Radiative decays of charmonium

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Based on "*Lattice QCD study of the radiative decays $J/\psi \to \eta_c \gamma$ and $h_c \to \eta_c \gamma$*"

D.Becirevic and F.Sanfilippo, arXiv:1206.1445, JHEP 1301 (2013) 028

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Summary

$J/\psi \to \eta_c \gamma$ radiative decay

1. $\Gamma\left(J/\psi \to \eta_c \gamma\right)$ experimental situation
2. Theoretical puzzle
3. Lattice computation

$h_c \to \eta_c \gamma$ radiative decay

1. $\Gamma\left(h_c \to \eta_c \gamma\right)$ lattice determination
2. Prediction for Γ_{h_c}

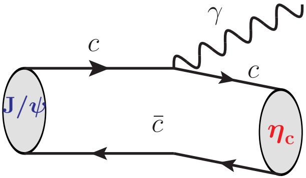
New: decays of radially excited states

Preliminary study of $\psi' \to \eta_c \gamma$ and $\eta_c\left(2S\right) \to J/\psi \gamma$

$$J/\psi \to \eta_c \gamma$$

radiative decays

$J/\psi \to \eta_c\gamma$ radiative decay

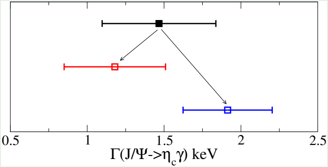


Current experimental situation is **unclear**

$\Gamma\left(J/\psi \to \eta_c\gamma\right)_{\rm PDG} = (\mathbf{1.58} \pm \mathbf{0.37})\,\mathbf{keV}$:

- Crystal Ball ('86): $(\mathbf{1.18} \pm \mathbf{0.33})$ **keV**
- CLEO ('09): $(\mathbf{1.91} \pm \mathbf{0.28} \pm \mathbf{0.03})$ **keV**

PDG heavily influenced by Crystal Ball



- KEDR (arXiv:1002.2071): $(\mathbf{2.17} \pm \mathbf{0.14} \pm \mathbf{0.37})$ **keV** (preliminary) final result expected this year
- BESIII will hopefully clarify the situation

$J/\psi \to \eta_c\gamma$ radiative decay

Theoretical predictions are **inconclusive**

- Dispersive bound from $\Gamma(\eta_c \to 2\gamma)$:
  $\Gamma(J/\psi \to \eta_c\gamma) < \mathbf{3.2\,keV}$ [M.A. Shifman, Z. Phys. C 6 ('80)]
- Two QCD sum rule calculations gave two different results:
  - $\sim \mathbf{(1.7 \pm 0.4)\ keV}$ [A.Y. Khodjamirian, Sov. J. Nucl. Phys. 39 ('84)]
  - $\sim \mathbf{(2.6 \pm 0.5)\ keV}$ [Beilin and Radyushkin, Nucl. Phys. B 260 ('85)]
- Potential non relativistic QCD
  $\mathbf{(1.5 \pm 1.0)\ keV}$ [N.Brambilla et al, PRD73 ('06)]
- Potential Quark Models:
  - $\sim \mathbf{3.3\,keV}$ [M.B Voloshin, Prog.Part.Nucl.Phys. 61 ('07)]
  - $\sim \mathbf{2.85\,keV}$ [E. Eichten et al., RMP80 ('08)]

Lattice QCD computations

- Quenched and single lattice spacing: $\mathbf{2.51(8)\,keV}$[J.J Dudek et al., PRD 79 ('09)]
- Unquenched but still single lattice spacing: $\mathbf{2.77\,(5)\ keV}$[Chen et. al, PRD 84 ('11)]

Both results obtained at large negative q^2's , then extrapolated to $q^2 = 0$

Lattice QCD

Desired features

Continuum: Several lattice spacings to take continuum limit

Renormalization: Non perturbative

Momentum: Work directly at $q^2 = 0$ to avoid the q^2 extrapolation

Unquenching: Include 2 physical light, strange and charm dynamical quarks

What we currently have...

Continuum: 4 different lattice spacings ($a \in [0.054; 0.100]$ fm)

Renormalization: Non perturbative (RI-MOM)

Momentum: Work at $q^2 = 0$ using twisted boundary conditions

Unquenching: Only 2 dynamical light quarks ($M_\pi \in [280; 500]$ MeV)

- Wilson regularization of QCD with twisted mass term (tmQCD)
- QCD gauge field configurations produced by ETM collaboration

Form factor computation

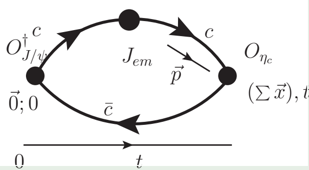
$$\Gamma\left(J/\psi \to \eta_c\gamma\right) \propto \left|\langle\eta_c\left|J^{em}\right|J/\psi\rangle\right|^2_{q^2=0}$$

Three point correlation functions

$$C^{(3)}(t)=\langle\sum_{\vec{x},\vec{y}} e^{-i\vec{p}\vec{x}} O_{\eta_c}(\vec{x},T)\, J_{em}(\vec{y},t) O^{\dagger}_{J/\psi}(\vec{0},0)\rangle$$

at intermediate times:

$$\overset{0\ll t\ll T}{\simeq} \frac{Z_{J\psi} Z_{\eta_c} \exp\left[\left(E_{\eta_c}-M_{J/\psi}\right)t\right]\left\langle \eta_c \left| J^{em}_j \right| (J/\psi)_i \right\rangle}{4 M_{J/\psi} E_{\eta_c}}$$



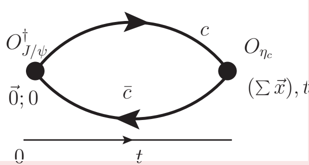
Two points correlation functions: used to remove the sources

$$C^{(2)}(t)=\sum_{\vec{x}}\langle O_{\eta_c}(\vec{x},t)\, O^{\dagger}_{\eta_c}(\vec{0},0)\rangle$$

at large times:

$$\overset{t\to\infty}{\simeq} {Z_{\eta_c}}^2 \exp(-E_{\eta_c} t)/2E_{\eta_c}$$

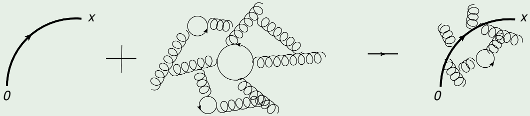
and similarly for J/ψ.



Two points correlation function computation

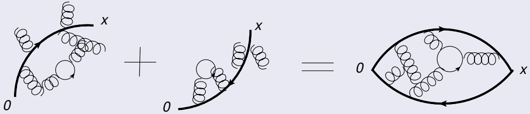
Compute full quark propagator

Solve discrete Dirac equation on gauge background: $D\,(\text{y,x}) \cdot S\,(x,0) = \delta_{y,0}$



- Discretized Dirac operator D embeds all non perturbative QCD dynamics
- So S is the fully non perturbative propagator

Combine 2 propagators with suitable Dirac structures



In practice

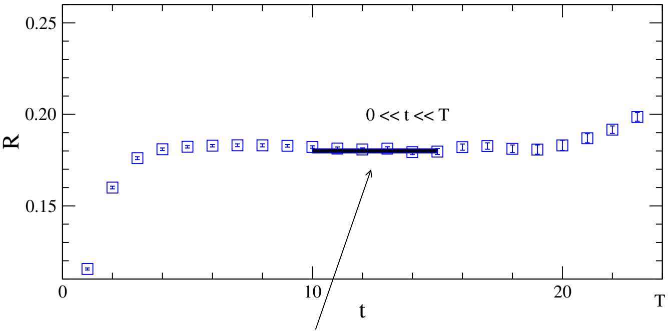
- D operator and the propagator S are large matrices ($\mathcal{O}\left(10^9\right)$ lines)
- Solving Dirac equation requires large amount of CPU resources

Matrix element $\langle \eta_c | J_j^{em} | (J/\psi)_i \rangle$ (example)

Combine with $C^{(3)}$ with Z and M determined from $C^{(2)}$

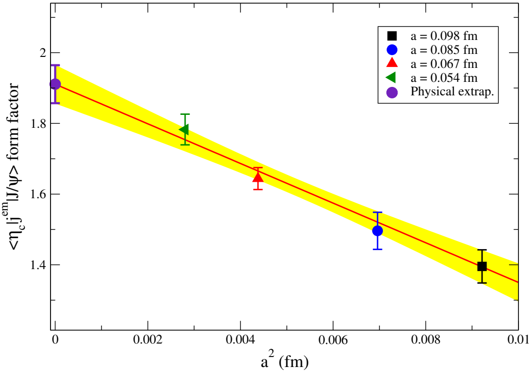
$$R(t) \equiv \frac{4 M_{J/\psi} E_{\eta_c} C_{ij}^{(3)}(t)}{Z_{J\psi} Z_{\eta_c} \exp\left[\left(E_{\eta_c} - M_{J/\psi}\right) t\right]} \overset{0 \ll t \ll T}{\simeq} \langle \eta_c | J_j^{em} | (J/\psi)_i \rangle$$

NB: very accurate determination of $\Delta M = M_{J/\psi} - M_{\eta_c} = 112(3)$ MeV



$\langle \eta_c | J_j^{em} | (J/\psi)_i \rangle$ on the plateau of $R(t)$

Continuum limit extrapolation



- Significant dependance on cut-off scale *a*
- No dependance on light (sea) quark mass

Our final result

- Putting everything together we get: $\Gamma(J/\psi \to \eta_c \gamma) = 2.58\,(13)\ \mathrm{keV}$
- Our value is clearly:
    - larger than Crystal Ball('86) 1.18(33) keV
    - compatible with CLEO('09) 1.91(30) keV, and KEDR('10) 2.17(40) keV

Recent developments

New lattice study

- HPQCD collabation (PRD86 (2012) 094501) soon after reported results:
  - using a totally different regularization ('HISQ')
  - including also dynamical strange quark (i.e. $\bar{s}s$ pairs creation)
- They show that $\langle \eta_c | J_j^{em} | (J/\psi)_i \rangle$ does not depend on m_s^{sea}
- Excellent agreement with our result in the continuum limit:

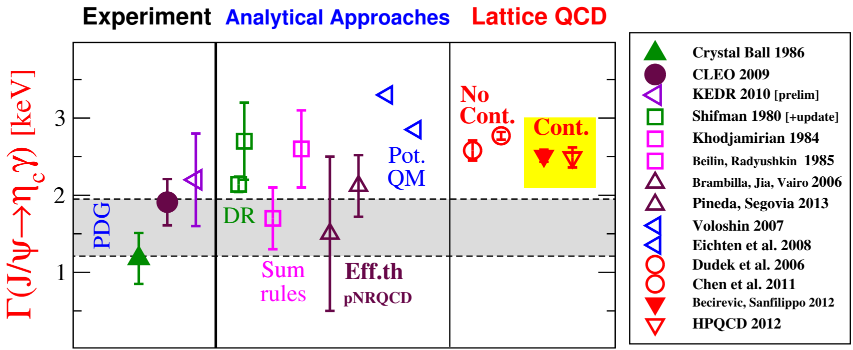
$$\Gamma_{J/\psi \to \eta_c \gamma} = 2.49\,(19)\ \mathrm{keV}$$

Improved pNRQCD determination

- Pineda & Segovia (PRD87, 2013) improved description of M1-decays
  - exact inclusion of the static potential in the low energy hamiltonian
  - resummation of large logs by means of RGE
- Cancellation of renormalon ambiguity $\rightarrow$ smaller uncertainty

$$\Gamma_{J/\psi \to \eta_c \gamma} = 2.14(40)\,\mathrm{keV}$$

Is the $J/\psi \to \eta_c\gamma$ puzzle solved?



- Two different lattice approaches give the same results in the continuum
- On the theory side the problem is solved
- This becomes a precision test of QCD
- The experimental situation needs to be clarified

$$h_c \to \eta_c \gamma$$

radiative decays

Facts about h_c meson

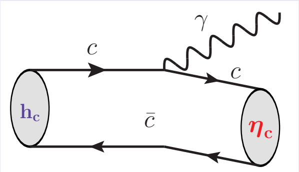
h_c: charmonium $J^{PC} = 1^{+-}$ state

- Elusive for many years
- Finally observed at CLEO (2005)
- BaBar confirmed (2008): $B \to X_{\bar{c}c} K^{(*)}$, through dominant $X_{\bar{c}c} \to \eta_c \gamma$
- BESIII (2010) observed $\psi' \to h_c \, (\to \eta_c \gamma) \, \pi^0$

PDG

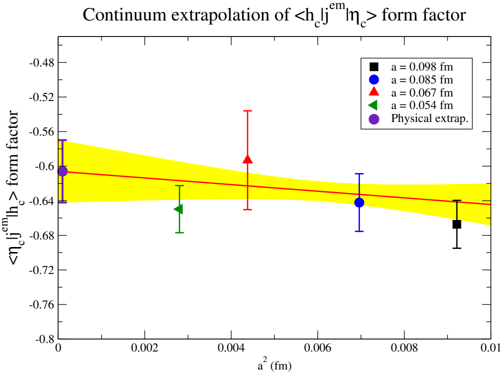
- $m_{h_c} = 3525.4(1)\,\mathrm{MeV}$
- $\Gamma\,(h_c) = 0.7(4)\,\mathrm{MeV}$
- $\mathcal{B}\,(h_c \to \eta_c \gamma) = 51\,(6)\,\%$

$\Gamma\,(h_c \to \eta_c \gamma) = 0.36(21)\,\mathrm{MeV}$



- No QCD based estimate for $\Gamma\,(h_c \to \eta_c \gamma)$
- At our study times, $\Gamma\,(h_c \to \eta_c \gamma)$ was still experimentally unknown!

$h_c \to \eta_c \gamma$ radiative decay



Result for the mass

$$m_{h_c}^{THIS} = 3542(32)\,\mathrm{MeV}$$
$$m_{h_c}^{PDG} = 3525.4(1)\,\mathrm{MeV}$$

✓ agrees very well

Partial width

$$\Gamma(h_c \to \eta_c\gamma)^{THIS} = 0.72(5)(2)\,\mathrm{MeV}$$
$$\Gamma(h_c \to \eta_c\gamma)^{PDG} = 0.36(21)\,\mathrm{MeV}$$

X quite in disagreement

- We need further experimental studies to assess the discrepancy
- Would be great if other theoretical study were studied as well
  - use other lattice discretization
  - improve pNRQCD for E1-transitions
  - try also QCD sum rule

Radiative decays

of

excited charmonium

Decays of radially excited charmomium (preliminary)

Radiative decays of an excited states to the ground state

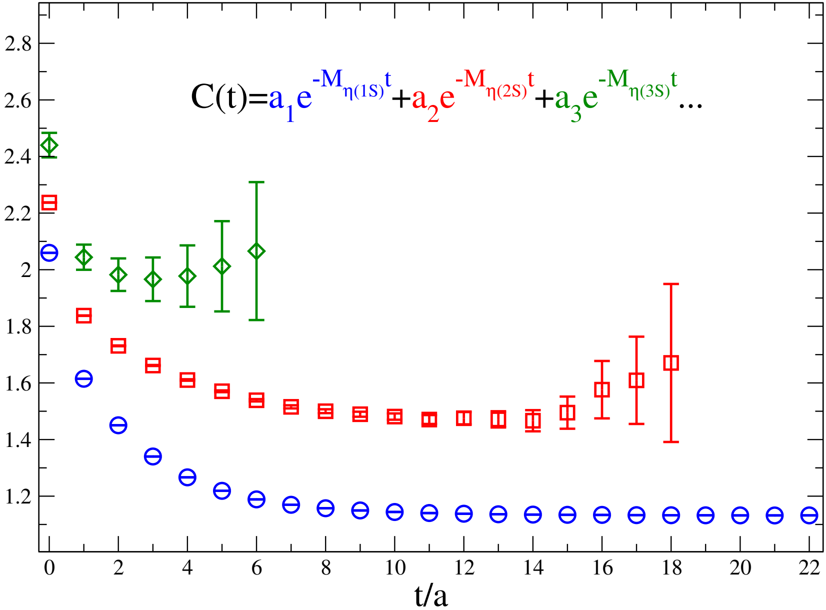
- Easier to measure for experimentalists
  - more energetic photons easier to recognize
  - more phase space w.r.t ground state decays
- Harder for theorists:
  - models and effective theories predictions are unreliable (very sensitive to high order relativistic correction)
  - lattice: reliable separation of excitation and ground state is difficult

Spectral decomposition of two points correlation function

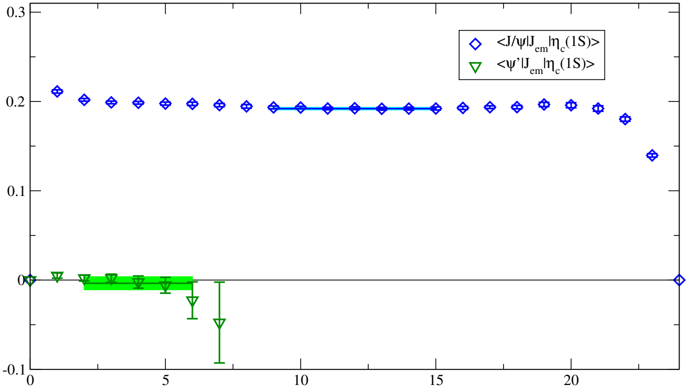
$$C_{2pt}(t) = a_1 e^{-M_1 t} + a_2 e^{-M_2 t} + a_3 e^{-M_3 t} + \ldots$$

- How to separate different states?
  $\rightarrow$ we use **several operators with the same quantum numbers**
- **Smeared** operators: operators with different spatial distribution
  $\rightarrow$ different couplings to states

Spectral decomposition of 2pts pseudoscalar corr. function



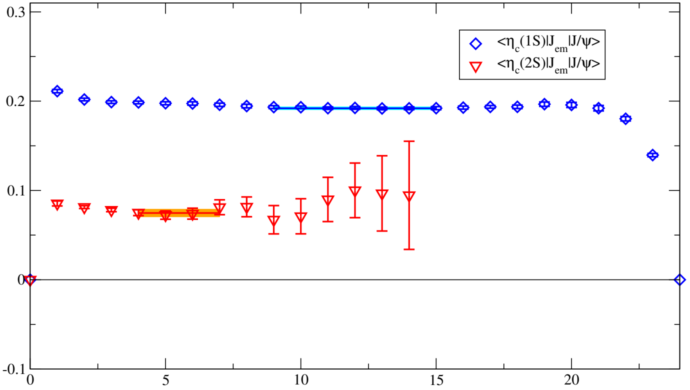
$\psi' \to \eta_c\,(1S)\,\gamma$



Form factor for $\psi' \to \eta_c\gamma$ very small: negligible decay width

- Compatible with findings of J.J Dudek et al, PRD 79 ('09)
- In line with experiments, that finds very small $\Gamma\,(\psi' \to \eta_c\gamma)$

$\eta_c(2S) \to J/\psi\gamma$



Form factor for $\eta_c(2S) \to J/\psi\gamma$ sizable: non negligible decay width

- Never explored in lattice before, never measured in experiments
- Caveat: no continuum limit

Conclusions

Results

First full determination of $J/\psi \to \eta_c \gamma$, $h_c \to \eta_c \gamma$ form factors:

- high statistics unquenched simulations
- continuum extrapolation under control
- non-perturbatively renormalized

Preliminary study of excited-to-ground state decays

Main message from Lattice QCD side

- Finally **assessed** theoretical estimate of $\Gamma\left(J/\psi \to \eta_c \gamma\right)$
- Found a (small) **discrepancy** for $\Gamma\left(h_c \to \eta_c \gamma\right)$
- Indication **of sizable** $\Gamma\left(\eta_c\left(2S\right) \to J/\psi \gamma\right)$

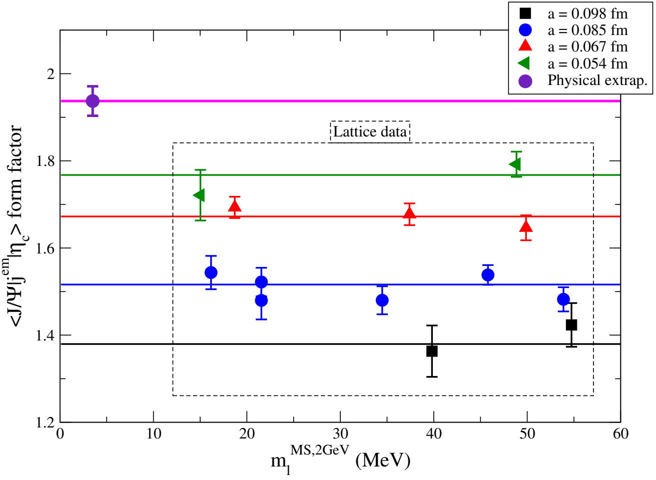
Main message for experimentalists

Radiative decays of charmonium could become a precision test of QCD but

- Indispensable to clarify $\Gamma\left(J/\psi \to \eta_c \gamma\right)$
- Improve the measurement of Γ_{h_c}
- Measure $\Gamma\left(\eta_c\left(2S\right) \to J/\psi \gamma\right)$

Backup slides

Dependence on light quark mass



Insensitive to variation of the light sea quark mass m_ℓ^{sea} (expected because $m_c^{val} \gg m_\ell^{sea}$)

Expected insensitivity to the dynamical strange quark ($m_c \gg m_s \gg m_\ell^{sea}$)

Can we do charm physics on current lattices?

Some back of the envelop calculation

- Lattice spacings: $a \sim 0.050 \div 0.100$ fm, $1/a \sim 2 \div 4$ GeV
- Charmed meson mass: $M_{D^\pm} = 1.87$ GeV, $M_{J/\Psi} = 3.1$ GeV

To study charm physics on such lattices seem questionable but...

Some deeper calculation

- In the free theory the cut off is given by $p_{max} = \pi/a \sim 6 \div 12$ GeV!
- Seems to be almost good also to study b quark...

Cutoff of interacting theory is unknown: only actual computations can teach us

How to keep the situation under control?

Having 4 different lattice spacing, and $\mathcal{O}(a)$ improved theory allows:

- to drop coarsest lattice spacing and check for stability of $a \to 0$ limit
- to assess the convergence $\propto a^2$ to the continuum limit: $\Phi^{latt} = \Phi^{cont.} + \Phi' a^2$

Determination of the charm quark mass

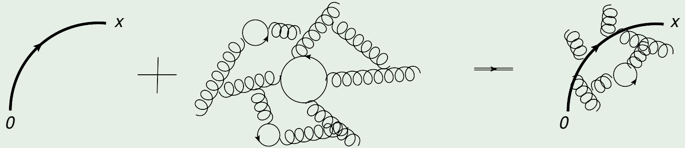
Wick contraction

$$C(\tau) = \sum_{\vec{x}} \left\langle O^{\dagger}(\vec{x},\tau)\, O\left(\vec{0},0\right) \right\rangle \underset{Wick}{=} \mathrm{Tr}\left[\Gamma S_l\left(\vec{x},\tau;\vec{0},0\right)\Gamma S_c\left(\vec{0},0;\vec{x},\tau\right)\right]$$

Quark propagator calculation

Solving Dirac equation on gauge background provides full quark propagator

$$D_q\,(\mathrm{y,x}) \cdot S_q\,(x,0) = \delta_{y,0} \qquad D_q = \left(\frac{1}{2\kappa} + K[U]\right)\mathbf{1} + i m_q \gamma_5 \tau_{\mathbf{3}}$$

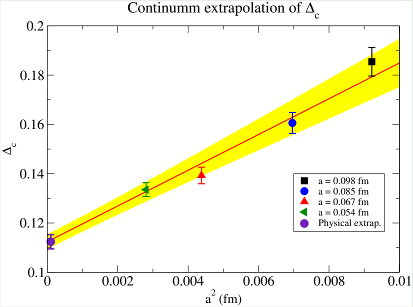


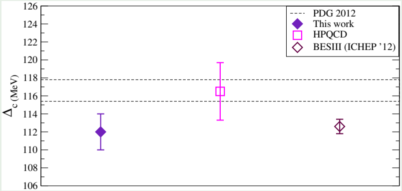
In practice

- D operator and the propagator S are large matrices ($\mathcal{O}\left(10^9\right)$ lines)
- Solving Dirac equation requires large amount of CPU resources

Other two precise tests of SM

Hyperfine splitting $\Delta_c = M_{J/\Psi} - M_{\eta_c}$





$J/\Psi \to e^+e^-$ decay constant