FLAG - the Flavour Lattice Averaging Group

v2

IPPP, Durham

04-07.09.2013

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UNIVERSITY OF Southampton

What is FLAG

Flavour Lattice Averaging Group –
“What’s currently the best lattice value for a particular quantity?”

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FLAG-1 *(Eur.Phys.J. C71 (2011) 1695)*:

- founded in 2009 as FLAVIANet Lattice Averaging Group (EU FP6 Marie Curie RTN)
- quantities: $m_{u,d}$, m_s, f_K/f_π, $f_+^{K\pi}(0)$, B_K, SU(2) and SU(3) LECs
- content:
  - summary of results
  - evaluation of FLAG quality criteria (colour coding)
  - averages or best values where possible
  - detailed summary of properties of individual simulations
  - lattice glossary

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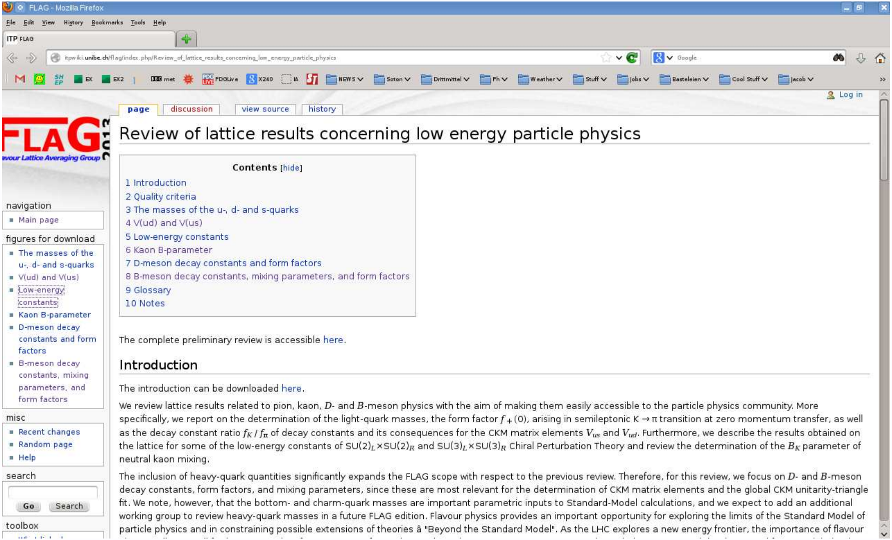
FLAG-2 *(web version: `http://itpwiki.unibe.ch/flag/`), version to be published is in preparation*:

- joined with `www.latticeaverages.org`
- geographically enlarged (America, Asia/Oceania, Europe)
- quantities: FLAG-1 + α_s, $f_{B_{(s)}}$, $f_{D_{(s)}}$, $B_{B_{(s)}}$ and B, D semileptonic decays
- data-deadline pre-lattice 2013

FLAG web



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Review of lattice results concerning low energy particle physics

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The complete preliminary review is accessible here.

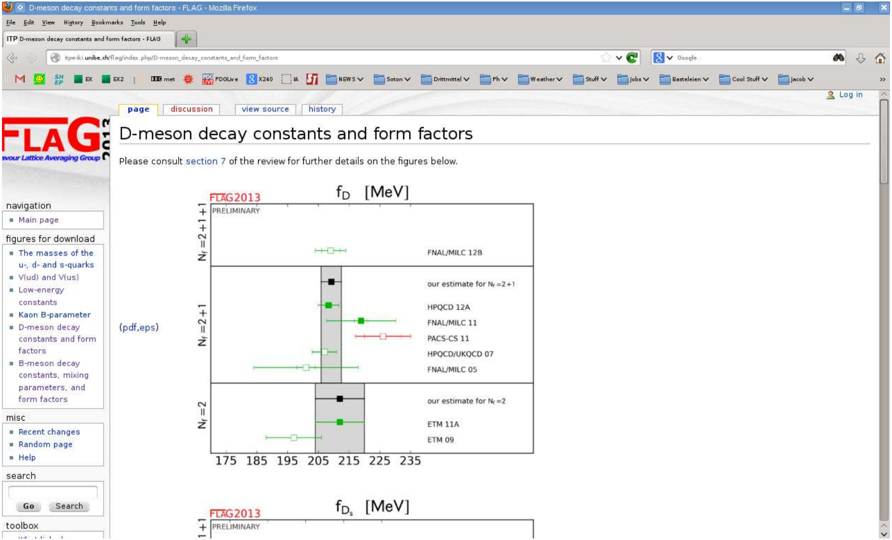
Introduction

The introduction can be downloaded here.

We review lattice results related to pion, kaon, D- and B-meson physics with the aim of making them easily accessible to the particle physics community. More specifically, we report on the determination of the light-quark masses, the form factor $f_+(0)$, arising in semileptonic $K \to \pi$ transition at zero momentum transfer, as well as the decay constant ratio f_K/f_π of decay constants and its consequences for the CKM matrix elements V_{us} and V_{ud}. Furthermore, we describe the results obtained on the lattice for some of the low-energy constants of $SU(2)_L \times SU(2)_R$ and $SU(3)_L \times SU(3)_R$ Chiral Perturbation Theory and review the determination of the B_K parameter of neutral kaon mixing.

The inclusion of heavy-quark quantities significantly expands the FLAG scope with respect to the previous review. Therefore, for this review, we focus on D- and B-meson decay constants, form factors, and mixing parameters, since these are most relevant for the determination of CKM matrix elements and the global CKM unitarity-triangle fit. We note, however, that the bottom- and charm-quark masses are important parametric inputs to Standard-Model calculations, and we expect to add an additional working group to review heavy-quark masses in a future FLAG edition. Flavour physics provides an important opportunity for exploring the limits of the Standard Model of particle physics and in constraining possible extensions of theories â "Beyond the Standard Model". As the LHC explores a new energy frontier, the importance of flavour

FLAG web


D-meson decay constants and form factors - FLAG - Mozilla Firefox
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PRELIMINARY

What is FLAG - collaboration

Advisory Board (AB):	S. Aoki, C. Bernard, C. Sachrajda
Editorial Board (EB):	G. Colangelo, H. Leutwyler, T. Vladikas, U. Wenger
Working Groups (WG):	
• Quark masses	L. Lellouch, T. Blum, V. Lubicz
• V_{us}, V_{ud}	A. Jüttner, T. Kaneko, S. Simula
• LEC	S. Dürr, H. Fukaya, S. Necco
• B_K	H. Wittig, J. Laiho, S. Sharpe
• α_s	R. Sommer, R. Horsley, T. Onogi
• $f_{B_{(s)}}$, $f_{D_{(s)}}$, B_B	A. El Khadra, Y. Aoki, M. Della Morte
• $B_{(s)}$, D semileptonic and radiative decays	R. Van de Water, E. Lunghi, C. Pena, J. Shigemitsu

Lattice QCD and systematic uncertainties

Define observables in terms of Euclidean *n*-pt functions

$$\langle 0|O(x_1,x_2,\cdots,x_n)|0\rangle = \frac{1}{\mathcal{Z}}\int \mathcal{D}[U,\psi,\bar{\psi}]O(x_1,x_2,\cdots,x_n)e^{-S_{\rm lat}[U,\psi,\bar{\psi}]}$$

Lattice QCD: Evaluate discretised path integral in finite volume by means of Monte Carlo integration

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Lattice QCD: Evaluate discretised path integral in finite volume by means of Monte Carlo integration

In practice need to cntrol a number of sources of systematic uncertainties:

- discretisation error (lattice spacing a)
  effects differ between heavy and light quarks, so currently FLAG uses different criteria
- finite volume errors (box size L)
- quark mass extrapolation
  (until recently mostly unphysically heavy quark masses)
- renormalisation, running
- heavy quark treatment

Lattice QCD and systematic uncertainties

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FLAG assesses in detail every computation and indicates its verdict for the "state-of-the-art- fulness" by assigning a color code

FLAG-2 criteria - light quark physics

- ★ when the systematic error has been estimated in a satisfactory manner and convincingly shown to be under control;
- ○ when a reasonable attempt at estimating the systematic error has been made, although this could be improved;
- ■ when no or a clearly unsatisfactory attempt at estimating the systematic error has been made.

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Chiral extrapolation:

- ★ $M_{\pi,\mathrm{min}} < 200$ MeV
- ○ 200 MeV $\leq M_{\pi,\mathrm{min}} \leq$ 400 MeV
- ■ $M_{\pi,\mathrm{min}} > 400$ MeV

Continuum extrapolation:

- ★ ≥3 lattice spacings, ≥ 2 points below 0.1 fm
- ○ 2 or more lattice spacings, at least 1 point below 0.1 fm
- ■ otherwise

Finite-volume effects:

- ★ $M_{\pi,\mathrm{min}}L > 4$ or at least 3 L
- ○ $M_{\pi,\mathrm{min}}L > 3$ and at least 2 L
- ■ otherwise

Renormalization (if applicable):

- ★ non-perturbative
- ○ 1-loop perturbation theory or higher with a reasonable estimate of truncation errors
- ■ otherwise

FLAG-2 criteria - heavy quark physics

- discretisation effects treated differently: $m_h = O(a^{-1})$
- lattice charm quarks: fully relativistic or effective theory based
- lattice bottom quark: effective theory based

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Heavy quark treatment:

✓ RHQ (tl $O(a)$ improved), NRQCD (tl. matched $O(1/m_h)$, improved through $O(a^2)$, HQET (including $1/m$ and leading cut-offs at $O(a^2)$), standard lattice actions ($O(a)$-improved)

Continuum extrapolation:

★ ≥3 lattice spacings & $a^2_{\rm max}/a^2_{\rm min} \geq 2$ & $D(a_{\rm min}) \leq 2\%$ & $\delta(a_{\rm min}) \leq 1$

○ Two or more lattice spacings & $a^2_{\rm max}/a^2_{\rm min} \geq 1.4$ & $D(a_{\rm min}) \leq 10\%$ & $\delta(a_{\rm min}) \leq 2$

■ otherwise

$D(a)$ relative difference between finest lattice data and continuum limit result

$\delta(a)$ deviation of finest lattice data relative to the statistical and systematic uncertainty of the calculation

FLAG-2 averages

If

- verdict ★ or ○
- NO single ■
- published/update

FLAG may provide an average.

Alternatively FLAG may provide a best value/range

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central value:

- add stat. and syst. error in quadrature
- take weighted average as central value
- error from solution of $\chi^2 \overset{!}{=} \chi^2_{\text{min}} + 1$
- PDG-inflation if necessary
- if final error smaller than any of the individual syst. errors, take the smallest syst. error as final error

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in the presence of correlations:

- construct covariance matrix assuming 100% correlation
- use Schmelling's *Phys.Scripta 51, 676 (1995)* prescription to compute error
- if necessary inflate error

If you decide use FLAG-averages

. . . PLEASE CITE THE ORIGINAL WORK!!!

- FLAG only only summarises lattice results for you - the years of hard work that lead to the original results needs to get credits!
- it's easy:
  - we explain every FLAG average in detail
  - collaborations are tagged, e.g. “MILC 10”, “RBC/UKQCD 12A”, . . .
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DATA IN THIS TALK IS STILL PRELIMINARY

CKM physics

$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

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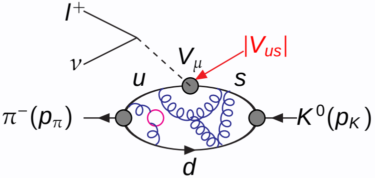
- f_π often input parameter
- leptonic and semileptonic decay
- B-mixing
- also: $B_K \to \epsilon_K$

Example: $|V_{us}|$ from semileptonic K-decay



$$\Gamma_{K\to\pi l\nu} = C_K^2 \frac{G_F^2 m_K^5}{192\pi^2} S_{\rm EW}(1 + \Delta_{SU(2)} + \Delta_{\rm EM})^2 \, I \, |f_+^{K\pi}(0)|^2 |V_{us}|^2$$

$$\langle \pi(p_\pi)|V_\mu(0)|K(p_K)\rangle = f_+^{K\pi}(q^2)(p_K + p_\pi)_\mu + f_-^{K\pi}(q^2)(p_K - p_\pi)_\mu$$

- I phase space integral (via FF shape from experiment)
- $S_{\rm EW}$ short distance EW corrections
- $\Delta_{SU(2)}$ iso-spin breaking corrections (χPT)
- $\Delta_{\rm EM}$ long distance EM corrections (χPT)

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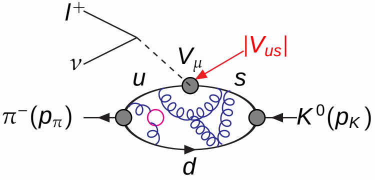

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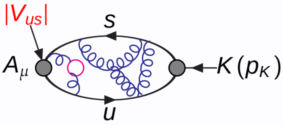
- all results from isospin symmetric, pure $N_f = 2, 2+1, 2+1+1$ QCD
- $D \to K, \pi$ decays treated similarly (full kinematical range accessible)
- $B \to D, K, \pi$ kinematical range restricted, heavy-quark treatment for b-quarks

$|V_{us}|$ from leptonic kaon decay



$$\Gamma(K \to \mu\bar{\nu}_\mu) = \frac{G_F^2}{8\pi} f_K^2 m_\mu^2 m_K \left(1 - \frac{m_\mu^2}{m_K^2}\right)^2 |V_{us}|^2$$

$$\left\langle 0|\bar{s}/\bar{d}\gamma_\mu\gamma_5 u|K/\pi(p)\right\rangle = i f_{K/\pi} p_\mu$$

$$\frac{\Gamma(K \to \mu\bar{\nu}_\mu)}{\Gamma(\pi \to \mu\bar{\nu}_\mu)} = \frac{|V_{us}|^2}{|V_{ud}|^2} \left(\frac{f_K}{f_\pi}\right)^2 \frac{m_K(1 - m_\mu^2/m_K^2)^2}{m_\pi(1 - m_\mu^2/m_\pi^2)^2} \times 0.9930(35)$$

(Marciano, Phys.Rev.Lett. 2004)

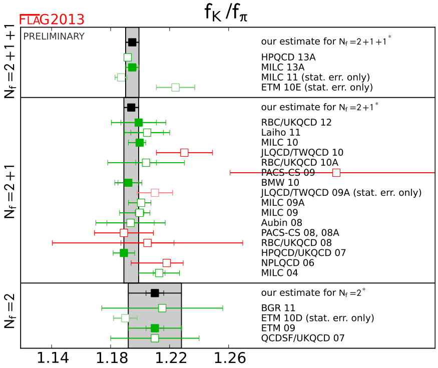
A FLAG example - the kaon sector

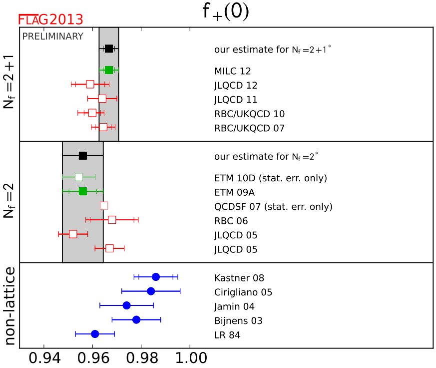
Collaboration	Ref.	N_f	publication status	chiral extrapolation	continuum extrapolation	finite volume errors	f_K/f_π	$f_{K^\pm}/f_{\pi^\pm}$
HPQCD 13A	[126]	2+1+1	P	★	○	★		1.1916(15)(16)
MILC 13A	[127]	2+1+1	A	★	○	★		1.1947(26)(37)
MILC 11	[9]	2+1+1	C	○	○	○		$1.1872(42)^{\dagger}_{\rm stat.}$
ETM 10E	[128]	2+1+1	C	○	○	○	$1.224(13)_{\rm stat}$	
RBC/UKQCD 12	[10]	2+1	A	★	○	★	1.199(12)(14)	
Laiho 11	[129]	2+1	C	○	○	○		$1.202(11)(9)(2)(5)^{\dagger\dagger}$
MILC 10	[130]	2+1	C	○	★	★		$1.197(2)(^{+3}_{-7})$
JLQCD/TWQCD 10	[131]	2+1	C	○	■	★	1.230(19)	
RBC/UKQCD 10A	[61]	2+1	A	○	○	★	1.204(7)(25)	
PACS-CS 09	[5]	2+1	A	★	■	■	1.333(72)	
BMW 10	[132]	2+1	A	★	★	★	1.192(7)(6)	
JLQCD/TWQCD 09A	[133]	2+1	C	○	■	■	$1.210(12)_{\rm stat}$	
MILC 09A	[23]	2+1	C	○	★	★		$1.198(2)(^{+6}_{-8})$
MILC 09	[22]	2+1	A	○	★	★		$1.197(3)(^{+6}_{-13})$
Aubin 08	[134]	2+1	C	○	○	○		1.191(16)(17)
PACS-CS 08, 08A	[4, 135]	2+1	A	★	■	■	1.189(20)	
RBC/UKQCD 08	[63]	2+1	A	○	■	★	1.205(18)(62)	
HPQCD/UKQCD 07	[136]	2+1	A	○	★	○	1.189(2)(7)	
NPLQCD 06	[137]	2+1	A	○	■	■	$1.218(2)(^{+11}_{-24})$	
MILC 04	[21]	2+1	A	○	○	○		1.210(4)(13)
BGR 11	[138]	2	A	★	■	■	1.215(41)	
ETM 10D	[116]	2	C	○	★	○	$1.190(8)_{\rm stat}$	
ETM 09	[139]	2	A	○	★	○	1.210(6)(15)(9)	
QCDSF/UKQCD 07	[140]	2	C	○	○	★	1.21(3)	

† Result with statistical error only from polynomial interpolation to the physical point. †† This work is the continuation of Aubin 08.

Table 9: Colour code for the data on the ratio of decay constants: f_K/f_π is the pure QCD SU(2)-symmetric ratio and $f_{K^\pm}/f_{\pi^\pm}$ is in pure QCD with the SU(2) isospin breaking applied after simulation.

A FLAG example - the kaon sector





	$f_{K^\pm}/f_{\pi^\pm}$	$f_+(0)$
$N_f=2+1+1$	1.195(3)(4)	
$N_f=2+1$	1.192(5)	0.9667(23)(33)
$N_f=2$	1.205(6)(17)	0.9560(57)(62)

- high precision
- many consistent results
- isospin on lattice needs to be included

Analyses (together with f_K/f_π)

Is the CKM matrix unitary? $|V_u|^2 \equiv |V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1$

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- experimental results

$$|V_{us}|f_+^{K\pi}(0) = 0.2163(5) \qquad f_K/f_\pi|V_{us}/V_{ud}| = 0.2758(5)$$

FLAVIA Kaon WG Eur. Phys. J. C 69, 399-424 (2010)

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FLAVIA Kaon WG Eur. Phys. J. C 69, 399-424 (2010)

- SM test
  - $f_+^{K\pi}(0)$ and $|V_{ud}|$ from experiment
  - f_K/f_π and $|V_{ud}|$ from experiment
  - $f_+^{K\pi}(0)$ and f_K/f_π from lattice

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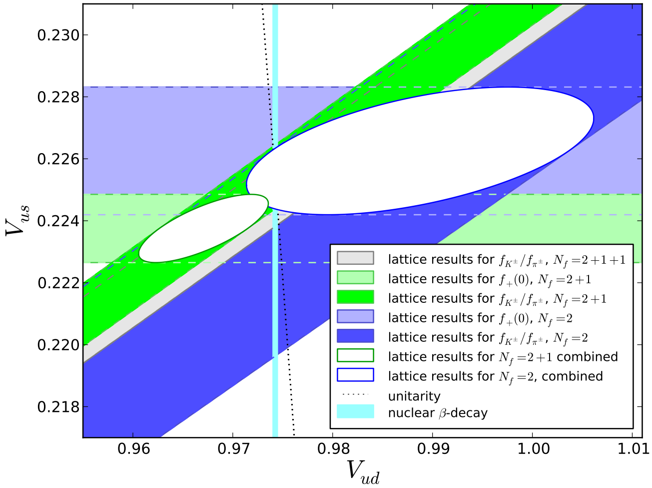
	$f_+^{K\pi}(0)$& $\lvert V_{ud}\rvert_{\rm exp}$	f_K/f_π & $\lvert V_{ud}\rvert_{\rm exp}$	combined$_{\rm no\lvert V_{ud}\rvert exp}$
$N_f = 2+1$	0.9992(6)	1.0000(6)	0.985(13)
$N_f = 2$	1.0004(10)	0.9989(16)	1.029(35)

FLAG tests the SM:



Analyses (together with f_K/f_π)

Use SM-unitarity

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$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1$$

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$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1$$

- ignore $|V_{ub}| = 4 \cdot 10^{-3}$, then 3 equations, 4 unknowns
- provide either
  - $f_+^{K\pi}(0) \rightarrow$ predict $|V_{ud}|$, $|V_{us}|$, f_K/f_π
  - $f_K/f_\pi \rightarrow$ predict $|V_{ud}|$, $|V_{us}|$, $f_+^{K\pi}(0)$

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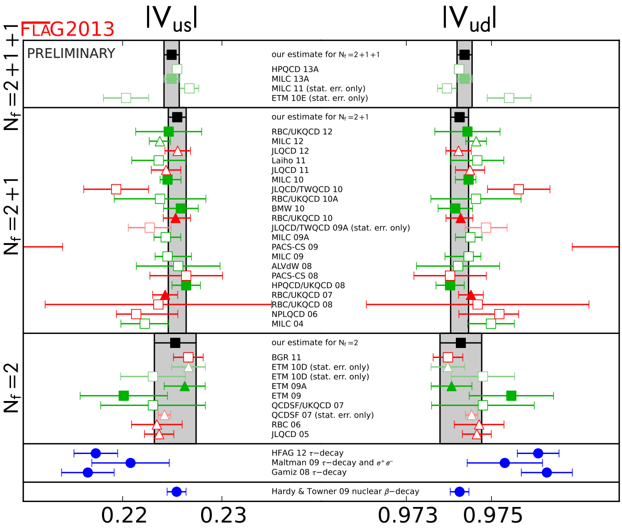
  - $f_K/f_\pi \rightarrow$ predict $|V_{ud}|$, $|V_{us}|$, $f_+^{K\pi}(0)$
- analysis within the SM (relying on the assumption of first-row unitarity)

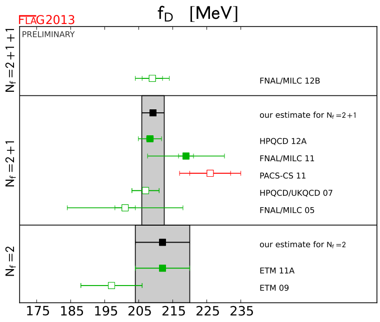
	$\|V_{us}\|$	$\|V_{ud}\|$	$f_+^{K\pi}(0)$	f_K/f_π
$N_f = 2+1$	0.2249(8)	0.97438(18)	0.9624(36)	1.194(4)
$N_f = 2$	0.2253(21)	0.97427(49)	0.9595(90)	1.192(12)

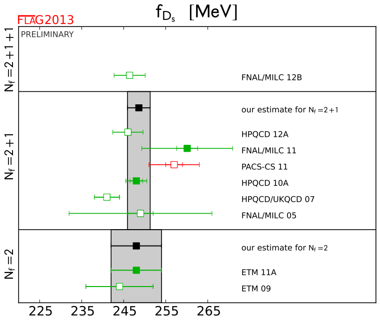
Analyses (together with f_K/f_π)

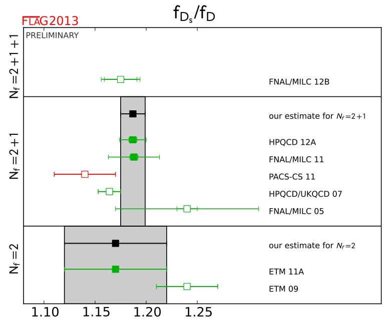


Heavy-light physics

$|V_{cd}|$, $|V_{cs}|$ - leptonic decay

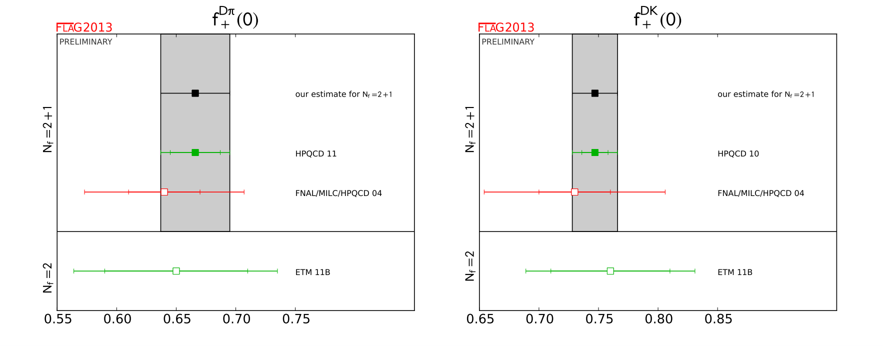






	f_D/MeV	f_{D_s}/MeV	f_{D_s}/f_D
$N_f = 2+1$	209.2(3.3)	248.6(2.7)	1.187(12)
$N_f = 2$	212(8)	248(6)	1.17(5)

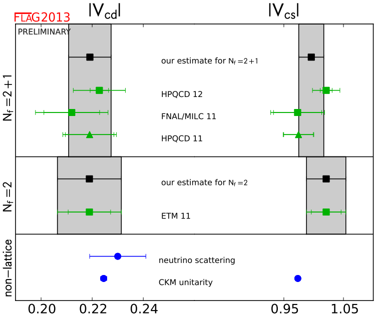
$|V_{cd}|$, $|V_{cs}|$ - leptonic decay



	$f_+^{D\pi}(0)$	$f_+^{DK}(0)$
$N_f = 2+1$	0.666(29)	0.747(19)

$|V_{cd}|, |V_{cs}|$

$$f_D|V_{cd}| = 46.60(1.98)\text{MeV} \quad f_{D_s}|V_{cs}| = 253.1(5.3)\text{MeV} \quad \textit{PDG}$$
$$f_+^{D\pi}(0)|V_{cd}| = 0.146(3) \quad f_+^{DK}|V_{cs}| = 0.728(5)\text{MeV} \quad \textit{HFAG}$$



	$\lvert V_{cd}\rvert$	$\lvert V_{cb}\rvert$	
$N_f = 2+1$	0.2191(83)	0.996(21)	(lept.+semilept. decay)
$N_f = 2$	0.2189(83)(94)	1.021(25)(21)	(lept. decay)

$|V_{cd}|$, $|V_{cs}|$

$$f_D|V_{cd}| = 46.60(1.98)\text{MeV} \quad f_{D_s}|V_{cs}| = 253.1(5.3)\text{MeV} \quad \textit{PDG}$$
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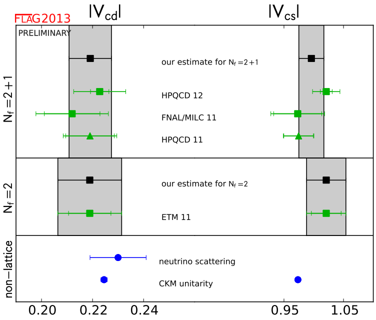
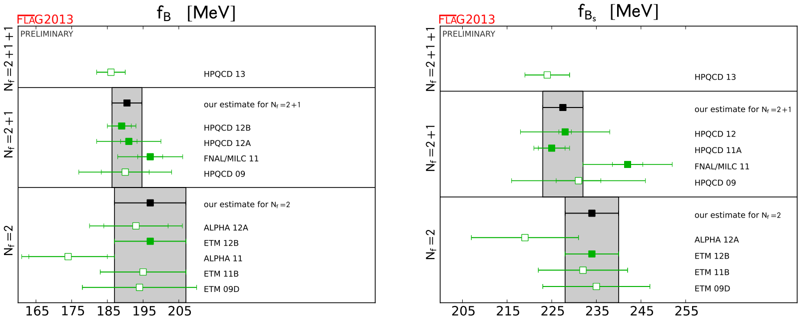


	$\lvert V_{cd}\rvert$	$\lvert V_{cb}\rvert$	
$N_f = 2+1$	0.2191(83)	0.996(21)	(lept.+semilept. decay)
$N_f = 2$	0.2189(83)(94)	1.021(25)(21)	(lept. decay)

2nd-row unitarity $\quad |V_{cd}|^2 + |V_{cs}|^2 + |V_{cb}|^2 - 1 = 0.04(6)$

f_B, f_{B_s}

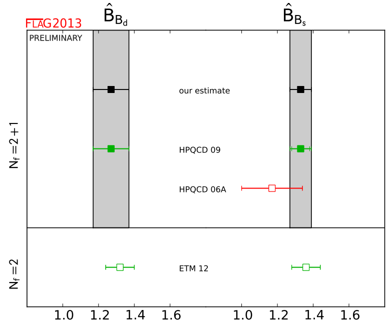


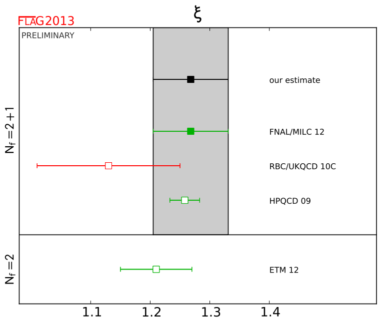
FLAG2013
f_B [MeV]
PRELIMINARY
$N_f=2+1+1$
HPQCD 13
$N_f=2+1$
our estimate for $N_f=2+1$
HPQCD 12B
HPQCD 12A
FNAL/MILC 11
HPQCD 09
$N_f=2$
our estimate for $N_f=2$
ALPHA 12A
ETM 12B
ALPHA 11
ETM 11B
ETM 09D
165 175 185 195 205
FLAG2013
f_{B_s} [MeV]
PRELIMINARY
$N_f=2+1+1$
HPQCD 13
$N_f=2+1$
our estimate for $N_f=2+1$
HPQCD 12
HPQCD 11A
FNAL/MILC 11
HPQCD 09
$N_f=2$
our estimate for $N_f=2$
ALPHA 12A
ETM 12B
ETM 11B
ETM 09D
205 215 225 235 245 255

B-mixing

$$B_{B_q}(\mu) = \frac{\langle \bar{B}^0_q | Q^q_R(\mu) | B^0_q \rangle}{\frac{8}{3} f^2_{B_q} m^2_B}$$

$\hat{B}_{B_q}$ RGI matrix element





$$\xi = \frac{f^2_{B_s} B_{B_s}}{f^2_{B_d} B_{B_d}} \quad N_f = 2+1 \rightarrow |V_{td}/V_{ts}|$$

B-mixing

$$B_{B_q}(\mu) = \frac{\langle \bar{B}^0_q | Q^q_R(\mu) | B^0_q \rangle}{\frac{8}{3} f^2_{B_q} m^2_B}$$

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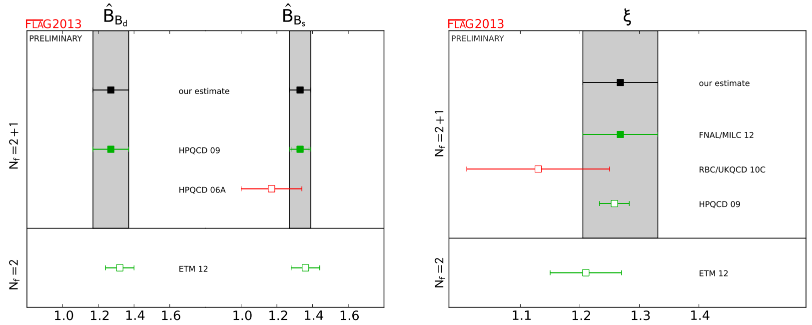
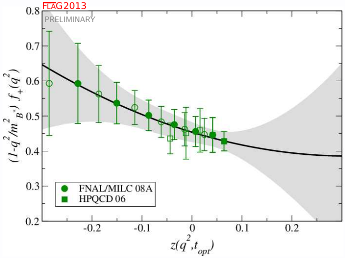
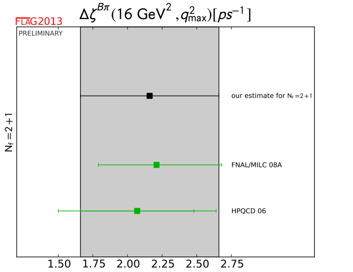


	f_{B_d}	f_{B_s}	$\hat{B}_{B_d}$	$\hat{B}_{B_s}$	ξ
$N_f = 2$	197(10)MeV	234(6) MeV			
$N_f = 2+1$	190.5(4.2)MeV	227.7(4.5)MeV	1.27(10)	1.33(6)	1.268(63)

$B \to \pi$



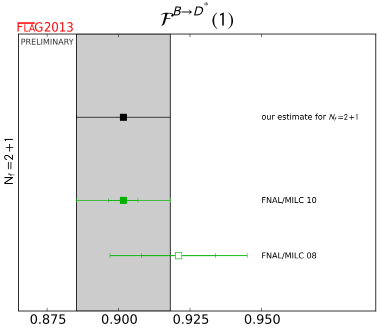


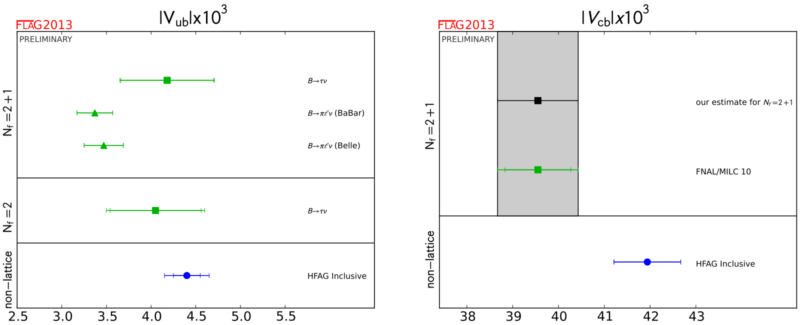
- kinematical reach on lattice very small
- FLAG combines suitably parameterised lattice and experimental data in range $16\text{GeV}^2 - q^2_{\max}$:

$$\Delta\zeta(16\text{GeV}^2, q^2_{\max}) = \frac{1}{|V_{ub}|^2} \int_{16\text{GeV}^2}^{q^2_{\max}} dq^2 \frac{d\Gamma}{dq^2}$$

$B \to D^*$

$$\frac{d\Gamma_{B^-\to D^{0,*} l\nu}}{d\omega} \propto |V_{cb}|^2 |\mathcal{F}(\omega)|^2 \qquad \omega = v_B \cdot v_{D^*}$$





	$\lvert V_{ub}\rvert$	$\lvert V_{cb}\rvert$	
$N_f = 2+1$	0.2191(83)	0.996(21)	(lept.+semilept. decay)
$N_f = 2$	0.2189(83)(94)	1.021(25)(21)	(lept. decay)

FLAG's CKM predictions – $N_f = 2+1$ exemplarily

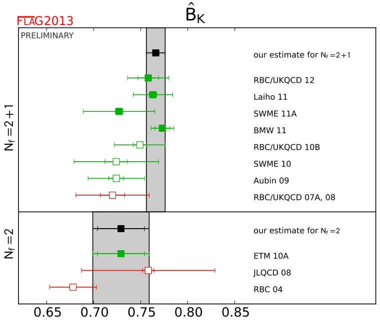
$$\begin{pmatrix} V_{ud} & 0.2238(11) & 0.00347(20)/0.00337(22) \\ 0.2191(83) & 0.996(21) & 0.00395(7)(5) \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

for the above:

- V_{us} from semileptonic decay + FLAVIANet (KTeV, Istra, Kloe, NA48)
- V_{ub} leptonic and semileptonic decay + Belle/BaBar
- V_{cd} leptonic and semileptonic decay + PDG, HFAG
- V_{cs} leptonic and semileptonic decay + PDG, HFAG
- V_{cb} semileptonic decay + HFAG

$\hat{B}_K$ - Neutral kaon mixing

$$B_K(\mu) = \frac{\langle \bar{K}^0 | Q_R^{\Delta S=2}(\mu) | K^0 \rangle}{\frac{8}{3} f_K^2 m_K^2}$$



N_f	$\hat{B}_K$	$B_K^{\overline{MS}}(2\text{GeV})$
$2+1$	0.766(10)	0.560(7)
2	0.729(25)(17)	0.533(18)(12)

FLAG Summary - PRELIMINARY

Quantity	$N_f = 2+1+1$	$N_f = 2+1$	$N_f = 2$
m_s(MeV)		93.8(2.4)	101(3)
m_{ud}(MeV)		3.42(9)	3.6(2)
m_s/m_{ud}		27.5(4)	28.1(1.2)
m_d(MeV)		4.68(14)(7)	4.8(15)(17)
m_u(MeV)		2.16(9)(7)	2.40(15)(17)
m_u/m_d		0.46(2)(2)	0.50(2)(3)
$f_+(0)$		0.9667(23)(33)	0.9560(57)(62)
f_{K^+}/f_{π^+}	1.195(3)(4)	1.192(5)	1.205(6)(17)
f_K(MeV)		156.3(0.8)	158.1(2.5)
f_π(MeV)		130.2(1.4)	
Σ(MeV)		265(17)	270(7)
F_π/F	1.0760(28)	1.0620(34)	1.0733(73)
$\bar{\ell}_3$	3.70(27)	2.77(1.27)	3.45(26)
$\bar{\ell}_4$	4.67(10)	3.95(35)	4.59(26)
$\hat{B}_K$		0.766(10)	0.729(25)(17)
$B_K^{\overline{\mathrm{MS}}}$(2 GeV)		0.560(7)	0.533(18)(12)

f_D(MeV)		209.2(3.3)	212(8)
f_{D_s}(MeV)		248.6(2.7)	248(6)
f_{D_s}/f_D		1.187(12)	1.17(5)
$f_+^{D\pi}(0)$		0.666(29)	0.65(6)(6)
$f_+^{DK}(0)$		0.747(19)	0.76(5)(5)
f_B(MeV)		190.5(4.2)	197(10)
f_{B_s}(MeV)		227.7(4.5)	234(6)
f_{B_s}/f_B		1.202(22)	1.19(5)
$f_{B_d}\sqrt{\hat{B}_{B_d}}$(MeV)		216(15)	
$f_{B_s}\sqrt{\hat{B}_{B_s}}$(MeV)		266(18)	
$\hat{B}_{B_d}$		1.27(10)	
$\hat{B}_{B_s}$		1.33(10)	
ξ		1.268(63)	
$\hat{B}_{B_s}/\hat{B}_{B_d}$		1.06(11)	

Summary

General comments:

- simulations of lattice QCD are now being carried with physically light (degenerate) *up*- and *down*-quarks
- for light quark physics the level of precision reached within this frame work forces us to now include isospin-breaking and EM effects in simulations
- we need more groups working on D and B; increased activity at Lattice 2013 visible

Summary

General comments:

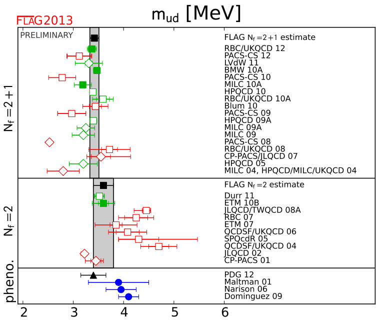
- simulations of lattice QCD are now being carried with physically light (degenerate) *up*- and *down*-quarks
- for light quark physics the level of precision reached within this frame work forces us to now include isospin-breaking and EM effects in simulations
- we need more groups working on D and B; increased activity at Lattice 2013 visible

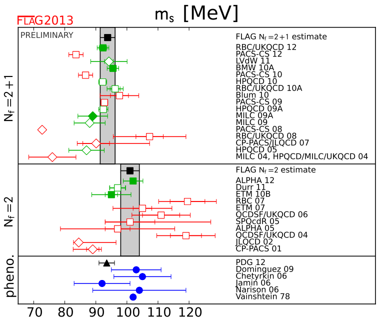
FLAG:

- publish this fall (data-deadline end of April 2013)
- content of FLAG-2 not covered here: $m_{u,d,s}$, α_s, low energy constants of $SU(2)$ and $SU(3)$ chiral Lagrangian
- future updates, possibly new quantities planned
- we hope it will be of use for you

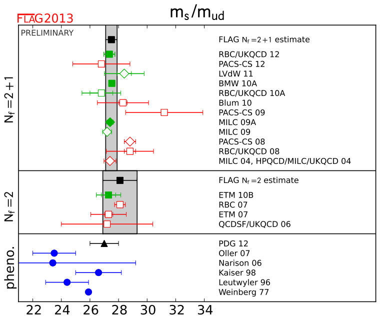
The research leading to these results has received funding from the European Research Council under the European Community's Seventh Framework Programme (FP7/2007-2013) ERC grant agreement No 279757

Light quark masses





Ratio of light quark masses



N_f	m_s/MeV	m_s/MeV	m_s/m_{ud}
$2+1$	93.8(1.5)(1.9)	3.42(6)(7)	27.46(15)(41)
2	103(3)	3.6(2)	28.1(1.2)