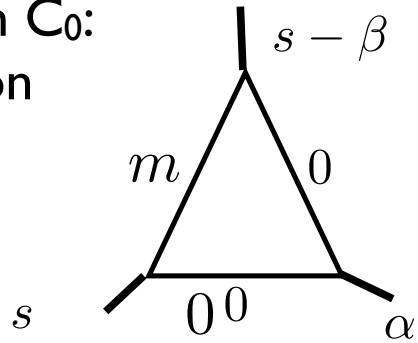
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For real singularities its relatively straightforward to answer not for complex ones!
- Found 4 ways to show/convince ourselves that one is present on PRS
  - 1) Kallen-Wightman paper '59 analytic properties three pts fcts (axiomatic approach)
  - 2) 6-dimensional projective geometry (did not do finally)
  - 3) deformation from non-complex case (tricky in case at hand)
  - 4) "invented method" using Feynman parameter integral (next slide)

Passarino-Veltman reduction  $C_0$ :

➤ study analytic continuation



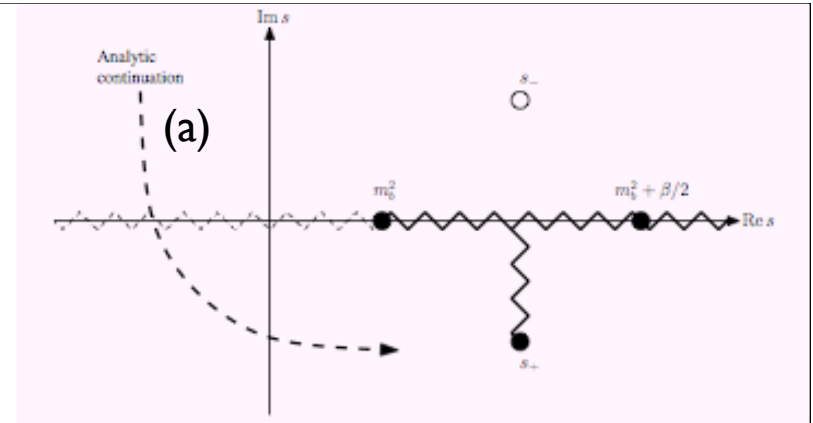
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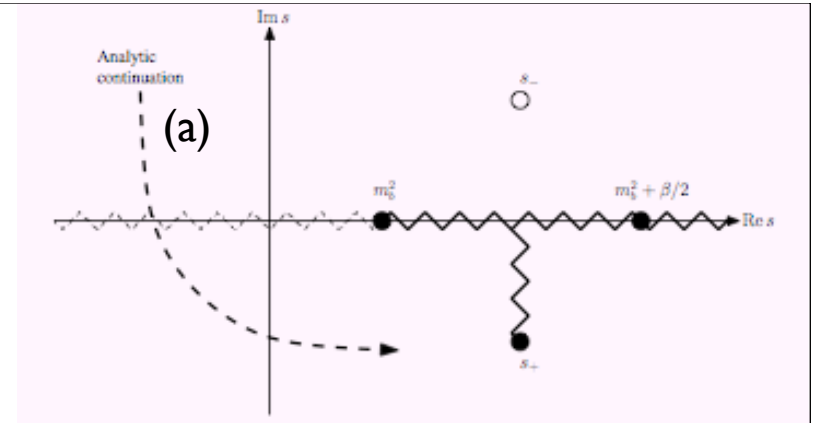
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❶ Real line:  $C_0(s) = C_0^F(s) \equiv \int_0^1 dx \int_0^{1-x} dy ((1-x-y)(xs+y(s-\beta)-m^2)....+xy\alpha+i0)^{-1}$

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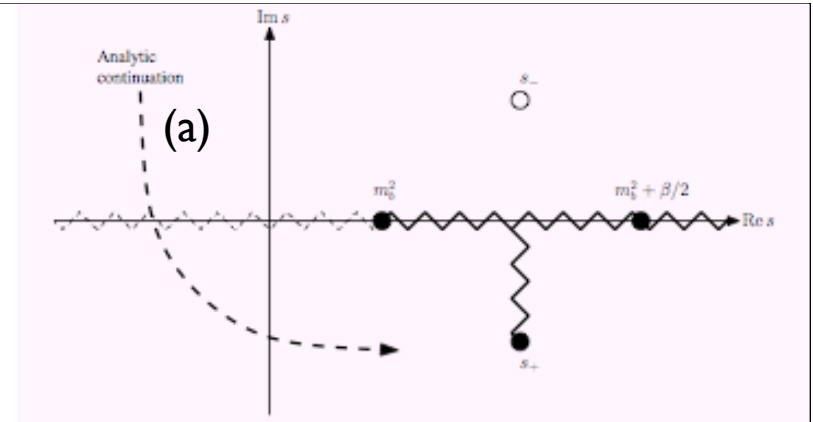
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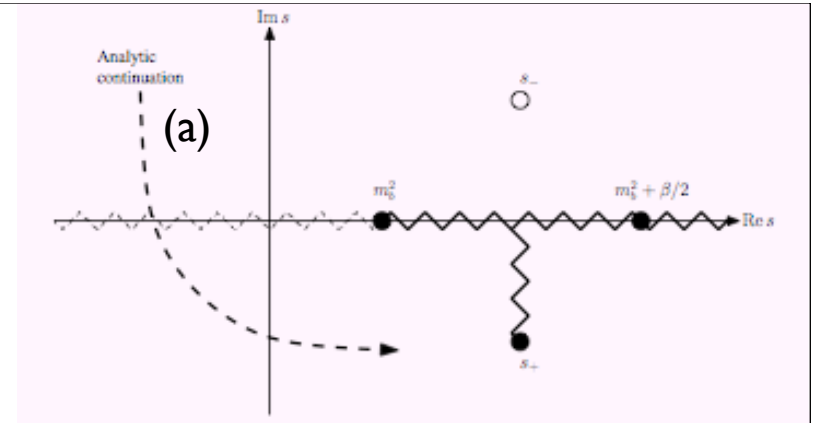
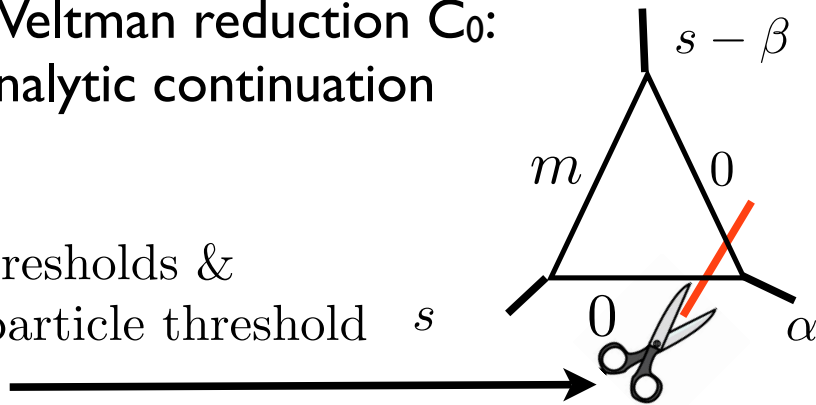


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$\rho/\omega$ -thresholds &  
 $(\bar{s}q)_{0\pm}$ -mutiparticle threshold  $s$



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$\text{Im}[s] \neq 0 : [C_0^F(s^*)]^* = C_0^F(s)$  Reflection principle

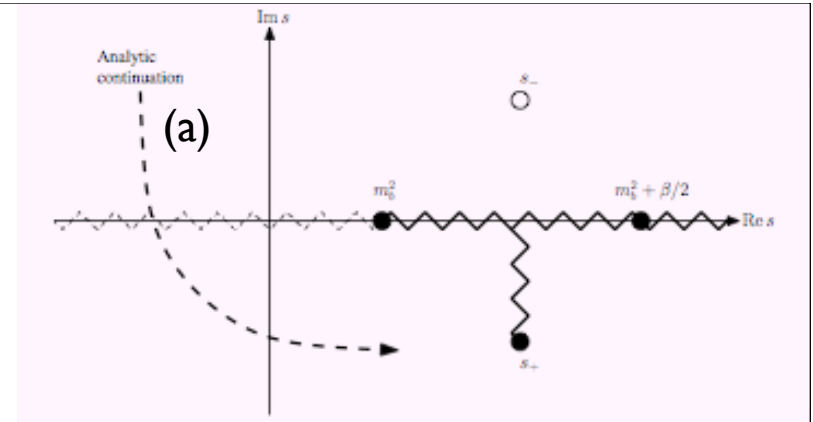
$$C_0(s) = \begin{cases} C_0^F(s) & \text{Im}[s] > 0 \\ C_0^F(s^*)^* + C_0^{\text{rem}}(s) & \text{Im}[s] < 0 \end{cases} \quad C_0^{\text{rem}}(s) = 2i \text{Im}[C_0^F(s)] \quad \text{Im}[s] = 0$$

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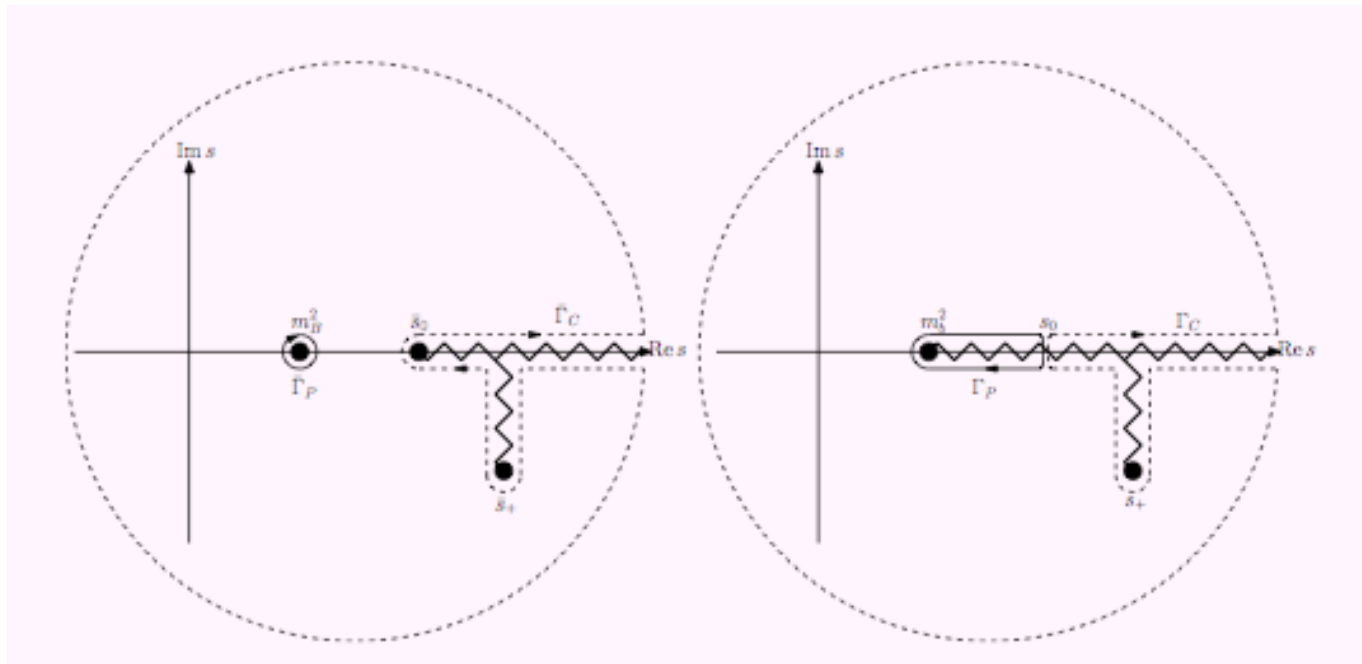
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❺ Analyze  $C_0^{\text{rem}}$  note  $\mathbf{s_+} \in \text{PRS}!! \Rightarrow$  Know how to choose path appropriately

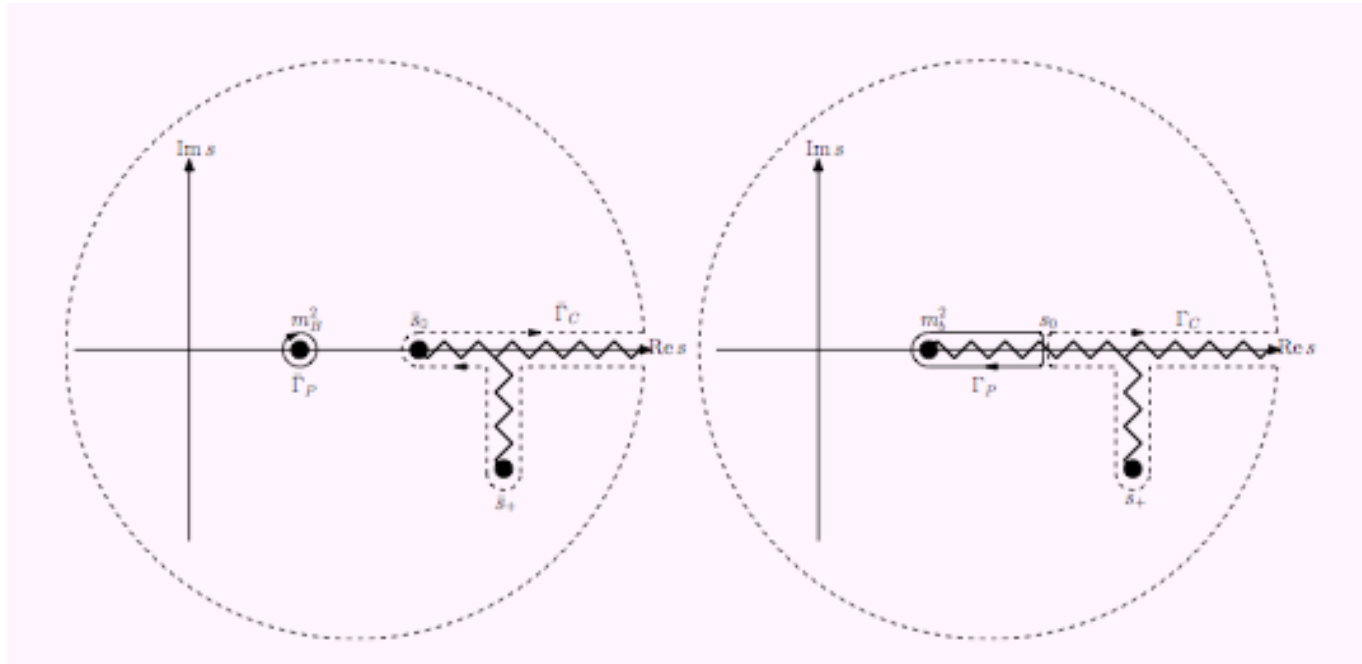




very briefly the analytic structure in full QCD and in partonic QCD

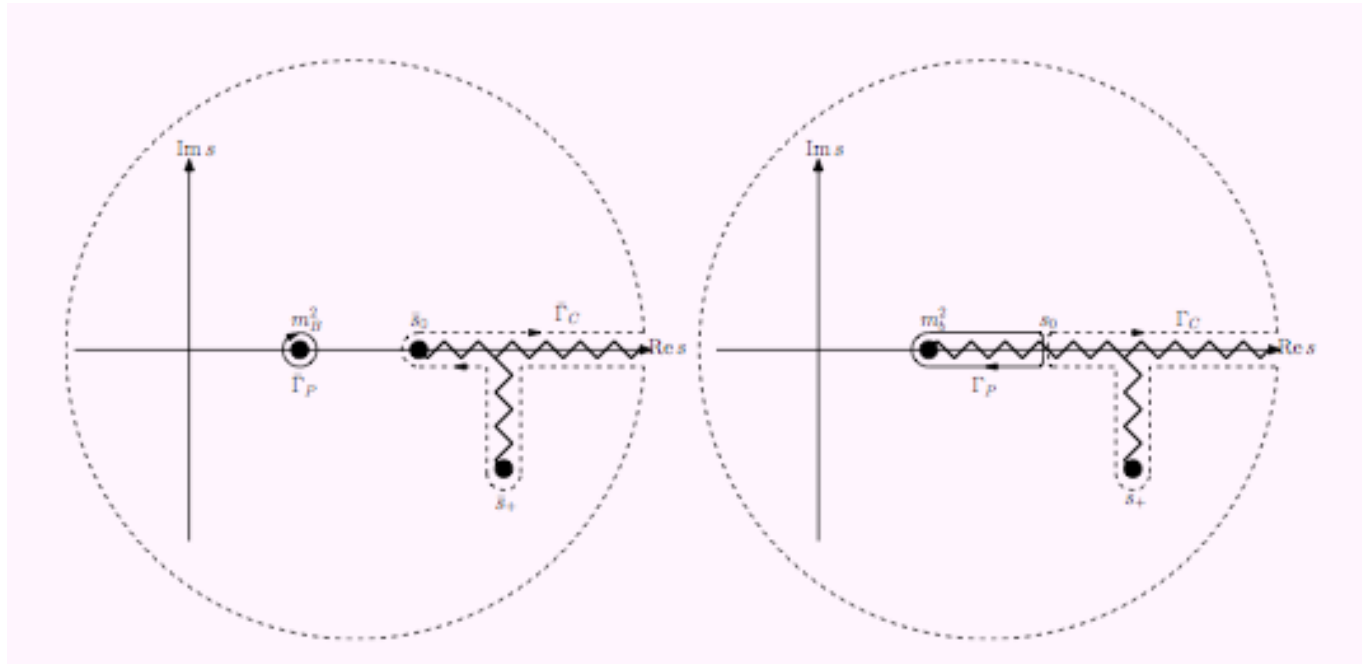


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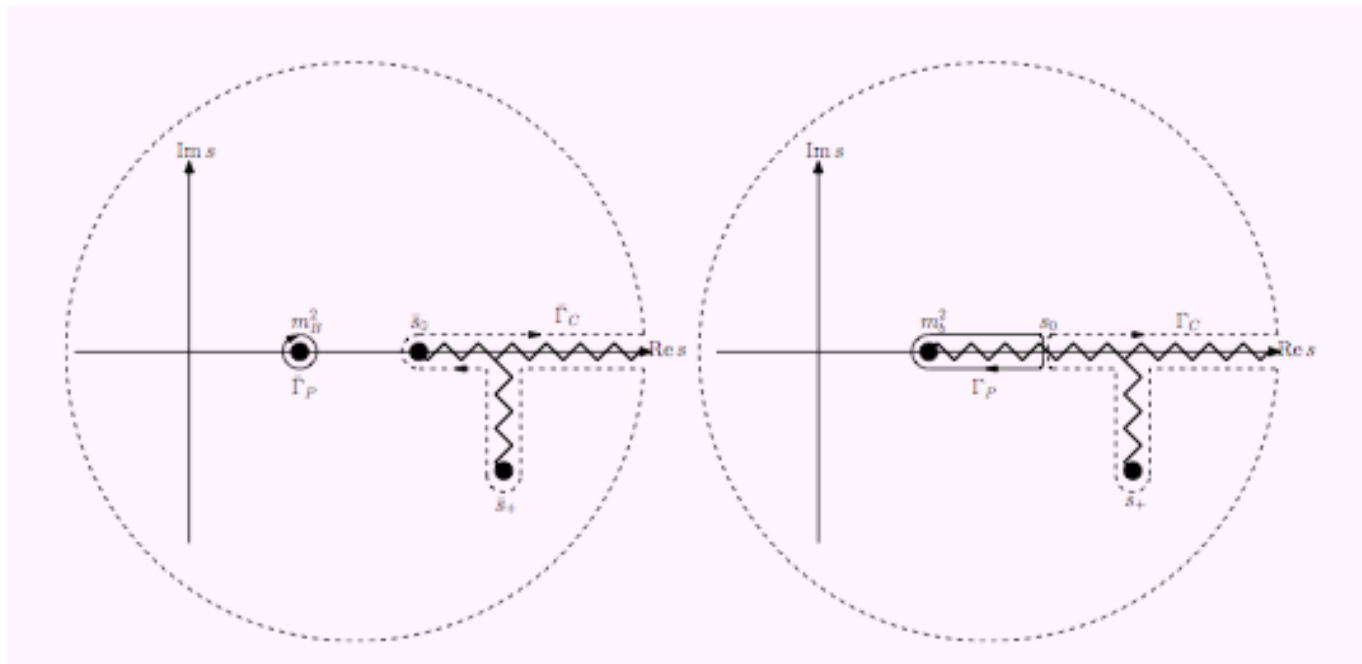
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**end of technical excursion**

# Selection rules

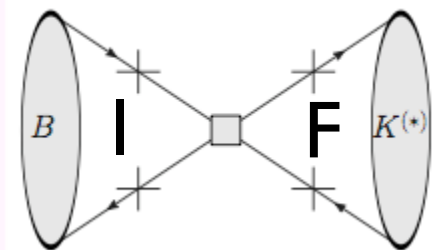
- **General:** a)  $B[0^-] \rightarrow K[0^-](\gamma^*[1^-] \rightarrow \ell[1^-]) \Rightarrow$  p-wave; i.e.  $l = 1$  ,  
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- **WA:** more stringent selection rules ground of Lorentz-invariance etc  
(at least at the level of the factorisable contribution)

		Twist	Operator $O_n^{WA}$									
			1	2	3	4	5	6	7	8	9	10
$B \rightarrow K$	cov. $(\alpha_s^0)$		$\times$	$\times$	$\times$		$\times$	$\times$	$\times$		$\times$	$\times$
	$\chi$ -even $(\phi_K)$	2								I,F		
	$\chi$ -odd $(\phi_{P,\sigma})$	3				I,F						
	cov. $(\alpha_s^n, n > 0)$		$\checkmark$	$\times$	$\times$	$\checkmark$	$\checkmark$	$\times$	$\times$	$\checkmark$	$\checkmark$	$\times$
$B \rightarrow K^*$	cov. $(\alpha_s^0)$		$\times$		$\times$				$\times$	$\times$		
	$\chi$ -even $(g_\perp^{(v)}, g_\perp^{(a)})$	3					I,F	I,F				
	$\chi$ -odd $(\phi_\perp)$	2		F		F					I	I
	$\chi$ -odd $(h_\parallel^{(t)}, h_\parallel^{(s)})$	3		F								I
	cov. $(\alpha_s^n, n > 0)$		$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$



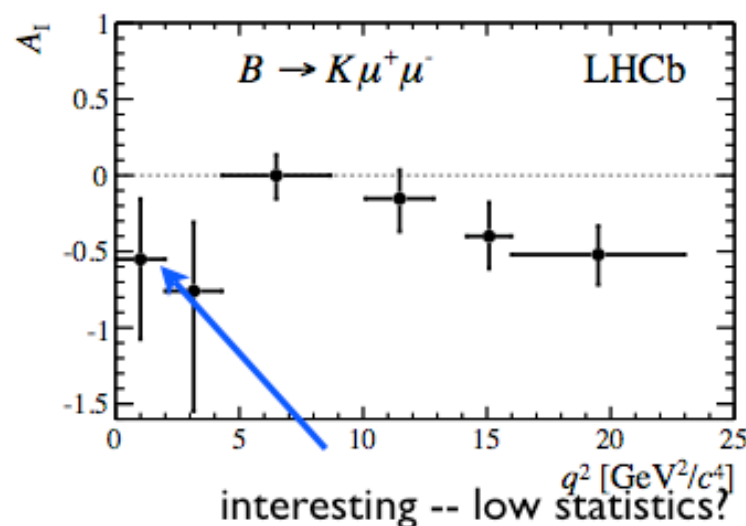
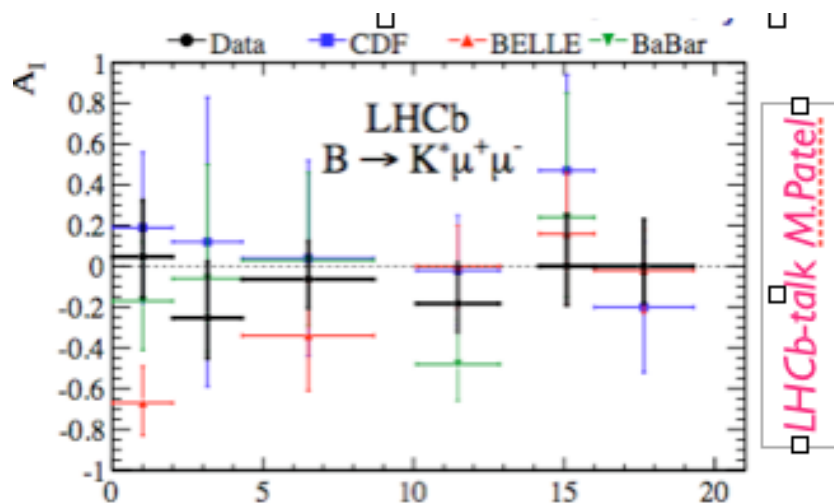
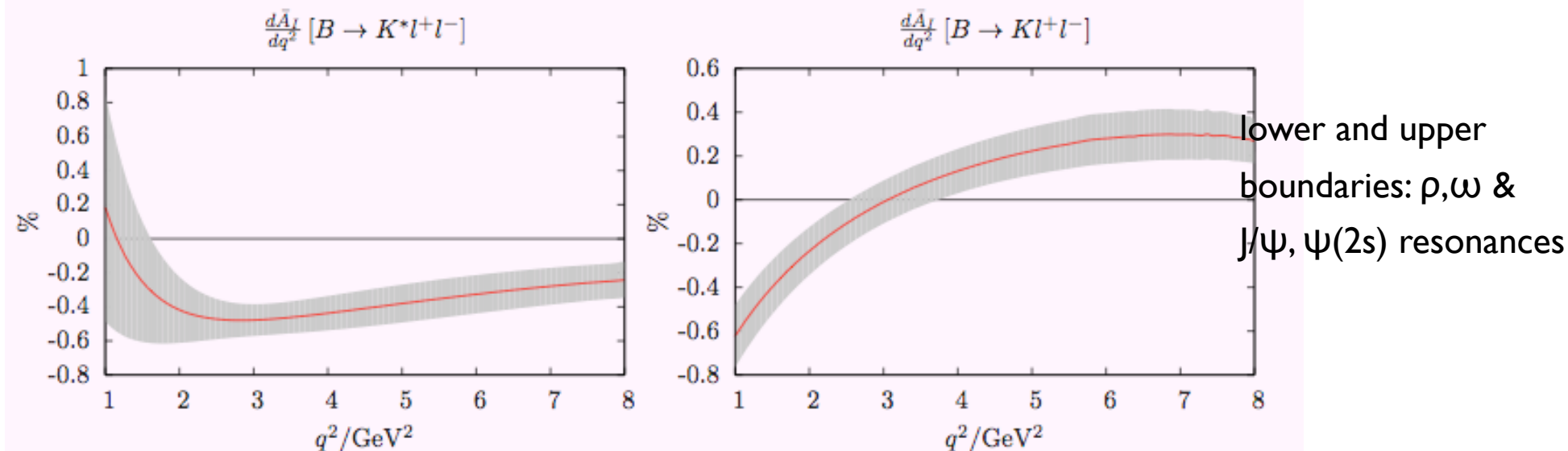
I(initial) & F(final)  
state radiation

“the K-meson contribution corresponds to the longitudinal part of  $K^*$ -meson”

true leading twist in the SM where V-A imposes  $a_6 = -a_8$ ; V+A not true

# Isospin asymmetries in the SM

- Are small for  $B \rightarrow K^{(*)} l l^-$  -- accidental sizeable tree-level WC double Cabibbo suppressed

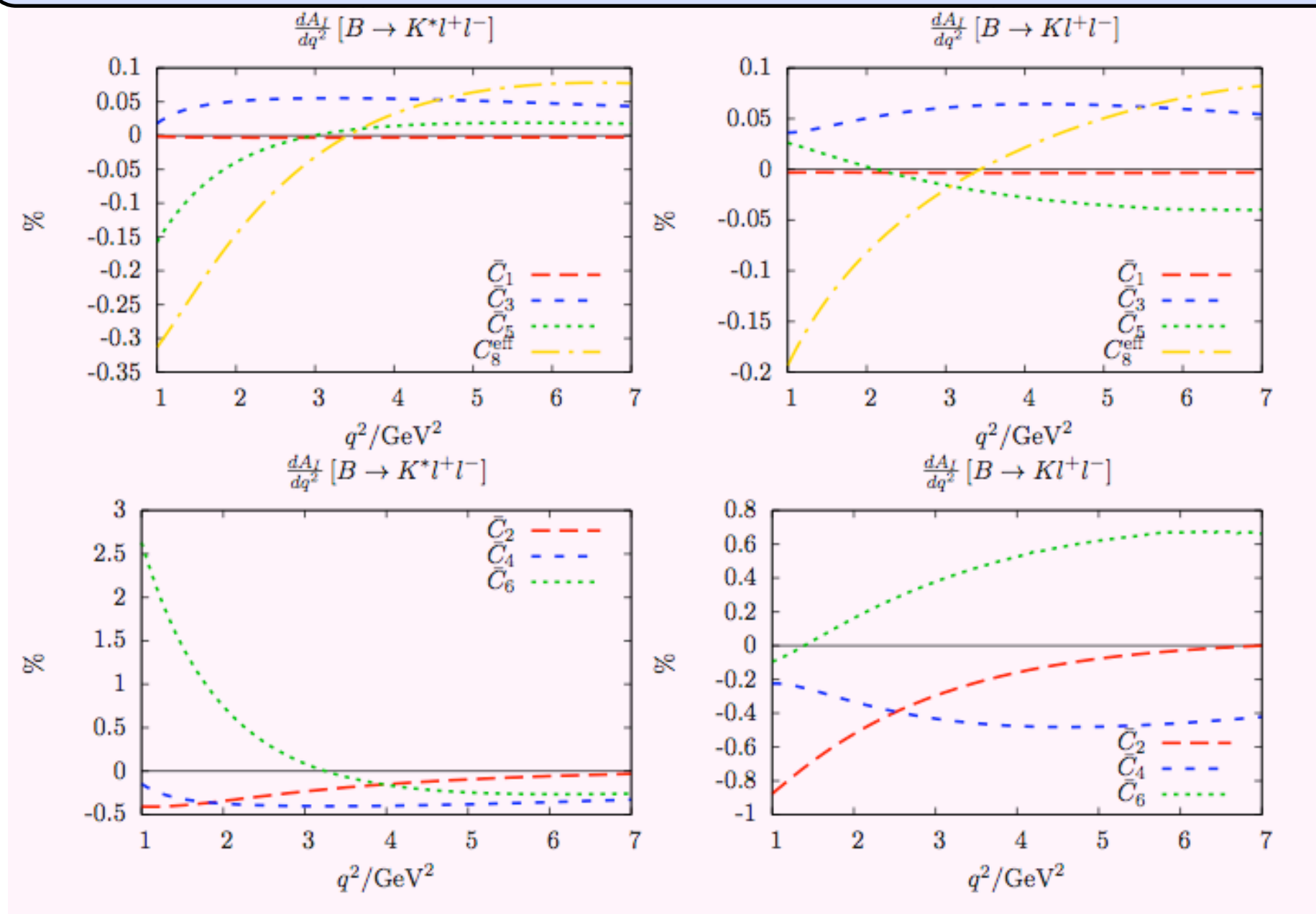


- Larger for  $B \rightarrow K^{(*)} \gamma$

$$\bar{a}_I(K^{*} \gamma)_{\text{HFAG}} = 5.2(2.6)\%, \quad \bar{a}_I(K^{*} \gamma)_{\text{LZ}} = 4.9(2.6)\% .$$



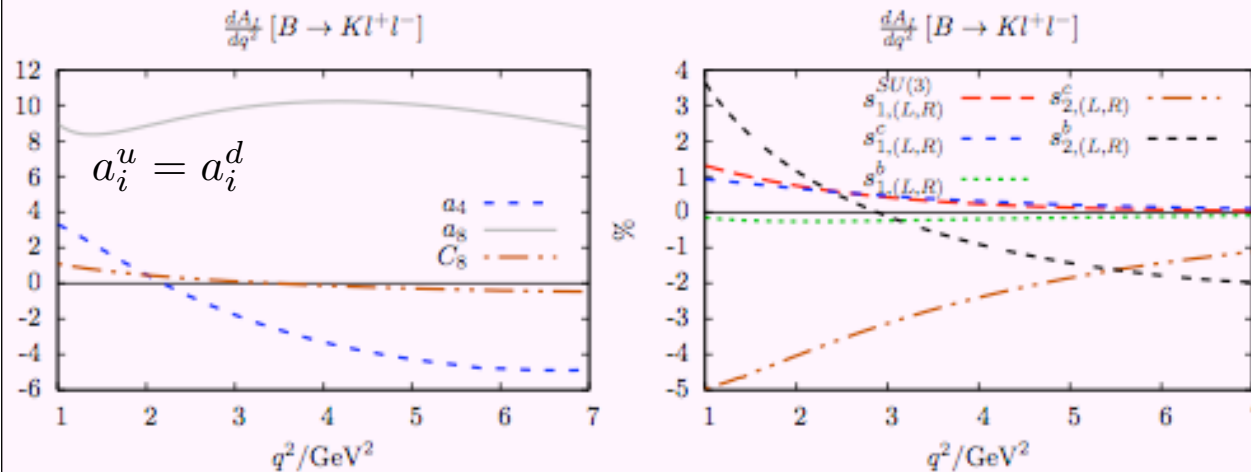
# The breakdown in the SM....



# Isospin asymmetries BSM

- After selection rules:  
still many operators!

	$C_8^{(')}$	WA	QLSS	total
$K^*$	2[1]	12[3]	10[3]	24[7]
$K$	1[1]	4[3]	5[3]	10[7]



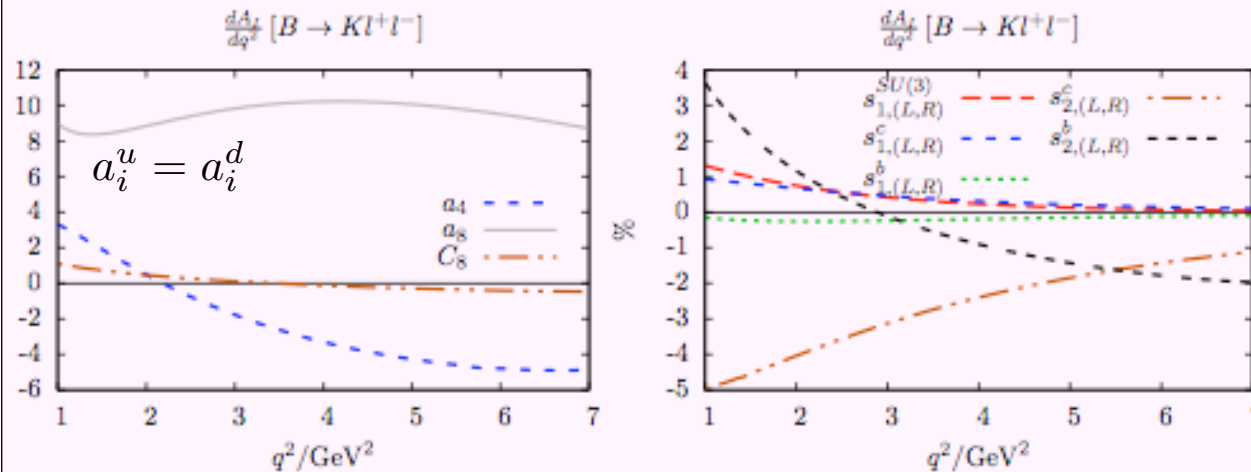
- $K^*$  even more “.. by the laws of probability cancellation ought to be the rule rather than the exception.”
- Or a new principle for flavour physics)
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can't be unlucky all the way

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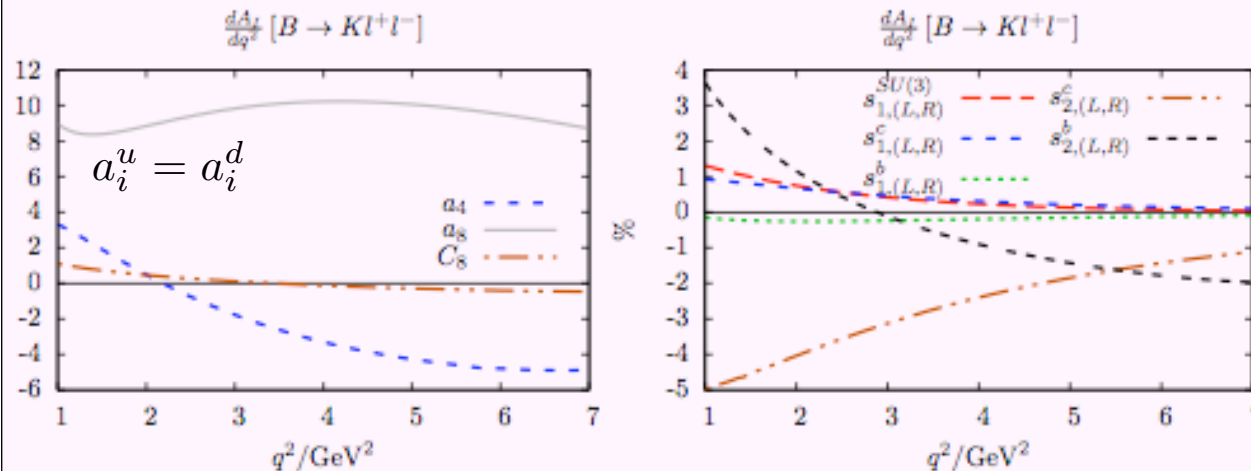
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Take  $a_i \neq 0$  others zero then to be within  $2\sigma$  of  $a_I^{0-}(K^* \gamma)_{\text{HFAG}} = 5.2(2.6)\%$ .

$$\begin{aligned}
 0 > a_2 > -3 \cdot 10^{-1}, \quad 0 > a_4 > -3 \cdot 10^{-1}, \quad 0 < a_5 < 5 \cdot 10^{-1} \\
 0 > a_6 > -7 \cdot 10^{-1}, \quad 0 < a_9 < 6 \cdot 10^{-2}, \quad 0 < a_{10} < 6 \cdot 10^{-2}.
 \end{aligned}$$

*indicative constraints*

# Comments on high $q^2$ (isospin)

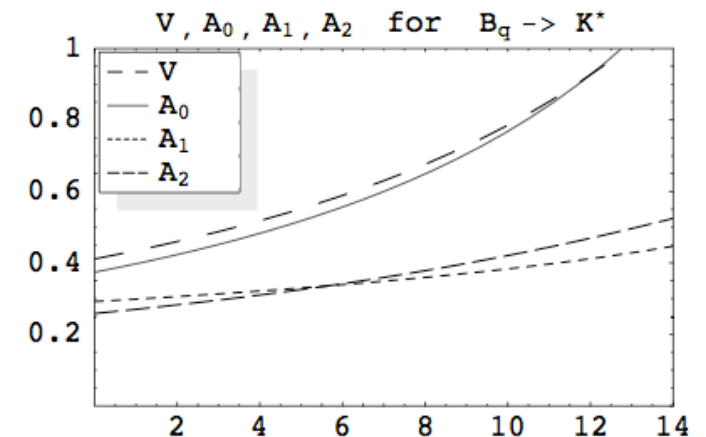
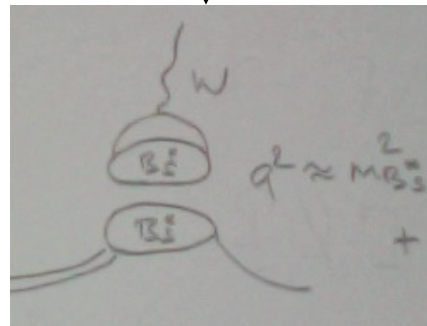
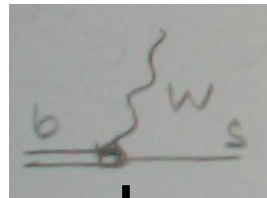
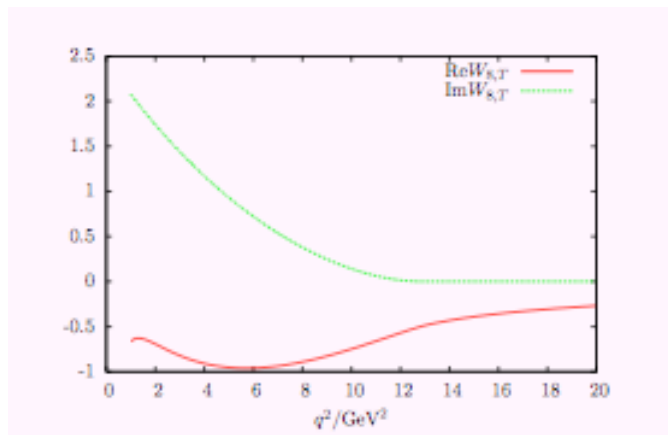
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- Isospin violation: - through photon ➤ enhanced through photon pole low  $q^2$   
➤ isospin asymmetry has to decrease (module conspiracy)  
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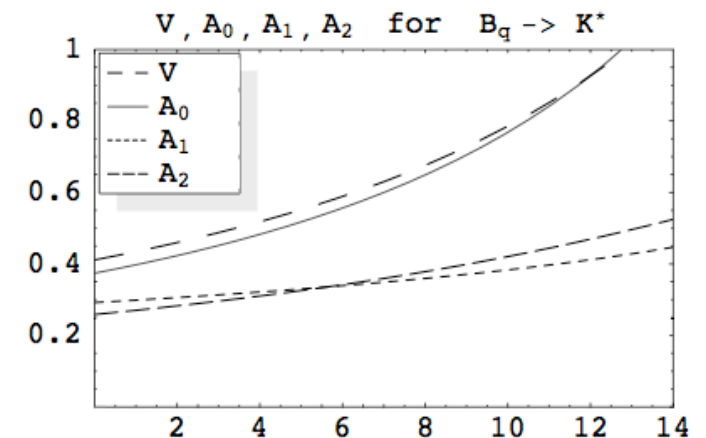
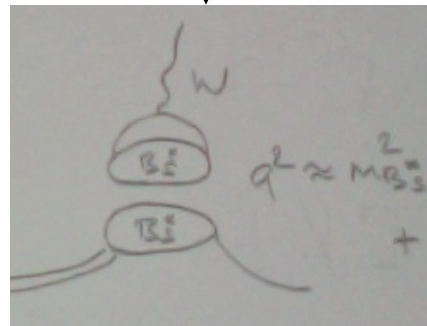
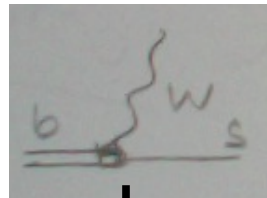
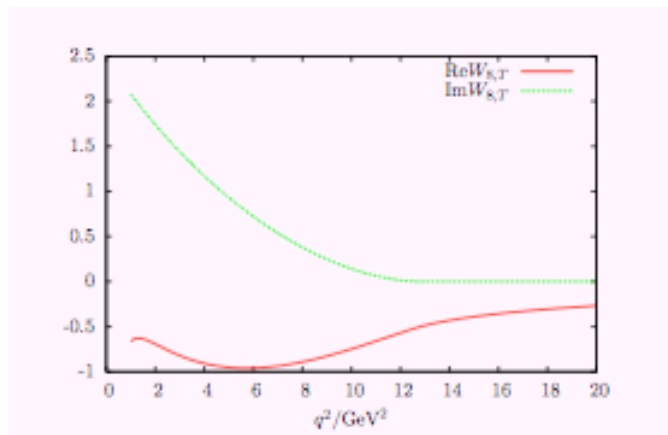
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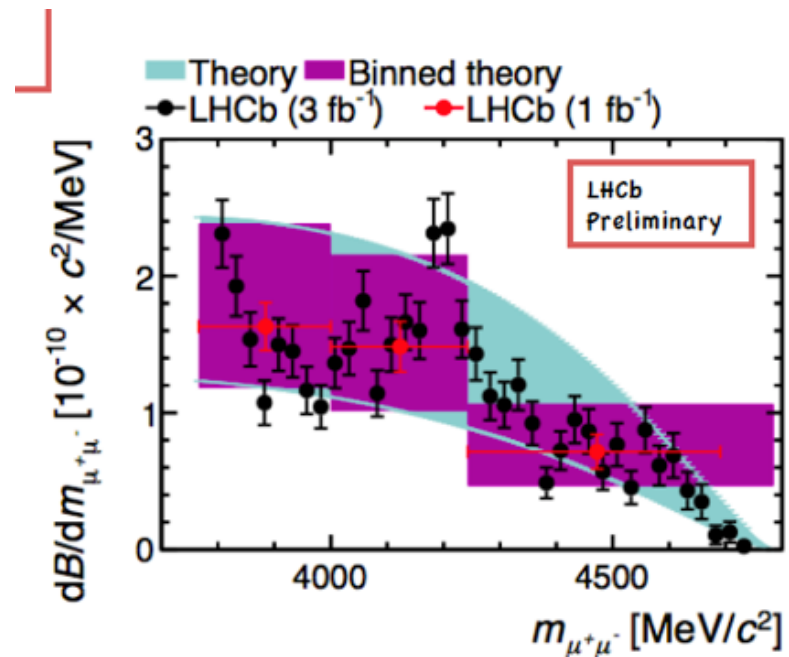


- ## 2) OPE language: form factors dim 3 operators isospin violation dim 5,6

• *Beylich, Buchalla, Feldmann'11*

# Comments on high $q^2$ --charm resonances

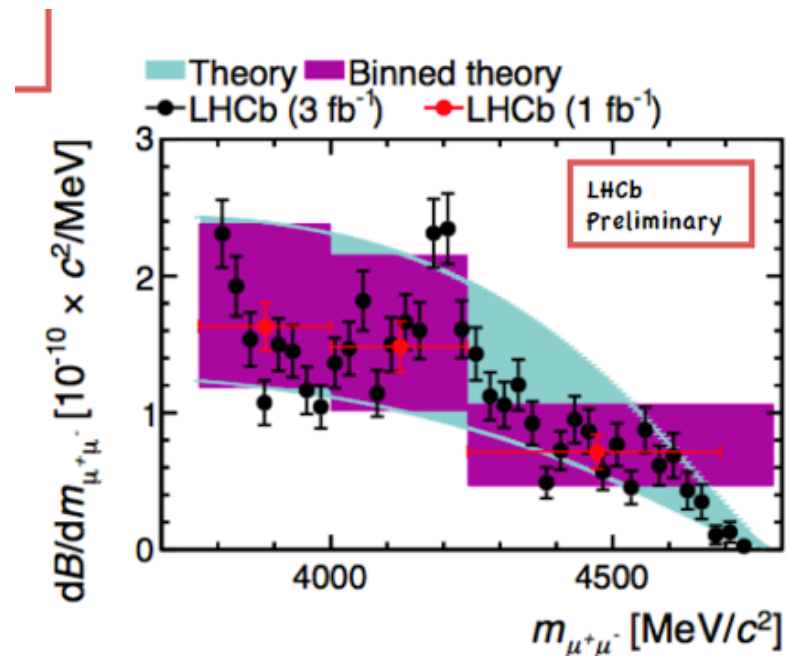
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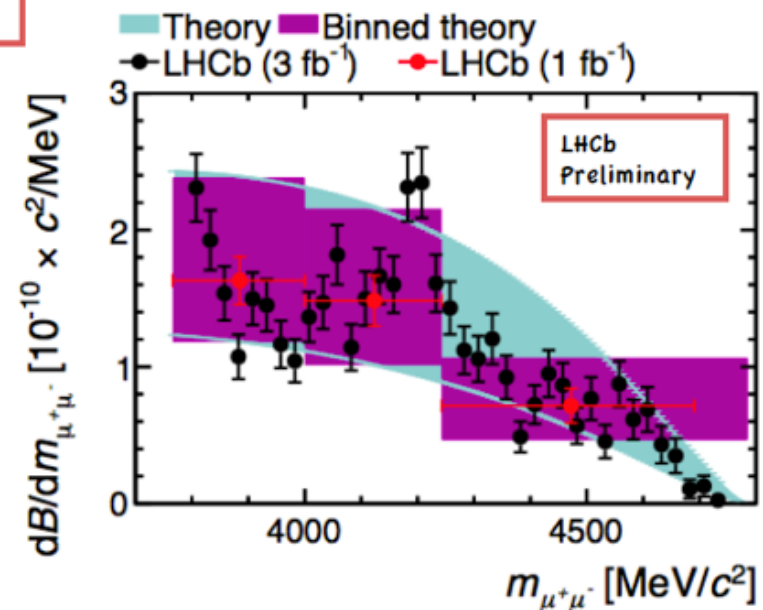
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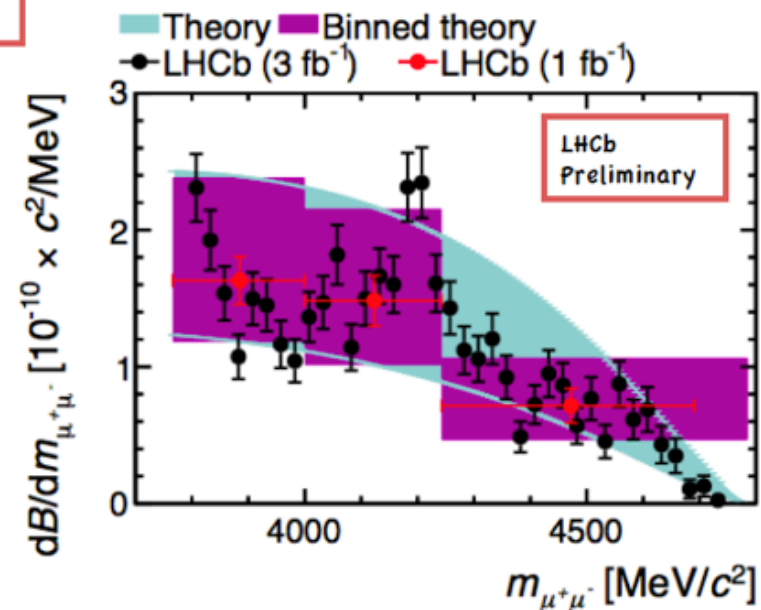
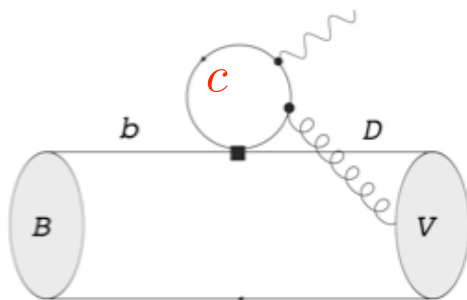
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- Important to estimate (high  $q^2$ ) non-factorizable contributions, which are helicity dependent e.g.



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- $D \rightarrow V\pi$  charm physics has potential -- need charged modes to check theory

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