Beyond the SM on the lattice

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Motivation

A **light composite Higgs**?

Light scalar from a strong sector:

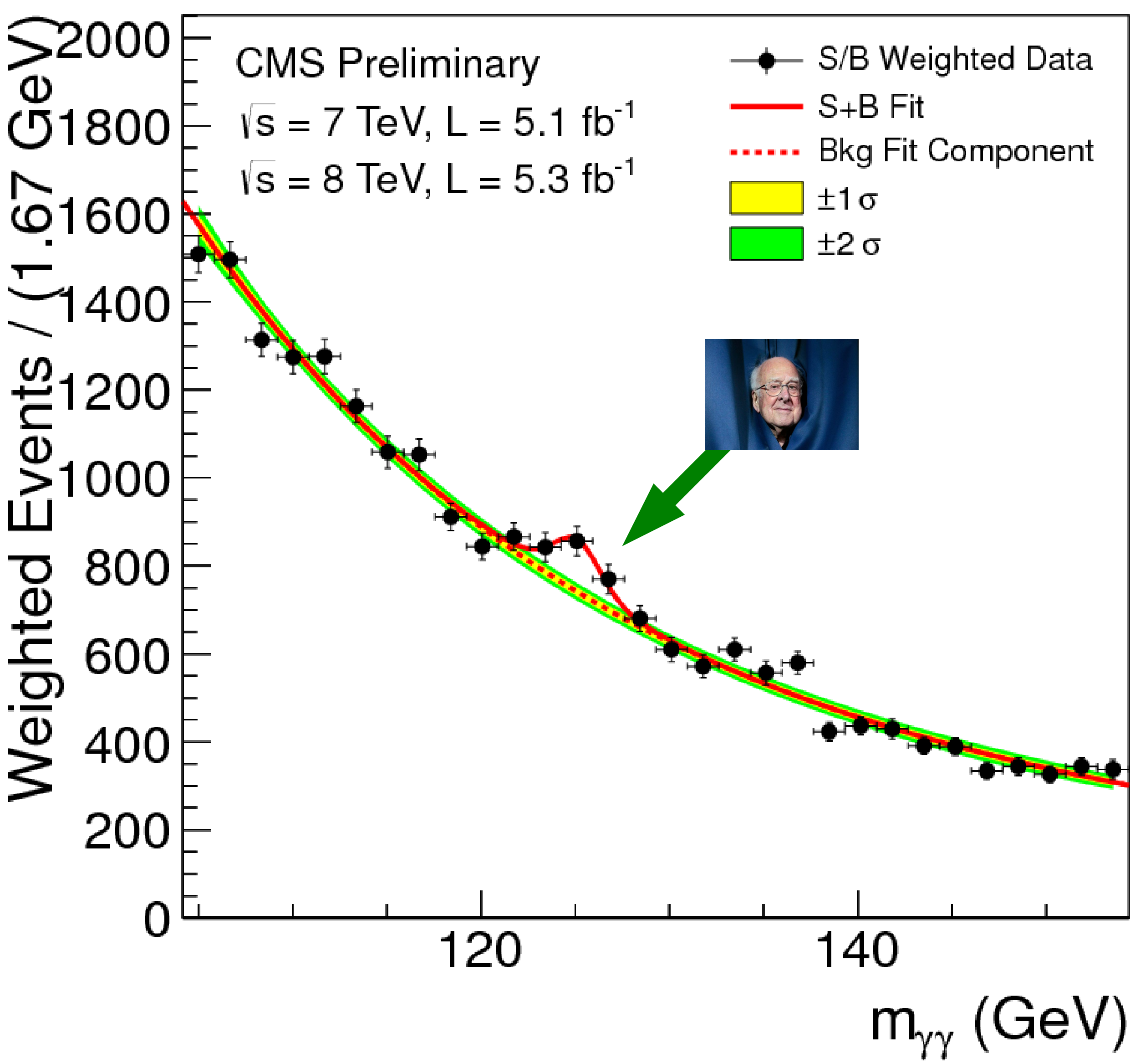
spontaneous symmetry breaking

1) global symmetry: G/H

2) dilatation invariance

Phenomenology from effective lagrangians

Physics encoded in LEC



EFT for NGB

$$\mathcal{L} = f^2 \mathrm{Tr}\,|D_\mu \Sigma|^2 + \dots\,, \qquad \Sigma = \Sigma_0\, \exp[i h^{\hat{a}} T^{\hat{a}}/f]$$

$$V(h) = 0 \qquad\qquad h \mapsto h + c$$

Higgs potential generated by the coupling to the SM fields

$$m_\rho \sim g_\rho f\,, \quad m_h \sim g_{\mathrm{SM}}\, v$$

$$m_h/m_\rho \sim \sqrt{\xi} = v/f$$

[Georgi & Kaplan, ...]

Vanilla technicolor: $\xi = 1$ **ruled out!**

non-QCD dynamics - walking technicolor [Holdom, Yamawaki] - more later

EFT for the dilaton

$$\mathcal{L} = \frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi + \kappa\varphi^4 + \ldots\,, \quad \varphi(x) = fe^{i\tau(x)/f}$$

$$V(\tau) = V_0 e^{4\tau} \qquad \tau(x) \mapsto \tau(e^\lambda x) + \lambda$$

$$e^{4\tau}\,(\partial)^m\left(e^{-\tau}\right)^m$$

Spontaneous breaking to Poincaré needs fine tuning! [Fubini 1976]

Explicit & anomalous breaking: $\Delta\mathcal{L} = \lambda\mathcal{O}_d\,, \quad \mu\frac{d}{d\mu}\lambda = \beta(\lambda) \neq 0$

$$V(\varphi) = \varphi^4 \sum_{n=0}^{\infty} c_N(\Delta_{\mathcal{O}})\left(\frac{\varphi}{f}\right)^{n(\Delta_{\mathcal{O}}-4)}$$

nearly marginal breaking needed for a light dilaton!

Hierarchies and scaling dimensions

Linearized RG flows in a neighbourhood of a fixed point

$$\mu \frac{d}{d\mu} g = (\Delta - D)\, g + O(g^2) \qquad\qquad g(\mu) = \left(\frac{\mu}{\Lambda_{\mathrm{UV}}}\right)^{\Delta - D} g(\Lambda_{\mathrm{UV}})$$

Associated IR scale:

$$\Lambda_{\mathrm{IR}} \sim g_0^{1/(D-\Delta)} \Lambda_{\mathrm{UV}} \qquad\qquad \begin{cases} D - \Delta = O(1) & g_0 \text{ must be tuned} \\ D - \Delta \ll 1 & \text{natural hierarchy} \end{cases}$$

Stable hierarchy related to *weakly* relevant operators. [Strassler 03, Sannino 04,Luty&Okui 04]

YM theory at the GFP is a limiting case:

$$\Lambda_{\mathrm{IR}} \sim \Lambda_{\mathrm{UV}} \exp\{-\frac{1}{\beta_0 g^2}\}$$

Global-singlet relevant operators (GSRO) require fine-tuning.

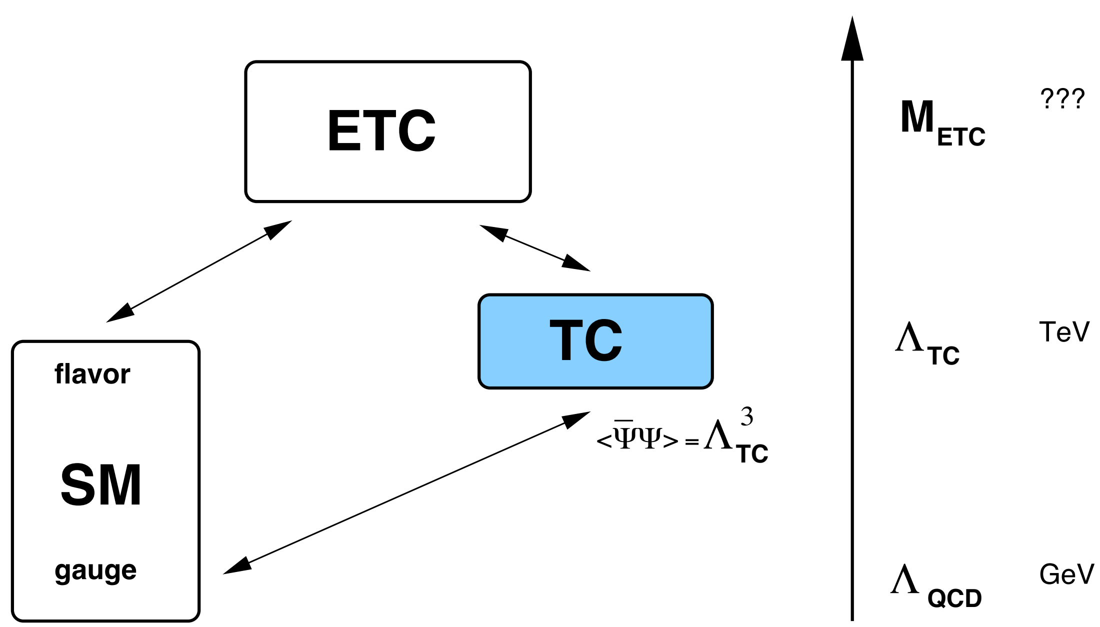
DEWSB from CFT

In the SM the elementary scalar generates a GSRO: $\boxed{\dim(H^\dagger H) \simeq 2}$

Strongly interacting CFT responsible for EWSB:

YM theory coupled to massless fermions

[Weinberg, Susskind]



Flavor sector - fermion masses

In the SM fermion masses are generated by Yukawa interactions

$$\mathcal{L}_Y = y^u \, H \bar{L} u_R + y^d \, H^\dagger \bar{L} d_R$$

dimension = 1+3 = 4

In models of DEWSB: scalar is composite [Dimopoulos et al 79, Eichten et al 1980]

$$\mathcal{L}_Y = \frac{y}{\Lambda_{\mathrm{UV}}^2} \bar{Q} Q \, \bar{q} q$$

dimension = 3+3 = 6

Tension with suppressing FCNC

$$\frac{f}{\Lambda_{\mathrm{UV}}^2} \bar{q} q \bar{q} q$$

dimension = 6

Walking TC

Alleviate the problem due to the **large** dimension of the composite scalar

Theory at the EW scale is **near** a non-trivial fixed point

Scaling dimension of the fermion bilinear is smaller

$$\mathrm{dim}(\bar{Q}Q) = 3 - \gamma$$

[Holdom,Yamawaki,Appelquist,Eichten,Lane]

Small dimension for H allows a better description of the flavor sector, BUT

$$\mathrm{dim}(H) \simeq 1 \quad \Longrightarrow \quad \mathrm{dim}(H^\dagger\, H) \simeq 2$$

In a strongly coupled theory we could have:

$$\mathrm{dim}(H) \text{ small}, \;\; \textbf{but} \;\; \mathrm{dim}(H^\dagger\, H) > 2\ \mathrm{dim}(H)$$

[Sannino 04, Luty 04, Rattazzi et al 08]

Simple illustration

$$C_K\,(\bar{s}_L\gamma^\mu d_L)\,(\bar{s}_L\gamma^\mu d_L)$$
$$C_D\,(\bar{c}_L\gamma^\mu u_L)\,(\bar{c}_L\gamma^\mu u_L)$$
$$C_B\,\left(\bar{b}_L\gamma^\mu d_L\right)\,\left(\bar{b}_L\gamma^\mu d_L\right)$$
$$C'_B\,\left(\bar{b}_L\gamma^\mu s_L\right)\,\left(\bar{b}_L\gamma^\mu s_L\right)$$

$$C_i = \frac{F_i}{\Lambda^2_{\mathrm{ETC}}}$$

Bounds on the ETC scale

K	10^3 TeV
D	1.5×10^3 TeV
B_d	0.21×10^3 TeV
B_s	0.03×10^3 TeV

SM fermion masses:

$$m_q \sim \frac{\langle \bar{Q}Q\rangle_{\mathrm{ETC}}}{\Lambda^2_{\mathrm{ETC}}} = \frac{1}{\Lambda^2_{\mathrm{TC}}}\left(\frac{\Lambda_{\mathrm{ETC}}}{\Lambda_{\mathrm{TC}}}\right)^{\gamma-2}\langle \bar{Q}Q\rangle_{\mathrm{TC}}$$

$$\boxed{\Lambda_{\mathrm{TC}} = 4\pi F, \quad \langle \bar{Q}Q\rangle_{\mathrm{TC}} = \Lambda^3_{\mathrm{TC}}}$$

$$m_q \simeq 1000\ \mathrm{GeV}\ (1.0 \times 10^{-3})^{2-\gamma}$$

$$\boxed{\begin{array}{ll} \gamma \simeq 1 & \Rightarrow m_b \\ \gamma \simeq 1.75 & \Rightarrow m_t \end{array}}$$

[Chivukula 12]

Couplings to SM

$$\mathcal{L} = \frac{M_V^2}{2} V_\mu^2 \left(1 + 2a\frac{S}{v} + b\frac{S^2}{v^2}\right) - m_f \bar{\psi}\psi(1 + c\frac{S}{v})$$

generic coupling of a scalar field - model independent

$$\mathrm{SM} : a = b = c = 1$$
$$\mathrm{SILH} : a = \sqrt{1-\xi},\ b = \sqrt{1-2\xi},\ c = \sqrt{1-\xi}$$
$$\mathrm{SILD} : a = b = \sqrt{\xi},\ c = \Delta\sqrt{\xi}$$

[Contino 10]

rescaled SM

dependence on the an. dim.

$$\Delta = 1 + \epsilon$$

deviation from the elementary scalar free field

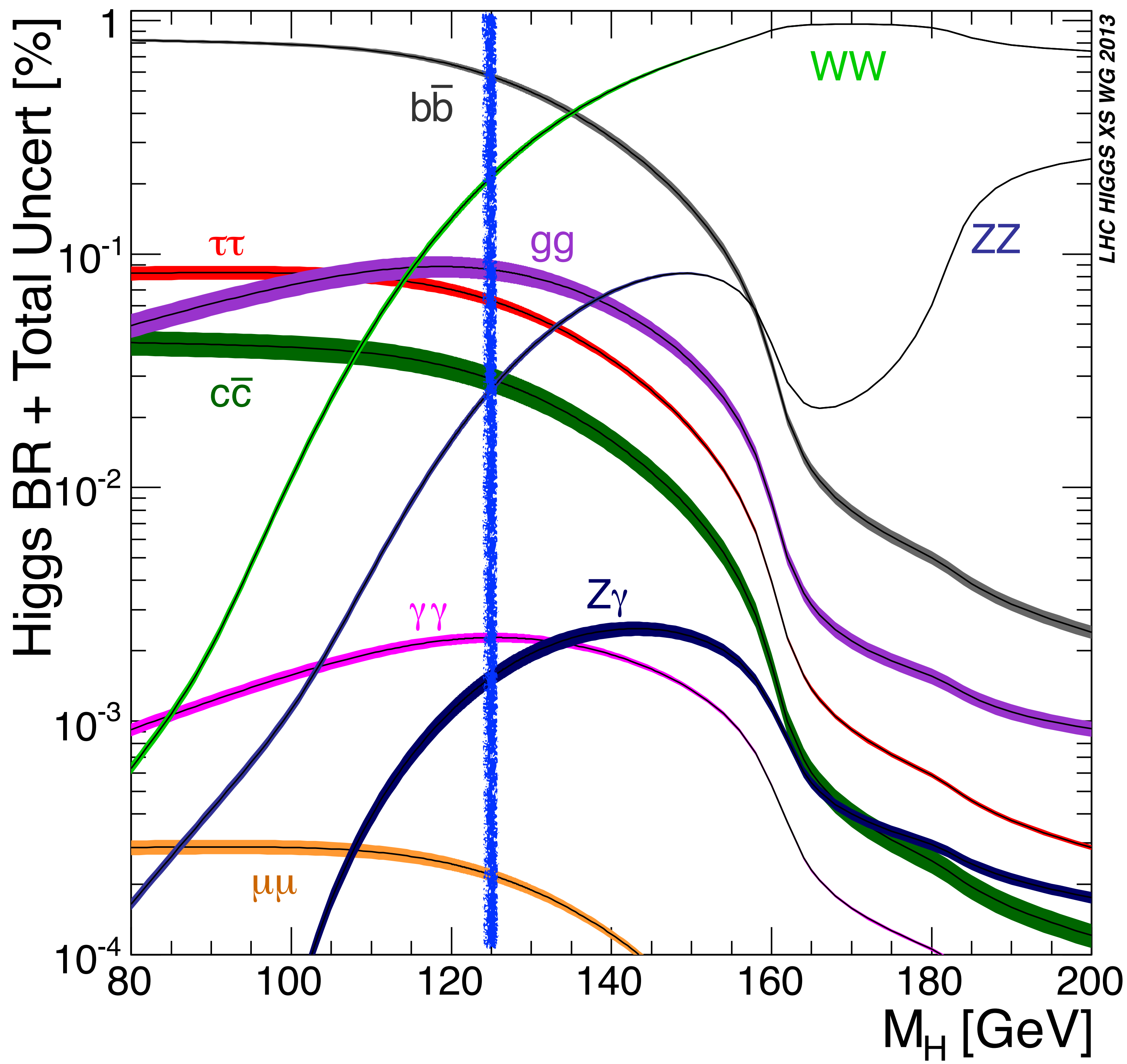
Decay modes - dilaton Higgs-like scalar

$$\eta_{XX} = \left(\frac{g_{\sigma XX}}{g_{\rm SM}}\right)^2$$

elementary

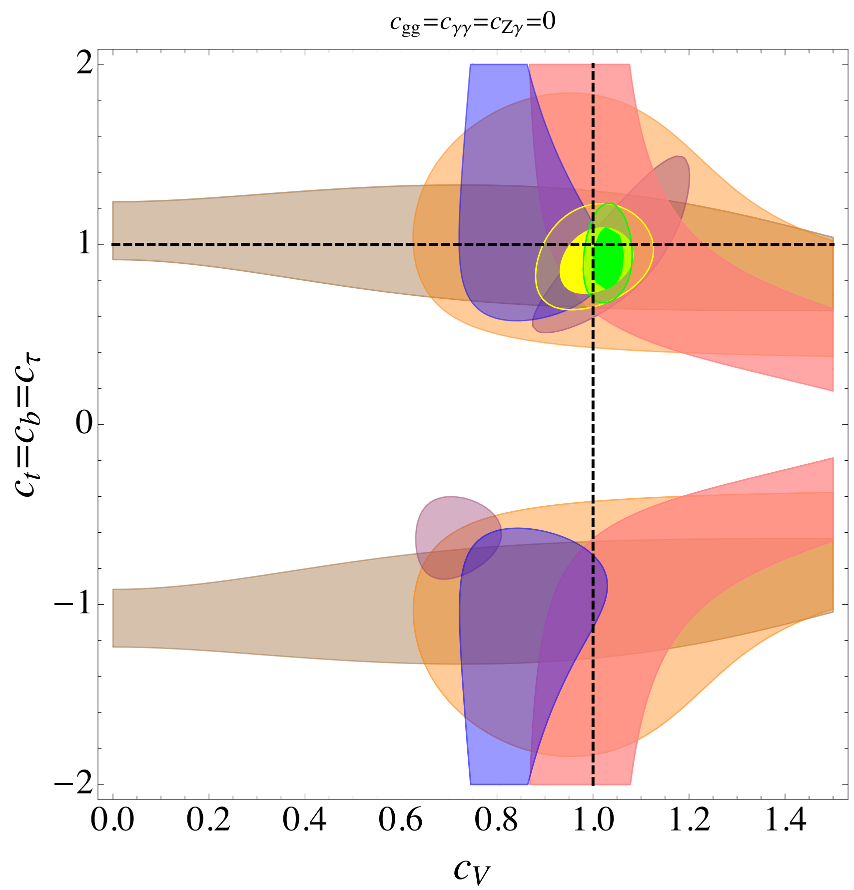
η_{bb}	$(\epsilon+1)^2\xi$
η_{WW}	ξ
$\eta_{\tau\tau}$	$(\epsilon+1)^2\xi$
η_{ZZ}	ξ
η_{gg}	$(\epsilon+1)^2\xi$
$\eta_{\gamma\gamma}$	$\xi\phi$

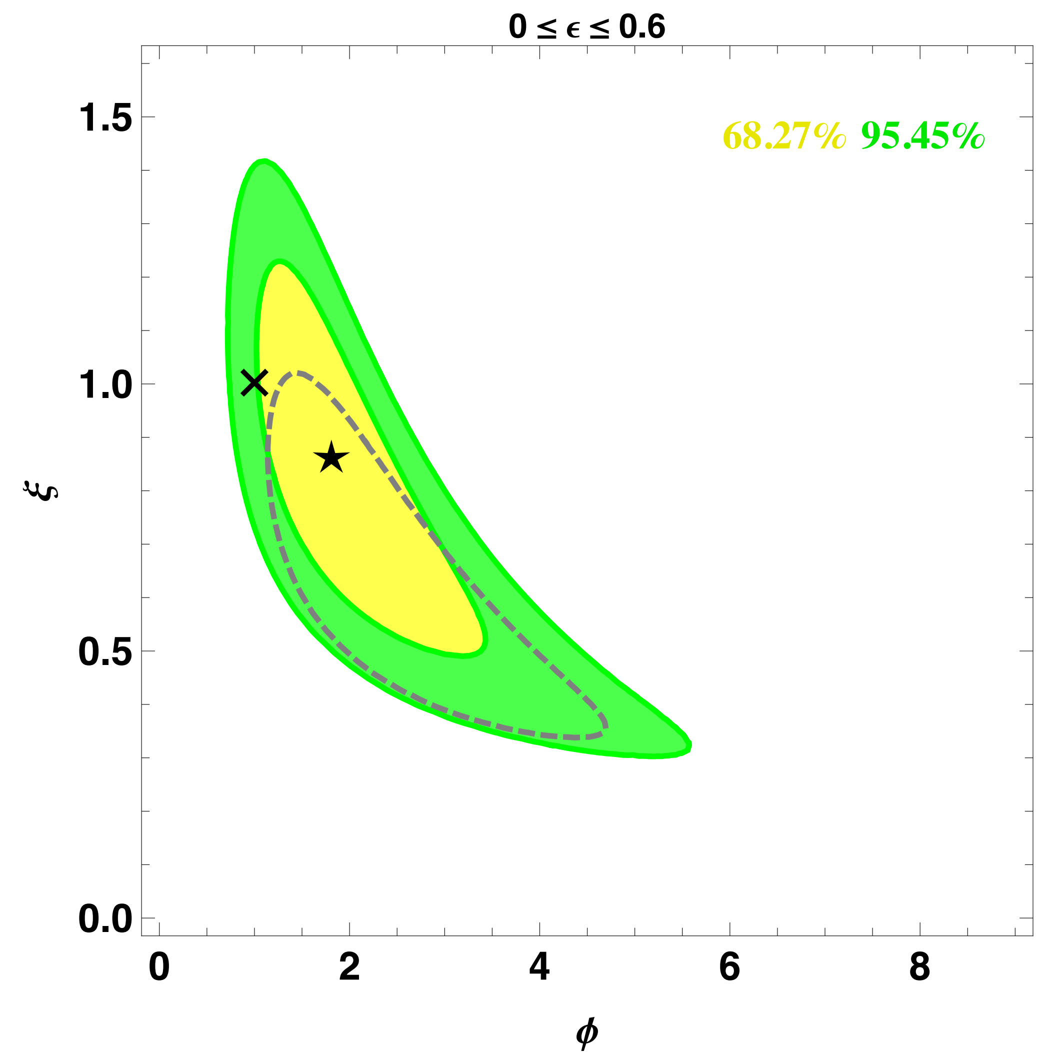
[Chacko et al 2012]



Best fit 2012/2013

[Chacko et al 2012, Falkowski et al 2013]

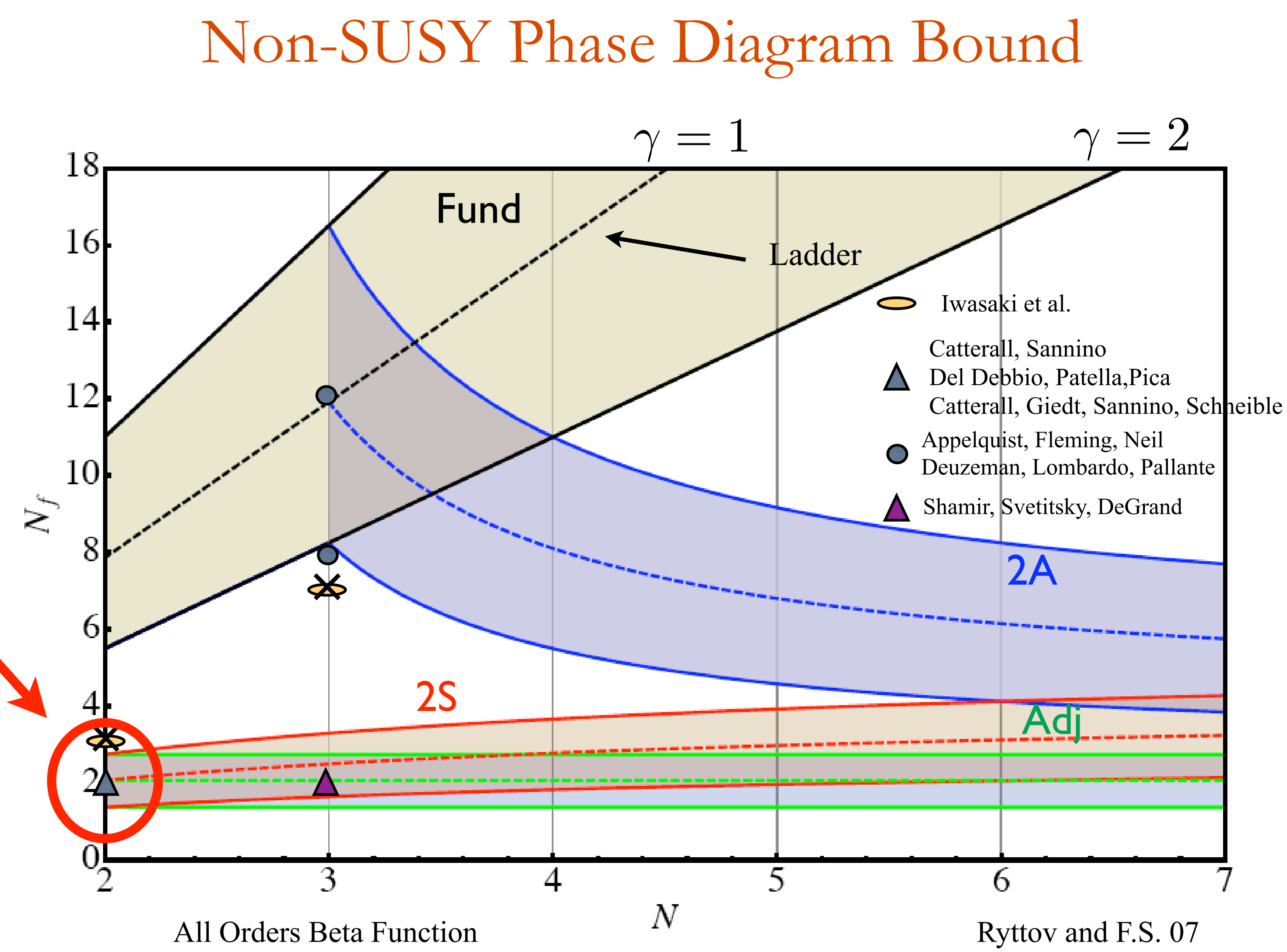




Phase diagram of SU(N) gauge theories

Use lattice tools to search for IRFPs in 4D SU(N) gauge theories

Light dynamical fermions are needed: results only in the last six years



Gauge/string duality [Nunez, Piai, Pomarol, Anguelova], RG [Braun, Gies], Dyson-Schwinger

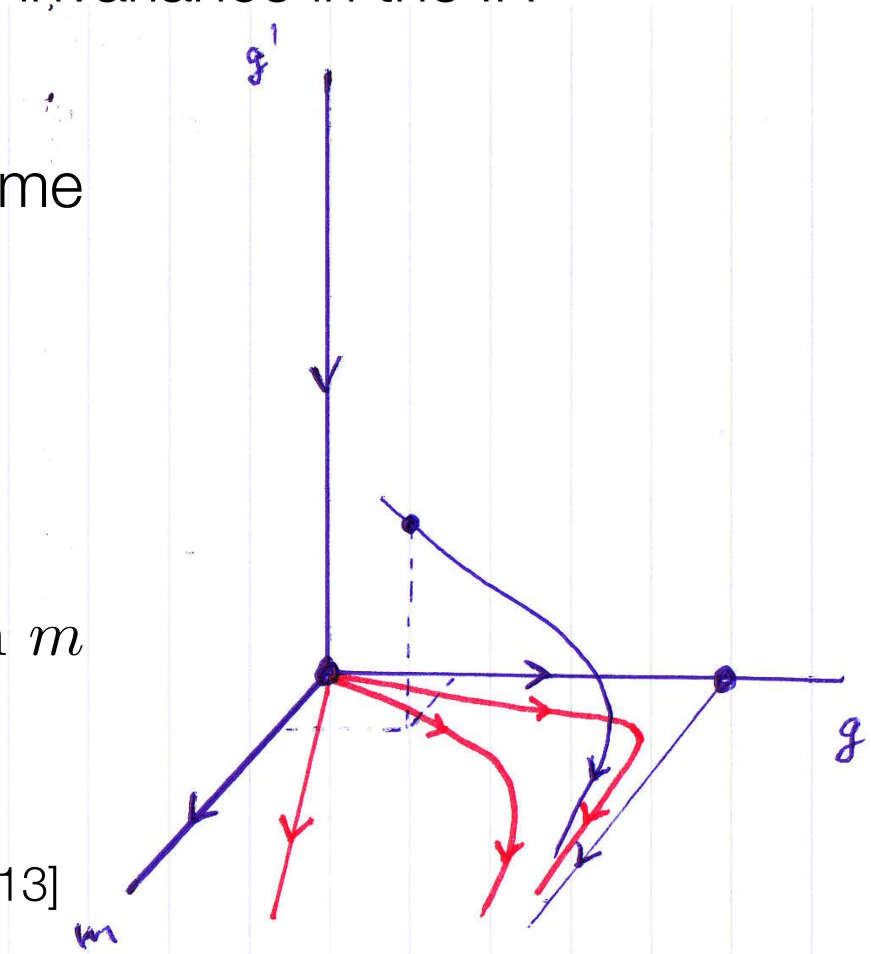
Mass-deformed CFT

- The identification of a CFT by numerical simulations is a difficult task
- No massive spectrum; power-law behaviour of correlators at large distances
- Numerical simulations are performed at *finite fermion mass*, and/or in a *finite-volume* box; both the mass and the finite volume break scale invariance in the IR
- Consider a CGT deformed by a mass term/finite volume
- Determine the scaling of physical observables

$$\mathcal{O} \sim m^{\eta_\mathcal{O}} + \text{higher order in } m + \text{terms analytic in } m$$

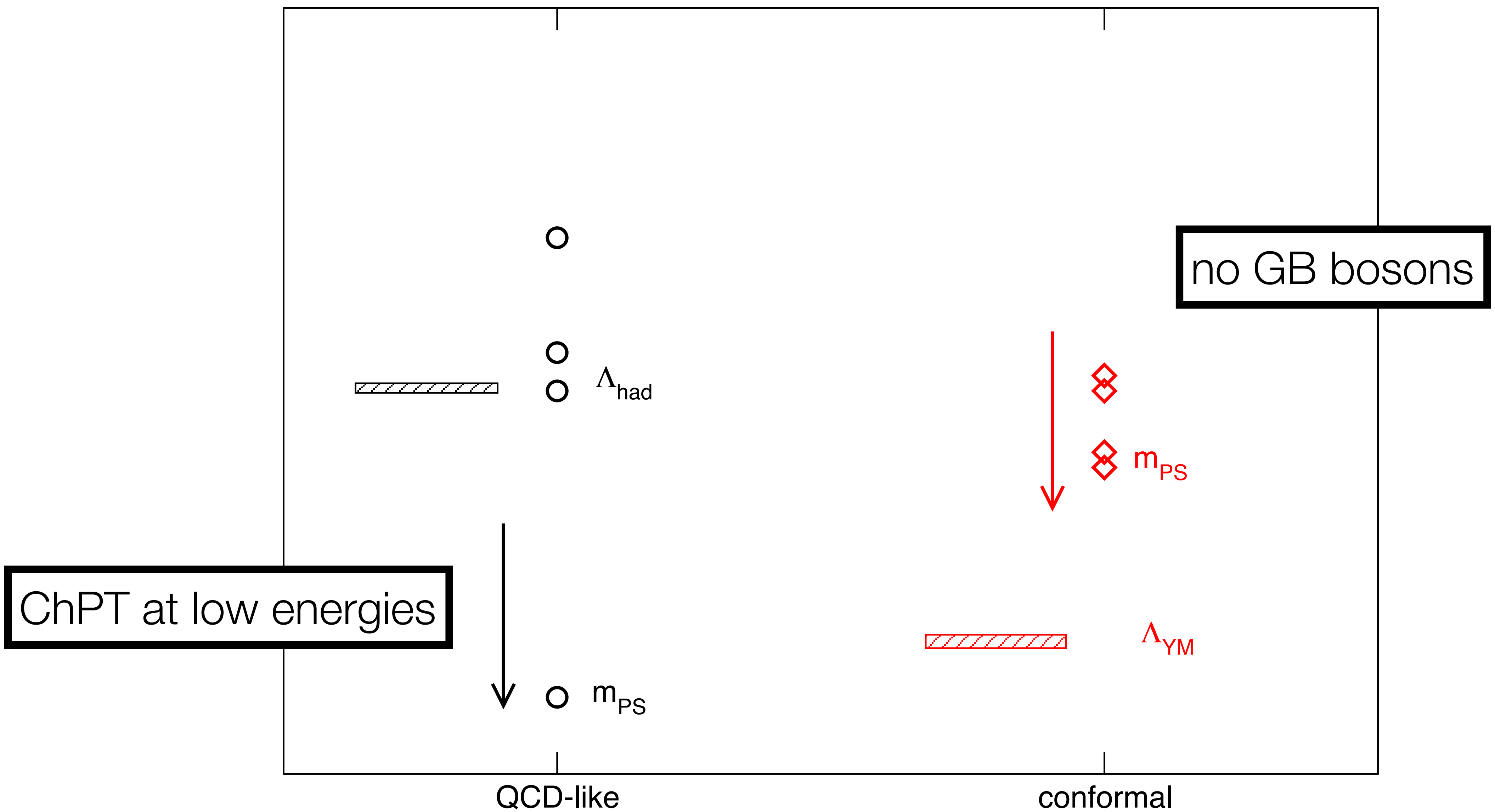
$$\boxed{M_H \propto \mu\, m^{\frac{1}{1+\gamma_*}}}$$

[LDD, Zwicky 10-13]



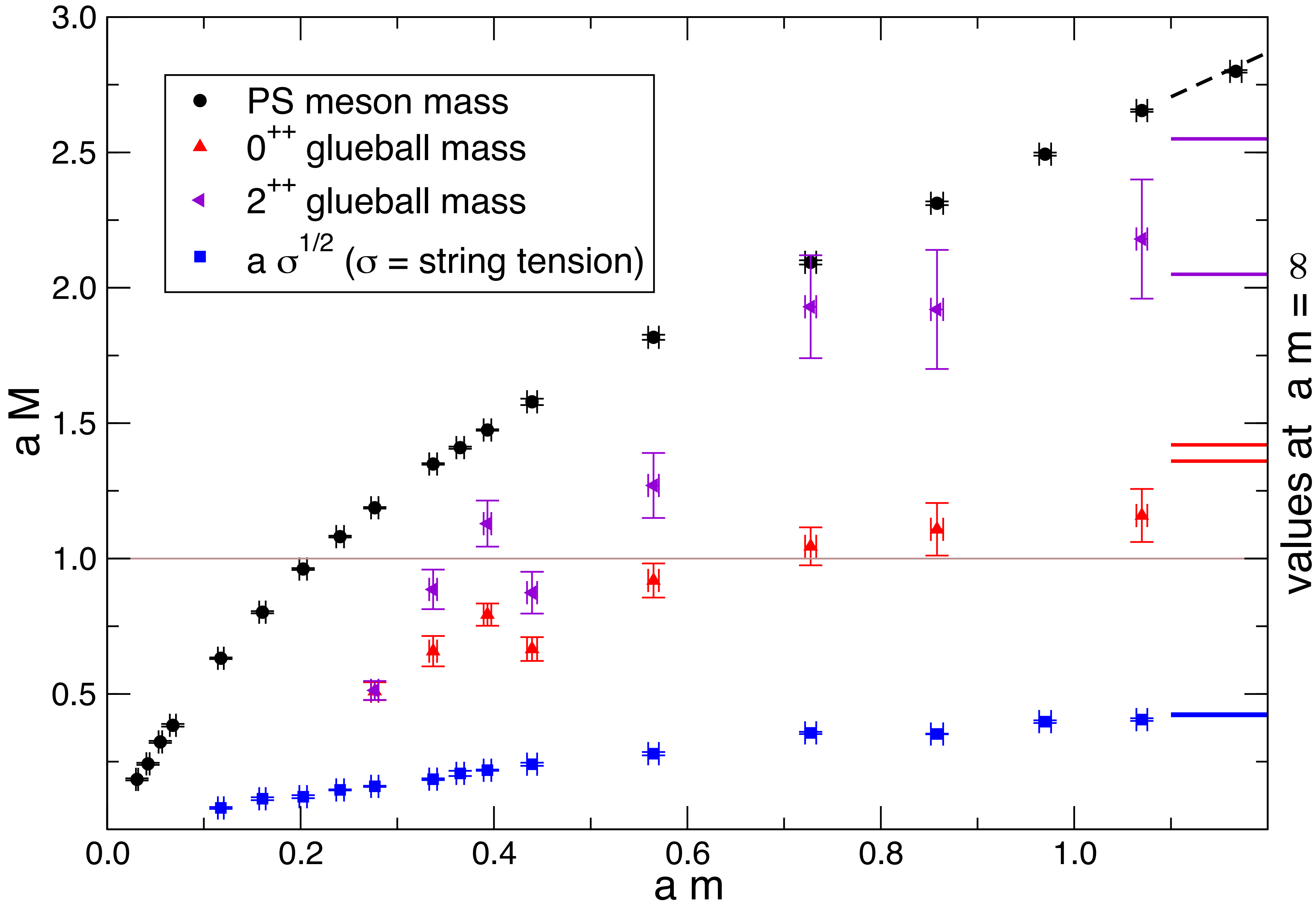
Conformal spectrum

- Different qualitative behaviours in the chiral limit



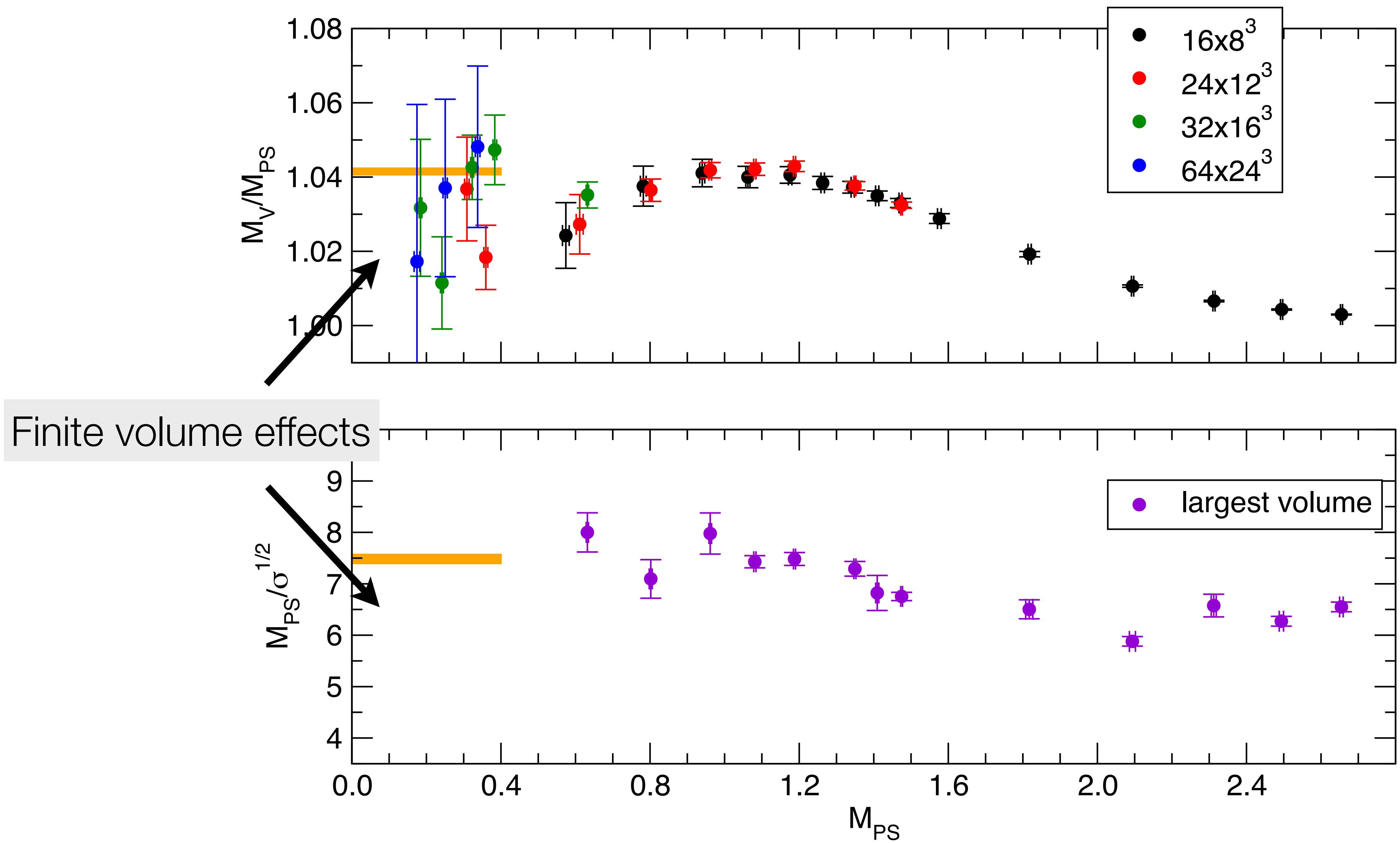
Spectrum for SU(2) + 2 adjoint fermions

- Overall picture: non-singlet meson states & glue



[LDD et al 09]

Finite volume effects?

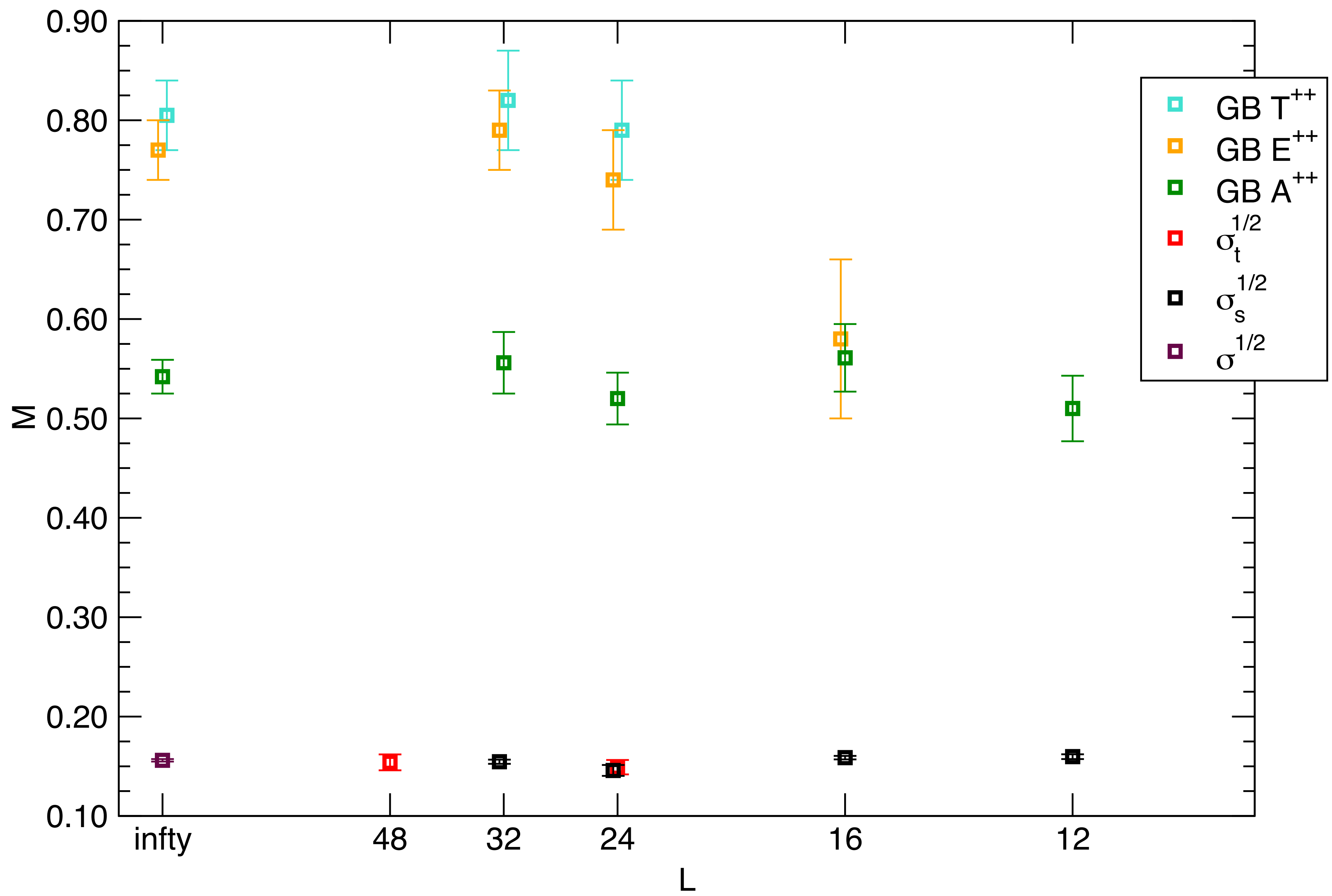


Qualitative evidence for a conformal spectrum

Need large lattices and small masses to control systematic errors

[LDD et al 11]

Larger volumes - heavier mass



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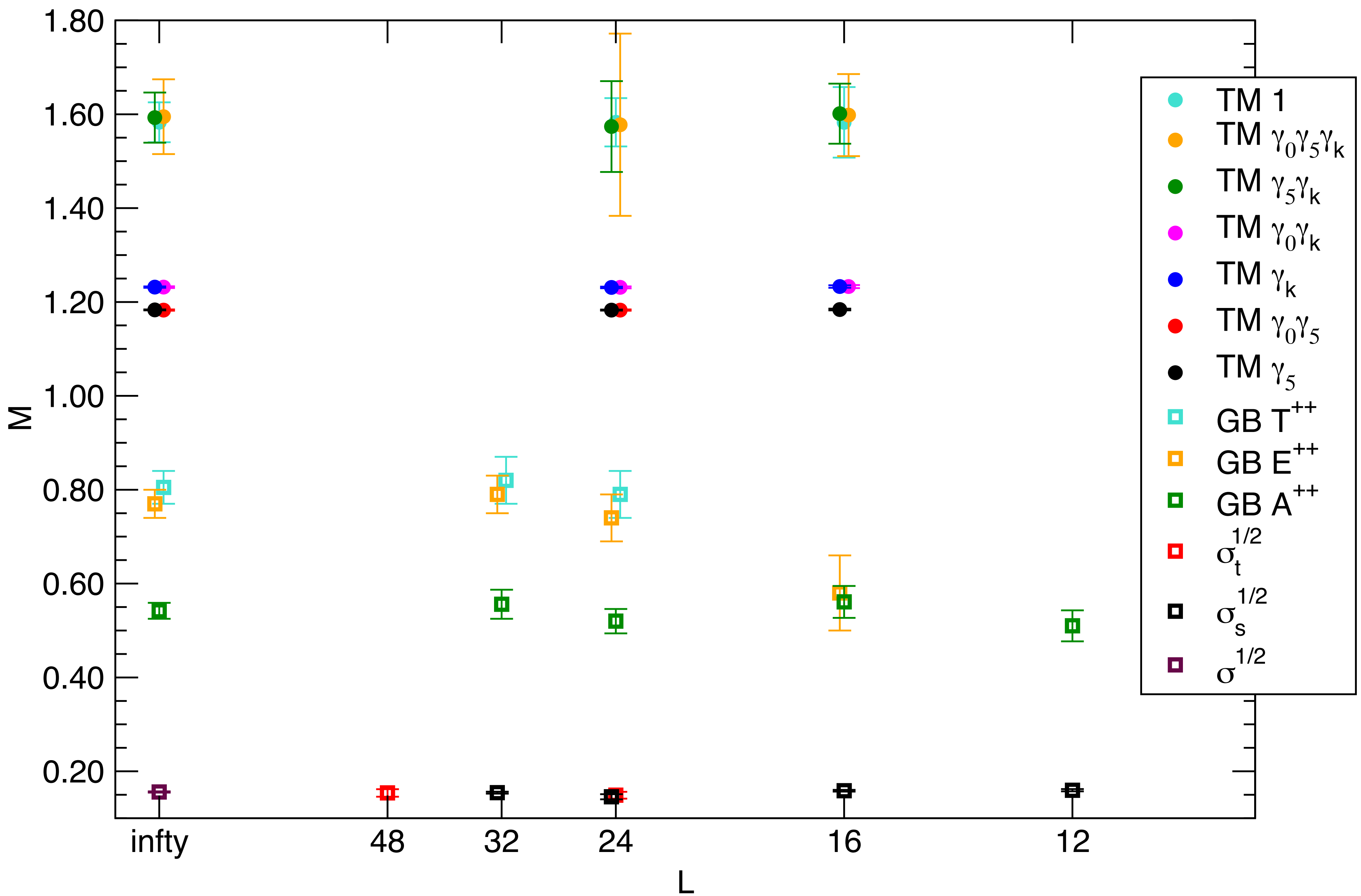

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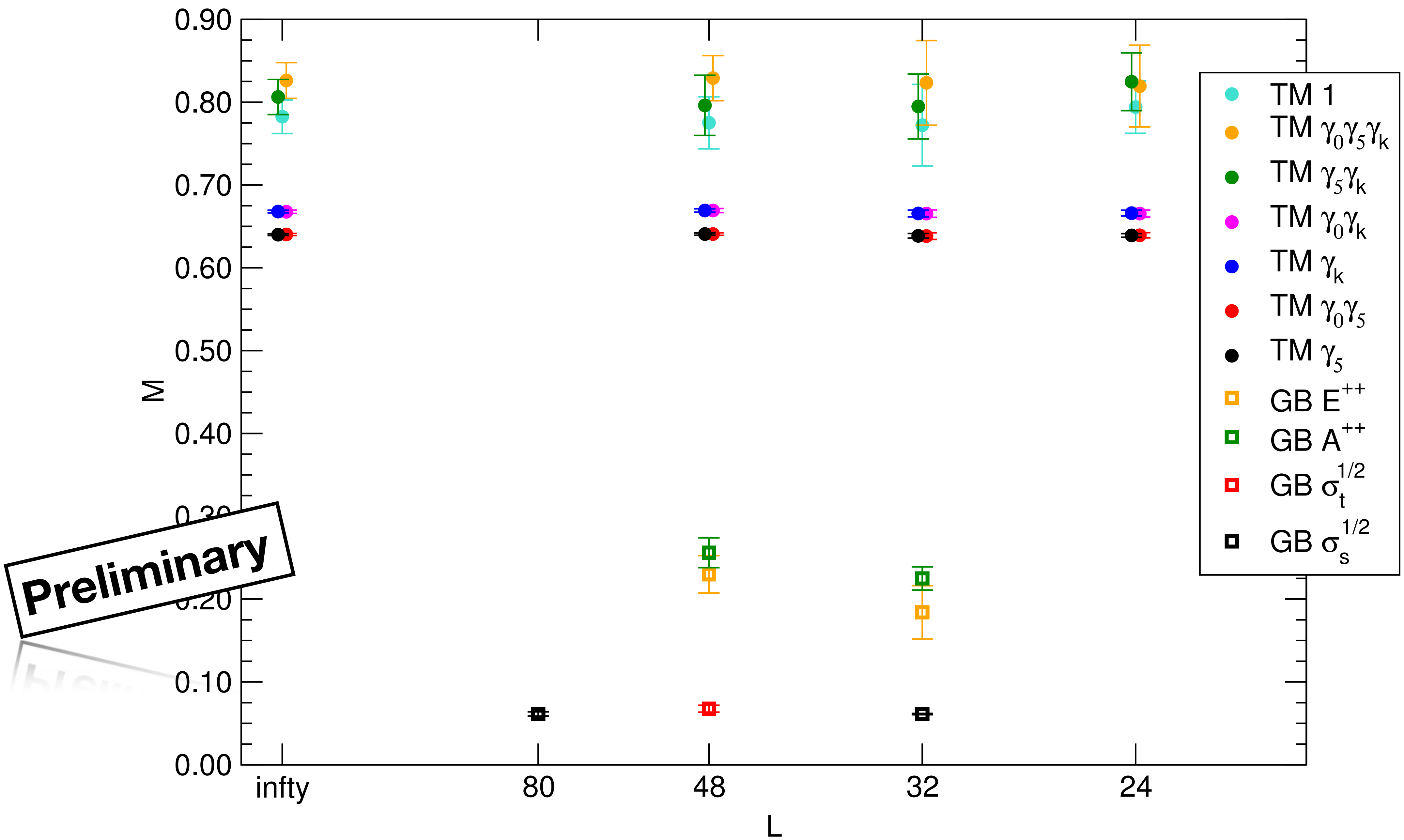
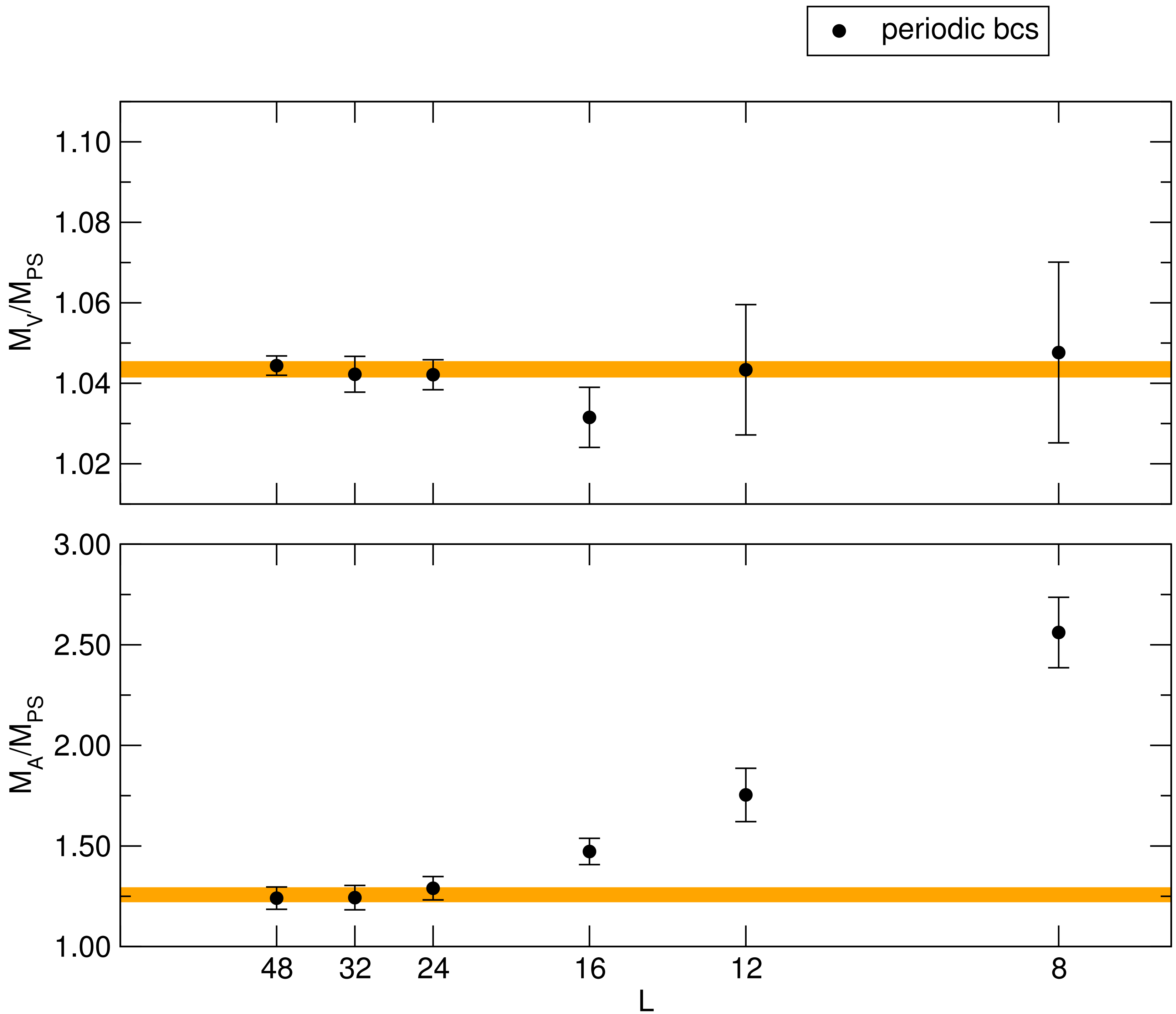


Mesonic mass ratios - lighter mass



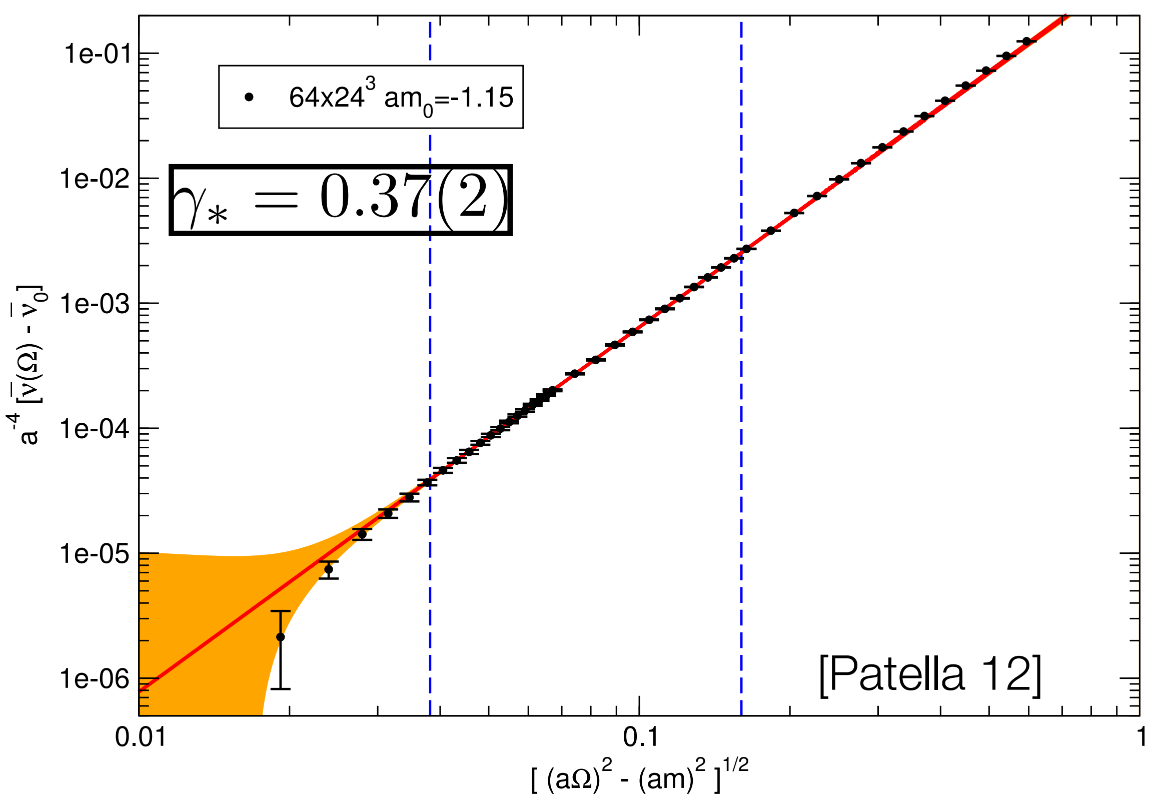
Dirac Eigenvalues

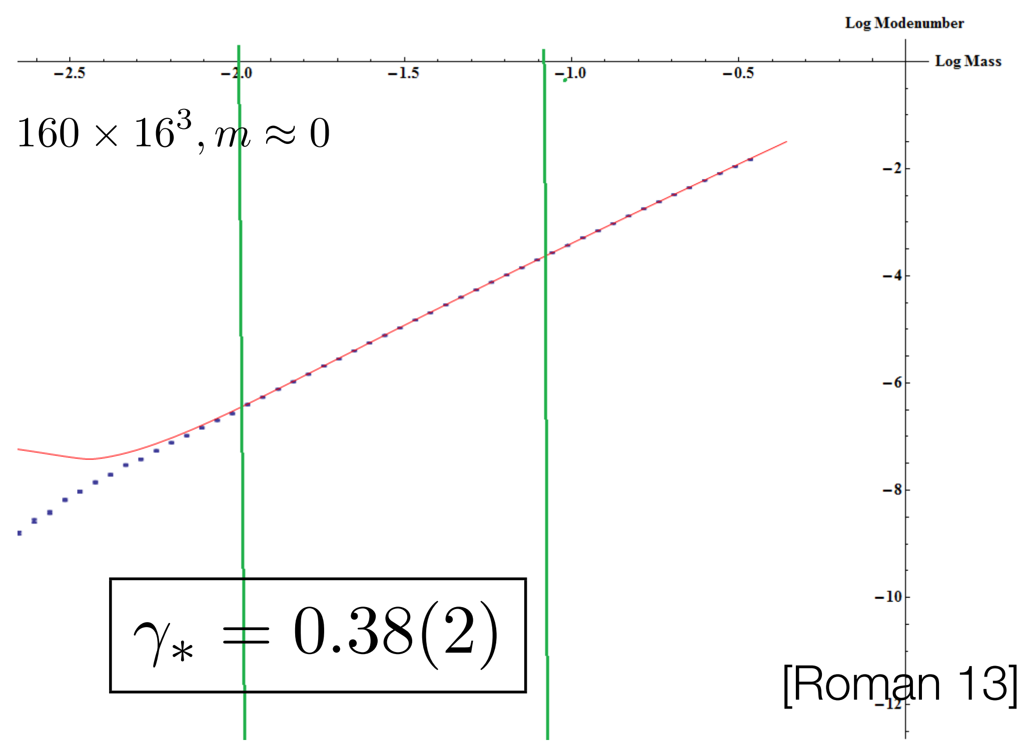
Scaling of the eigenvalue density:

$$\langle \bar{q}q \rangle \overset{m\to 0}{\sim} m^{\eta_{\bar{q}q}} \quad \Leftrightarrow \quad \rho(\lambda) \overset{\lambda\to 0}{\sim} \lambda^{\eta_{\bar{q}q}} \; .$$

[DeGrand 09, LDD & Zwicky 10, Patella 12]

Measure the mode number of $D^\dagger D + m^2 \longrightarrow \nu(M,m) = C + \left(M^2 - m^2\right)^{2/(1+\gamma_*)}$





Finite-size scaling

FSS for the masses in the spectrum:

$$\boxed{M_H = L^{-1} f(x)} \qquad\qquad x = L^{y_m} m$$

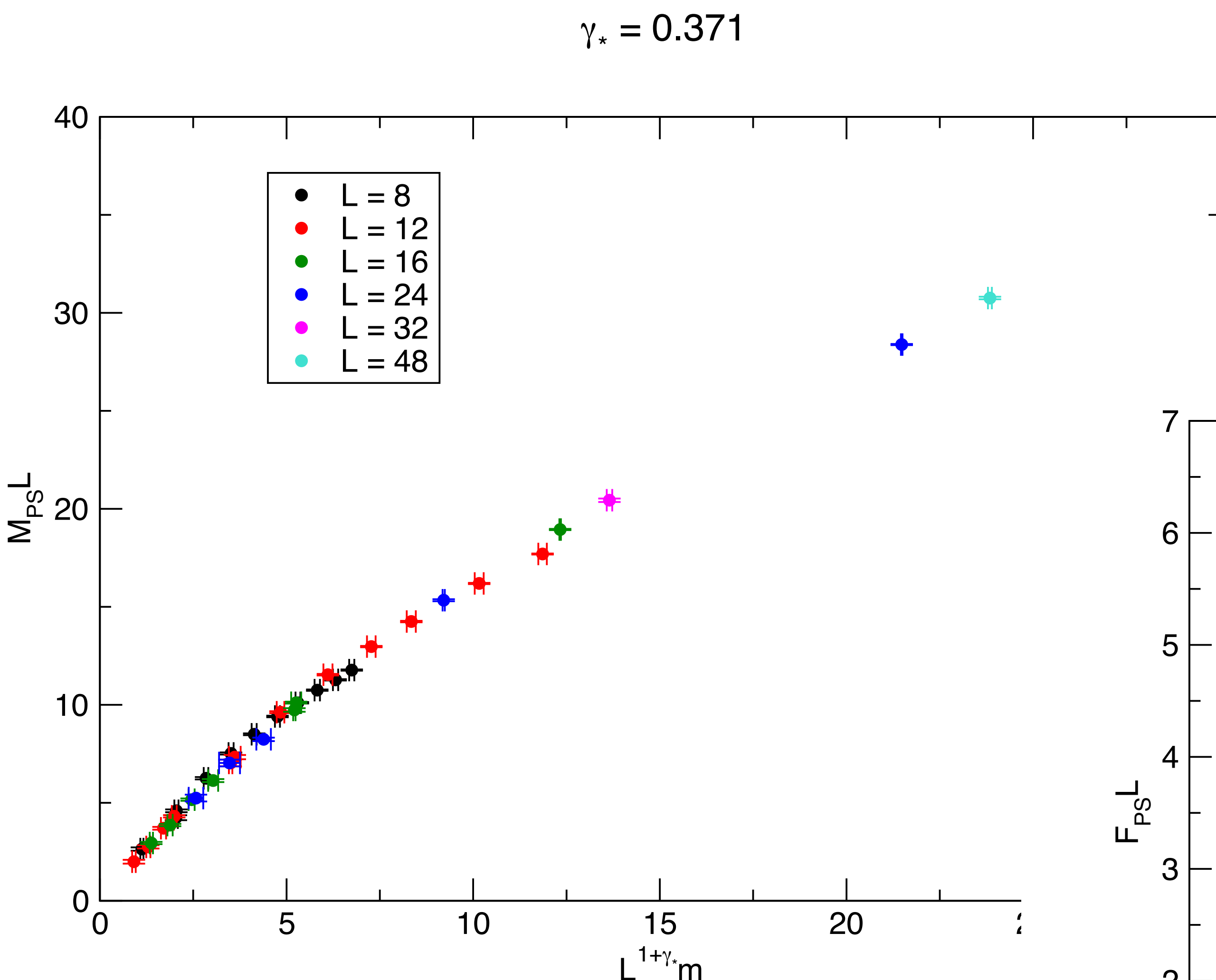
In order to recover the correct scaling with m at **infinite volume**:

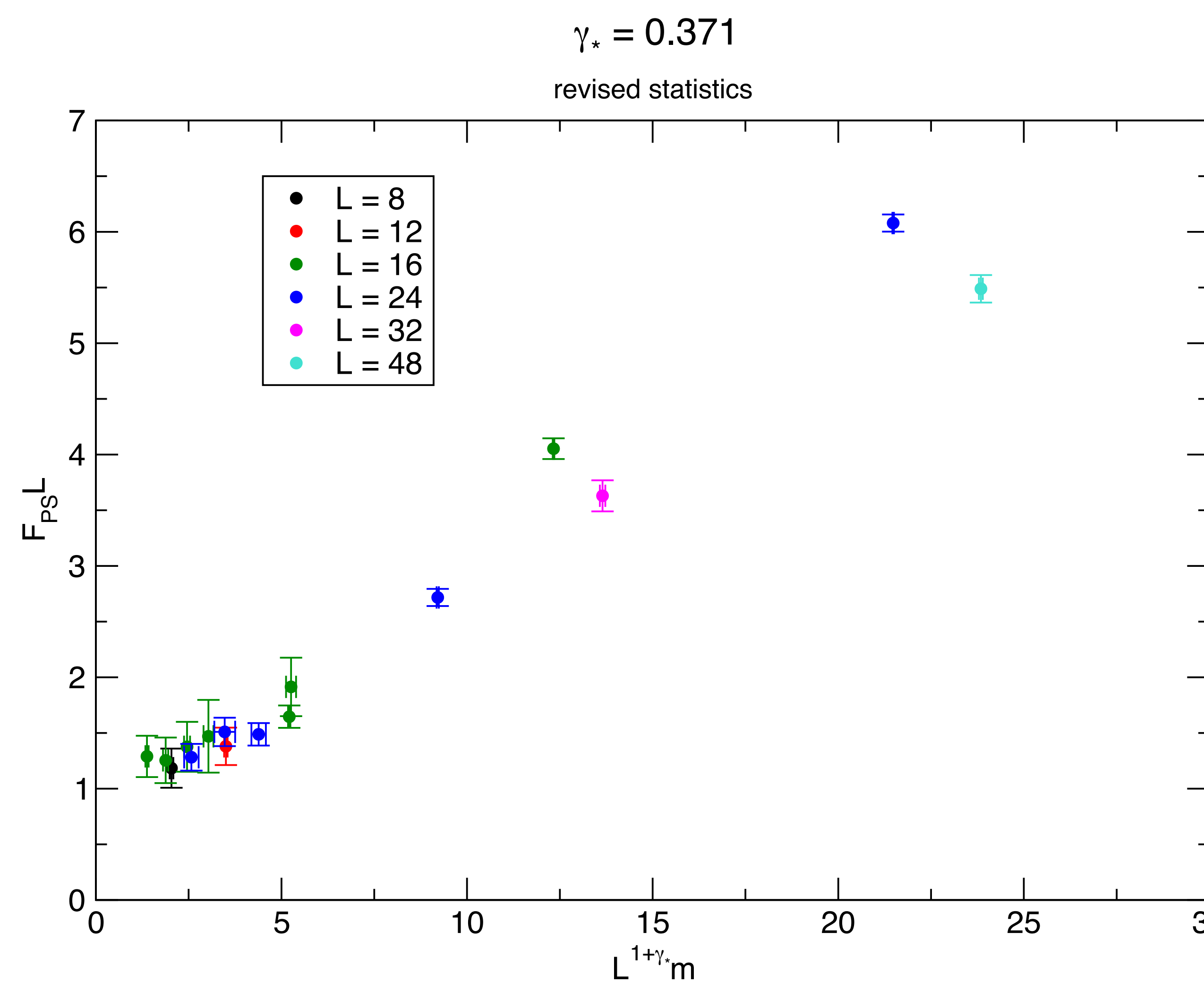
$$f(x) \sim x^{1/y_m}\,, \quad \text{as } x \to \infty$$

If we go to the massless limit, at **fixed** volume and cut-off, the masses of the states in the spectrum of the theory saturate and scale as:

$$M_H \propto L^{-1}$$

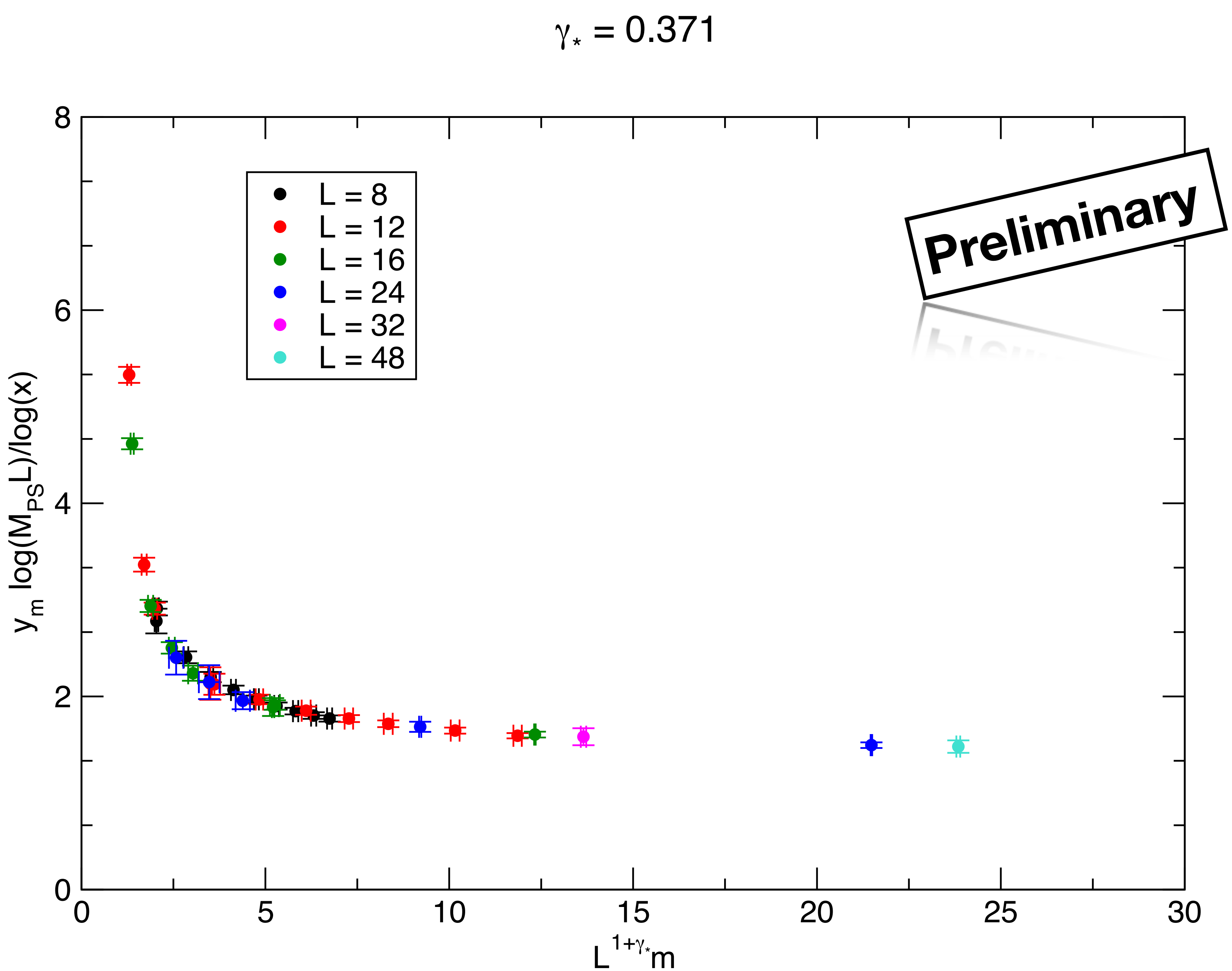
FSS - example





Preliminary

FSS - asymptotic behaviour



Conclusions

Spectrum in the mesonic sector is under control - confirm our earlier observations

Data are **consistent** with conformal scaling

Lighter states in the gluonic sector are difficult (variational method, centre symmetry)

Eigenvalues of Dirac operator yield the best determination of the anomalous dimension

FSS compatible with lattice data

Consistent picture in agreement with a IRFP

Other models more phenomenologically appealing?